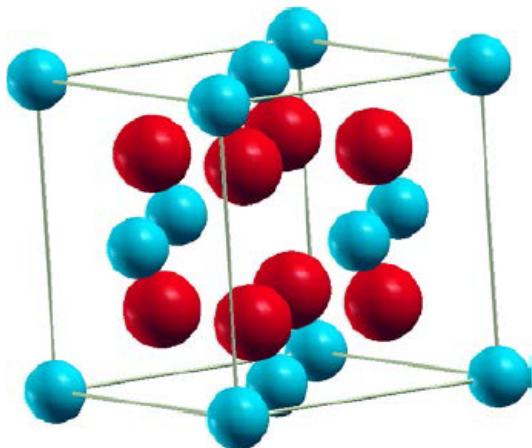


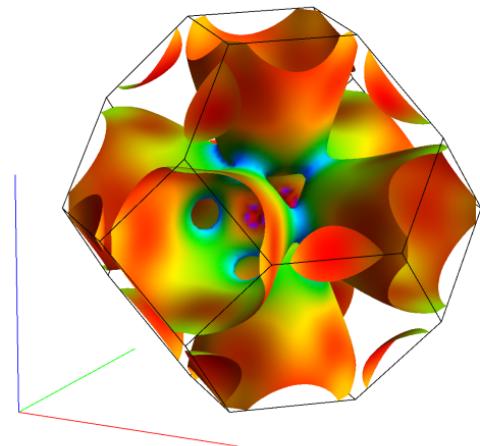
# Struktura elektronowa a efektywność zjawisk konwersji energii w złożonych strukturach krystalicznych

**Janusz TOBOŁA**

KFMS WFIS AGH



$$\mathbf{G} = \mathbf{G}_0 + \mathbf{G}_0 V \mathbf{G}$$



## PLAN

### Energy conversion effects in solids

fundamental- quantum electrodynamics & thermodynamics

practical - search for energy recovering, storage and saving

complex phenomena – *thermoelectrics, magnetocalorics, ion-batteries*

### *Ab initio* modelling: electronic structure (KKR & KKR-CPA)

electronic structure of realistic disordered systems

relativistic effects (spin-orbit interaction in transport)

electron transport via Boltzmann approach

densities of states vs. character of charge/discharge curves

### Applications to thermoelectric materials

Effect of **band convergence** in Mg<sub>2</sub>(Si-Sn-Ge) alloys

Effect of **spin-orbit coupling** on TE in p-doped Mg<sub>2</sub>X

Effect of **bands alignment** in half-Heusler phases

### Applications to Li-/Na-ion battery systems

Electronic structure of NaxCoO<sub>2</sub> and particular role of O vacancies

Electronic structure & magnetism of „fully disordered” Lix(Ni-Co-Mn)O<sub>2</sub>

# Collaboration in TE materials

**S. KAPRZYK, K. KUTORASINSKI, B. WIENDLOCHA, P. ZWOLENSKI**

*AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, KRAKOW, Poland*

**C. CANDOLFI, Y. BOUYRIE, A. DAUSCHER, B. LENOIR, H. SCHERRER**

*Institut Jean Lamour CNRS, Universite de Lorraine, Campus Artem, NANCY, France*

**T. KYRATSI, A.U. KHAN**

*University of Cyprus, Department of Mechanical and Manufacturing Engineering, NICOSIA, Cyprus*

# Collaboration in 'battery' materials

**J. MOLENDA, A. MILEWSKA, D. BASTER, L. KONDRAKCI, B. GEDZIOWSKI**

*AGH University of Science and Technology, Faculty of Energy and Fuels, KRAKOW, Poland*

**M. RYBSKI**

*AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, KRAKOW, Poland*

*Partly supported by the Polish National Science Center (NCN)*

**Project MAESTRO DEC-2011/02/A/ST3/00124**

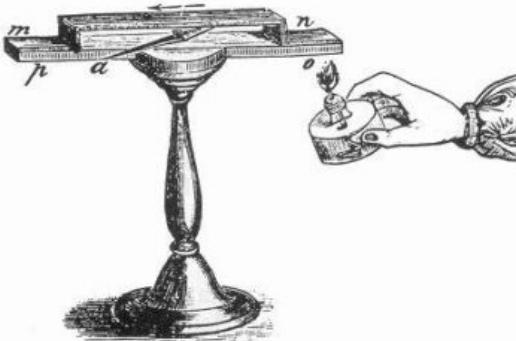
# Thermoelectric properties

## search for optimum

Improvement of figure of merit



Geometry of the devices



Physical properties of the system



*Carnot limit*  
COOLING ELEMENTS  
 $COP = (T_H - T_C)(\gamma - 1)(T_C + \gamma T_H)^{-1}$   
POWER GENERATORS  
 $\eta = (\gamma T_C - T_H)[(T_H - T_C + (\gamma + 1)]^{-1}$   
 $\gamma = (1 + ZT)^{1/2}$



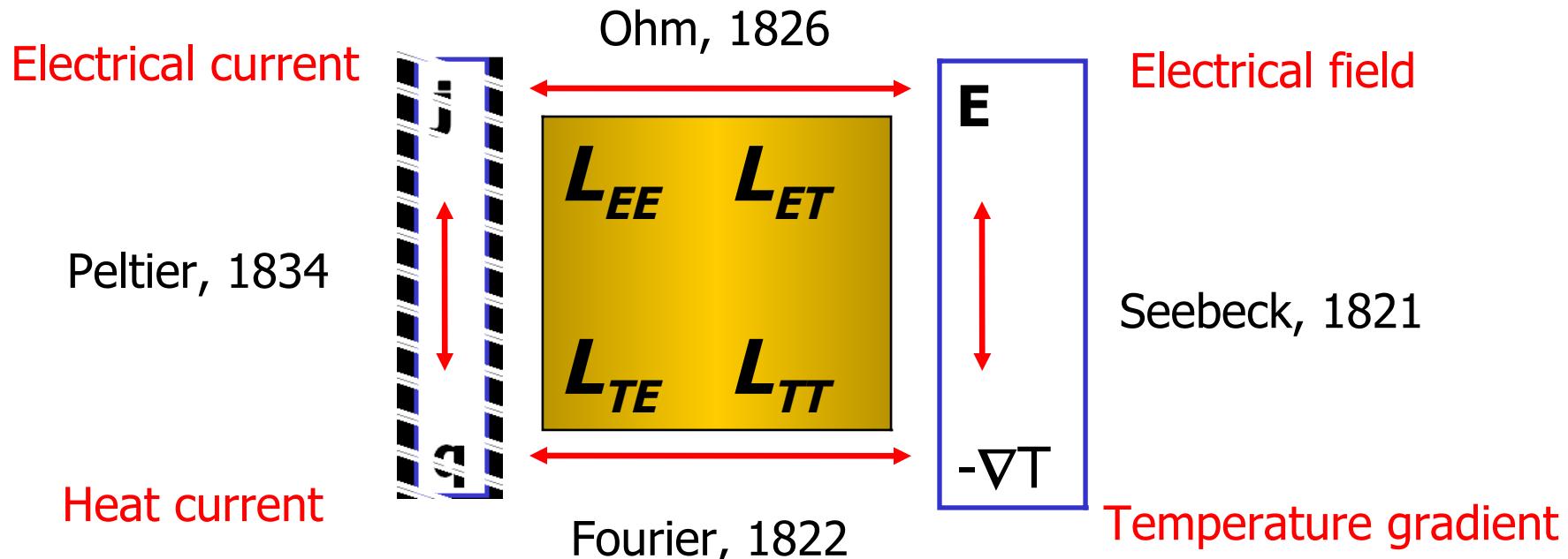
A.F. Ioffe

$$ZT = \frac{S^2 \sigma}{K}$$

Lorentz factor

Thermal conductivity  
(phonons /electrons)

# Thermoelectric „tetragon”



$$S T = \Pi$$

(Thomson-Kelvin-Onsager)

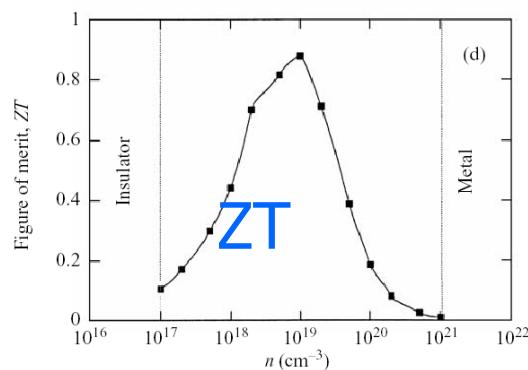
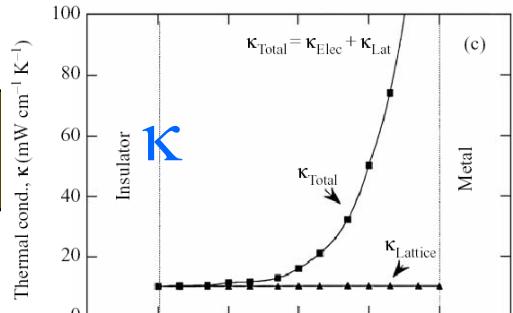
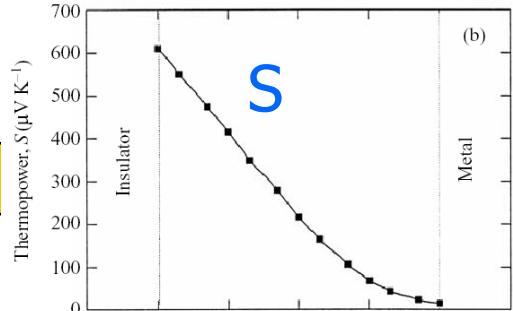
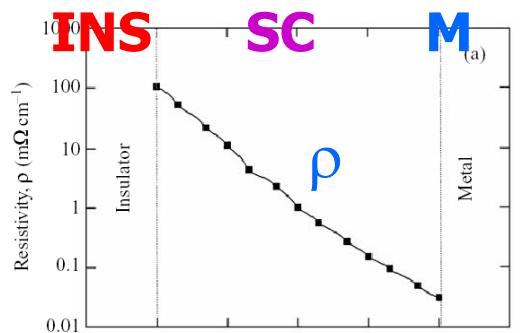
$$L_{ET} = L_{TE}/T$$

$$\kappa/\sigma \approx L_0 T$$

(Wiedemann-Franz,  $L_0$  Lorentz number)  $\kappa \approx -L_{TT}$

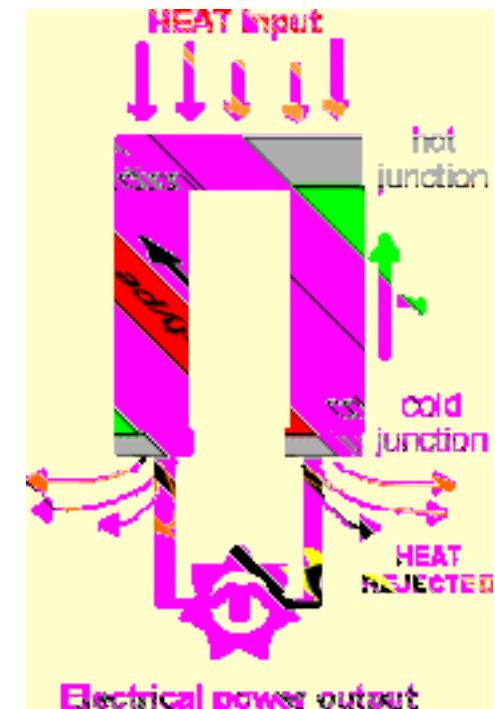
Volta (1800), Ampere (1820), Faraday (1831), Gauss (1832), ...

## Resistivity

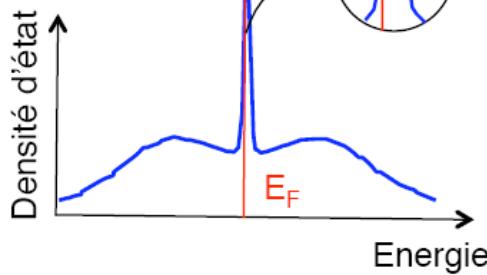


# Thermoelectric properties

$$ZT = \frac{S^2 \kappa}{K}$$

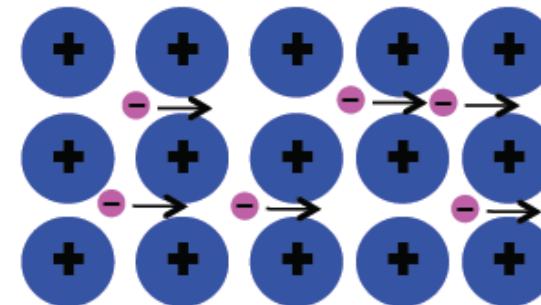
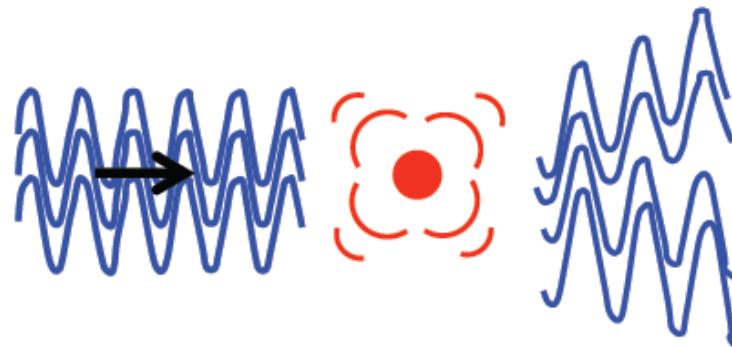


# Concepts of ZT improvement

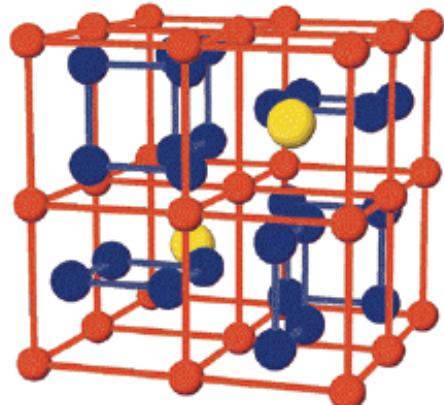


**Sharp DOS (heavy fermions, QC, low dimension).**

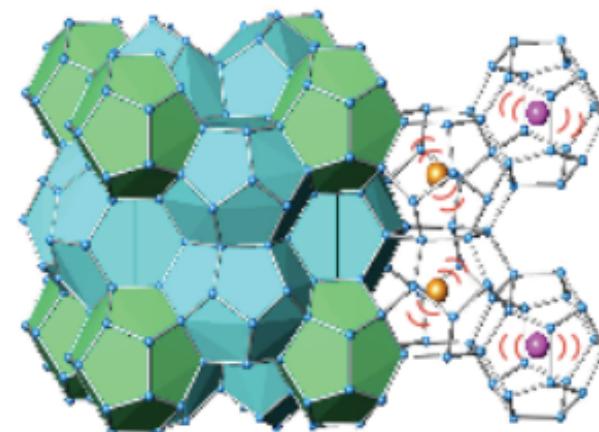
PGEC – „phonon glass“ + „electron crystal“ (Slack, '95)



more complex structures + specific vibrations (rattling, phonon, magnon, ...)



