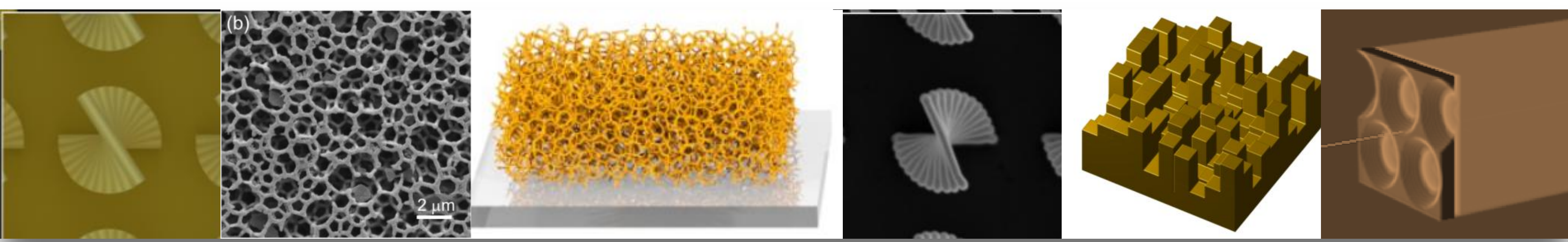
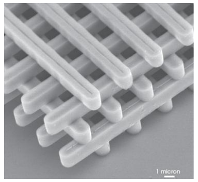
Metamateriały optyczne – projekt, symulacja i wykonanie

Jakub Haberko
KFMS, WFiIS AGH

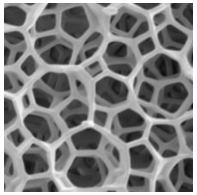


Seminarium Wydziałowe WFiIS, 24.11.2017

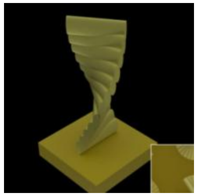
Plan prezentacji



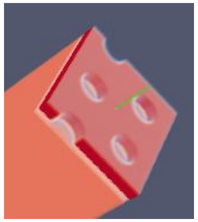
- Nieco o kryształach fotonicznych
- Izotropowe metamateriały z przerwą fotoniczną



- Makarony – skręcone taśmy sposobem na zmianę polaryzacji światła
- Metalowe szerokopasmowe płytki ćwierćfalowe



- Pokrycia antyodbiciowe na czole światłowodu
- Struktury o asymetrycznej transmisji
- Warstwy antyodbiciowe dla ogniw fotowoltaicznych



- Podsumowanie

Metamateriał optyczny – co to takiego?

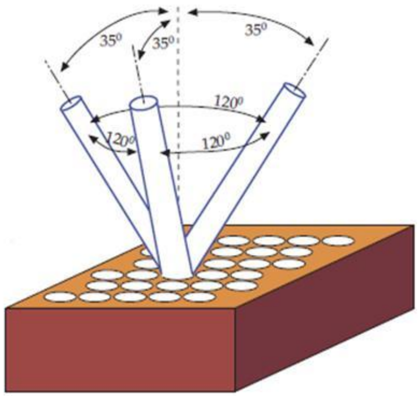
Metamateriał optyczny

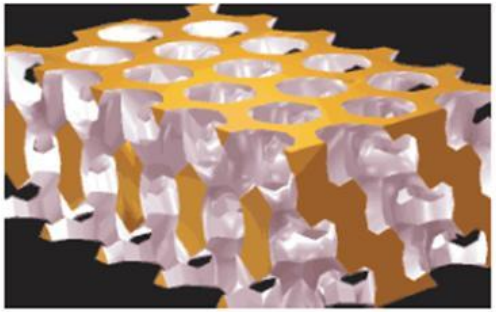
- Materiał wytworzony sztucznie
- Posiada nietypowe lub trudne do uzyskania właściwości optyczne
- Zbudowany z elementów (metaatomów) o rozmiarach mniejszych niż długość światła, dla którego ma funkcjonować

W. Cai, V. Shalaev *Optical metamaterials. Fundamentals and applications,* Springer, 2010

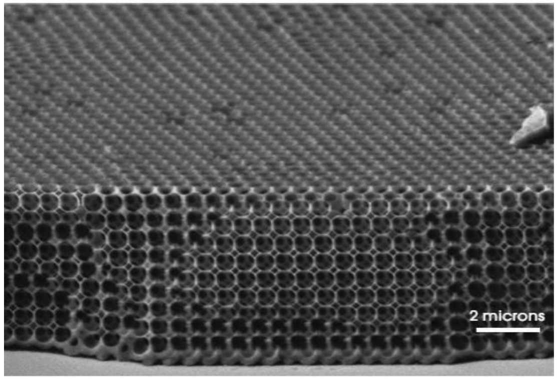
Trójwymiarowe kryształy fotoniczne

Yablonovite (1991)



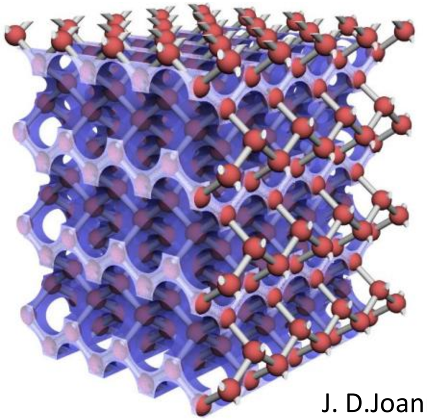


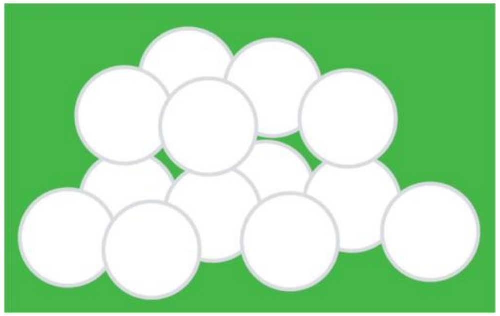
Odwrócony opal (2001)



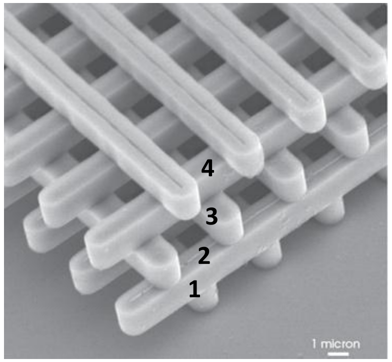
Pełna przerwa fotoniczna dla wystarczająco dużego kontrastu Δε

Kule powietrzne na sieci diamentu (2001)





Stos drewna (1994)

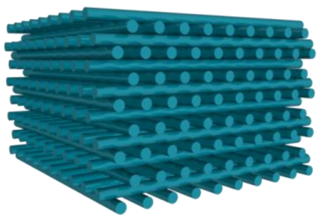


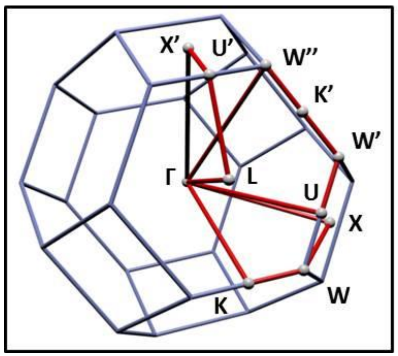
J. D.Joannopoulos , *Molding the Flow of Light*, Princeton Univ. Press,2nd ed. 2008

Materiały nieuporządkowane: struktury superjednorodne

Kryształy fotoniczne

3D

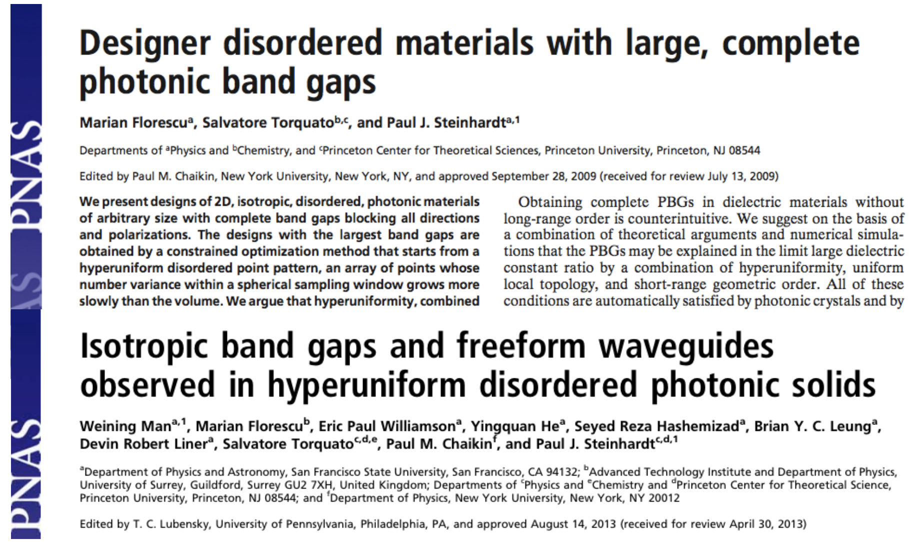




wavelength λ/μm

Problem: **przerwa nie jest izotropowa!**

Nieuporządkowane struktury fotoniczne



PNAS

Designer disordered materials with large, complete photonic band gaps

Marian Florescu[a], Salvatore Torquato[b,c], and Paul J. Steinhardt[a,1]

Departments of [a]Physics and [b]Chemistry, and [c]Princeton Center for Theoretical Sciences, Princeton University, Princeton, NJ 08544

Edited by Paul M. Chaikin, New York University, New York, NY, and approved September 28, 2009 (received for review July 13, 2009)

We present designs of 2D, isotropic, disordered, photonic materials of arbitrary size with complete band gaps blocking all directions and polarizations. The designs with the largest band gaps are obtained by a constrained optimization method that starts from a hyperuniform disordered point pattern, an array of points whose number variance within a spherical sampling window grows more slowly than the volume. We argue that hyperuniformity, combined

Obtaining complete PBGs in dielectric materials without long-range order is counterintuitive. We suggest on the basis of a combination of theoretical arguments and numerical simulations that the PBGs may be explained in the limit large dielectric constant ratio by a combination of hyperuniformity, uniform local topology, and short-range geometric order. All of these conditions are automatically satisfied by photonic crystals and by

PNAS

Isotropic band gaps and freeform waveguides observed in hyperuniform disordered photonic solids

Weining Man[a,1], Marian Florescu[b], Eric Paul Williamson[a], Yingquan He[a], Seyed Reza Hashemizad[a], Brian Y. C. Leung[a], Devin Robert Liner[a], Salvatore Torquato[c,d,e], Paul M. Chaikin[f], and Paul J. Steinhardt[c,d,1]

[a]Department of Physics and Astronomy, San Francisco State University, San Francisco, CA 94132; [b]Advanced Technology Institute and Department of Physics, University of Surrey, Guildford, Surrey GU2 7XH, United Kingdom; Departments of [c]Physics and [e]Chemistry and [d]Princeton Center for Theoretical Science, Princeton University, Princeton, NJ 08544; and [f]Department of Physics, New York University, New York, NY 20012

Edited by T. C. Lubensky, University of Pennsylvania, Philadelphia, PA, and approved August 14, 2013 (received for review April 30, 2013)

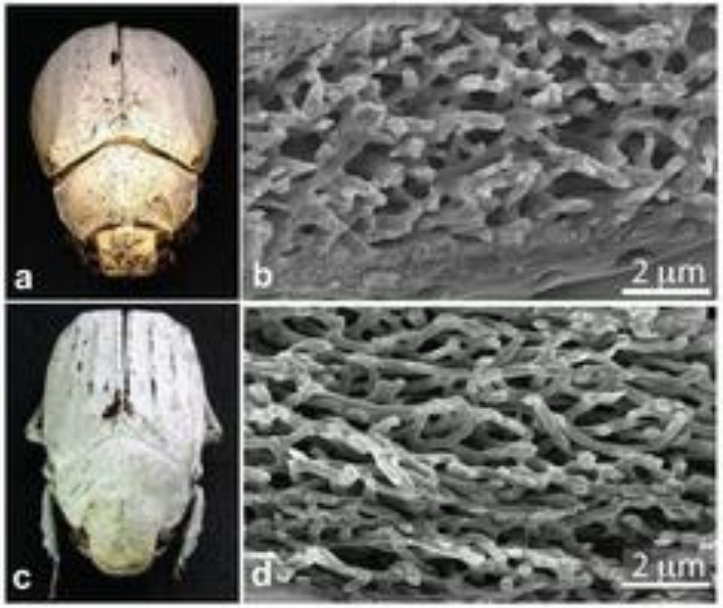
Trzy warunkei konieczne istnienienia przerwy fotonicznej:

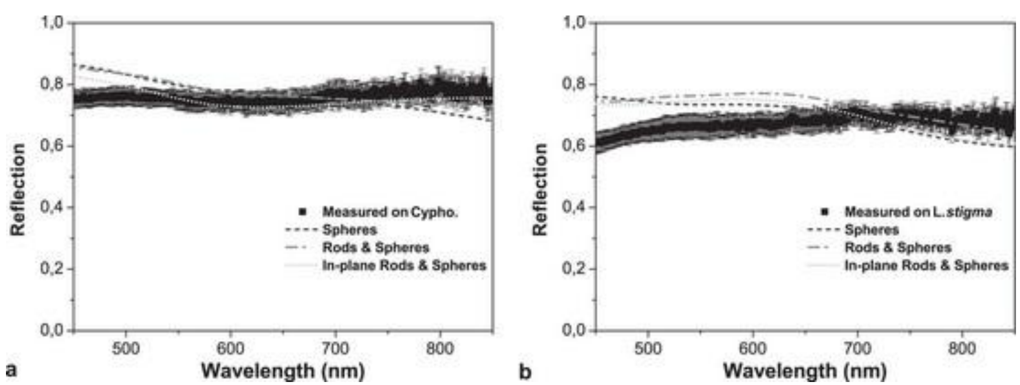
- Superjednorodność
- Jednorodna lokalna topologia
- Geometryczny porządek bliskiego zasięgu

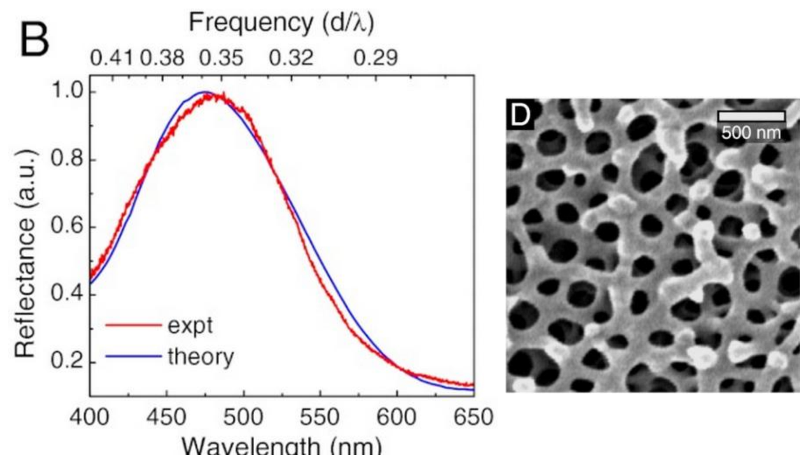
Porządek dalekiego zasięgu nie jest konieczny

→ Izotropowa przerwa?

Nieuporządkowane materiały fotoniczne w naturze







Bright-White Beetle Scales Optimise Multiple Scattering of Light, M. Burresi, L. Cortese, L. Pattelli, M. Kolle, P. Vukusic, D. S. Wiersma, U. Steiner, S. Vignolini, Nature Scientific Reports,2014

Amorphous diamond-structured photonic crystal in the feather barbs of the scarlet macaw, H. Yin, B. Dong, X. Liu, T. Zhan, L. Shi, J. Zi, E. Yablonovitch, PNAS, 2012

Superjednorodne układy punktów

Definicja:

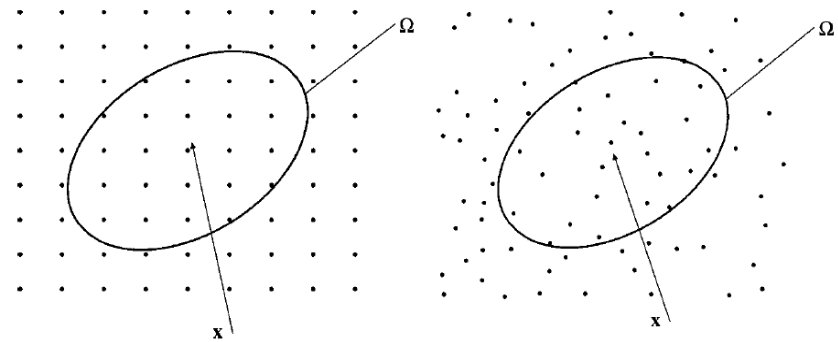
Układ punktów jest superjednorodny, jeżeli wariancja liczby punktów

$$\sigma^2(R) = \langle N^2 \rangle - \langle N \rangle^2$$

W sferycznym oknie o promieniu *R* rośnie wolniej niż objętość okna dla dużych *R*

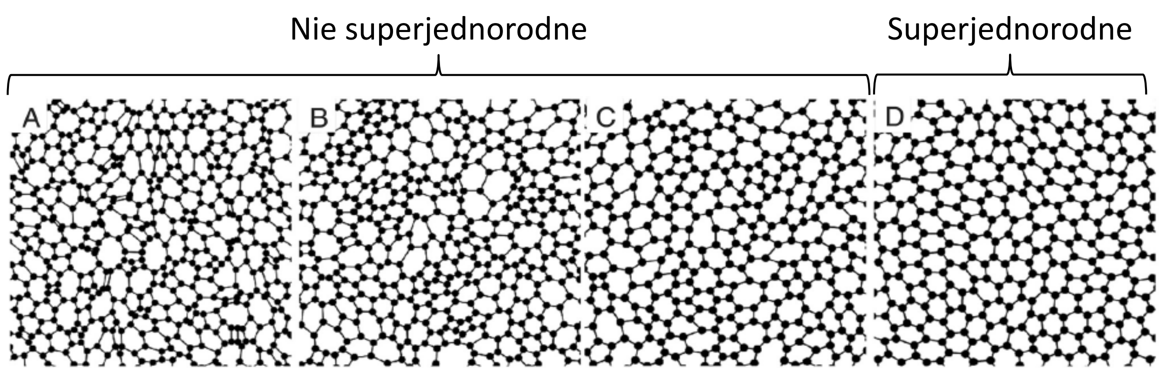
$$\Updownarrow$$

$$S(k) \xrightarrow[k \to 0]{} 0$$



S. Torquato, F. H. Stillinger, *Local density fluctuations, hyperuniformity, and order metrics* Phys. Rev. E 68, 041113 (2003)

Fluktuacje gęstości o dużej skali przestrzennej znikają



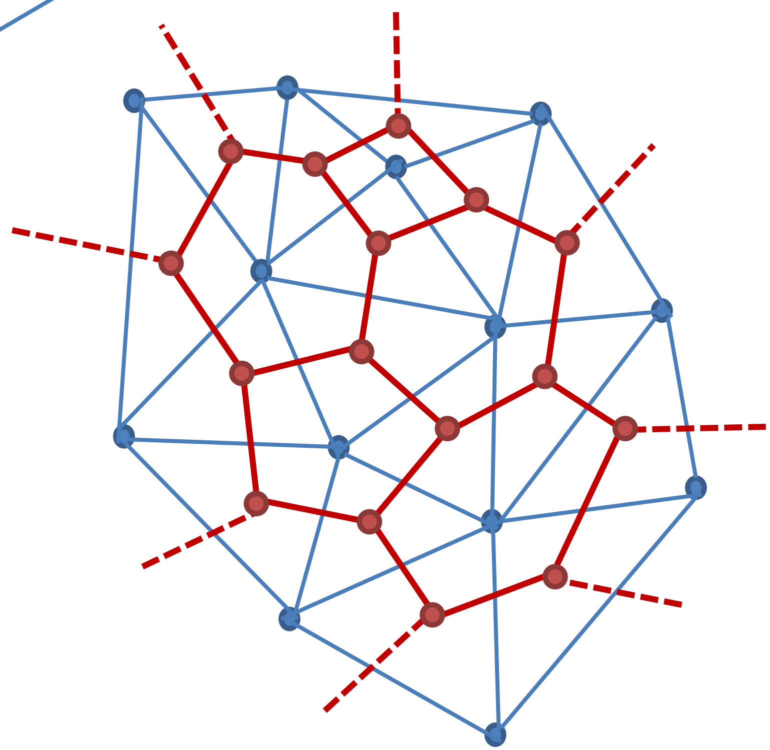
M. Florescu, S. Torquato, P. J. Steinhardt, PNAS, vol. 106 (2009) no. 49 20658–20663

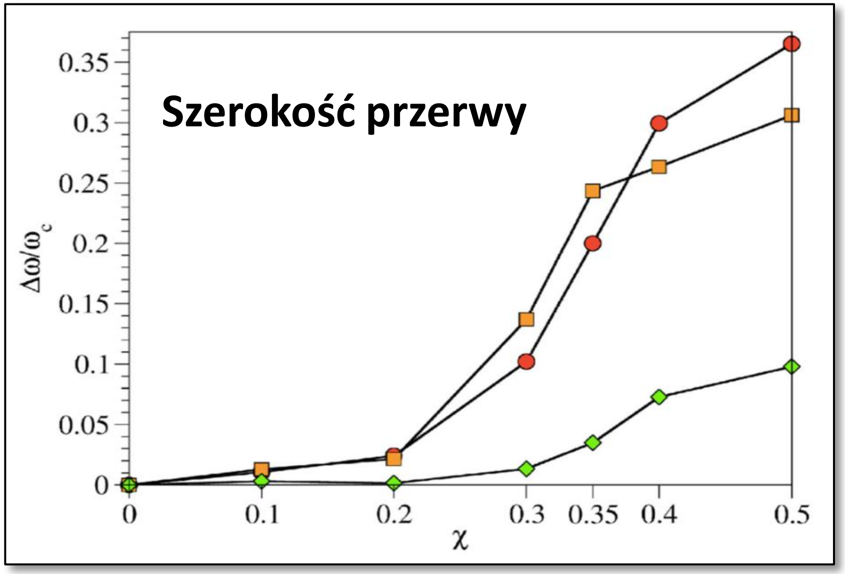
Konsekwencja:

Wszystkie krystaliczne i kwazikrystaliczne układy punktów są superjednorodne

Generowanie sieci superjednorodnych w 2D

- Stworzenie superjednorodnego układu punktów
- Trangulacja Delone → trójwalentna sieć (w 2D)
- Znalezienie środków trójkątów
- Umieszczenie dielektrycznych walców
- Połączenie walców ścianami



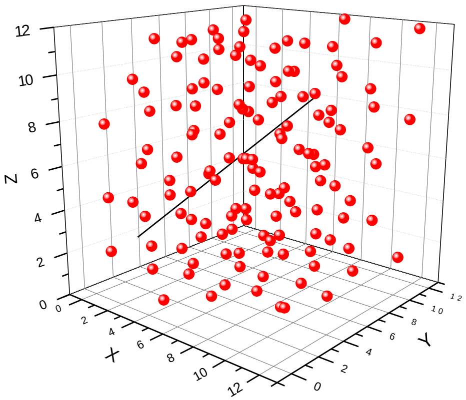


Symulacje komputerowe pokazują, że taka sieć powinna posiada przerwę fotoniczną

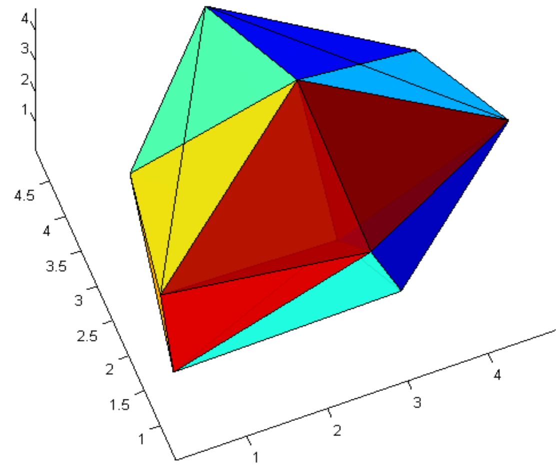
Designer disordered materials with large, complete photonic band gaps, M. Florescu, S. Torquato, P. J. Steinhardt, PNAS, 106, 49 ,2009

Generowanie sieci superjednorodnych w 3D

Superjednorodny układ punktów w 3D



Triangulacja Delone

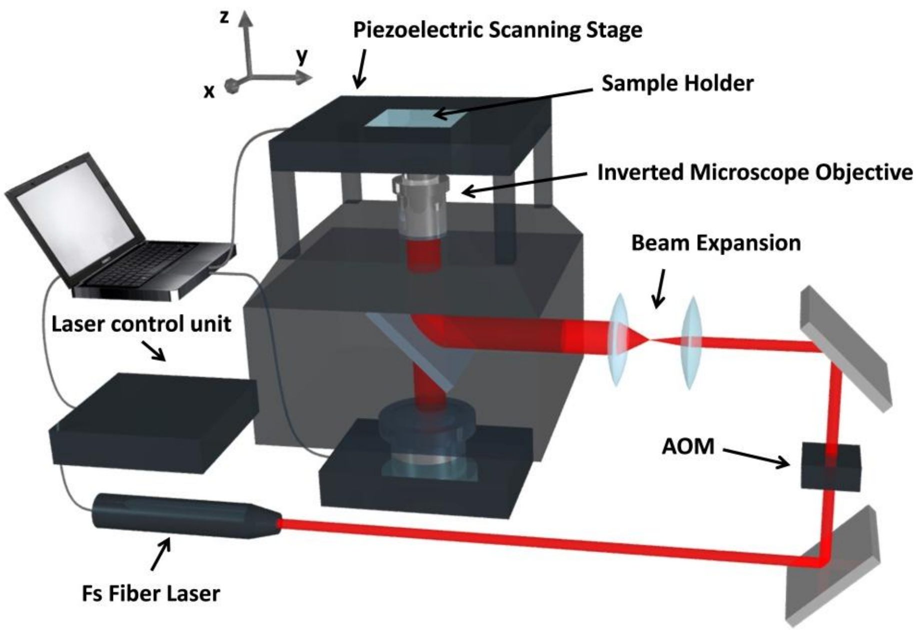


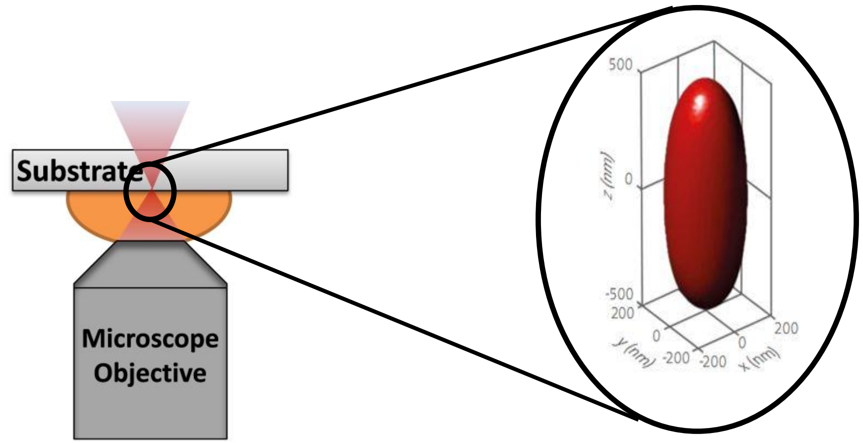
Coordinates of points from:

- Ch. Song, P. Wang, H. A. Makse, *A phase diagram for jammed matter,* Nature, vol. 453 (2008), 629
- *http://lev.ccny.cuny.edu/~hmakse/soft_data.html*

Czy da się wytworzyć takie struktury o skali porównywalnej z długością światła widzialnego? Np. w krzemie n>3? Czy da się zaobserwować przerwę?

Direct Laser Writing
Trójwymiarowa nanolitografia laserowa

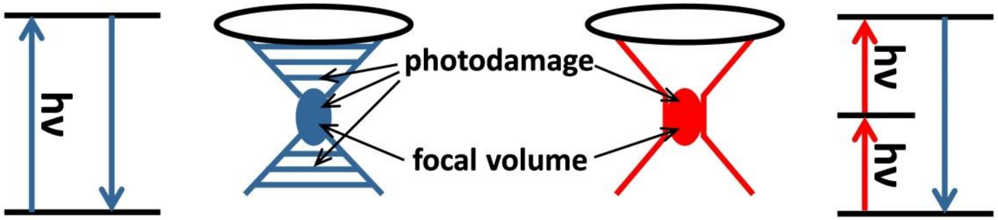




Zogniskowana wiązka laserowa 780 nm

„Pióro" w kształcie elipsoidy

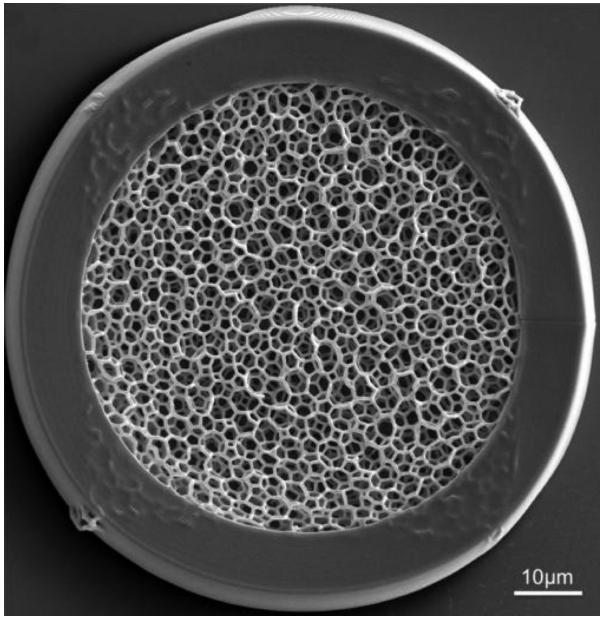
Absorpcja dwufotonowa w polimerowym fotorezyście

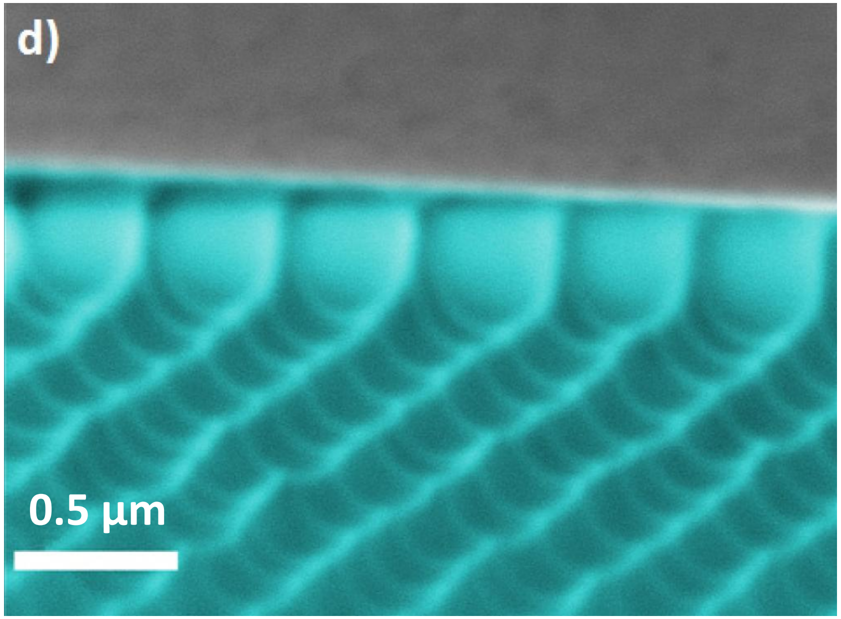


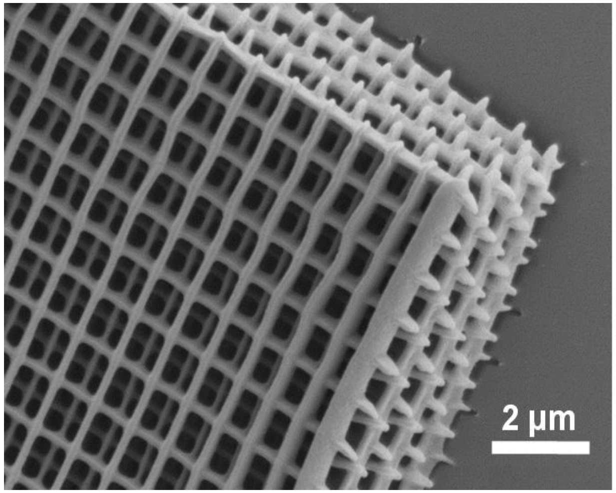
- Rozdzielczość: ~150 nm w poprzed wiązki
  ~ 450 nm wzdłuż wiązki
- Fotorezyst: przezroczysty @ 780 nm