skutterudites

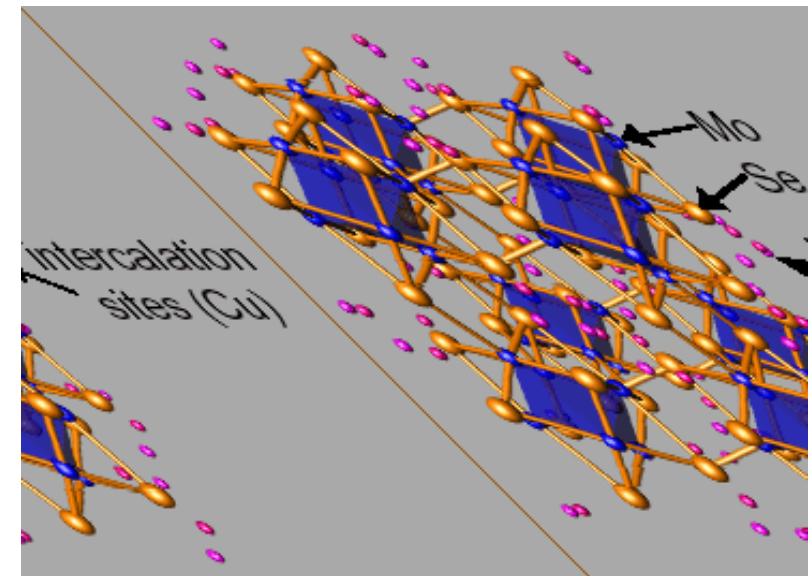
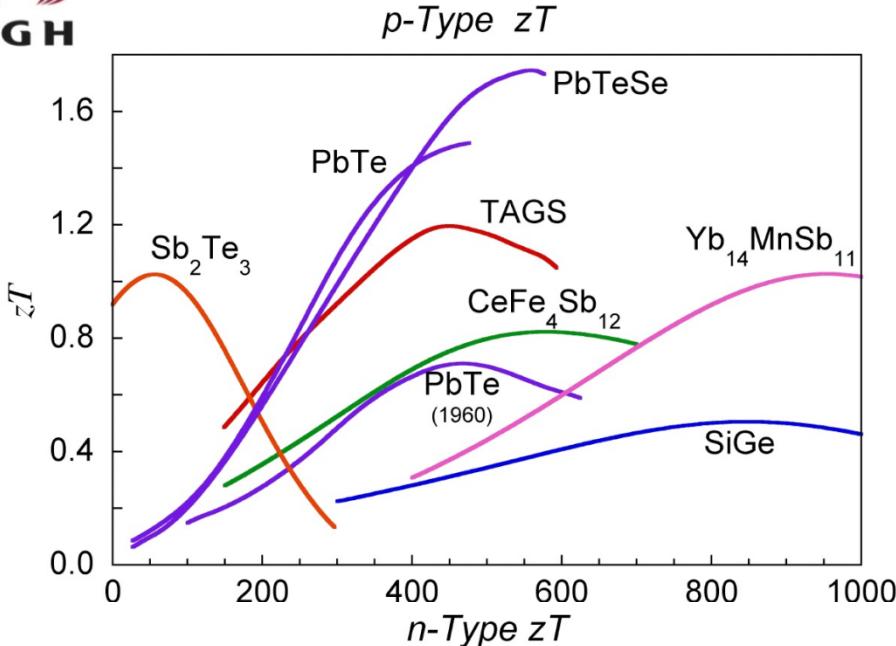


clathrates

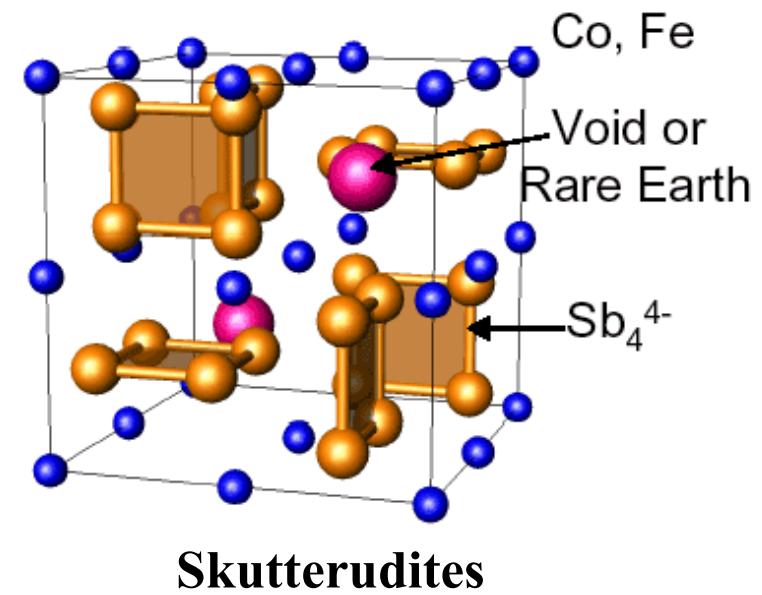
7

From Silke Paschen (2008)

# Thermoelectric materials



Chevrel phases



# Mildred Dresselhaus

From Wikipedia, the free encyclopedia

„Queen of nanoscience”

**Mildred Dresselhaus**<sup>[1]</sup> (née Spiewak; November 11, 1930 – February 20, 2017),<sup>[2]</sup> known as the "queen of carbon science",<sup>[3]</sup> was the first female Institute Professor and professor emerita of physics and electrical engineering at the Massachusetts Institute of Technology.<sup>[4]</sup> Dresselhaus won numerous awards including the Presidential Medal of Freedom, the National Medal of Science, the Enrico Fermi Award and the Vannevar Bush Award.



Mildred Dresselhaus at the White House in 2012

Born	Mildred Spiewak November 11, 1930 Brooklyn, New York, U.S.
Died	February 20, 2017 (aged 86) Cambridge, Massachusetts, U.S.
Nationality	American
Fields	Applied physics
Institutions	Cornell · MIT
Alma mater	Hunter College · Cambridge University · Harvard University · University of Chicago

## Early life and education

Mildred was born Mildred Spiewak on November 11, 1930, in Brooklyn, the daughter of Ethel (Teichtheil) and Meyer Spiewak, who were Polish Jewish immigrants.<sup>[5][6]</sup>

## Effect of quantum-well structures on the thermoelectric figure of merit

L. D. Hicks

*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

M. S. Dresselhaus

*Department of Electrical Engineering and Computer Science and Department of Physics,  
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

(Received 3 December 1992)

## Thermoelectric figure of merit of a one-dimensional conductor

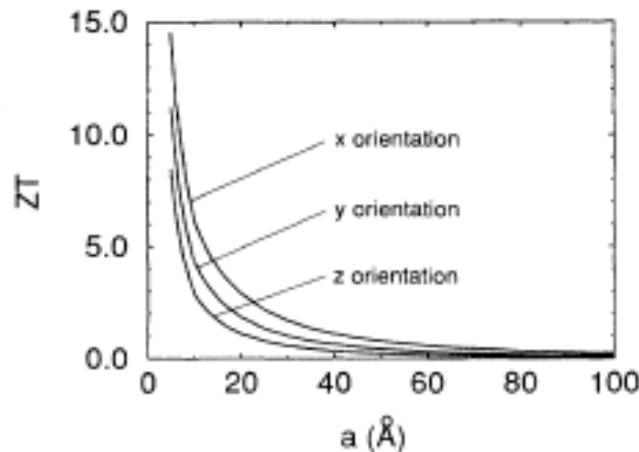
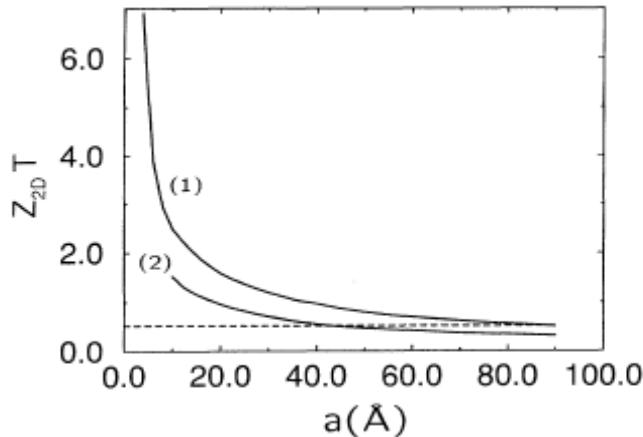
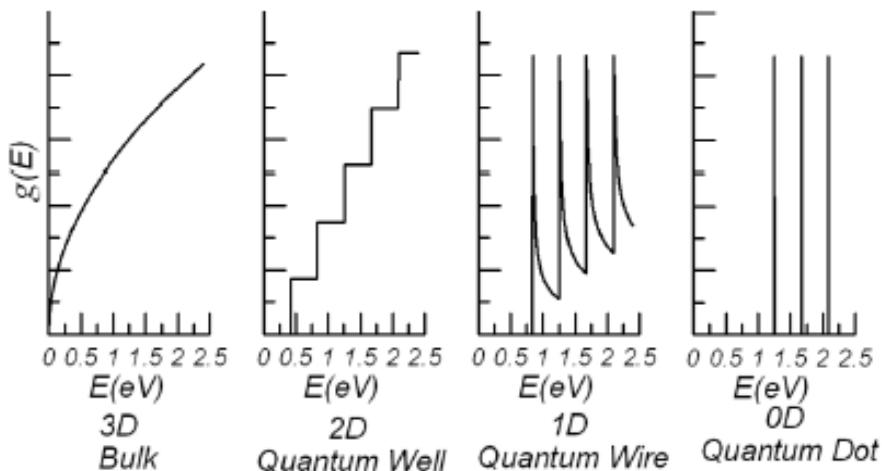
L. D. Hicks

*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

M. S. Dresselhaus

*Department of Electrical Engineering and Computer Science and Department of Physics,  
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

(Received 29 March 1993)

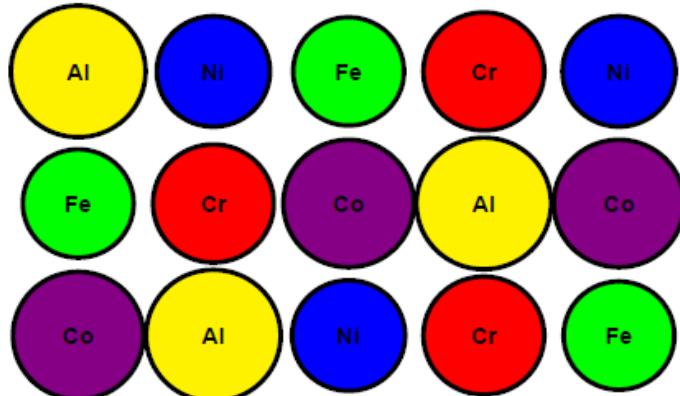


## Mott's formula (thermopower)

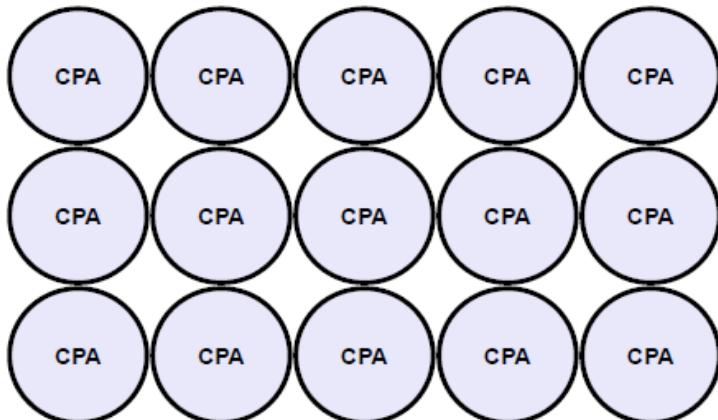
$$S = \frac{\pi^2}{3} \frac{k_B}{q} k_B T \left\{ \frac{d[\ln(\sigma(E))]}{dE} \right\}_{E=E_F}$$

# KKR-CPA method (S. Kaprzyk 1948-2018)

Disordered alloys: periodic - Coherent Potential Approximation (CPA):



CPA “trick”



## CPA condition



$$G^{CP} = c_A G_A + c_B G_B + c_C G_C + \dots + c_N G_N$$

CPA crystal consists of ‘disordered’ nodes arranged with translation symmetry of cell and mimics alloys, defects, etc.

KKR-CPA code allows for treat many atoms on disordered sites ( $N > 10$ ) solved self-consistently.

*Muffin-tin* potential is used due to CPA condition, defined for spherical potentials.

$$T_{k'\sigma'L',k\sigma L}^{CP} = \frac{1}{N} \sum_{\mathbf{k} \in BZ} [\tau_{CP}^{-1} - B(E, \mathbf{k})]_{k'\sigma'L',k\sigma L}^{-1}$$

# KKR-CPA method for disordered alloys

Korringa-Kohn-Rostoker with coherent potential approximation

$$G(E) = \sum_{s=(+,-)} \sum_{k=1}^K \int_{V_k} d^3r \langle s, \mathbf{r} + \mathbf{a}_k | G(E) | s, \mathbf{r} + \mathbf{a}_k \rangle.$$

Bansil, Kaprzyk, Mijnarends, Tobola,  
Phys. Rev. B (1999) conventional KKR

Full GF

Stopa, Kaprzyk, Tobola, J.Phys.CM (2004)  
novel formulation of KKR

$$\langle s', \mathbf{r}' + \mathbf{a}_{k_{CP}} | G^{A(B)}(E) | s, \mathbf{r} + \mathbf{a}_{k_{CP}} \rangle$$

$$= - \sum_{\sigma L} J_{\sigma L}^{A(B)}(s' \mathbf{r}') Z_{\sigma L}^{A(B)}(s \mathbf{r})$$

$$+ \sum_{\sigma' L', \sigma L} Z_{\sigma' L'}^{A(B)}(s' \mathbf{r}') T_{k_{CP} \sigma' L', k_{CP} \sigma L}^{A(B)} Z_{\sigma L}^{A(B)}(s \mathbf{r})$$

$$G(E) = - \frac{d}{dE} \left\{ \frac{1}{N} \sum_{\mathbf{k} \in BZ} \text{Tr} \ln [G_0^{-1}(E, \mathbf{k}) + D^{(j)} - D_{CP}]^{-1} \right\}$$

$$- \frac{d}{dE} \{ c_A \text{Tr} \ln [\Psi_A^{-1} G^A] + c_B \text{Tr} \ln [\Psi_B^{-1} G^B]$$

$$- \text{Tr} \ln G^{CP} \} + \frac{d}{dE} \left\{ \sum_{k \neq k_{CP}} \text{Tr} \ln [\Psi^{(k)}] \right\}, \quad (2.22)$$

Lloyd formula

Kaprzyk et al. Phys. Rev. B (1990)

$$\langle s', \mathbf{r}' + \mathbf{a}_k | G(E) | s, \mathbf{r} + \mathbf{a}_k \rangle$$

$$= - \sum_{\sigma L} J_{\sigma L}^{(k)}(s' \mathbf{r}') Z_{\sigma L}^{(k)}(s \mathbf{r}) \delta_{kk'}$$

$$+ \sum_{\sigma' L', \sigma L} Z_{\sigma' L'}^{(k')}(s' \mathbf{r}') T_{k' \sigma' L', k \sigma L}^{CP} Z_{\sigma L}^{(k)}(s \mathbf{r})$$

$$T_{k' \sigma' L', k \sigma L}^{CP} = \frac{1}{N} \sum_{\mathbf{k} \in BZ} [\tau_{CP}^{-1} - B(E, \mathbf{k})]_{k' \sigma' L', k \sigma L}^{-1}$$

$$\text{CPA} \quad c_A T^A + c_B T^B = T^{CP}.$$

Density  
of states

$$N(E) = - \frac{1}{\pi} \text{Im} \int_{-\infty}^E dE G(E)$$

Fermi energy  $N(E_F) = Z$

# Boltzmann equation



Electron system described by distribution function  $f$  in the  $(\mathbf{r}, \mathbf{k})$  space.