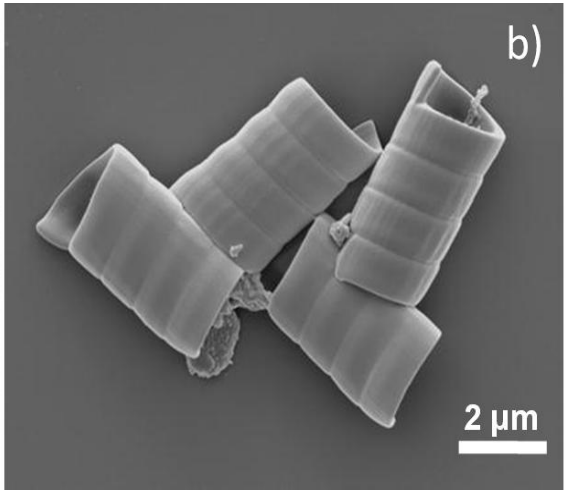
Images taken from: http://mtstandard.com/lifestyles/health-med-fit/nail-polish-controversial---some-dermatologists-say-use-of/article_4d91f49e-9b6d-11e2-a5d2-001a4bcf887a.html

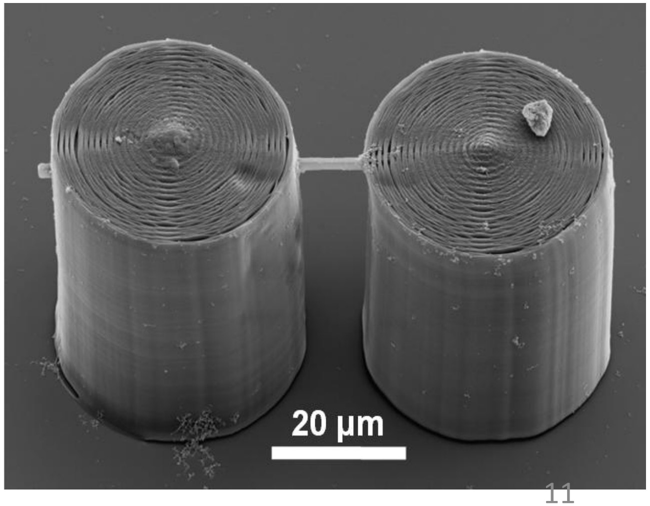
Direct Laser Writing











Symulacje komputerowe – metoda FDTD

- Rozwiązywanie równań Maxwella w dziedzinie czasu dla zadanej struktury (ε(**x**), μ(**x**))

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_o}; \ \nabla \cdot \vec{B} = 0; \ \nabla \times \vec{E} = -\frac{\partial B}{\partial t}; \ \nabla \times \vec{B} = \mu_o \left(\vec{J} + \varepsilon_o \frac{\partial \vec{E}}{\partial t} \right)$$

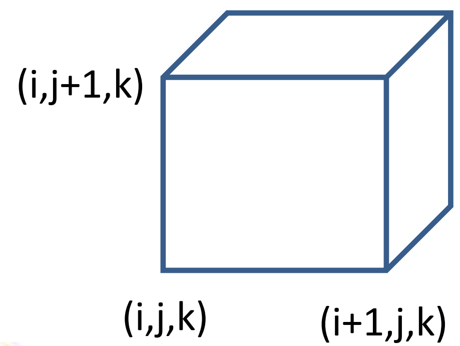
- Obliczenia prowadzone na siatce w czasie i przestrzeni. Dyskretyzacja równań Maxwella:

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t}$$

$$\Downarrow$$

$$\frac{E_y^n(i+1,j,k) - E_y^n(i,j,k)}{\Delta x} - \frac{E_x^n(i,j+1,k) - E_x^n(i,j,k)}{\Delta y} = -\mu \frac{H_z^{n+1/2}(i,j,k) - H_z^{n-1/2}(i,j,k)}{\Delta t}$$

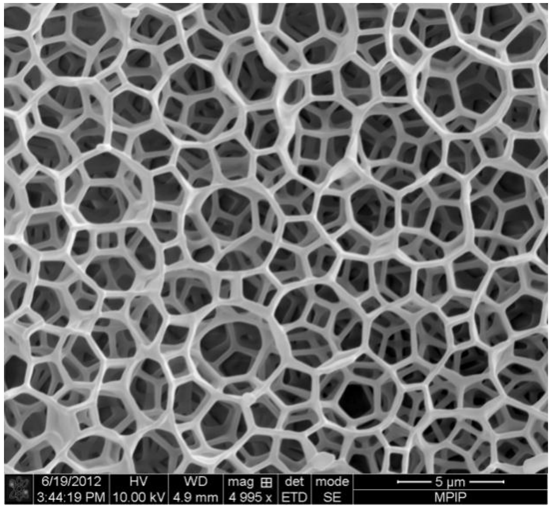
Gdzie *n* – czas



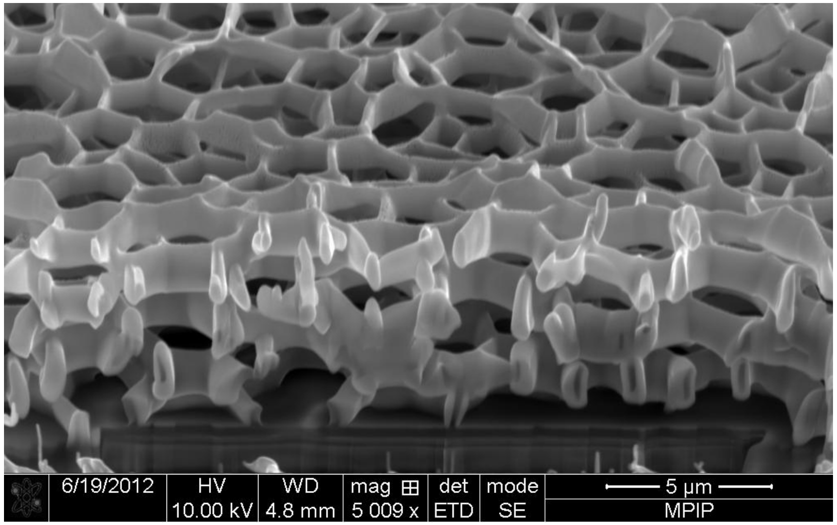
Sieć Yee

Polimerowe sieci superjednorodne

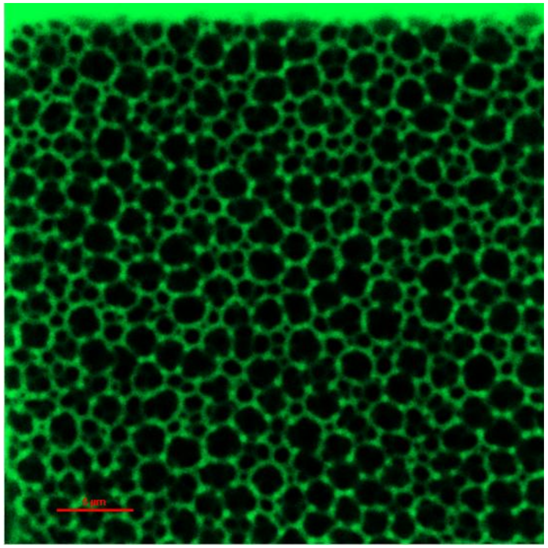
SEM

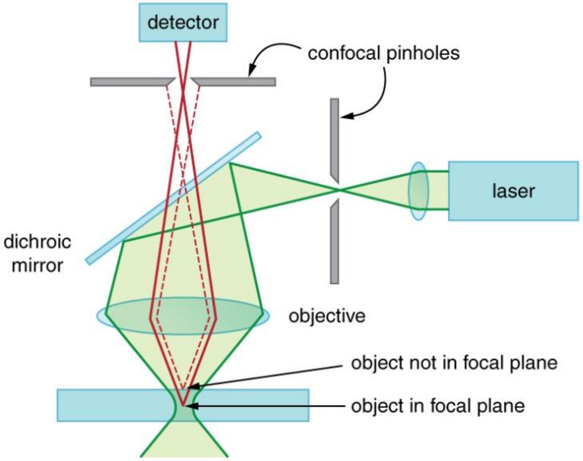


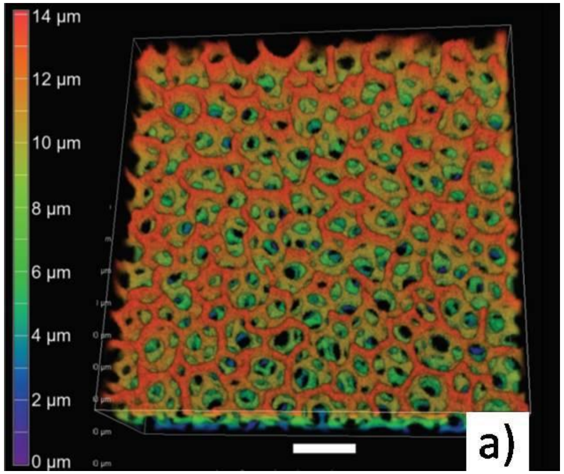
FIB-SEM



Mikroskopia konfokalna

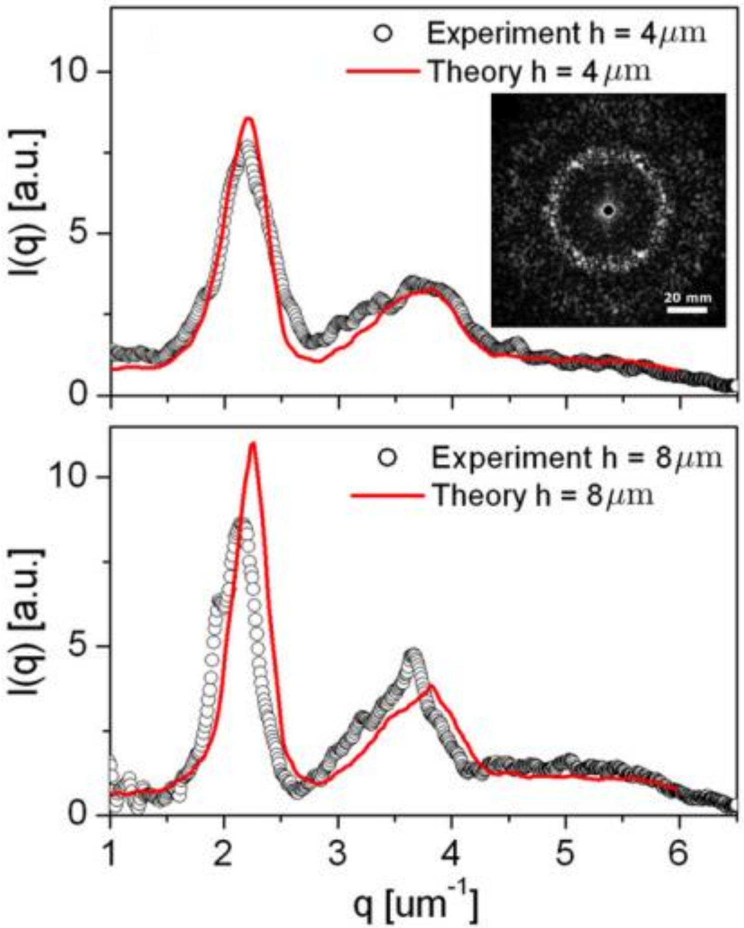




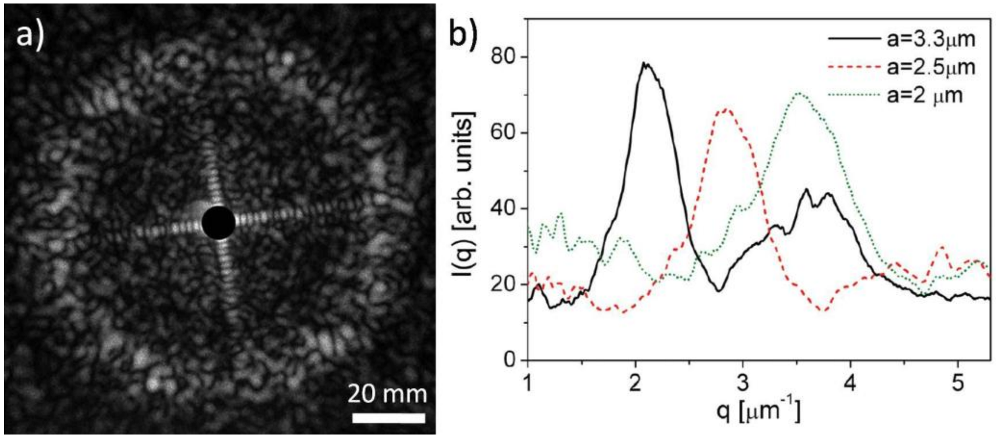


Polimerowe sieci superjednorodne

Badanie sieci metodą dyfrakcji światła widzialnego



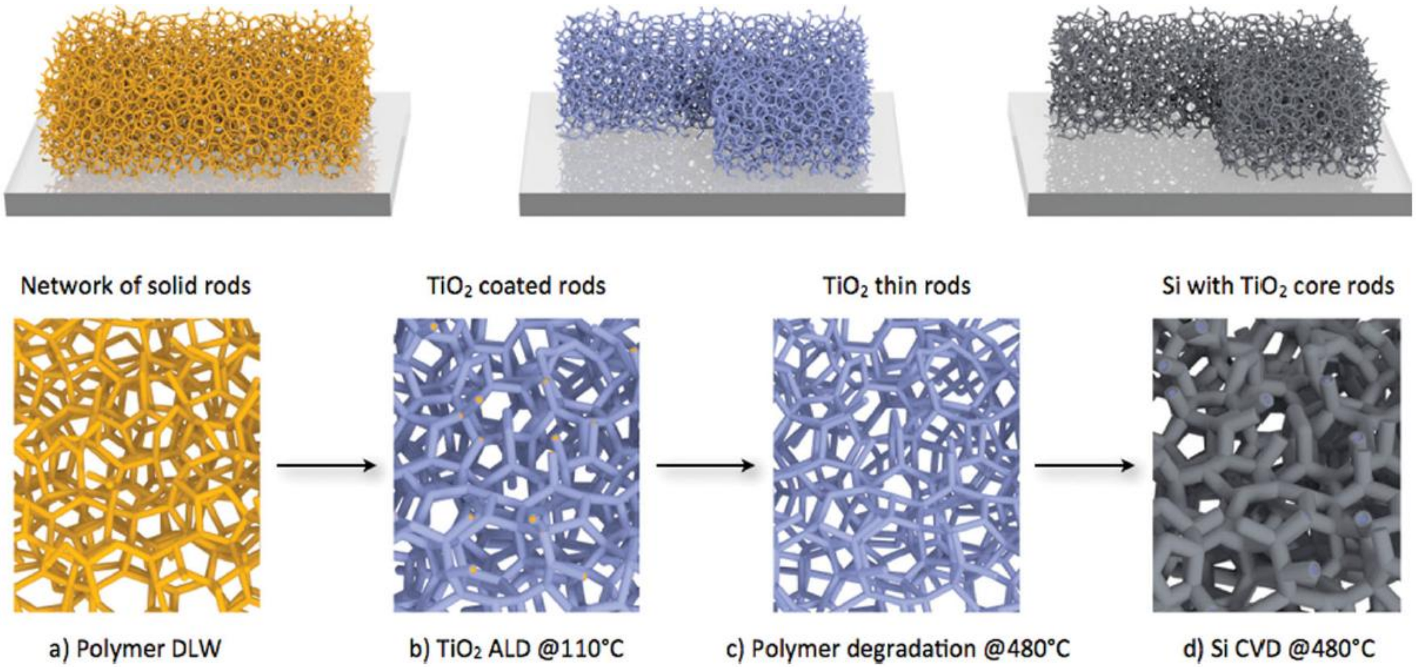
Miniaturyzacja sieci

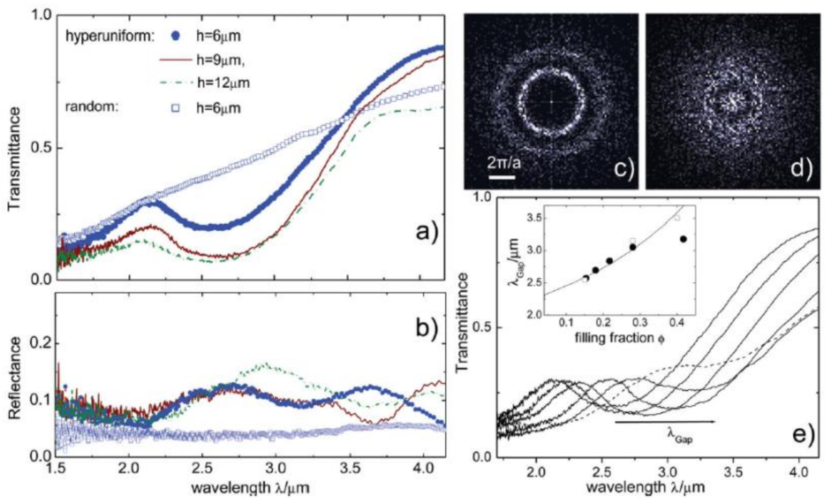


Problem:

Zbyt niski współczynnik załamania polimeru (~1.5) – konieczna zamiana na Si lub TiO_2

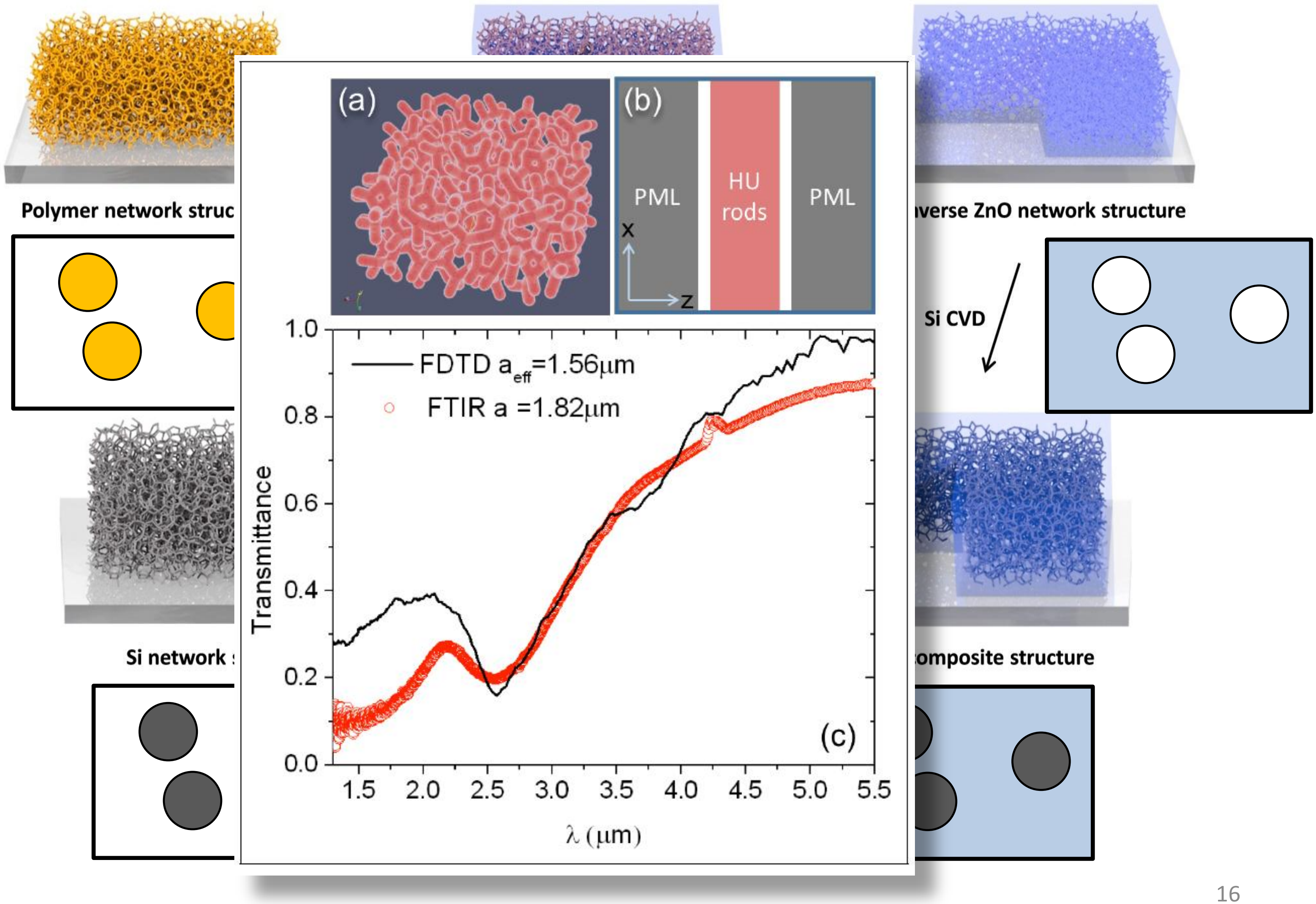
Pojedyncza inwersja do Si





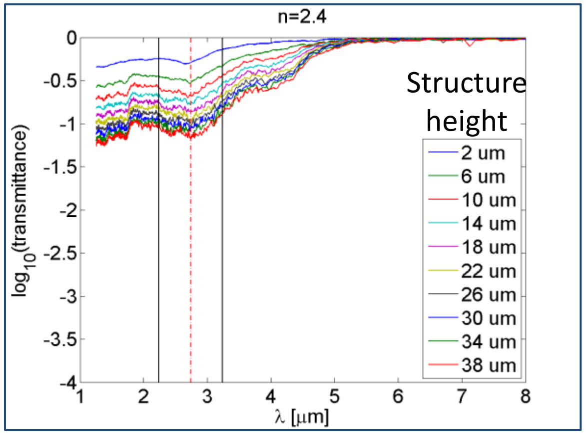
N. Muller , J. Haberko , C. Marichy , F. Scheffold, Adv. Opt. Mater. 2014, 2, 115–119

Podwójna inwersja do Si

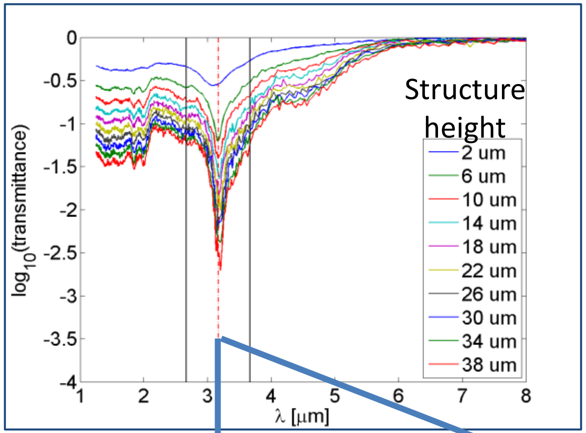


Polimerowe sieci superjednorodne - symulacje

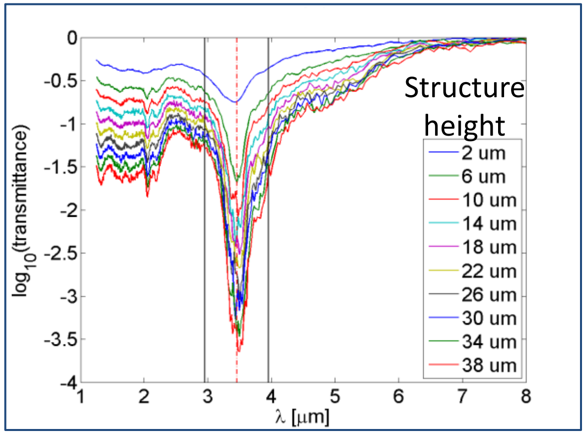
n = 2.4



n = 3.0

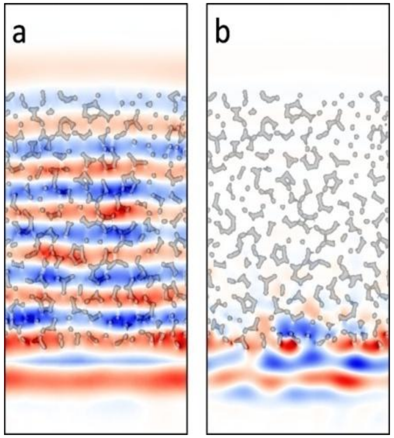


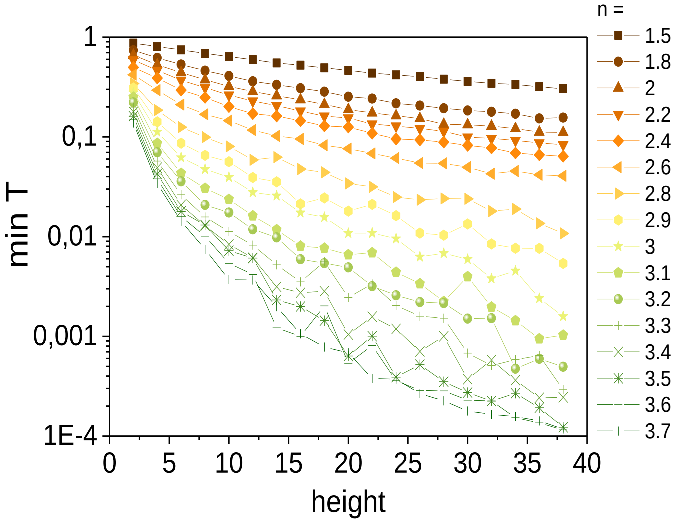
n = 3.4

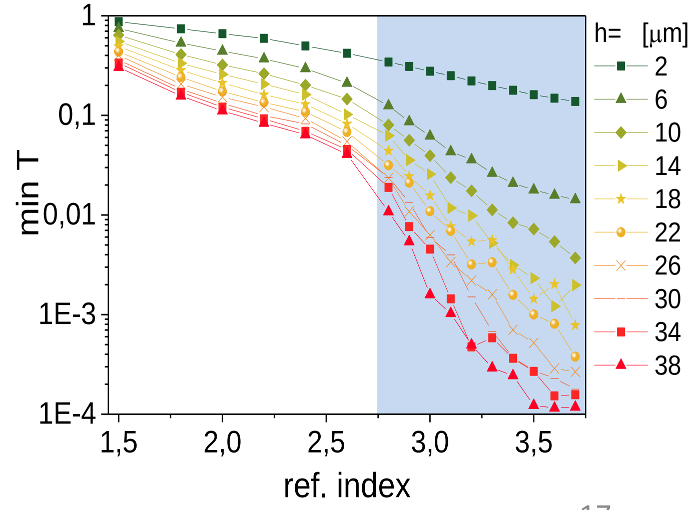


Długość fali:

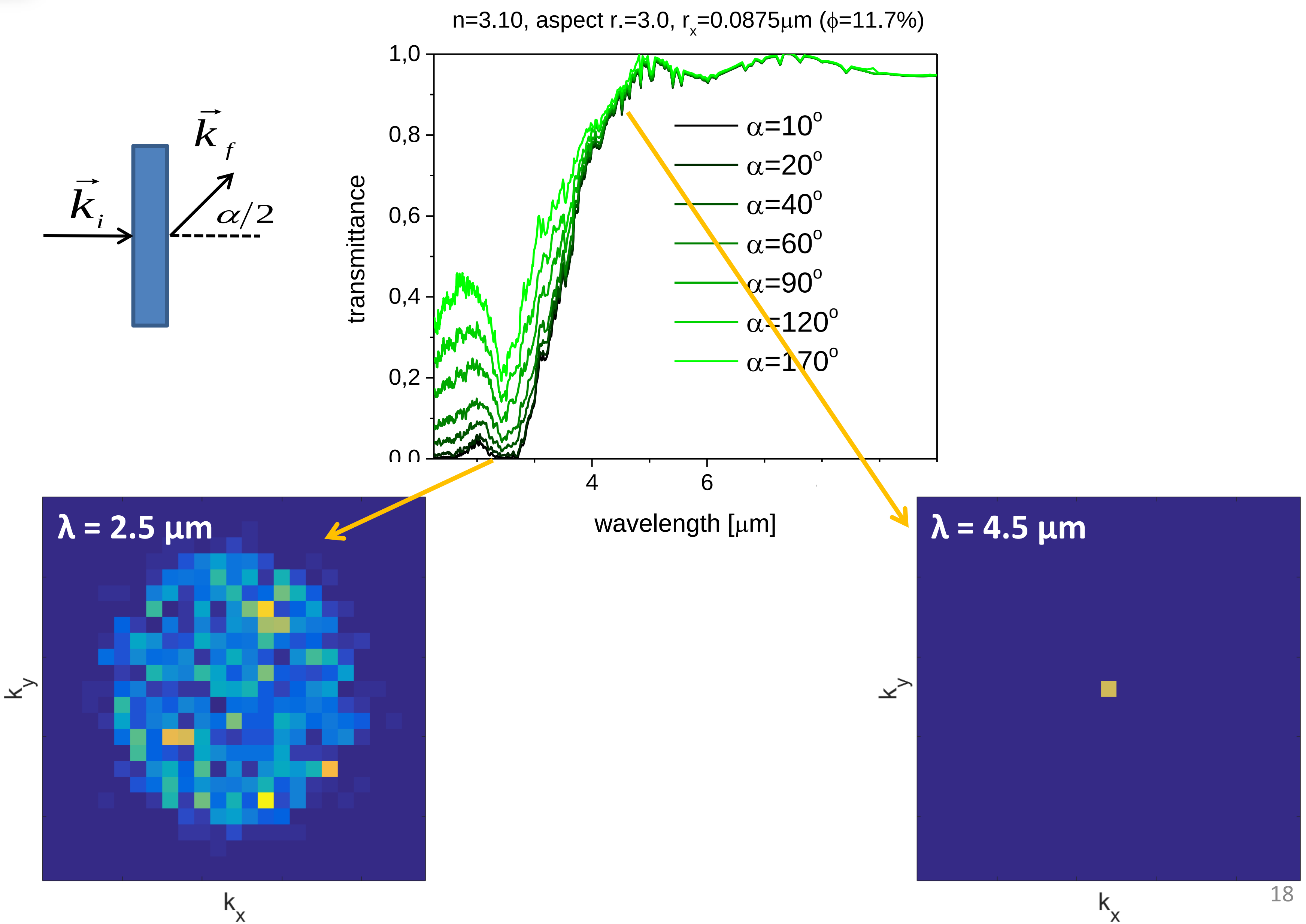
Poza przerwą	W obrębie przerwy







Polimerowe sieci superjednorodne - symulacje



Polimerowe sieci superjednorodne - symulacje

Fitowanie symulowanych krzywych transmisji

Sieć można traktować jako złożenie:

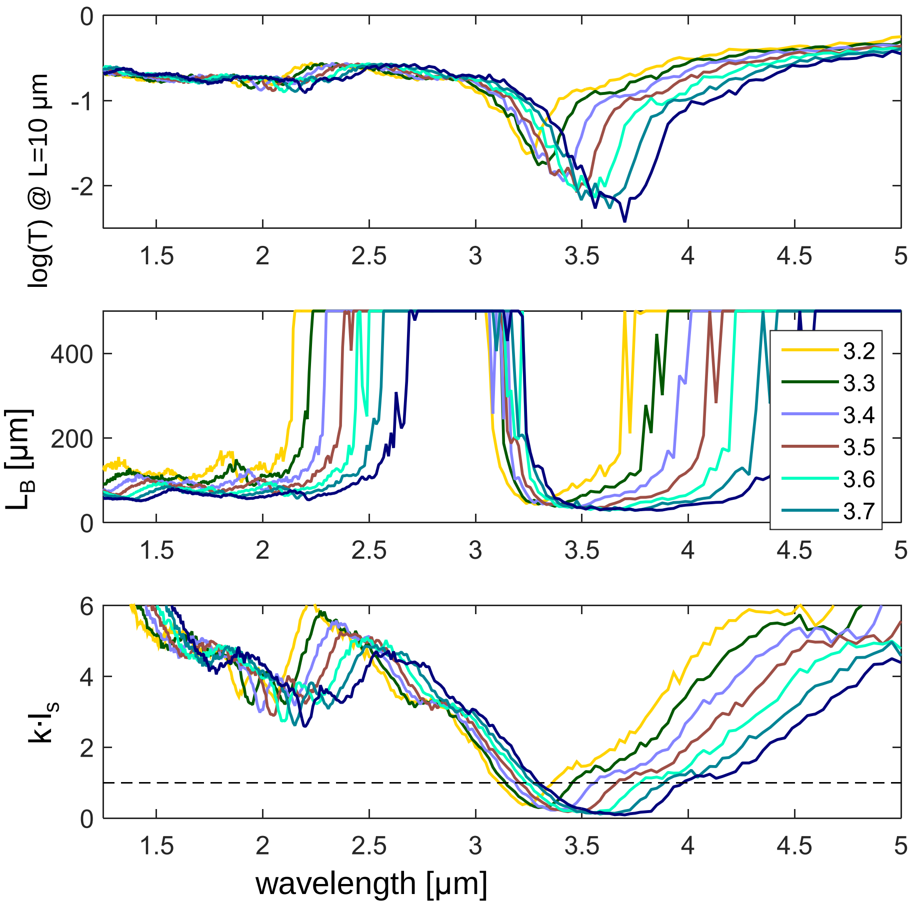
- W pełni nieuporządkowanej płytki materiału o grubości *L:* $T_{slab}(L) = \frac{\frac{5}{3}\frac{l_s}{L}}{\left(1+\frac{4}{3}\frac{l_s}{L}\right)}$
- Materiału z przerwą: $T_{BG}(L) = \cosh^{-2}\left(\frac{2L}{L_B}\right)$

gdzie l_s – średnia droga rozpraszania,

L_B – „Bragg length”

$$T_{hu} = \frac{\frac{5}{3}\frac{l_s}{L}}{\left(1+\frac{4}{3}\frac{l_s}{L}\right)\cosh^2\left(\frac{2L}{L_B}\right)}$$