$$\frac{1}{4\pi^3} f(k, r, t)$$

Fermi-Dirac function in equilibrium state

Electron density current

$$J(r, t) = \frac{e}{4\pi}$$

Transport equation

$$\frac{df}{dt} = -\frac{dk}{dt} \cdot \nabla_l$$

— —

Stationary condition

$$\frac{\partial f}{\partial t} = 0$$

time-independent forces

Collision integral  $\left( \frac{\partial f}{\partial t} \right)_{coll}$

Describes e-e scatterings/collisions , probability of exit outside the  $d\mathbf{k}dr$  volume

$$\left( \frac{\partial f}{\partial t} \right)_{coll} = . \quad \text{---}$$

Relaxation time approximation

# Electron transport coefficients

$$\sigma_e = \mathcal{L}^{(0)},$$

$$S = -\frac{1}{eT} \frac{\mathcal{L}^{(1)}}{\mathcal{L}^{(0)}},$$

$$\kappa_e = \frac{\mathcal{L}^{(2)}}{e^2 T} - \frac{\mathcal{L}^{(1)} \mathcal{L}^{(1)}}{e^2 T \mathcal{L}^{(0)}}$$

$$L(T) = \frac{\kappa_e(T)}{\sigma(T)T} \quad \text{Wiedemann-Franz-Lorenz}$$

## Onsager-related functions

$$\mathcal{L}^{(\alpha)} = \int d\mathcal{E} \left( -\frac{\partial f}{\partial \mathcal{E}} \right) (\mathcal{E} - \mu)^\alpha \sigma(\mathcal{E})$$

## Transport functions (in general tensors)

$$\sigma(\mathcal{E}) = e^2 \sum_n \int \frac{d\mathbf{k}}{4\pi^3} \tau_n(\mathbf{k}) \mathbf{v}_n(\mathbf{k}) \otimes \mathbf{v}_n(\mathbf{k}) \delta(\mathcal{E} - \mathcal{E}_n(\mathbf{k}))$$

**Electrical conductivity**

**Seebeck coefficient (thermopower)**

**Electronic thermal conductivity**

$$L = \frac{\kappa_e}{\sigma T}$$

$$PF = S^2 \sigma$$

$$ZT = \frac{S^2 \sigma T}{\kappa_e + \kappa_l}$$

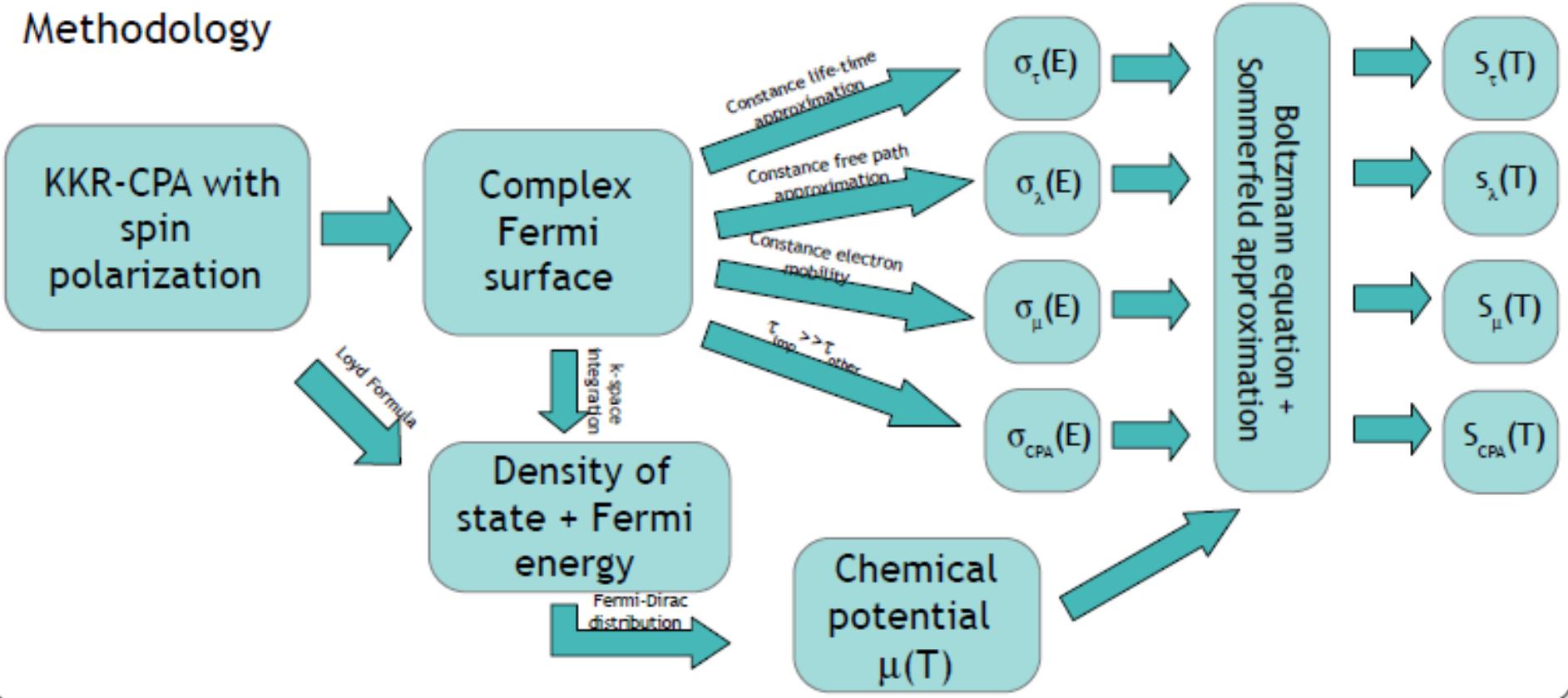
$$L(T, n)$$

$$PF(T, n)$$

$$ZT(T, n)$$

# Boltzmann transport & KKR-CPA calculations of complex energy bands and thermopower

## Methodology

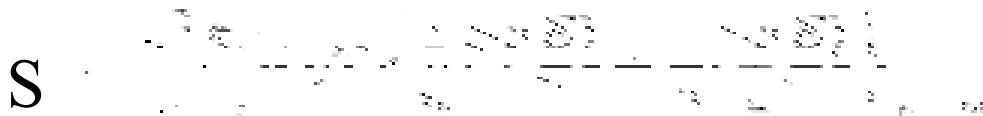


K. Kutorasinski, Ph.D. Thesis (2014)

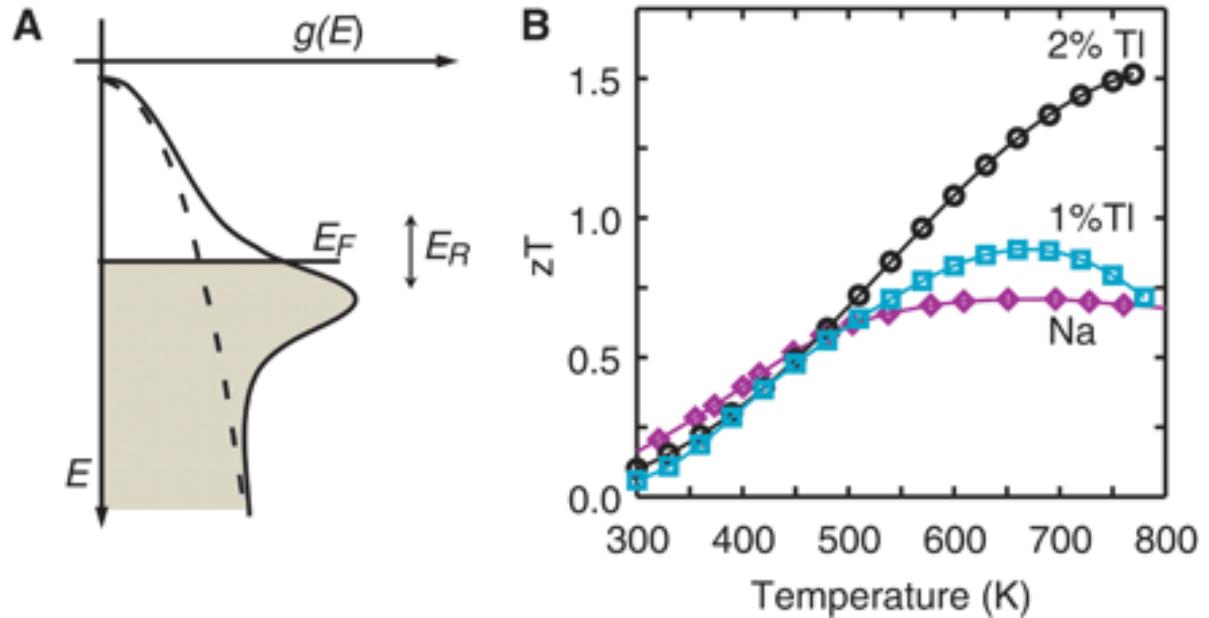
## Different approximations used

- (1)  $\tau = \text{const}$ ; (2)  $\lambda = \text{const}$ ; (3)  $\mu = \text{const}$ ; (4) CPA (velocity + life-time);

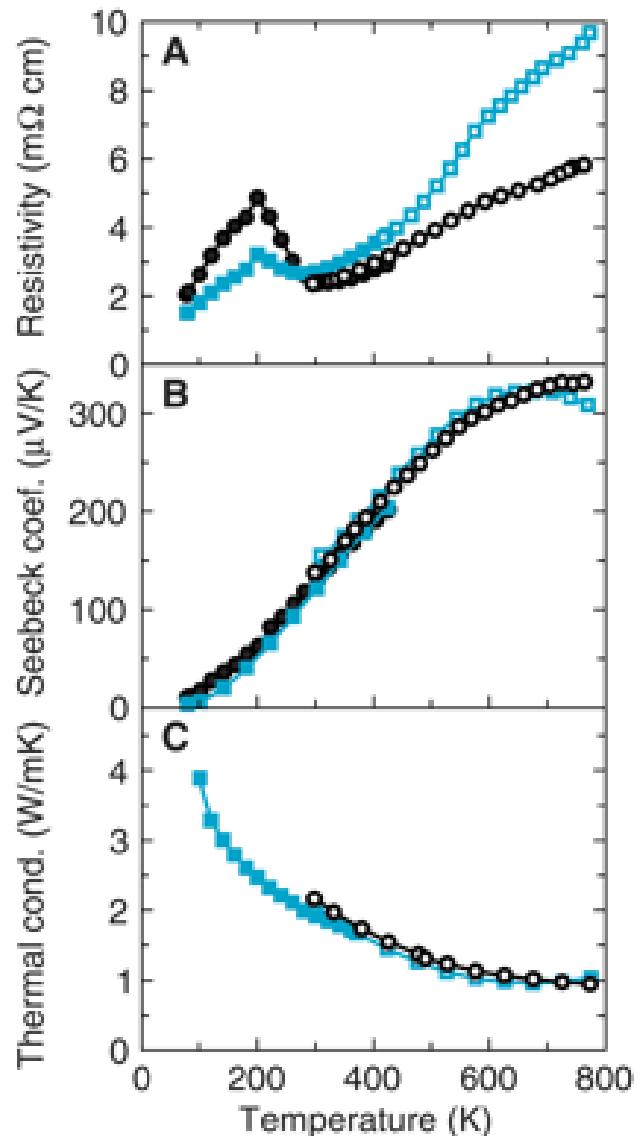
# Enhancement of TE efficiency in PbTe (distortion of electronic DOS)



Mott's formula for thermopower



J. Heremans et al., Science 321 (2008) 544



# Resonant levels (RL) in TE materials:

RL: increase in thermopower for given  $n$ , large PF and  $zT$   
**PbTe:Tl** Heremans et al,  
*Science 321 (2008) 544*

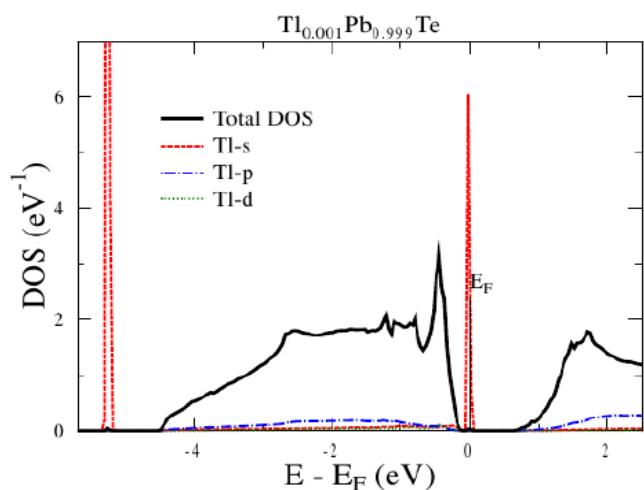
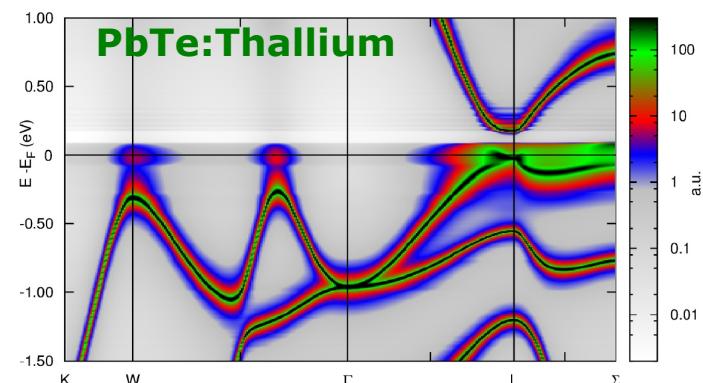
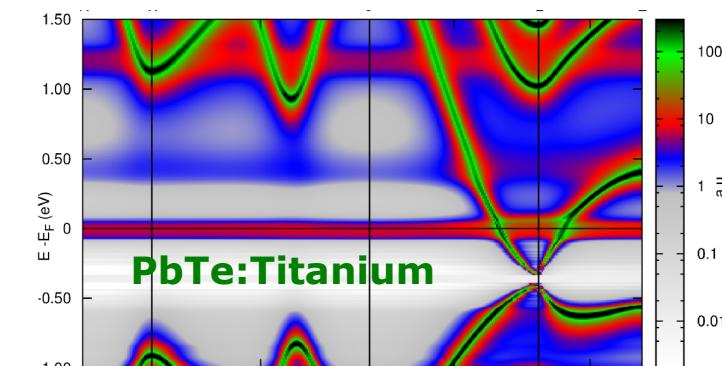
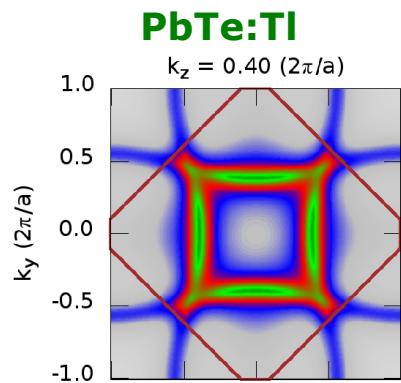
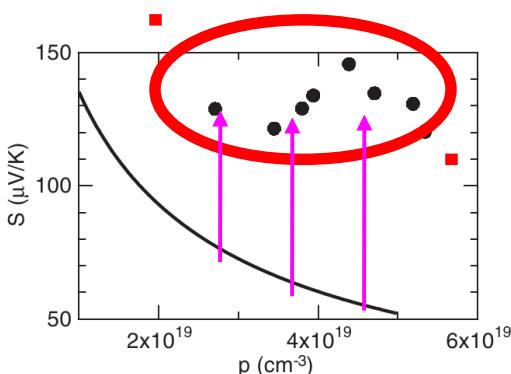
## Band structure results:

sharp peak in DOS from RL

resonance from  $s$  electrons!