Zależność parametrów fitu od współczynnika załamania



Polimerowe sieci superjednorodne - symulacje

Zależność parametrów fitu od kształtu beleczek

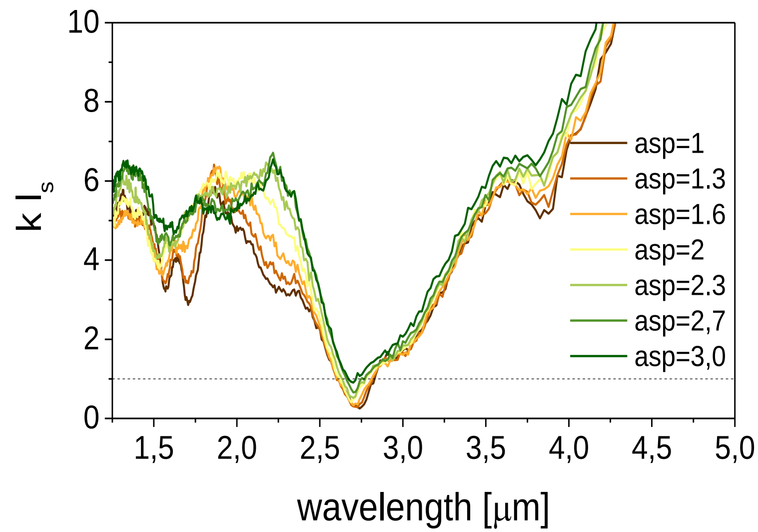
Modyfikacja kształtu woksela litograficznego

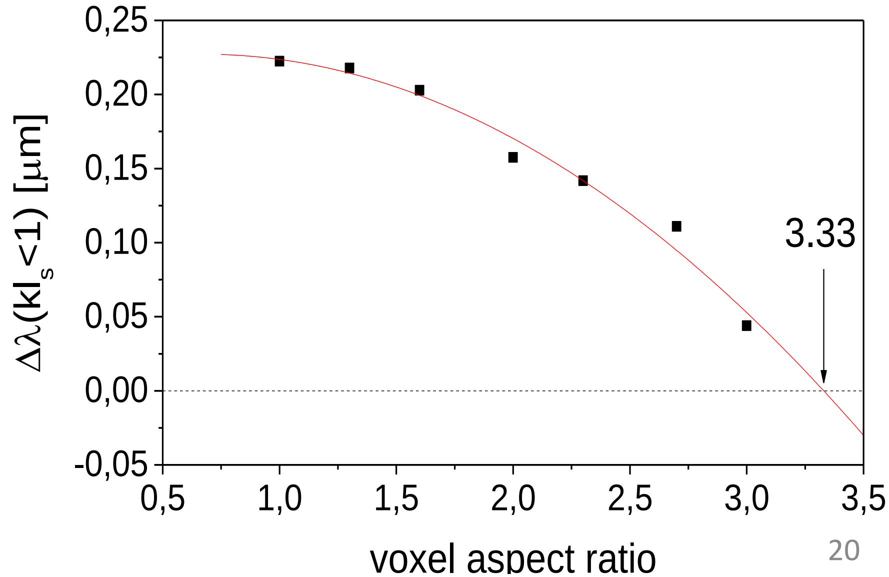
Beleczka o przekroju kołowym

Wsp. aspektu woksela litograficznego: 1.0

Beleczka o przekroju eliptycznym

Wsp. aspektu woksela litograficznego: ~2,5





Układy superjednorodne - publikacje

Optics Express 21(1), 1057–1065 (2013)

Fabrication of mesoscale polymeric templates for three-dimensional disordered photonic materials

Jakub Haberko[1,2] and Frank Scheffold[1,*]

PHYSICAL REVIEW A **88**, 043822 (2013)

Direct laser writing of three-dimensional network structures as templates for disordered photonic materials

Jakub Haberko,[1,2] Nicolas Muller,[1] and Frank Scheffold[1,*]

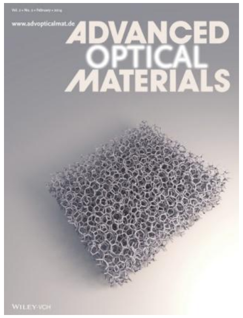
[1] *Physics Department and Fribourg Center for Nanomaterials, University of Fribourg, Chemin du Musée 3, 1700 Fribourg, Switzerland*

[2] *Faculty of Physics and Applied Computer Science, AGH University of Science and Technology, aleja Mickiewicza 30, 30-059 Krakow, Poland*

Materials Views
www.MaterialsViews.com

ADVANCED OPTICAL MATERIALS
www.advopticalmat.de

Adv. Optical Mater. **2014**, *2*, 115–119



Silicon Hyperuniform Disordered Photonic Materials with a Pronounced Gap in the Shortwave Infrared

*Nicolas Muller, Jakub Haberko, Catherine Marichy, and Frank Scheffold**

Układy superjednorodne - publikacje

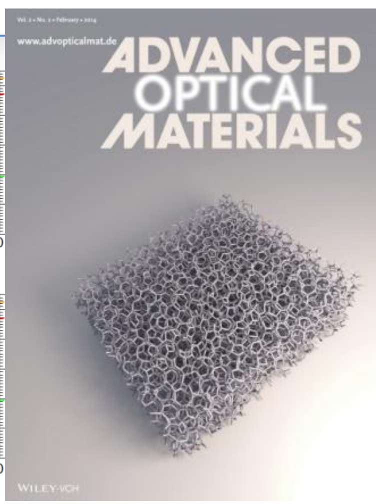
PRL **117,** 053902 (2016) PHYSICAL REVIEW LETTERS week ending 29 JULY 2016

Role of Short-Range Order and Hyperuniformity in the Formation of Band Gaps in Disordered Photonic Materials

Luis S. Froufe-Pérez,[1] Michael Engel,[2,3] Pablo F. Damasceno,[2,3] Nicolas Muller,[1] Jakub Haberko,[4] Sharon C. Glotzer,[2,3,5] and Frank Scheffold[1]

[1]*Department of Physics, University of Fribourg, CH-1700 Fribourg, Switzerland*
[2]*Department of Chemical Engineering, University of Michigan, Ann Arbor, Michigan 48109, USA*
[3]*Biointerfaces Institute, University of Michigan, Ann Arbor, Michigan 48109, USA*
[4]*Faculty of Physics and Applied Computer Science, AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland*
[5]*Department of Materials Science and Engineering, University of Michigan, Ann Arbor, Michigan 48109, USA*

(Received 29 January 2016; published 27 July 2016)



Photonic hyperuniform networks obtained by silicon double inversion of polymer templates

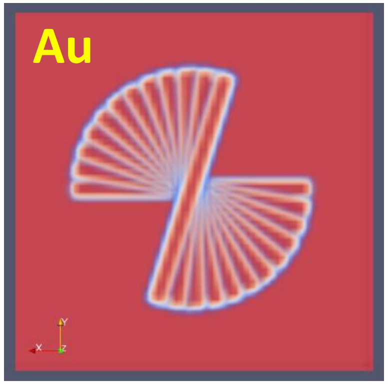
NICOLAS MULLER[1,*], JAKUB HABERKO[2], CATHERINE MARICHY[1,3], AND FRANK SCHEFFOLD[1]

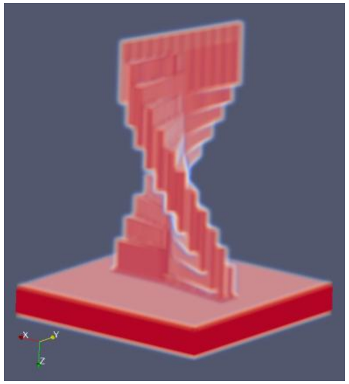
[1]*Publications Department, OSA-The Optical Society, 2010 Massachusetts Avenue NW, Washington D.C., 20036*
[2]*Faculty of Physics and Applied Computer Science, AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland*
[3]*Université de Lyon, Laboratoire des Multimatériaux et Interfaces, Villeurbanne Cedex, France, 69622*
[*]*Frank.Scheffold@unifr.ch*

Compiled October 9, 2016

Optica 4(3), 361–366 (2017)

Metalowe makarony

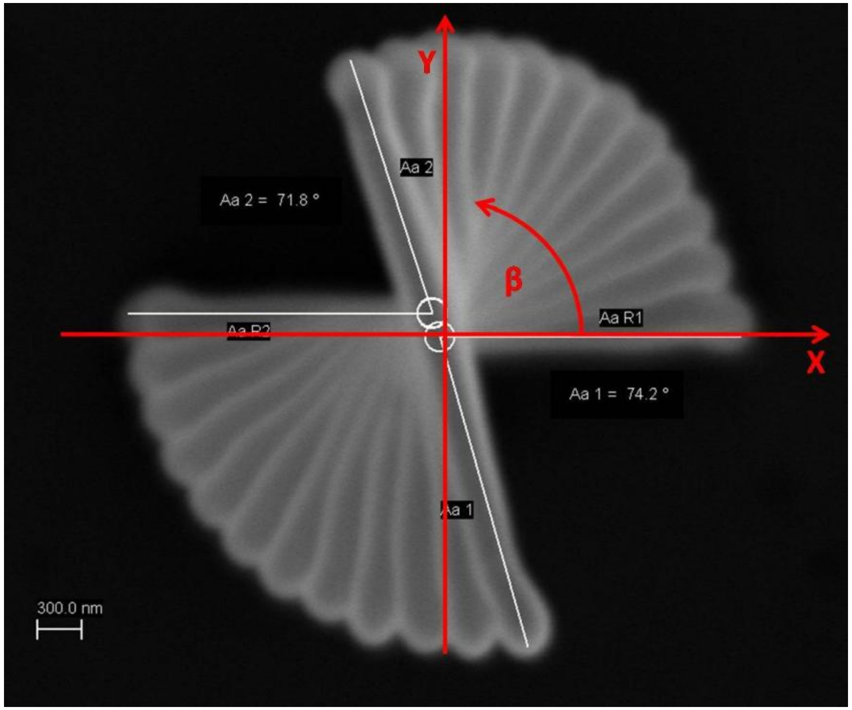


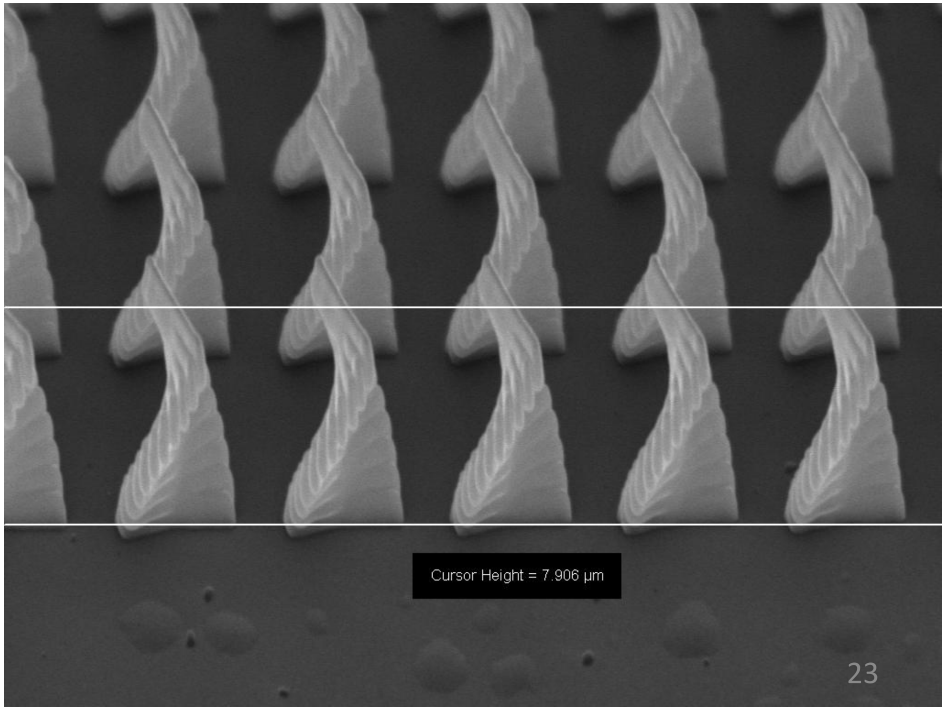


Czy taka struktura może zmieniać polaryzację światła?

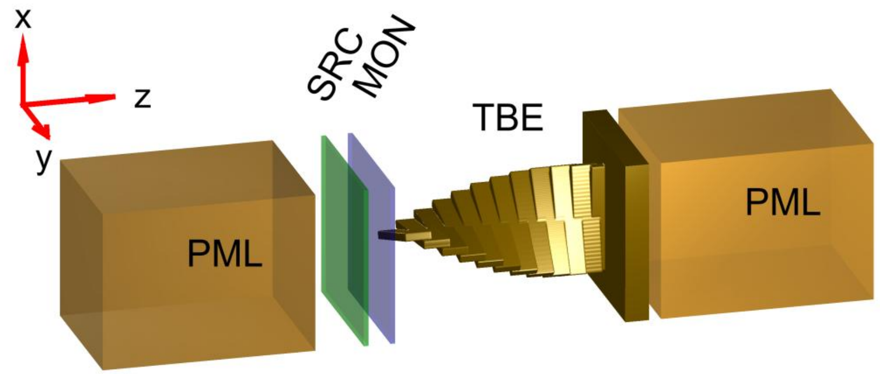
Parametry geometryczne makaronu:

- L – period
- d – średnica
- h – wysokość



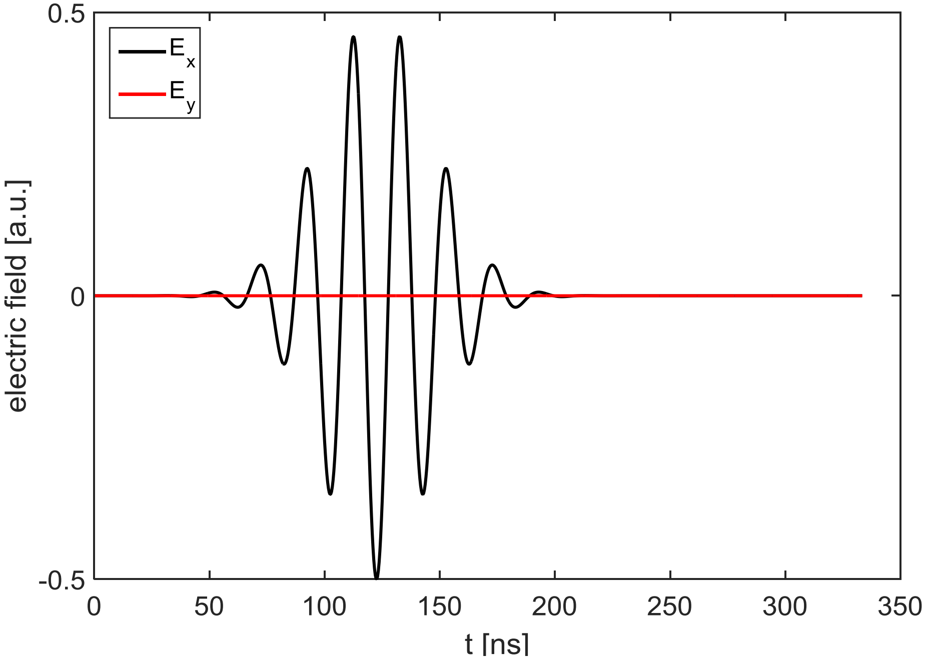


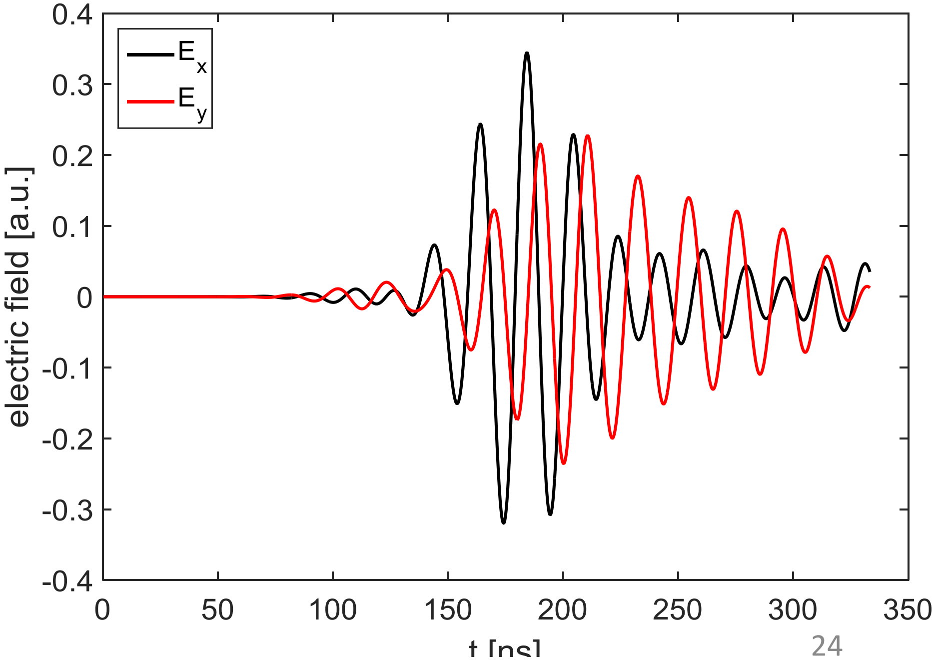
Metalowe makarony – polaryzacja fali odbitej