strongly disturbed bands

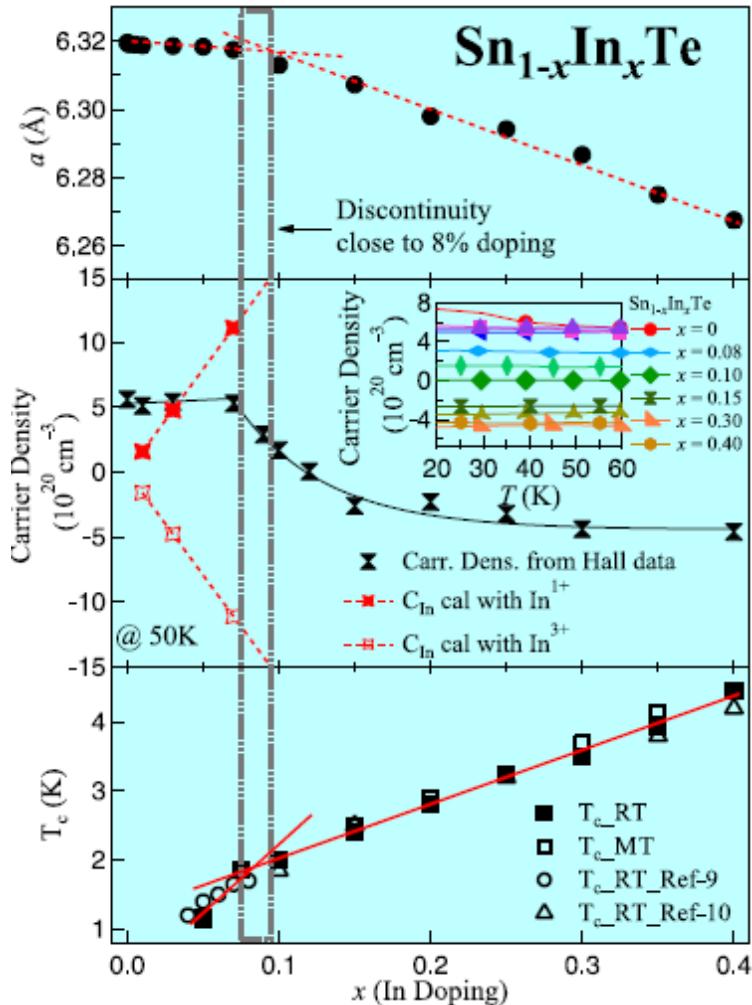
Fermi Surface: increase in number of states taking part in TE effects



B. Wiendlocha, PRB 88 (2013) 205205.

# Unusual physical properties of SnTe

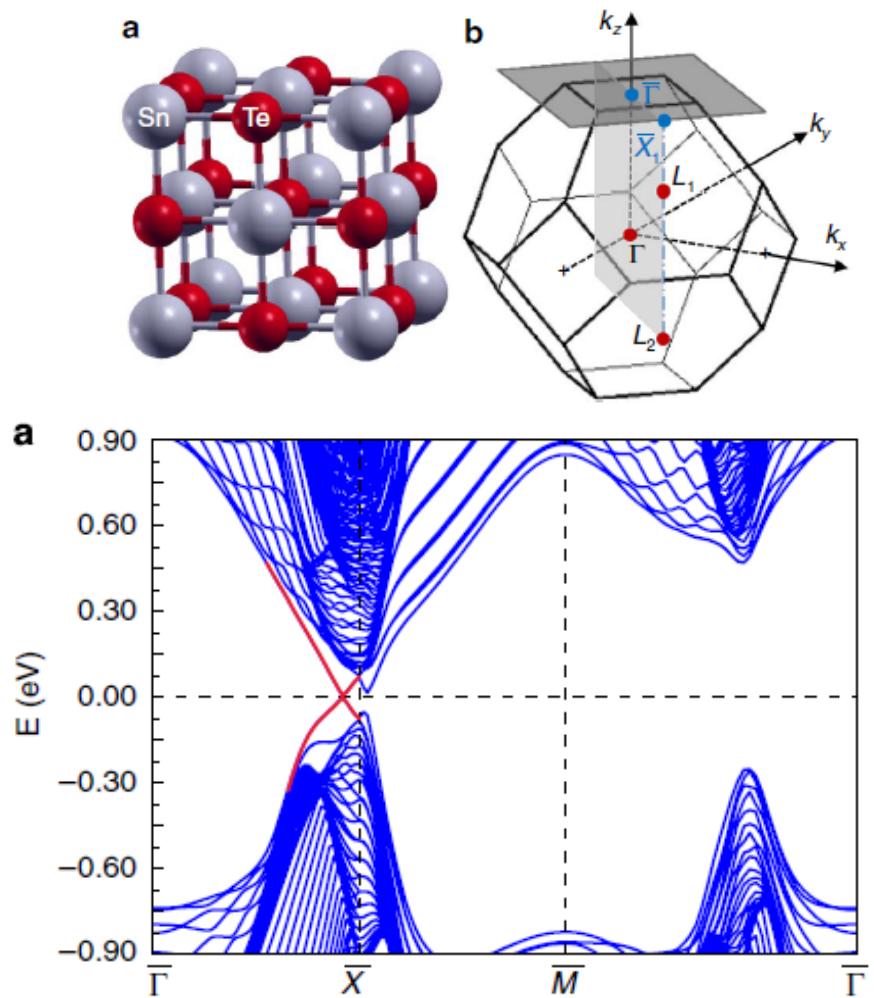
## Superconductivity



Haldolaarachchige et al., Phys. Rev. 93 (2016)  
024520

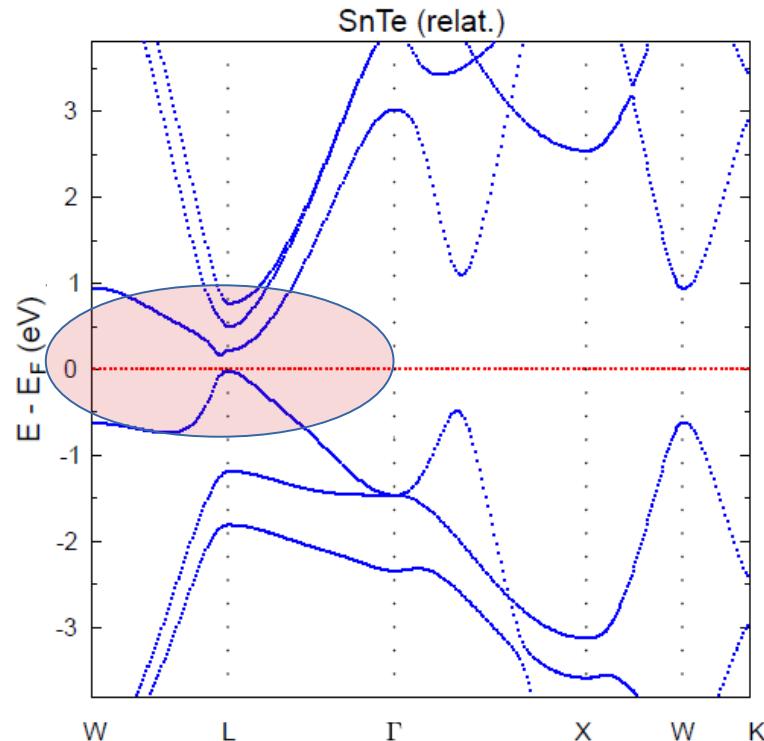
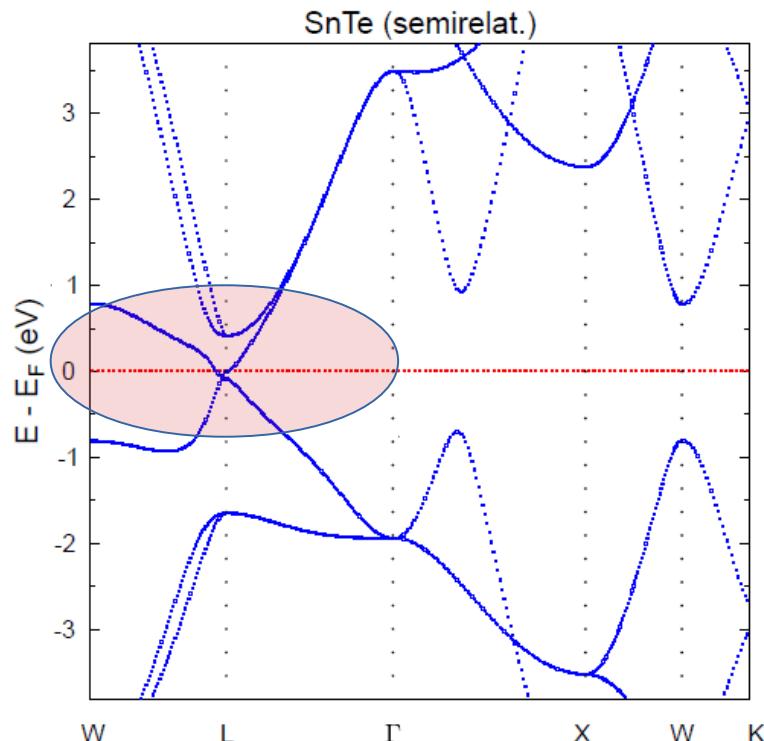
P. B. Allen and M. L. Cohen, Phys. Rev. 177, 704 (1969).

## Topological crystalline insulator

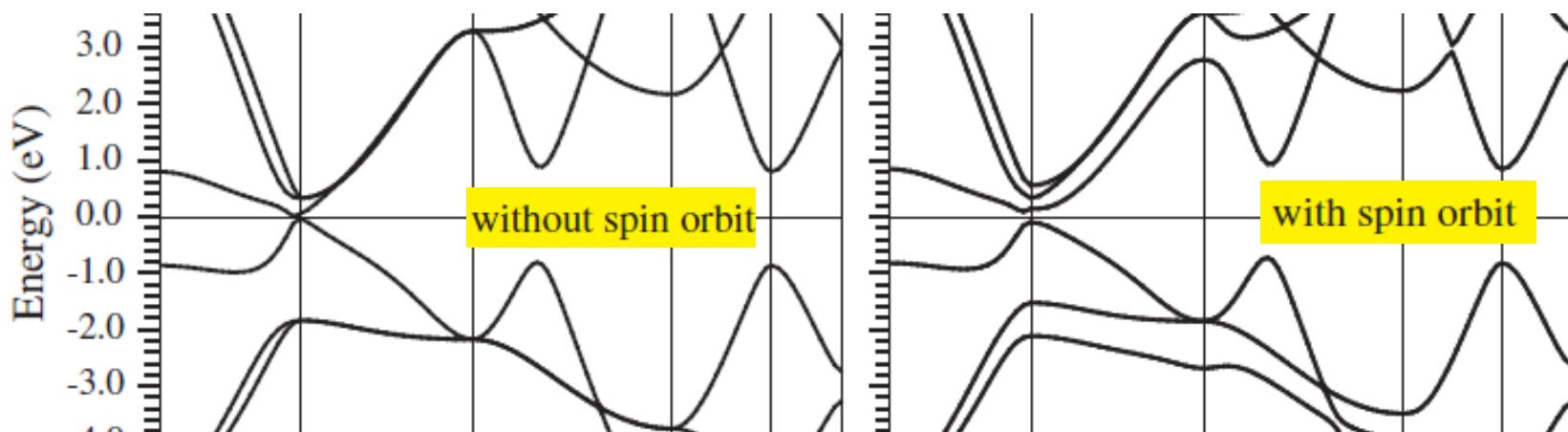


Hsieh et al., Nature Comm. (2016): DOI:10.1038/ncomms1969

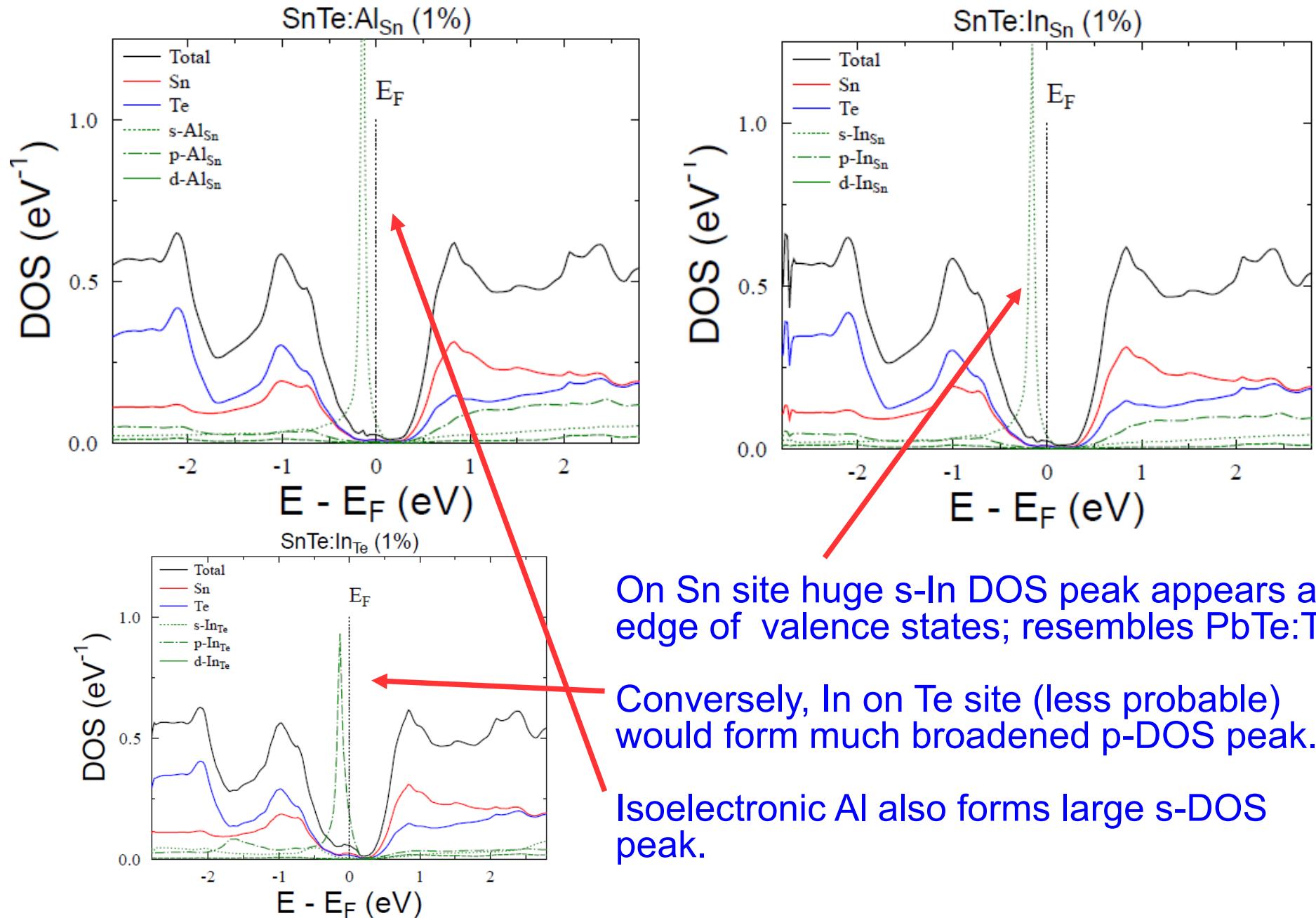
# KKR (*relat.* vs. *semirelat.*) dispersion curves in SnTe