TBE – twisted band element (skręcona taśma)

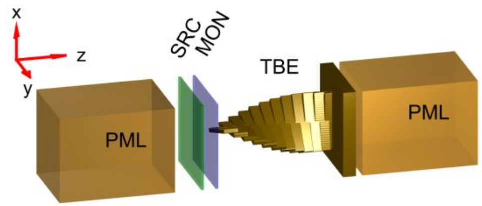
Kształt impulsu padającego i po odbiciu od struktury

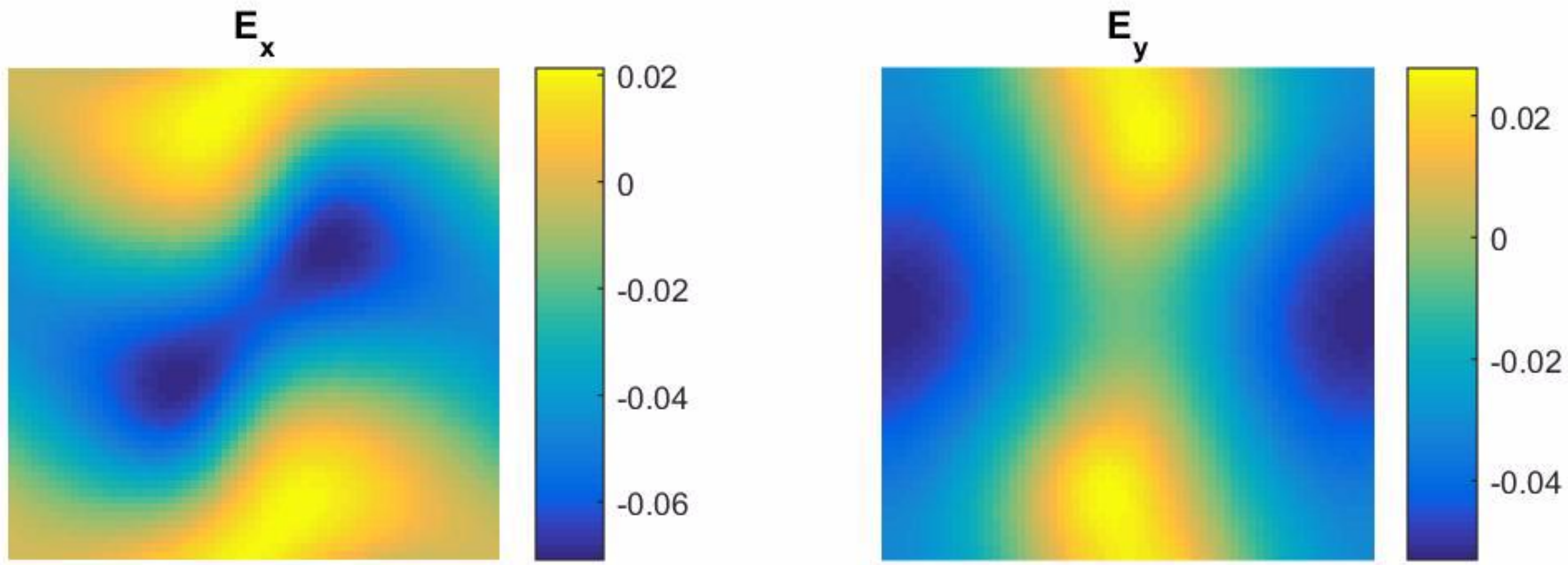




Metalowe makarony – polaryzacja fali odbitej

Ewolucja składowych natężenia pola elektrycznego w płaszczyźnie monitora (MON)





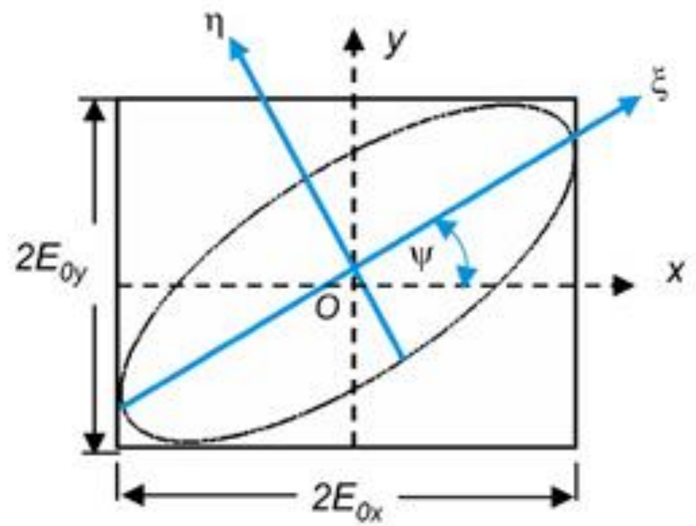
Metalowe makarony – stan polaryzacji po odbiciu

Eliptyczność

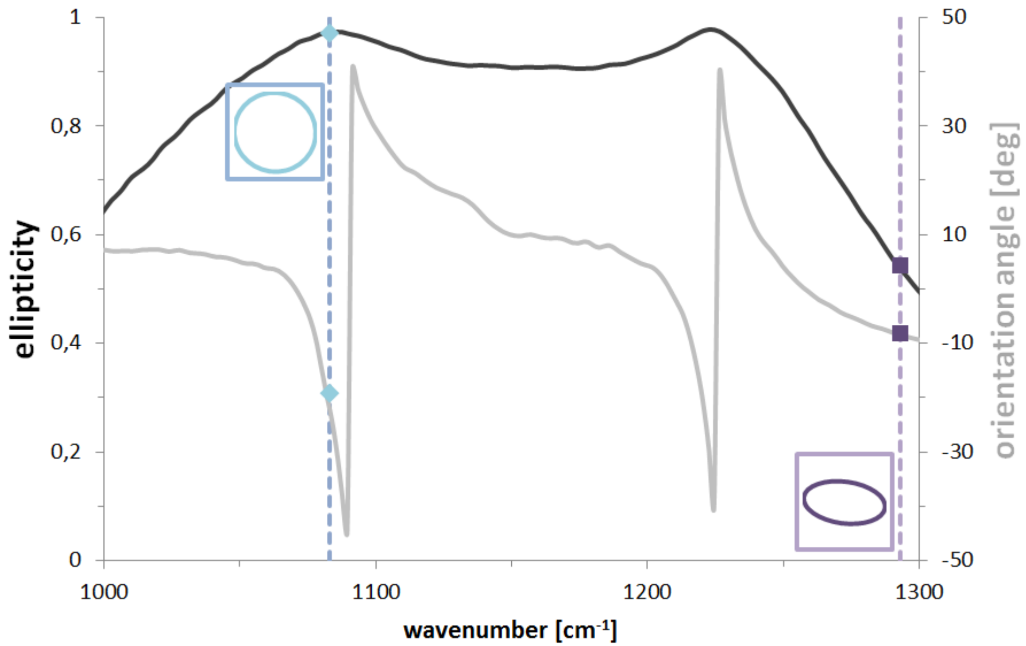
$$\tan(2\psi) = \frac{2E_{0x}E_{0y}}{E_{0x}^{\;2} - E_{0y}^{\;2}}\cos\delta$$

Orientacja

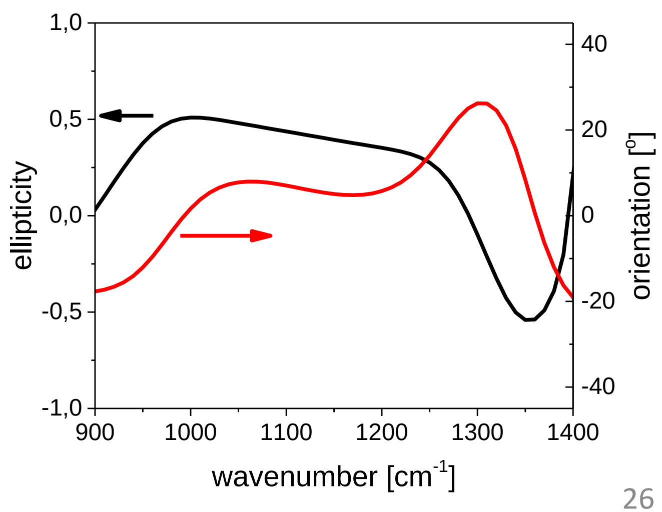
$$\sin(2\chi) = \frac{2E_{0x}E_{0y}}{E_{0x}^{\;2} + E_{0y}^{\;2}}\sin\delta$$



Eksperyment



Symulacje



Metalowe makarony – publikacja

Applied Physics B (2017) 123:285
https://doi.org/10.1007/s00340-017-6864-y

Applied Physics B
Lasers and Optics

CrossMark

Light polarization management via reflection from arrays of sub-wavelength metallic twisted bands

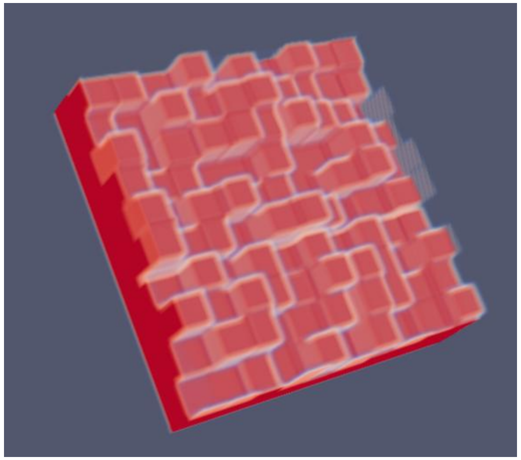
M. Nawrot[1] · J. Haberko[2] · Ł. Zinkiewicz[1] · P. Wasylczyk[1]

Received: 12 September 2017 / Accepted: 9 November 2017

Abstract
With constant progress of nano- and microfabrication technologies, photolithography in particular, a number of sub-wavelength metallic structures have been demonstrated that can be used to manipulate light polarization. Numerical simulations of light propagation hint that helical twisted bands can have interesting polarization properties. We use three-dimensional

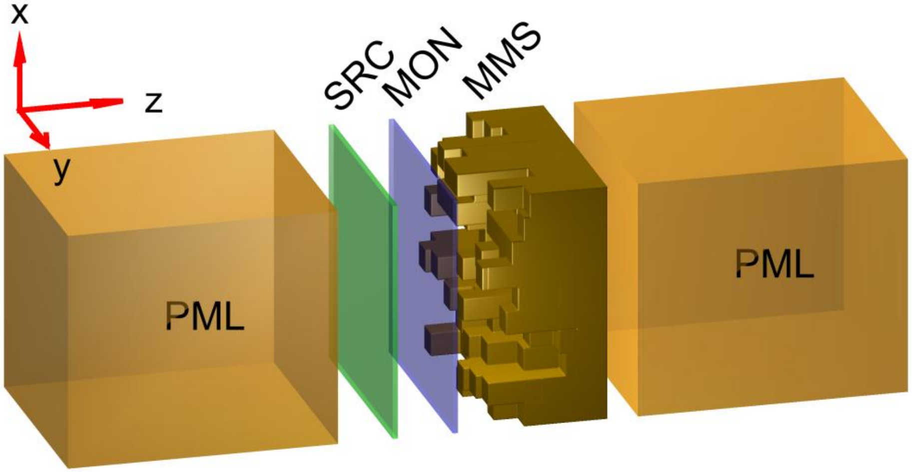
Losowe piksele

Idea: modyfikowanie stanu polaryzacji fali e-m poprzez odbicie od metalicznej „macierzy pikseli”

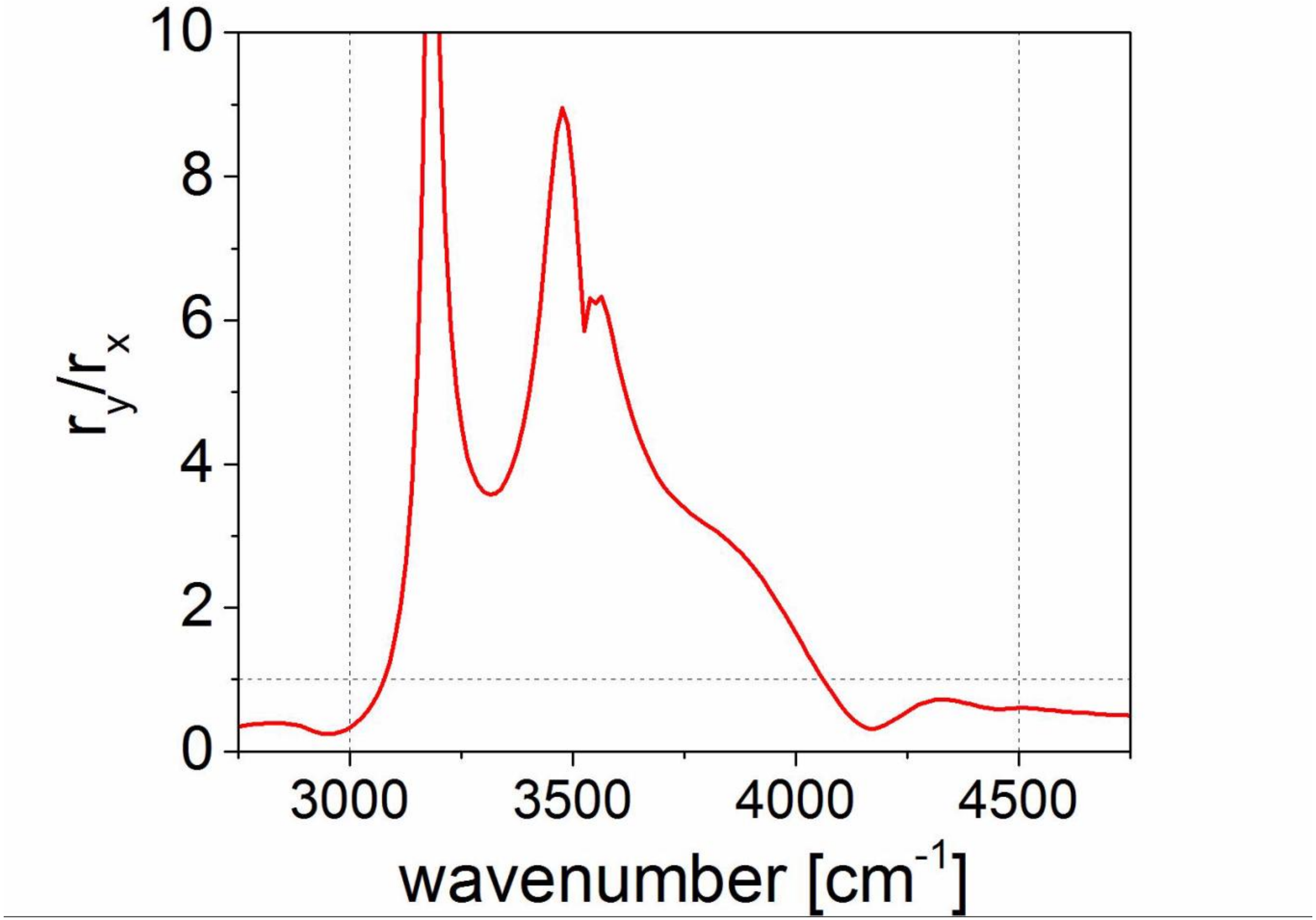


Procedura optymalizacyjna:

- Wygeneruj losową macierz pikseli
- Wykonaj symulację dla tej struktury
- Oblicz jak dobrze realizuje założenia (wylicz funkcję fitness)
- Zmodyfikuj poprzednią macierz
- Znów wykonaj symulację i oblicz funkcję fitness
- Jeśli zmodyfikowana struktura jest lepsza, to zachowaj ją, jeśli nie to odrzuć
- I tak dalej…



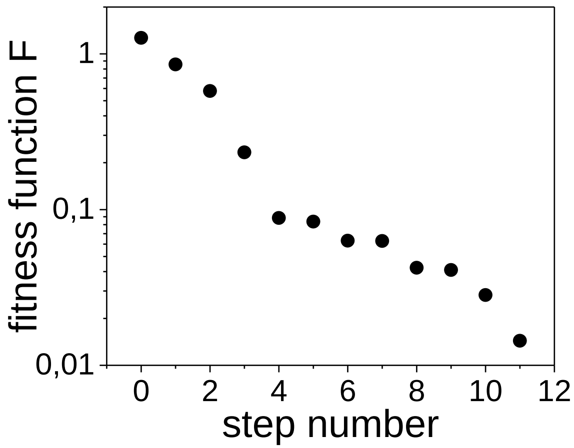
Losowe piksele – przebieg optymalizacji



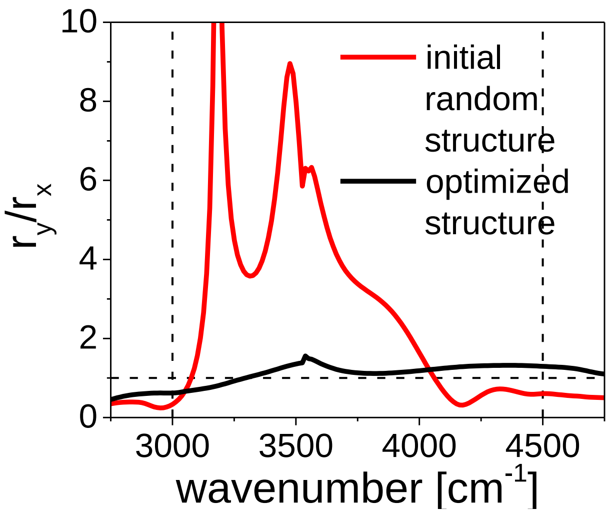
Losowe piksele – płytka półfalowa

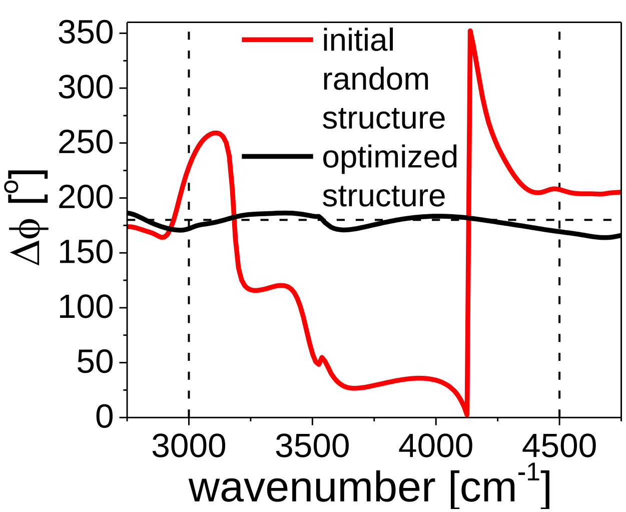
Funkcja oceny jakości danej struktury *F*

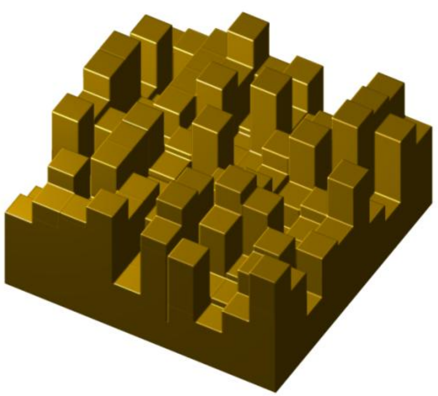
$$F_{\lambda/2} = \frac{1}{N}\sum_{i=1}^{N}\left[\left(r_x(f_i) - r_y(f_i)\right)^2 + w\left(\Delta\varphi(f_i) - 180^o\right)^2\right]$$



$$\langle\Delta\phi\rangle = 179^o,\ \sigma_{\Delta\phi} = 5^o$$



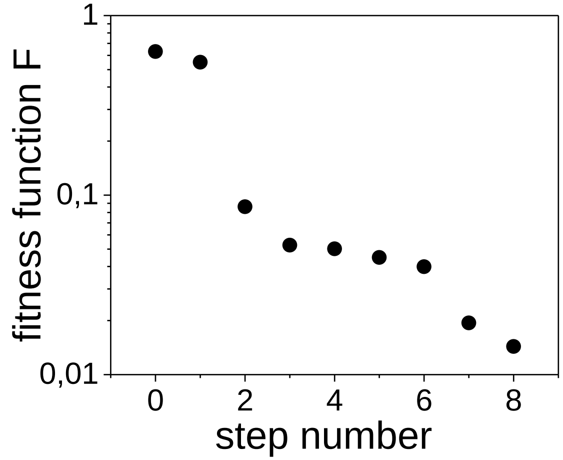




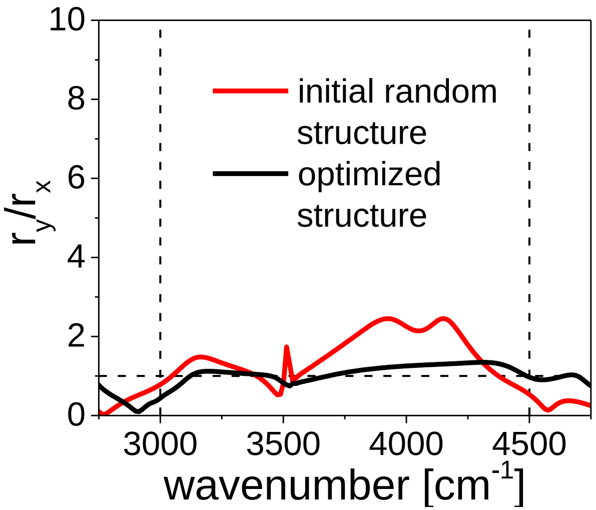
Losowe piksele – płytka ćwierćfalowa

Funkcja oceny jakości danej struktury *F*

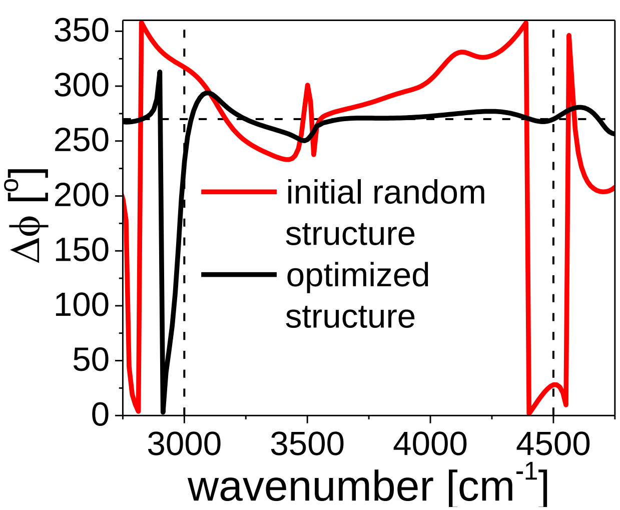
$$F_{\lambda/4} = \frac{1}{N}\sum_{i=1}^{N}\left(r_x(f_i) - r_y(f_i)\right)^2 + \frac{w}{N}\min\left[\sum_{i=1}^{N}\left(\Delta\varphi(f_i) - 90^o\right)^2, \sum_{i=1}^{N}\left(\Delta\varphi(f_i) - 270^o\right)^2\right]$$

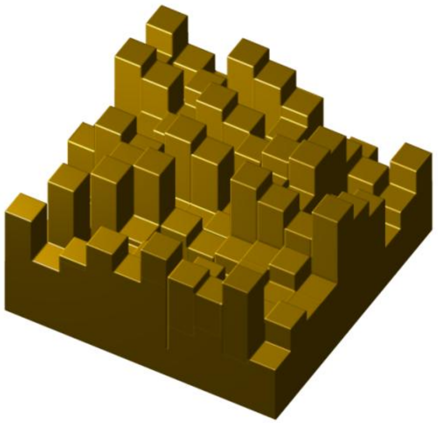


$\langle r_y/r_x \rangle = 1.09,\ \sigma_{r_x/r_y} = 0.20$



$\langle \Delta\phi \rangle = 271^o,\ \sigma_{\Delta\phi} = 9^o$





Losowe piksele – publikacja

OPTICS: 22659 Model 5G pp. 1–4 (col. fig: NIL)

ARTICLE IN PRESS

Optics Communications xx (xxxx) xxx–xxx

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Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Optics Communications

Reflecting metallic metasurfaces designed with stochastic optimization as waveplates for manipulating light polarization

Jakub Haberko [a,*], Piotr Wasylczyk [b]

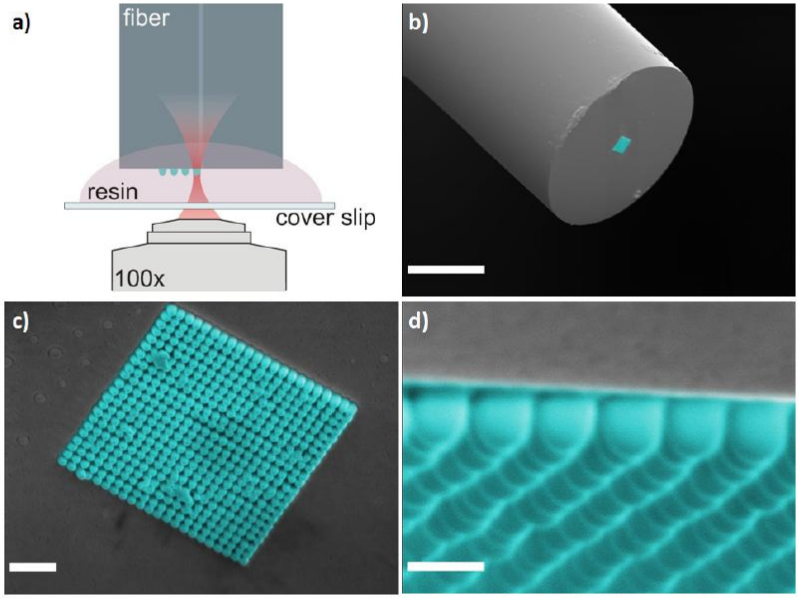
[a] *AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. Mickiewicza 30, 30-059 Krakow, Poland*

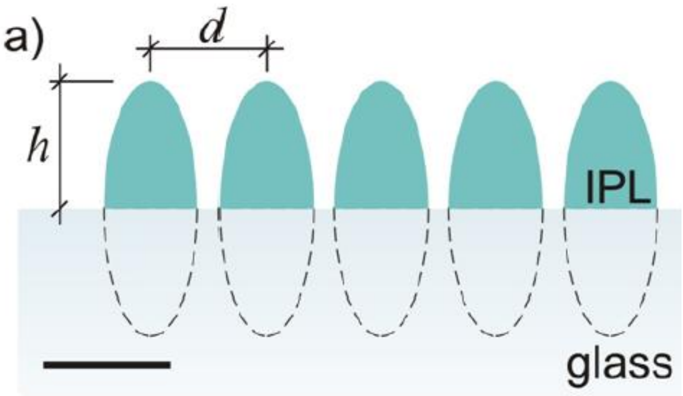
[b] *Photonic Nanostructure Facility, Institute of Experimental Physics, Faculty of Physics, University of Warsaw, ul. Pasteura 5, 02-093 Warszawa, Poland*

A R T I C L E I N F O

A B S T R A C T

Pokrycia antyrefleksyjne na końcu światłowodu





Pokrycia antyrefleksyjne – publikacja

Optics Express 22(10), 12545–12550 (2014)

Microstructured gradient-index antireflective coating fabricated on a fiber tip with direct laser writing

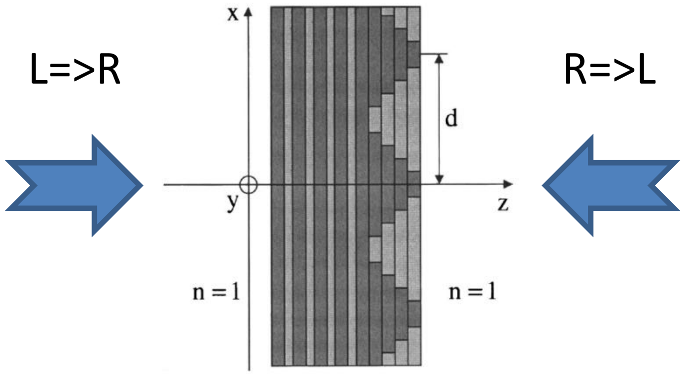
Maciej Kowalczyk,[1] Jakub Haberko,[2] and Piotr Wasylczyk[1,*]

[1]Photonic Nanostructure Facility, Institute of Experimental Physics, Faculty of Physics, University of Warsaw, ul Hoża 69, 00-681 Warszawa, Poland
[2]AGH University of Science and Technology, Faculty of Physics and Applied Computer Science,Al. Mickiewicza 30, 30-059 Kraków, Poland
[*]piotr.wasylczyk@fuw.edu.pl

Abstract: We present a simple broadband gradient-index antireflective coating, fabricated directly on a single mode telecom fiber tip. A regular array of hemi-ellipsoidal protrusions significantly reduce the Fresnel reflection from the glass-air interface. The parameters of the structure were

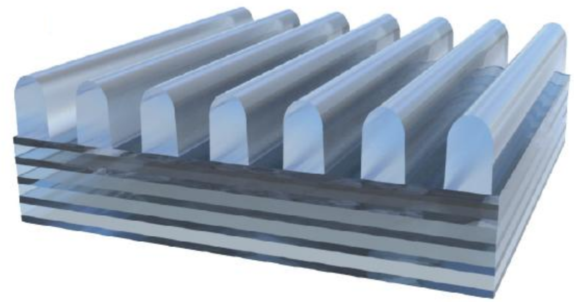
Układy o asymetrycznej transmisji dla jednej polaryzacji

Struktura, której **transmisja** w zakresie optycznym **zależy od kierunku propagacji**

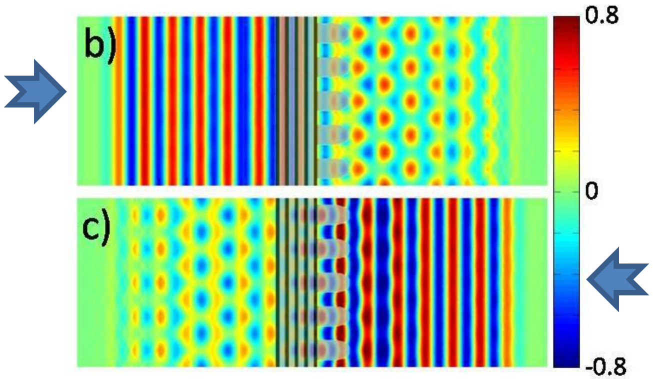


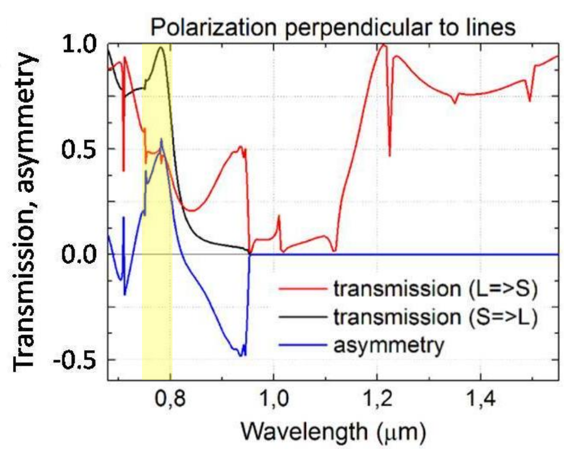
Pomysł za: Mandatori et al.
J. Opt. Soc. Am. B Vol. 24, No. 3 (2007)

Wersja uproszczona
(electron beam deposition + DLW)