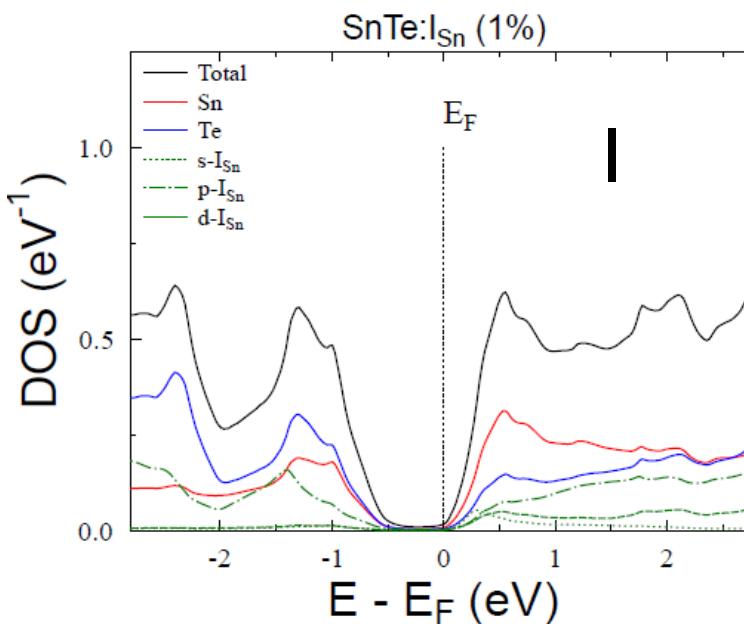
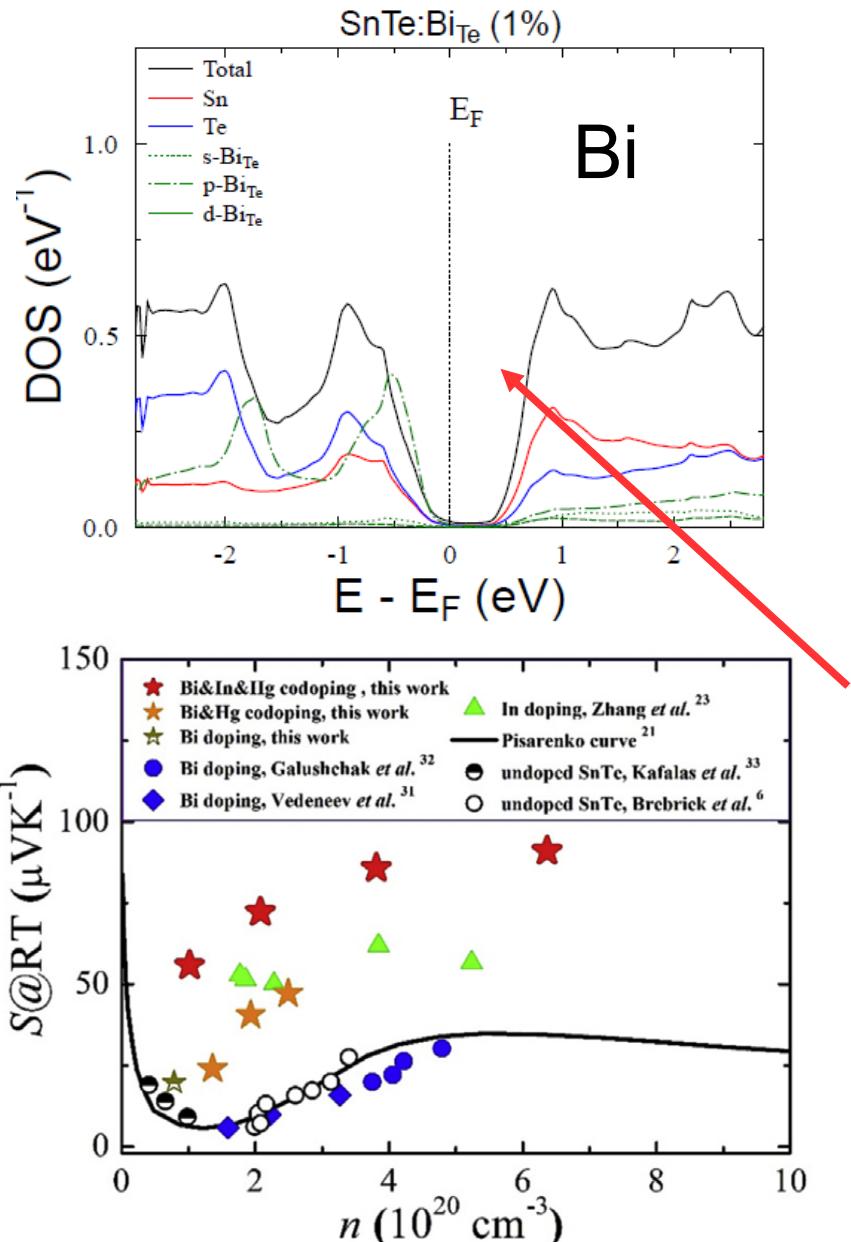
*Littlewood et al. PRL 105 (2010) 086404*



# DOS of In impurity in SnTe



# DOS of conventional dopants in SnTe



Bi on Te site → p-type

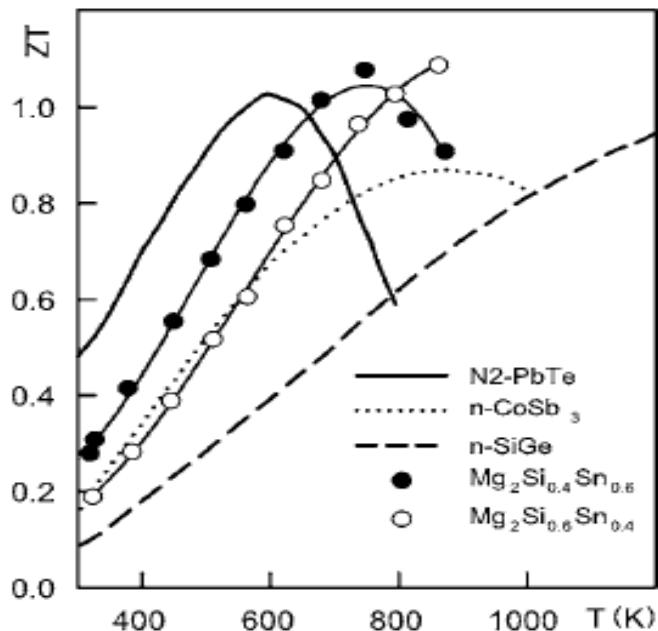
I on Sn site → n-type

depending on substituted sites some elements behave as conventional dopants, not disturbing DOS, only shifting in rigid band manner:

# $Mg_2X$ ( $X = Si, Ge, Sn$ ) as thermoelectrics

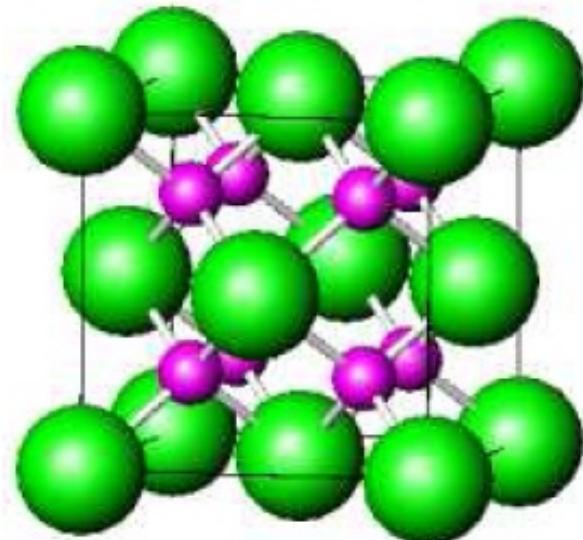
$$ZT = \frac{S^2 \sigma}{K}$$

„Anti-fluorite” ( $CaF_2$ ) structure



Zaitsev et al., Phys. Rev. B 74 (2006) 045207

Light, non-toxic, rather cheap, large  
 $ZT$  of undoped compounds & alloys

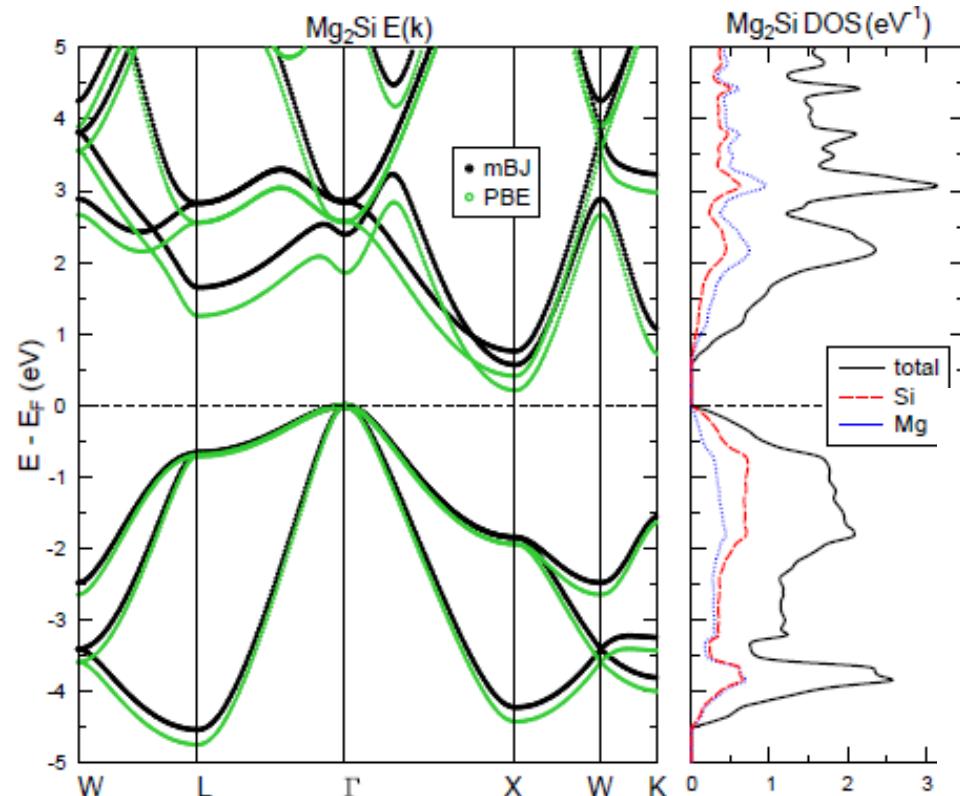


**Mg:**  $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$   
 $\frac{3}{4}, \frac{3}{4}, \frac{3}{4}$

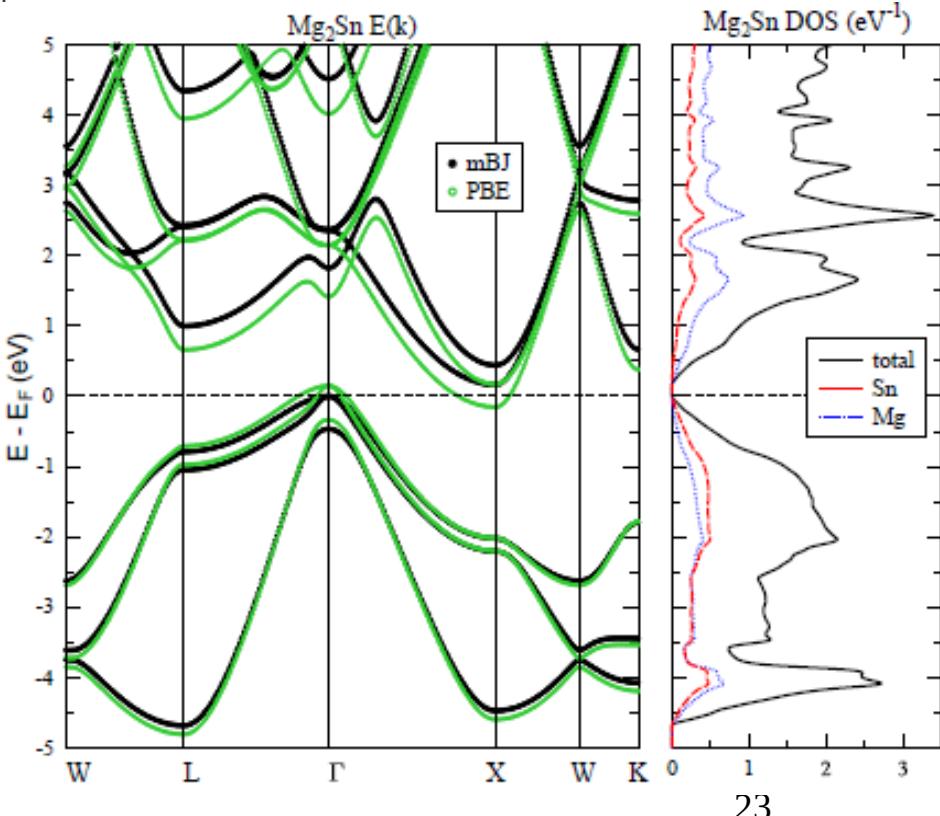
**X:** 0, 0, 0, 0

**Si, Ge, Sn**

# More realistic treatment of “band gap problem”

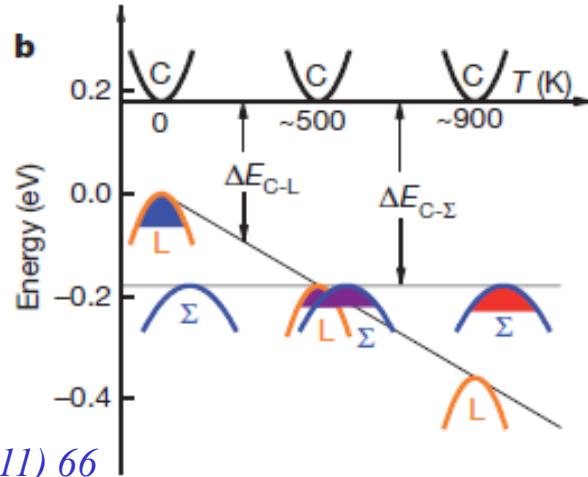
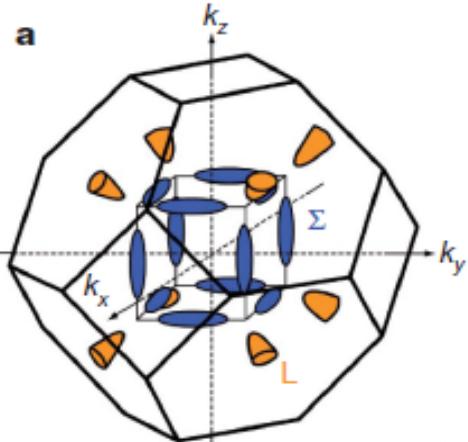


Application of more sophisticated exchange-correlation potential improves exp-theory agreement for band gap.



BUT the overall bands shape remains quite similar -> important results for transport calculations based on LDA results

# „Engineering“ of band degeneracy to improve $zT$



Pei et al., Nature 473 (2011) 66

Pb(Te-Se)

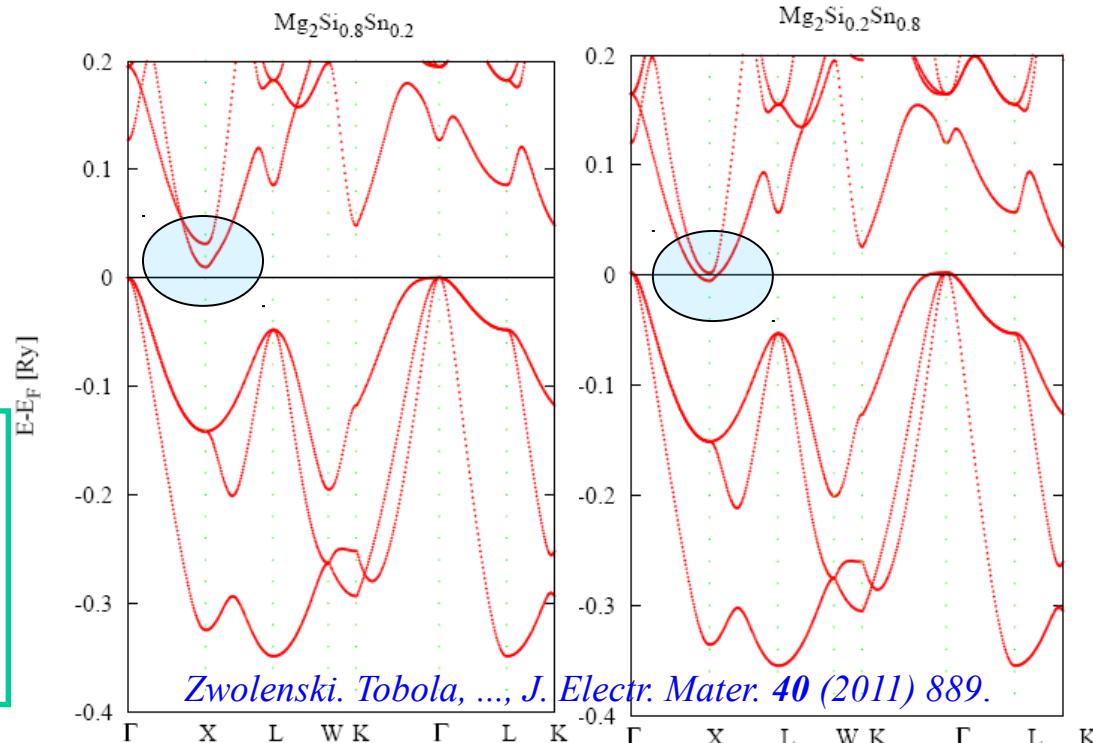
- similar thermopower but higher electrical conductivity (more carriers) BUT p-type

$\sim \text{Mg}_2\text{Si}_{0.4}\text{Sn}_{0.6}$

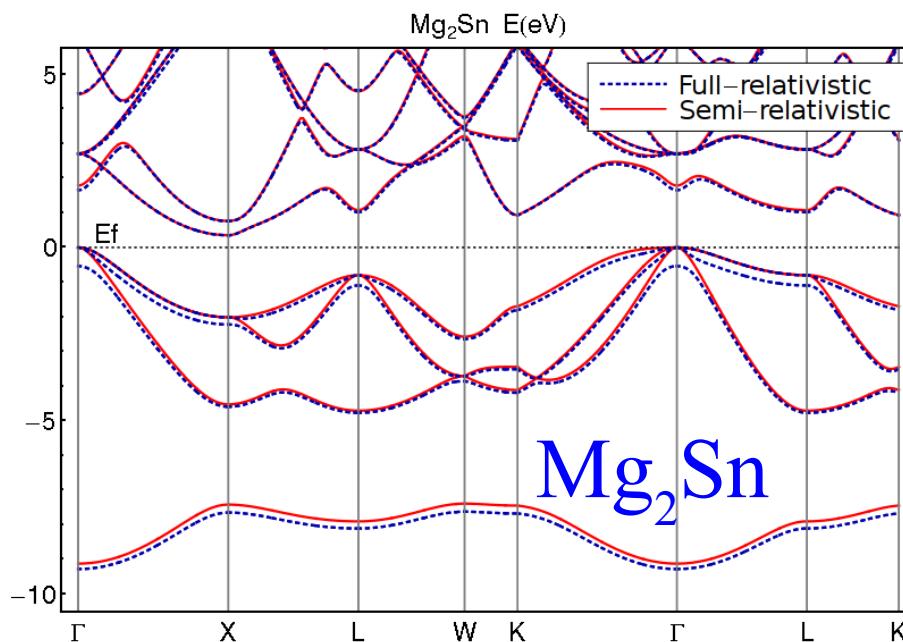
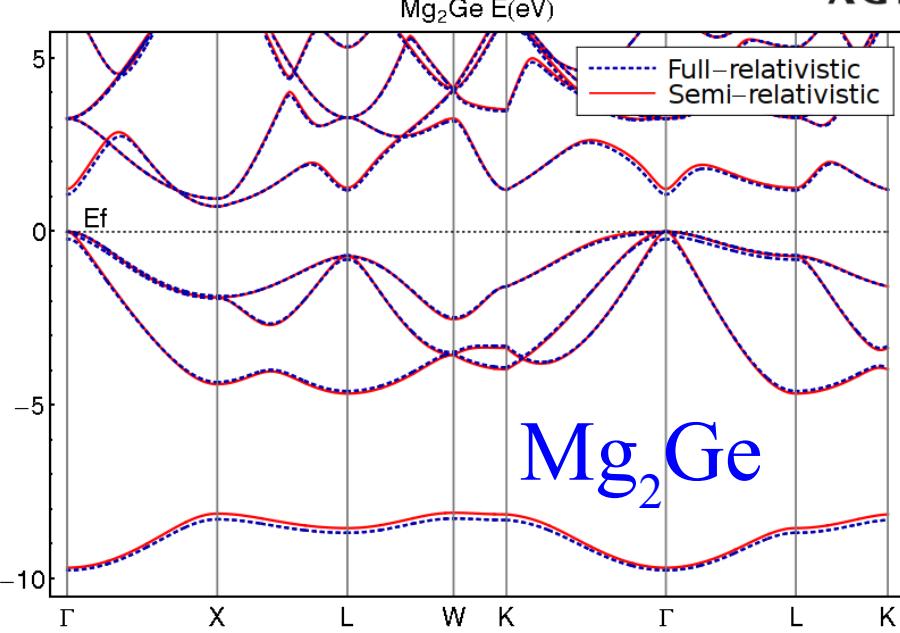
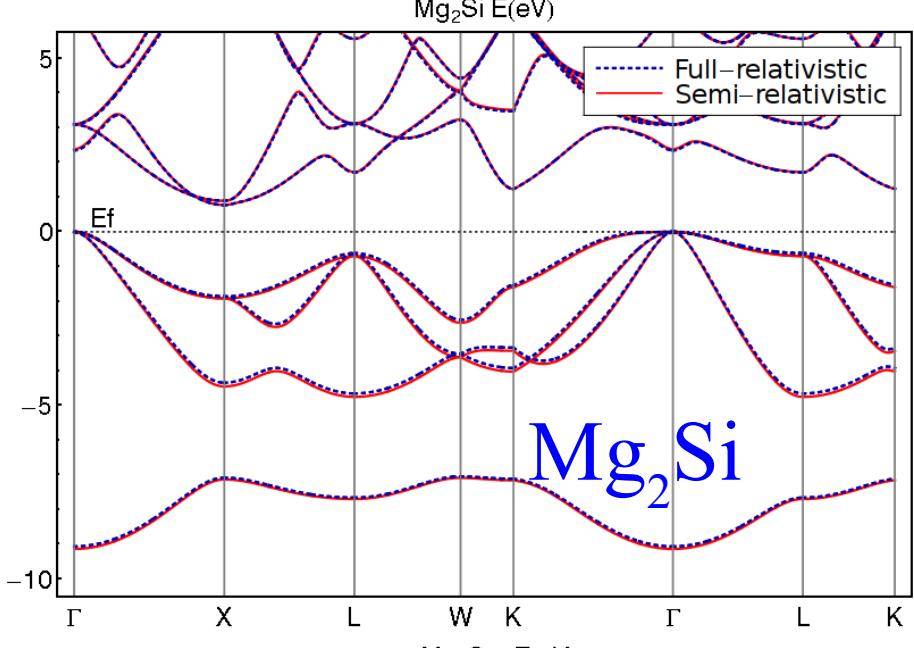
- Achievement of **HIGH** band degeneracy – general strategy to improve TE properties in bulk materials (**n-type**)

Mg<sub>2</sub>(Si-Sn)

- shrinking of band gap and mutual shift of conduction bands near X : **not effect** of Si/Sn disorder, rather due to important variation of lattice constant



# Convergence of conduction bands in Mg<sub>x</sub>X



Conduction band degeneracy tends to increase thermopower in n-doped systems and is expected in Mg<sub>2</sub>(Si-Sn) alloys !

# Complex energy Fermi surface

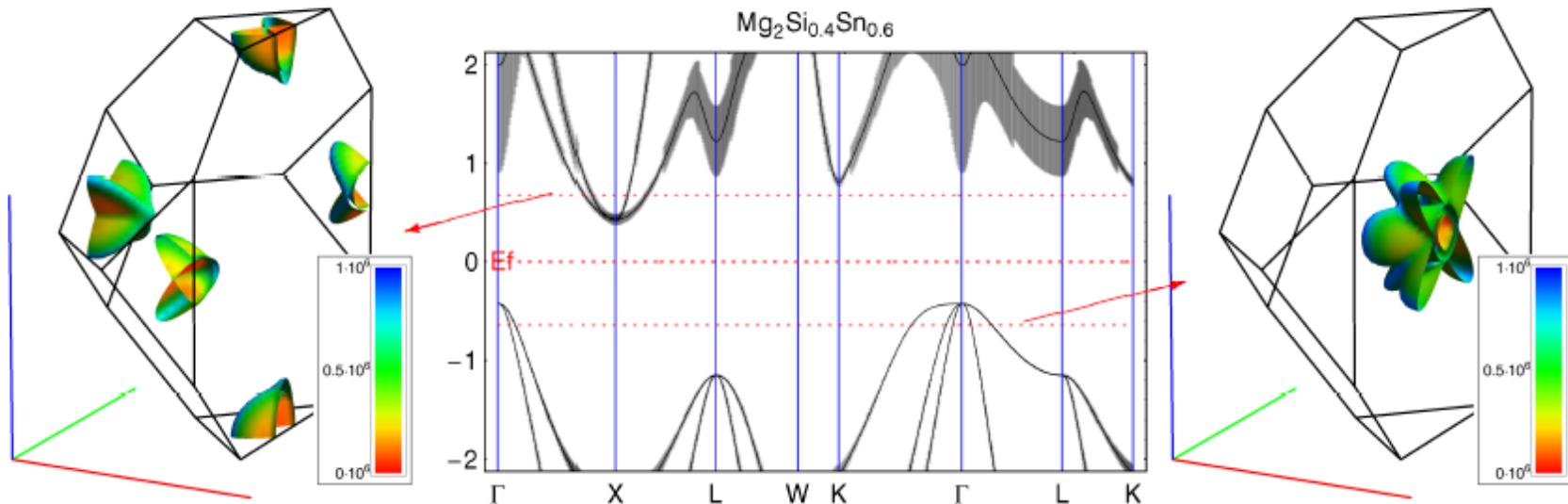


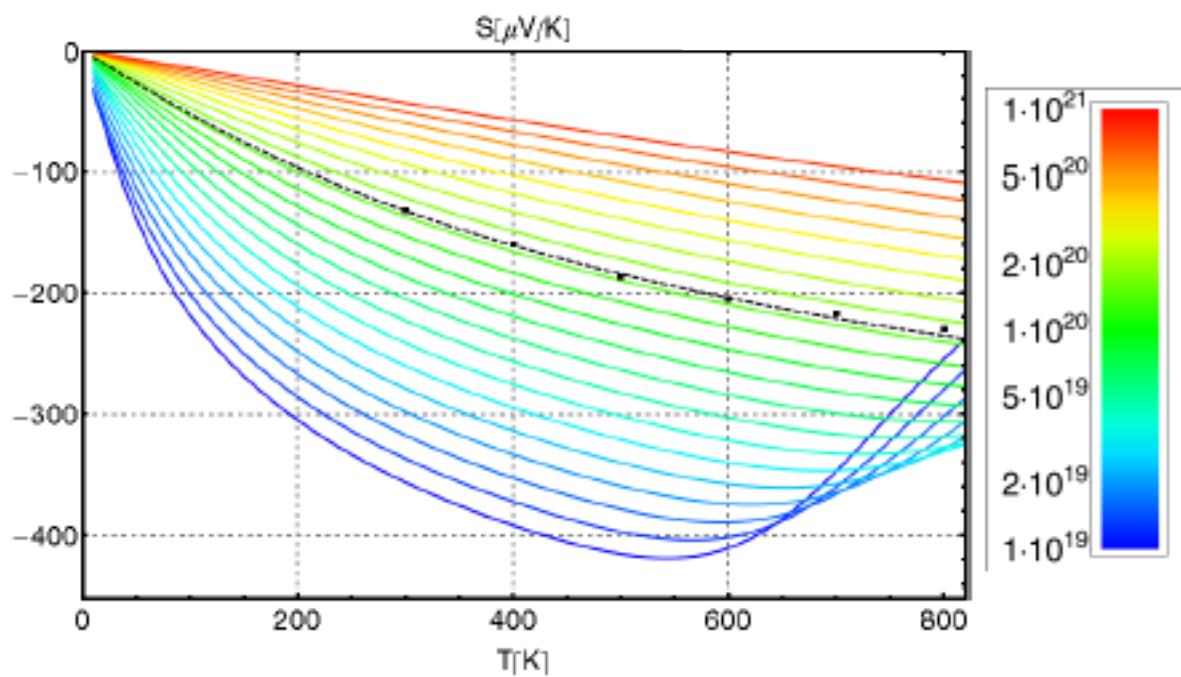
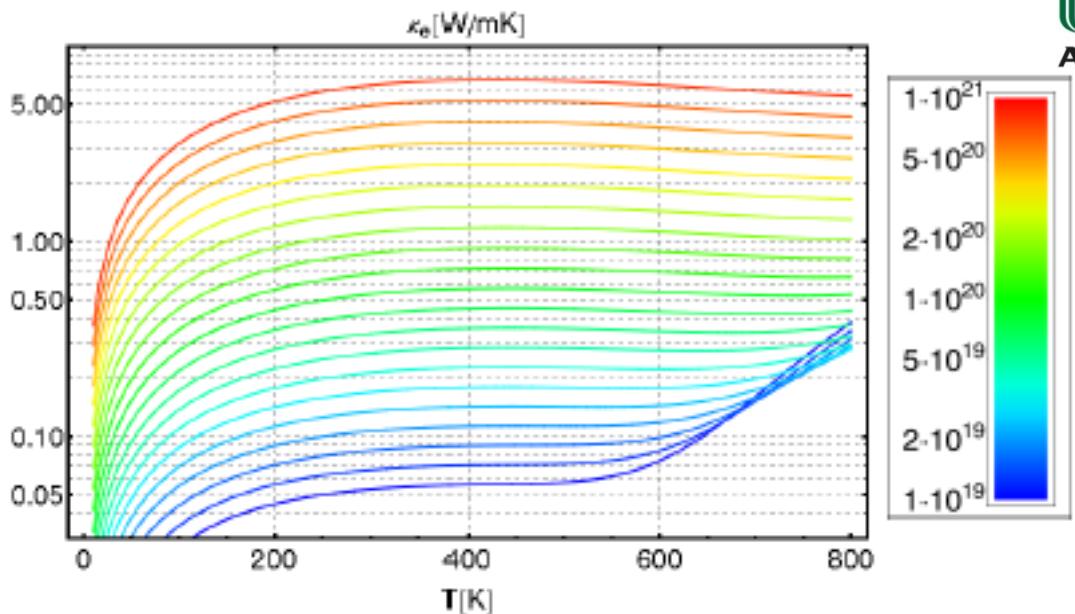
FIG. 1. Bands  $\text{Mg}_2\text{Si}_{1-x}\text{Sn}_x$ . Shadows around bands represent imaginary part of energy, enlarged 100 times to make them visible