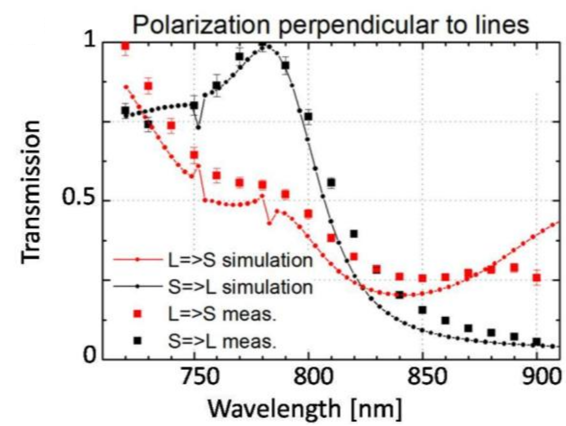


Rozkład natężenia pola elektrycznego

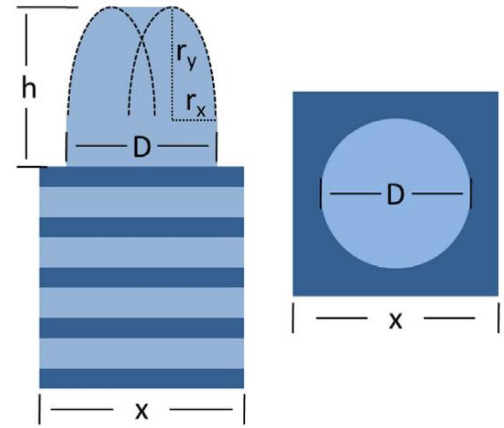




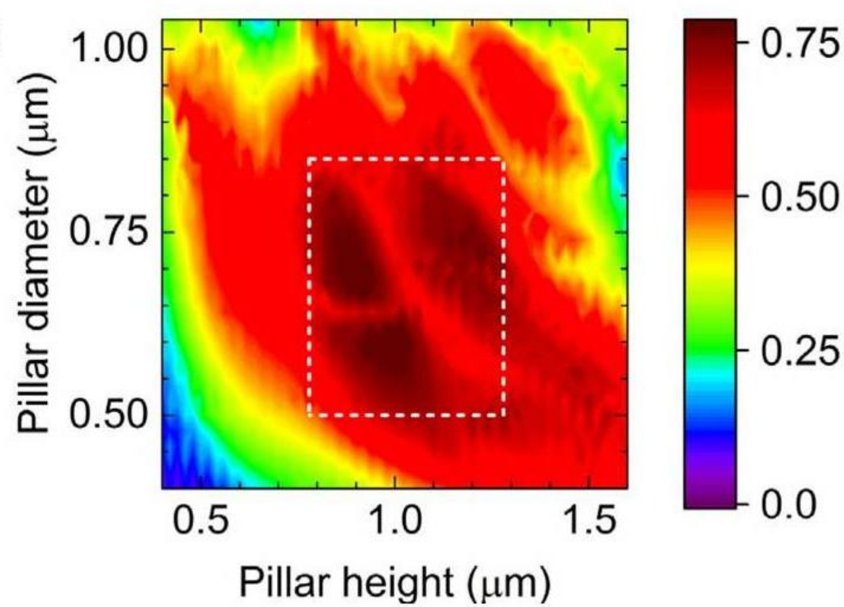
Eksperyment vs. symulacje



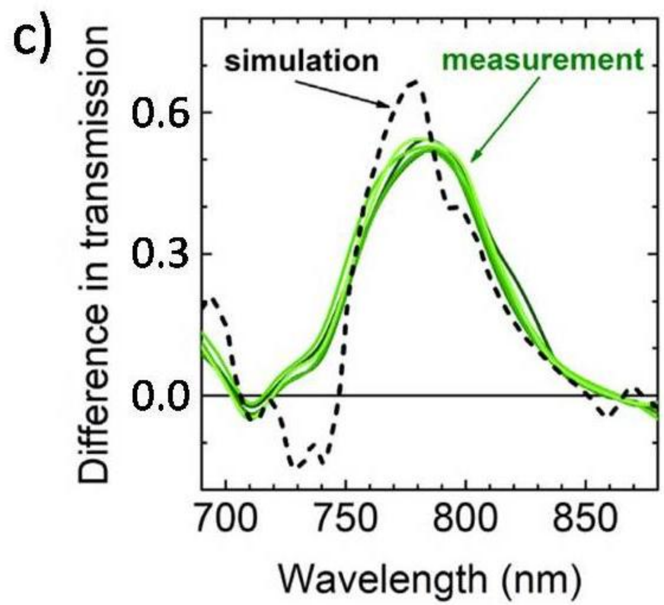
Układy o asymetrycznej transmisji niezależnej od polaryzacji



Optymalizacja



Eksperyment vs. symulacje



Układy o asymetrycznej transmisji – publikacje

Optics Express 23(4), 4206–4211 (2015)

Highly asymmetric near infrared light transmission in an all-dielectric grating-on-mirror photonic structure

[1] Photoni

[2] A

A

q

d

Optical Materials 73 (2017) 484–488

Contents lists available at ScienceDirect

Optical Materials

journal homepage: www.elsevier.com/locate/optmat

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Polarization-independent asymmetric light transmission in all-dielectric photonic structures

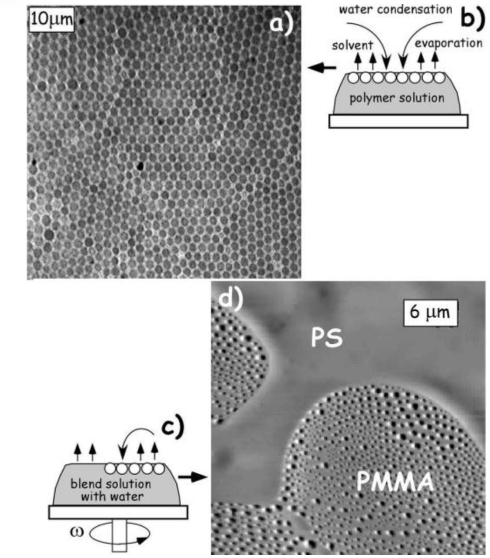
CrossMark

Lukasz Zinkiewicz [a,*], Michal Nawrot [a], Jakub Haberko [b], Piotr Wasylczyk [a]

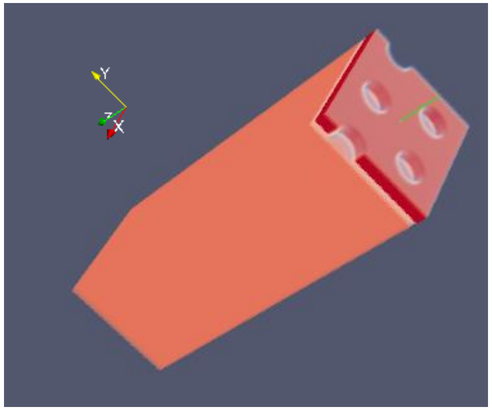
[a] Photonic Nanostructure Facility, Institute of Experimental Physics, Faculty of Physics, University of Warsaw, ul. Pasteura 5, 02-093, Warszawa, Poland

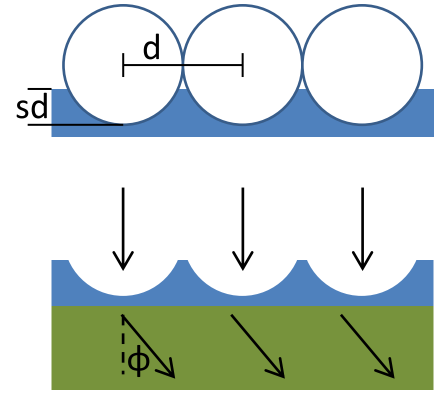
[b] AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, Al. Mickiewicza 30, 30-059, Krakow, Poland

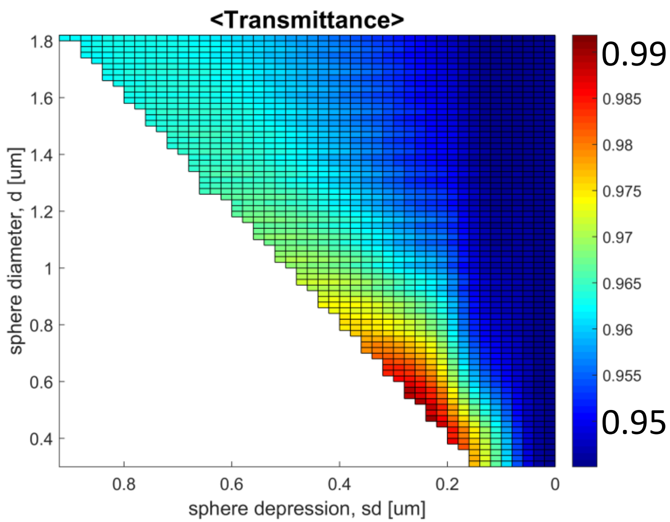
Warstwy antyrefleksyjne do organicznych ogniw fotowoltaicznych

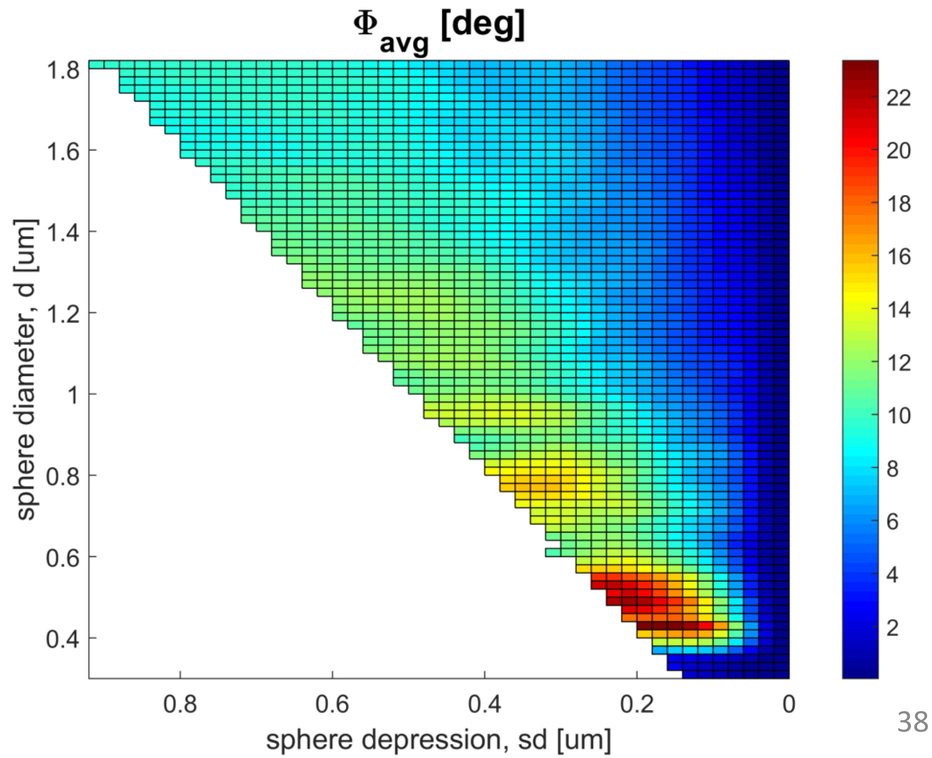


A. Budkowski et al., Acta Phys Pol. A 115(2) 435 (2009)



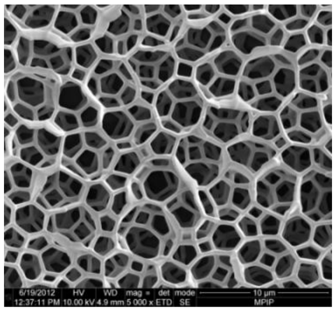


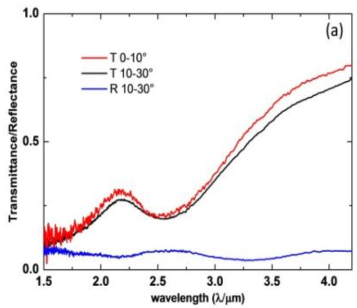


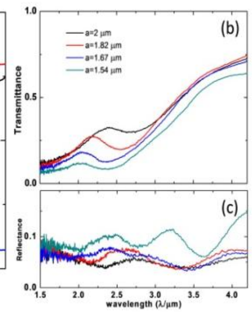


Podsumowanie

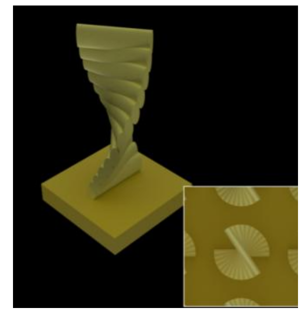
Izotropowe sieci z przerwą fotoniczną

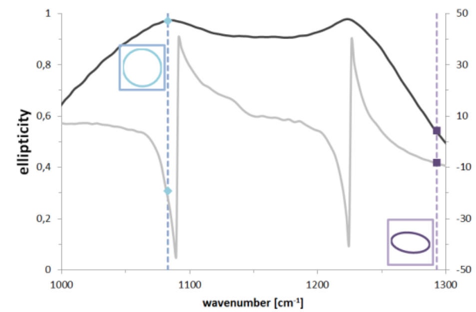




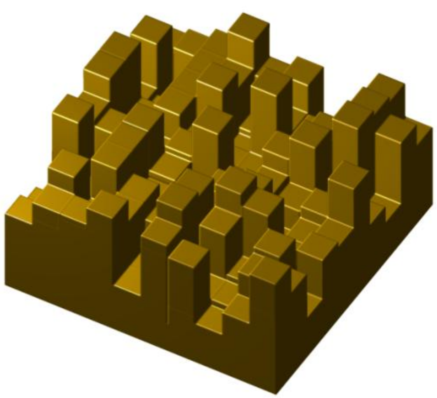


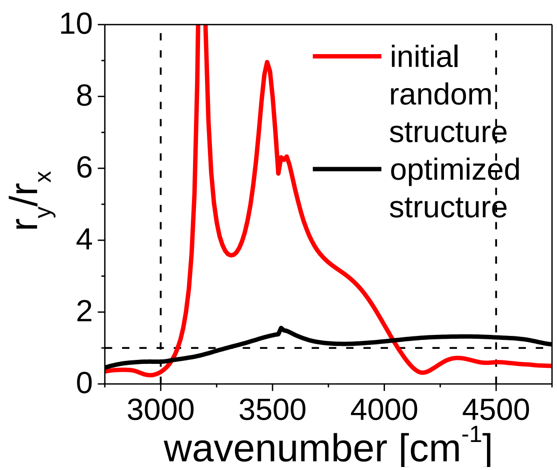
Metalowe makarony polaryzujące światło

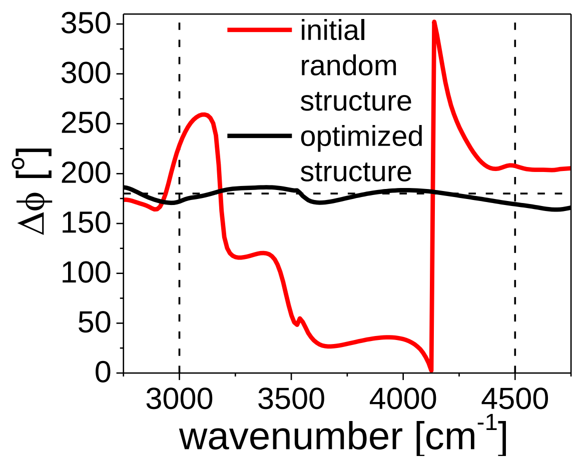




Stochastyczne płytki pół- i ćwierćfalowe

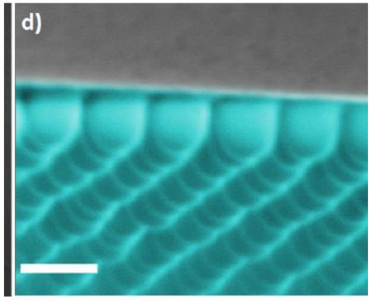




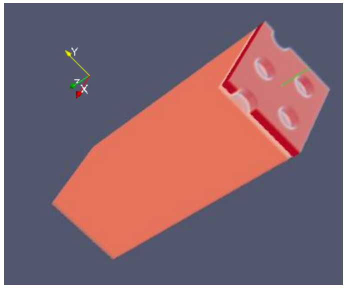


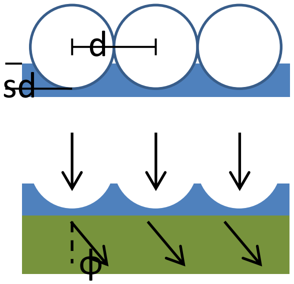
Podsumowanie

Pokrycia anyodbiciowe na światłowody

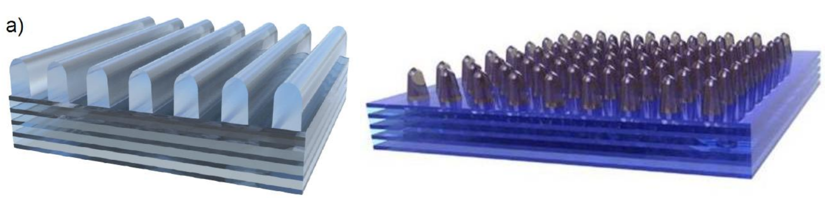


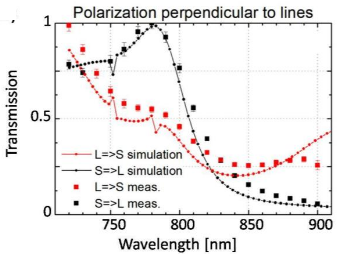
Pokrycia anyodbiciowe dla fotowoltaiki





Układy o asymetrycznej transmisji





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Dziękuję za uwagę

Metale w MEEPie

Model Lorentza-Drudego:

$$\varepsilon(\omega, \mathbf{x}) = \left(1 + \frac{i \cdot \sigma_D(\mathbf{x})}{\omega}\right) \left[\varepsilon_\infty(\mathbf{x}) + \sum_n \frac{\sigma_n(\mathbf{x}) \cdot \omega_n^2}{\omega_n^2 - \omega^2 - i\omega\gamma_n}\right],$$

$$= \left(1 + \frac{i \cdot \sigma_D(\mathbf{x})}{2\pi f}\right) \left[\varepsilon_\infty(\mathbf{x}) + \sum_n \frac{\sigma_n(\mathbf{x}) \cdot f_n^2}{f_n^2 - f^2 - i f \gamma_n / 2\pi}\right],$$

σ_n – względna „siła” rezonansu
ω_n – częstość rezonansu
γ_n – współczynnik tłumienia