*Kutorasinski et al. (JT),  
Phys. Rev. B 87 (2013)195205.*

	$a$ (Å)	$E_g^{\text{LDA}}$ (eV)		$E_g^{\text{exp}}$ (eV)
		SR	FR	
$\text{Mg}_2\text{Si}$	6.336	0.32	0.33	0.78
$\text{Mg}_2\text{Ge}$	6.385	0.21	0.23	0.72
$\text{Mg}_2\text{Sn}$	6.765	-0.17	-0.25	0.35

# Electronic thermal conductivity

$$\kappa_e = \frac{\mathcal{L}^{(2)}}{e^2 T} - \frac{\mathcal{L}^{(1)} \mathcal{L}^{(1)}}{e^2 T \mathcal{L}^{(0)}}$$



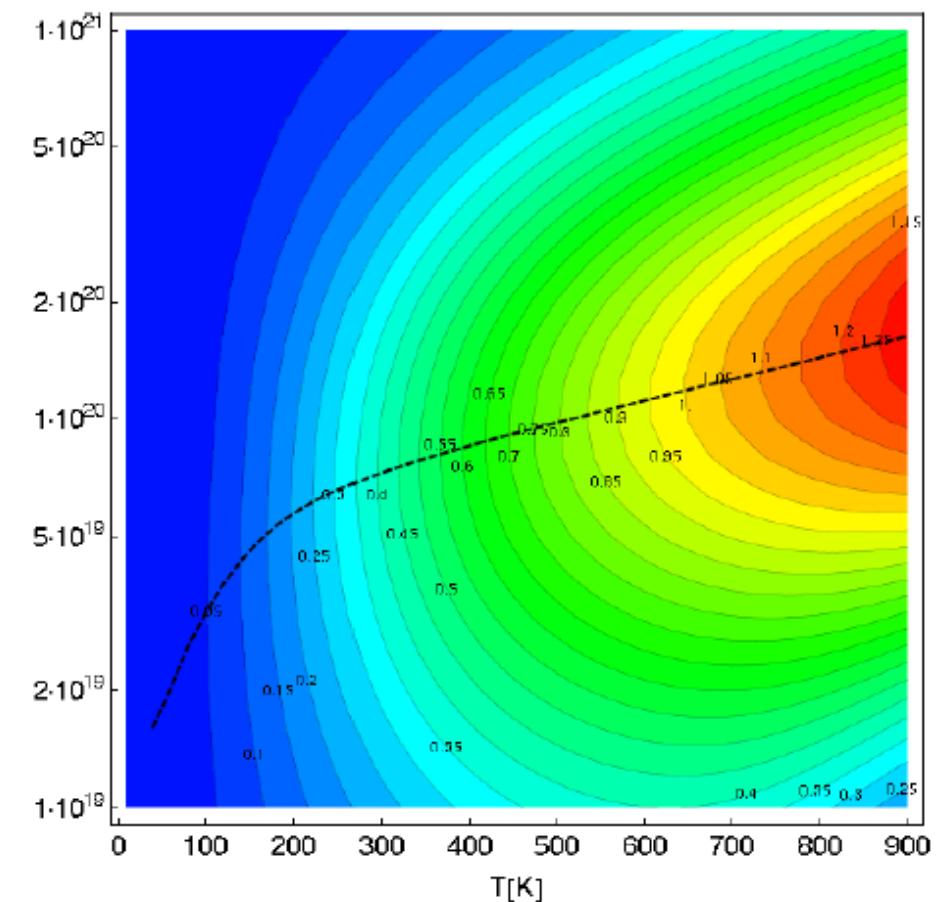
Seebeck coefficient

$$S = -\frac{1}{eT} \frac{\mathcal{L}^{(1)}}{\mathcal{L}^{(0)}},$$

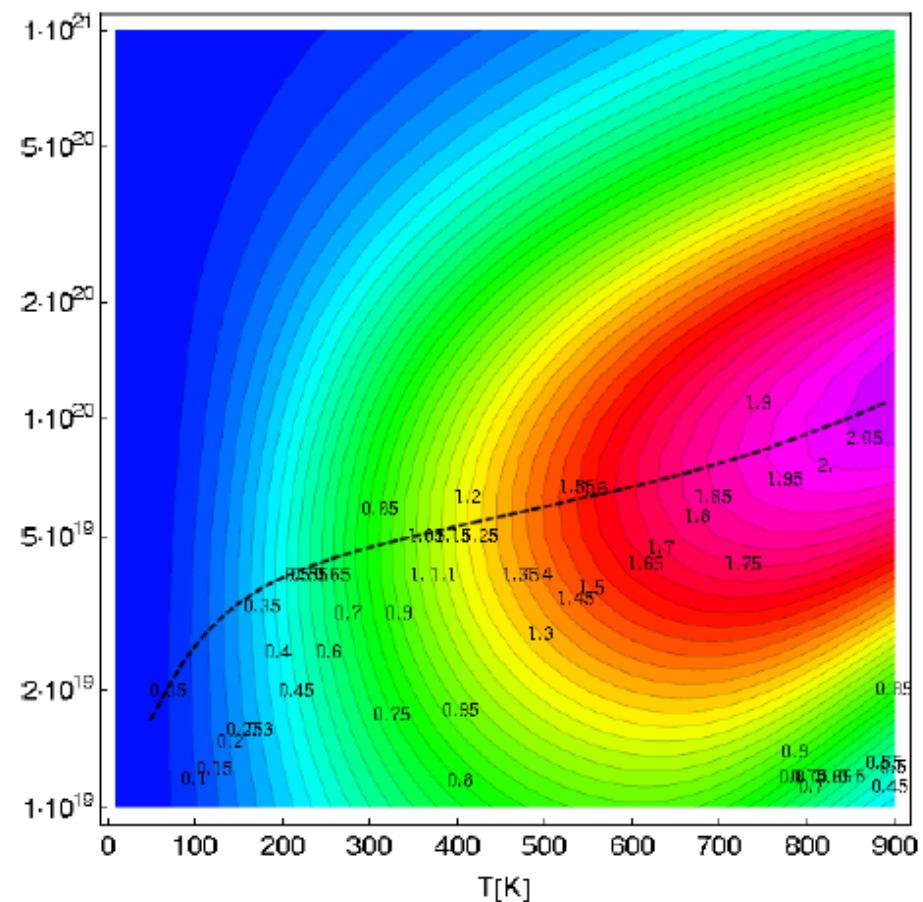
TEST for relaxation time approach

# ZT vs. n/p & T

lattice thermal conductivity as parameter



$$\kappa_l = 1.25 \text{ W/mK}$$



$$0.5 \text{ W/mK}$$

# Relativistic KKR calculations

Full form of Dirac equation including four components

$$\left( \beta mc^2 + c(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3) \right) \psi(x, t) = i\hbar \frac{\partial \psi(x, t)}{\partial t}$$

$$\begin{pmatrix} (mc^2 - E + e\phi) & c\sigma \cdot (p - \frac{e}{c}A) \\ -c\sigma \cdot (p - \frac{e}{c}A) & (mc^2 + E - e\phi) \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

More readable: non-relativistic approach of Dirac equation

$$H = H_0 + H_{\text{kinetic}} + H_{\text{so}} + H_{\text{Darwinian}}$$

$$H_{\text{so}} = \frac{1}{2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right) \left( \frac{g_s}{2m_e^2 c^2} \right) \frac{\vec{L} \cdot \vec{S}}{r^3}$$

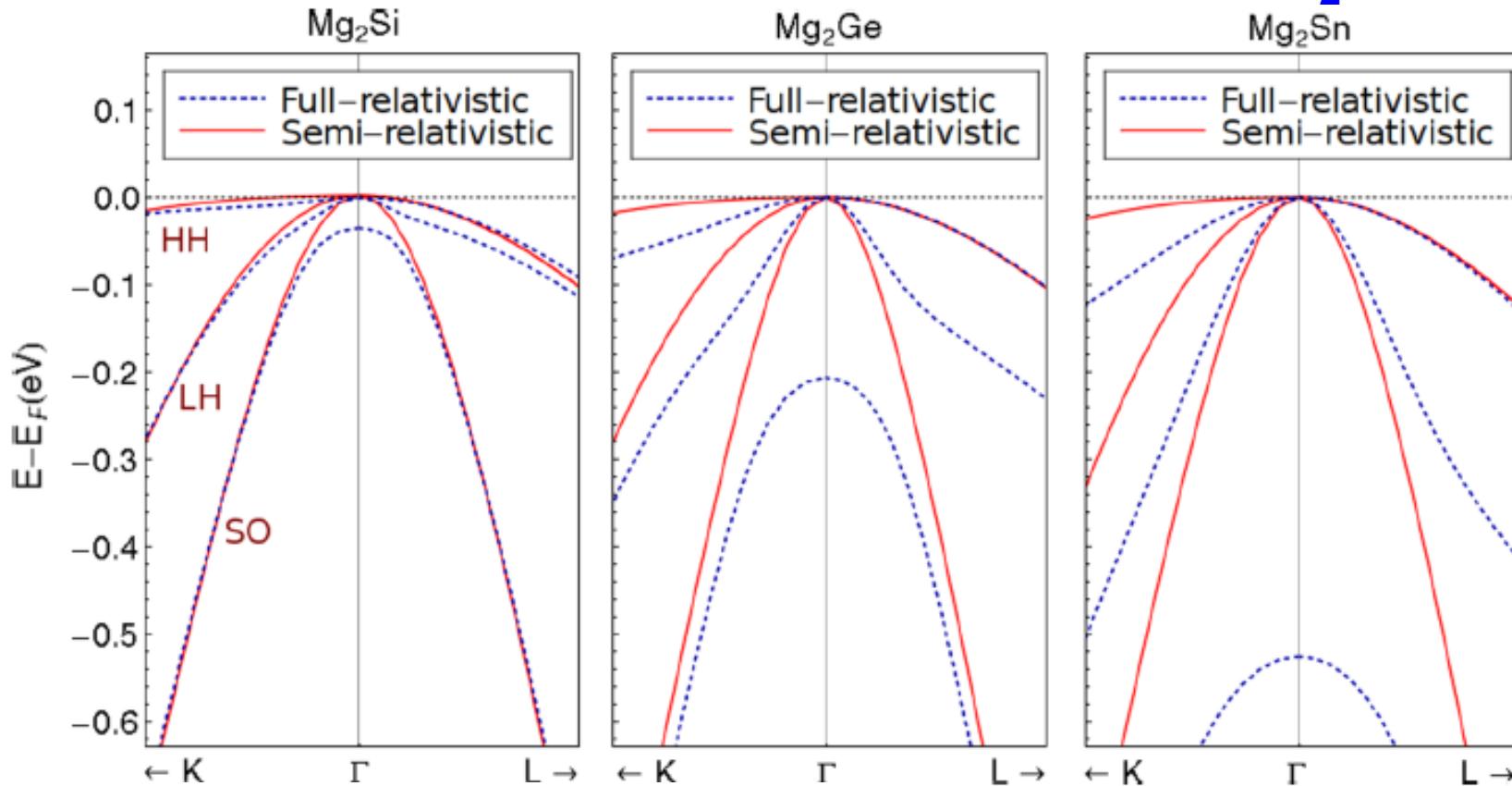
$$H_{\text{kinetic}} = -\frac{p^4}{8m_e^3 c^2}$$

$$\langle H_{SO} \rangle = \frac{{E_n}^2}{m_e c^2} \left( n \frac{j(j+1) - l(l+1) - \frac{3}{4}}{l(l+\frac{1}{2})(l+1)} \right)$$

$$H_{\text{Darwinian}} = \frac{\hbar^2}{8m_e^2 c^2} 4\pi \left( \frac{Ze^2}{4\pi\epsilon_0} \right) \delta^3(\vec{r})$$

**SO splitting**  $\sim \frac{Z^4}{n^3(j+1/2)} 10^{-5} \text{ eV}$

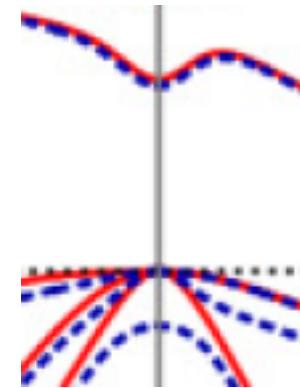
# Effect of S-O on electronic bands in $Mg_2X$



	$Mg_2Si$	$Mg_2Ge$	$Mg_2Sn$
Calculated (meV)	36	208	525
Measured (meV)	$30^a$	$200^a$	$480^a, 600^b$

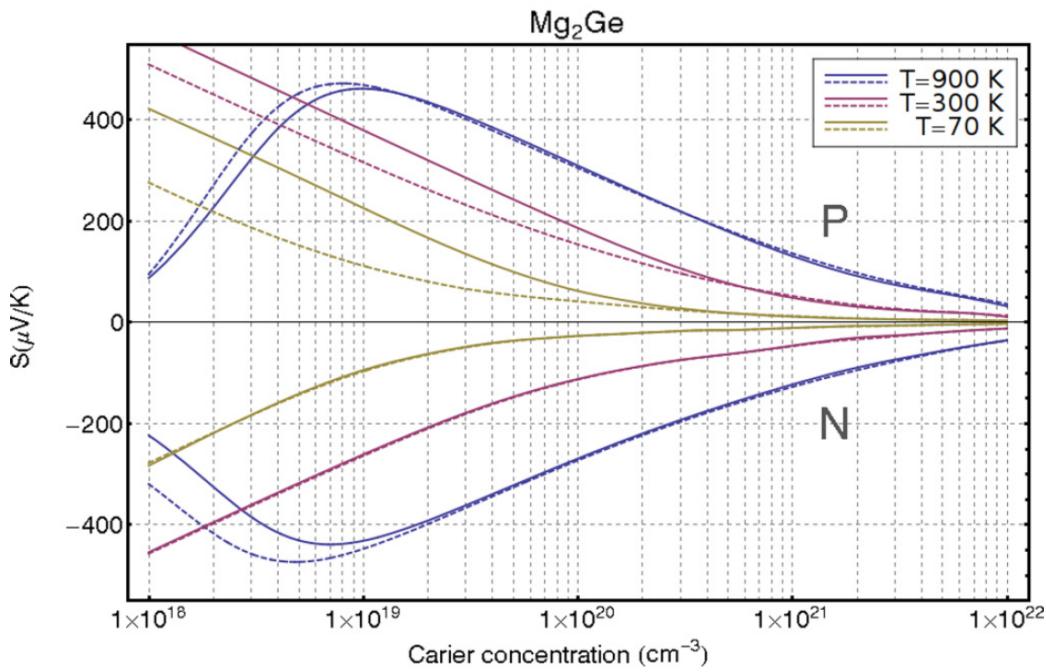
<sup>11</sup> F. Vazquez, A. R. Forman, and M. Cardonna *Phys. Rev.*, vol. 176, p. 905, 1968

<sup>12</sup> L. A. Lott and D. W. Lynch, *Phys. Rev.* 141, 681 (1965)



effect of S-O:  
on CB – negligible  
on VB – HUGE !

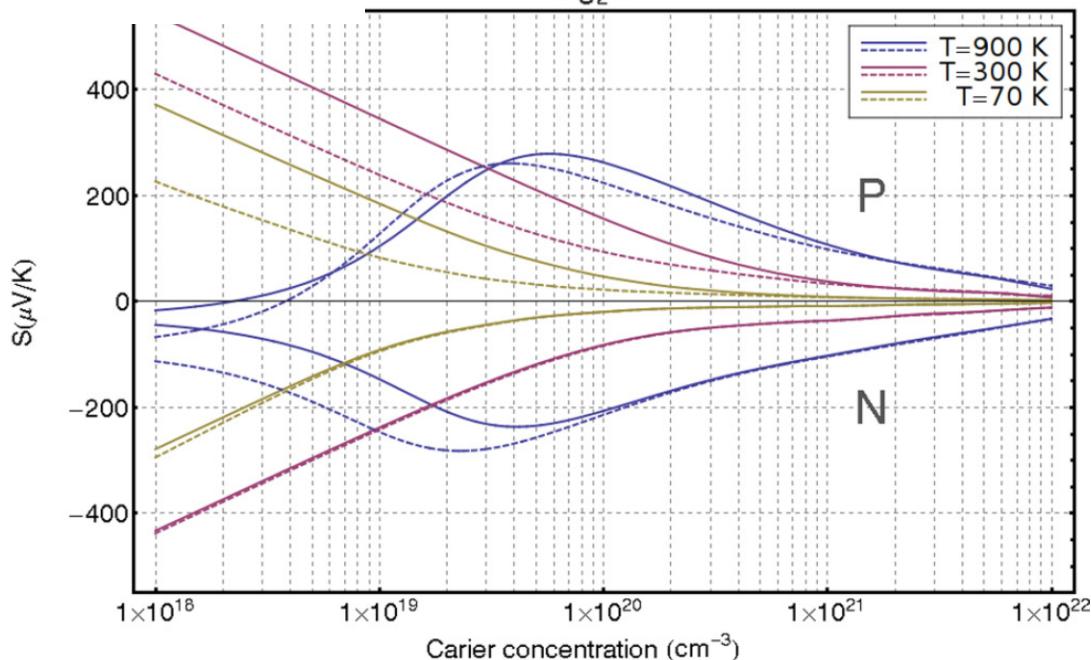
# Seebeck coefficient



$\text{Mg}_2\text{Ge}$

$\text{Mg}_2\text{Sn}$

$\text{Mg}_2\text{Sn}$



For p-type:  
**S-O decreases thermopower**

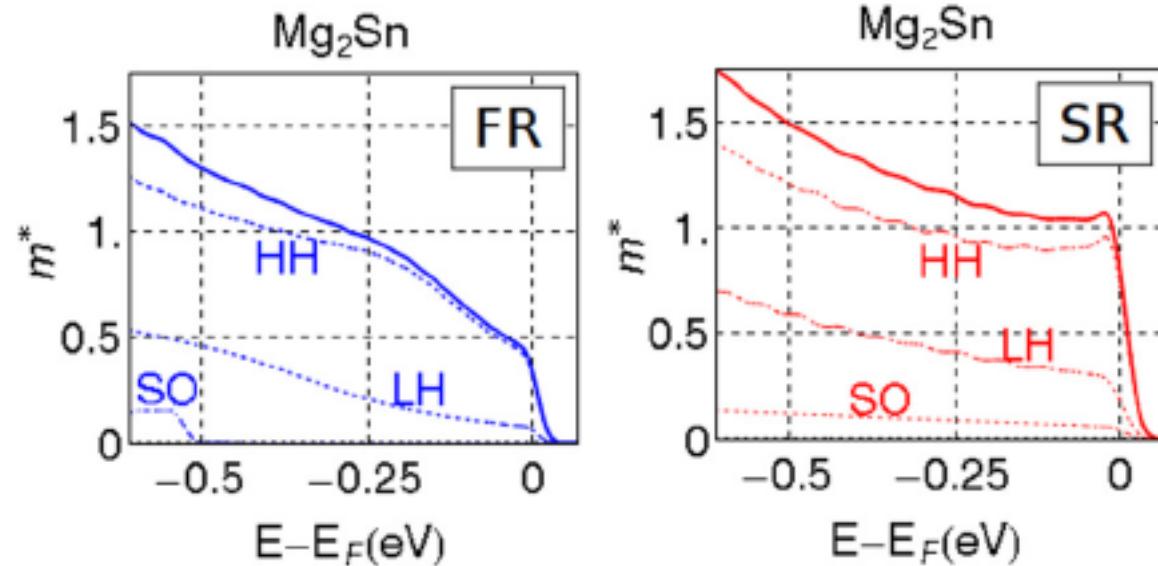
For n-type:  
paradoxally S-O increases  
thermopower due to **weakened  
bipolar effect**

# Net effect of S-O on Seebeck coefficient

TABLE II: Seebeck coefficient at  $n=10^{20} \text{ cm}^{-3}$  for different temperature and compound in  $\text{Mg}_2\text{X}$ .

	Si			Ge			Sn			
	70 K	300 K	900 K	70	300 K	900 K	70 K	300 K	900 K	
n-type	$S_{\text{FullRell}} (\mu\text{V}/\text{K})$	-33.	-139.	-291.	-27.	-113.	-273.	-20	-84	-214.
	$S_{\text{SemiRell}} (\mu\text{V}/\text{K})$	-34.	-141.	-290.	-27.	-112.	-269.	-19.	-82.	-207.
	$S_{\text{FR}}/S_{\text{SR}} (-)$	98%	99%	100%	100%	101%	101%	103%	102%	104%
p-type	$S_{\text{FullRell}} (\mu\text{V}/\text{K})$	77.	208.	329.	41.	154.	305.	22.	94.	224.
	$S_{\text{SemiRell}} (\mu\text{V}/\text{K})$	65.	191.	314.	62.	189.	310.	47.	156.	263.
	$S_{\text{FR}}/S_{\text{SR}} (-)$	118%	109%	105%	67%	83%	98%	48%	60%	85%

## Bands' contribution to hole effective mass

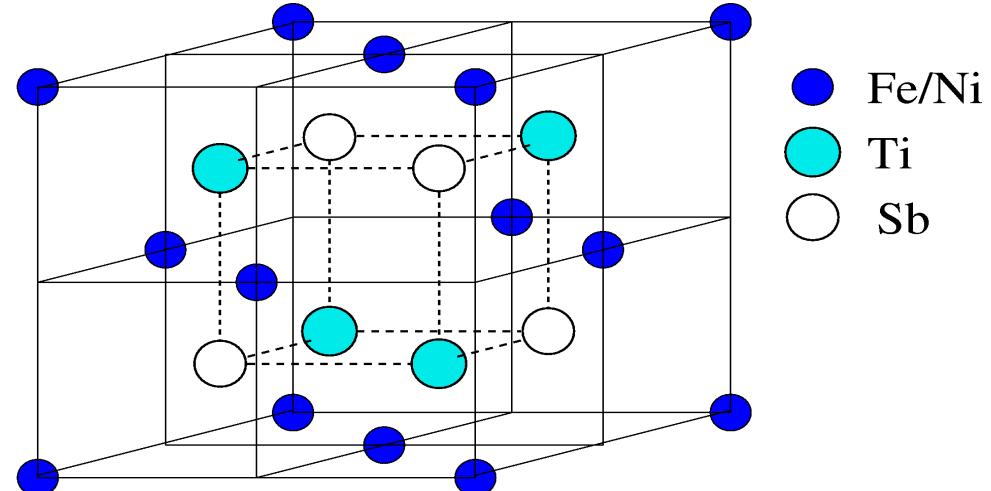


# Heusler phases $X_2YZ$ , $XYZ$ (1903)

## Structure $DO_3$

**Fm $3m$**  (type  $Fe_3Al$ )

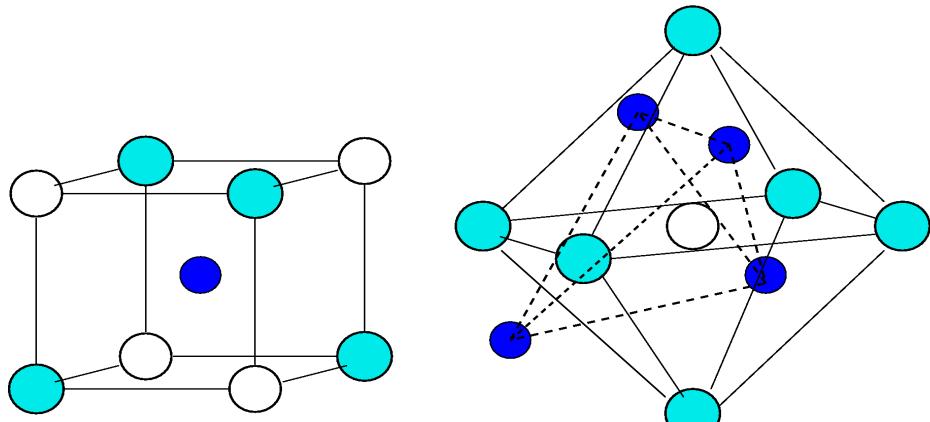
$X1: (0,0,0); (1/2,1/2,1/2)$   
 $X2: (3/4,3/4,3/4)$   
 $Z: (1/4,1/4,1/4)$



## Normal Heusler $L2_1$

**Fm $3m$**  (type  $Cu_2MnAl$ )

$X: (0,0,0); (1/2,1/2,1/2)$   
 $Y: (3/4,3/4,3/4)$   
 $Z: (1/4,1/4,1/4)$



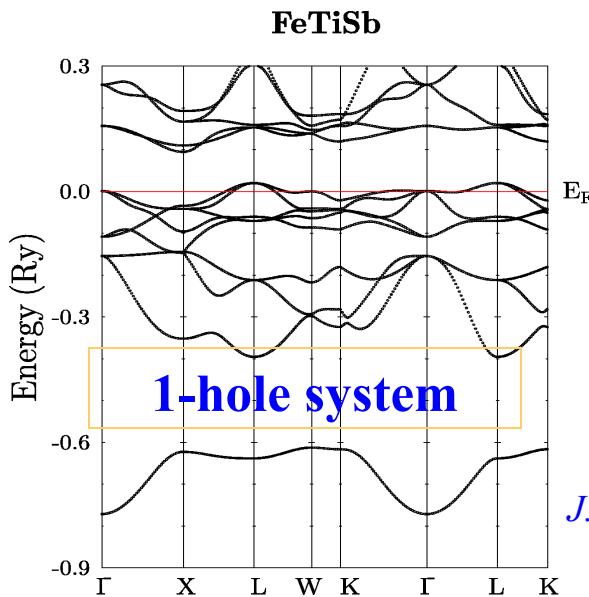
## Half-Heusler $C1_b$

**F-43m** (type  $AgMgAs$ )

$X: (0,0,0) \quad 4a$   
 $Y: (3/4,3/4,3/4) \quad 4d$   
 $Z: (1/4,1/4,1/4) \quad 4c$

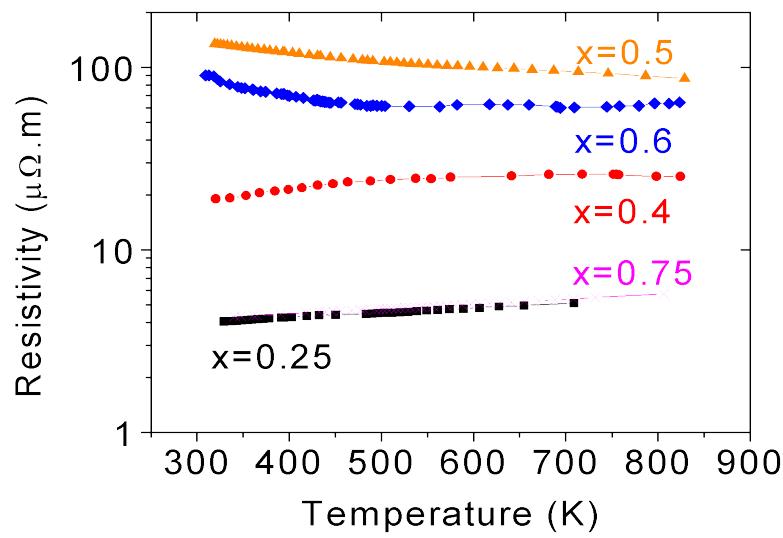
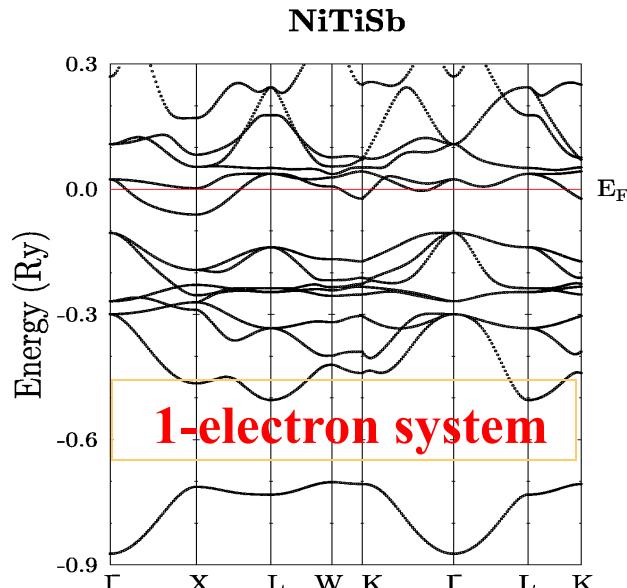
**Crystal stability  
orbitals  $sp^3, d$**

# Metal–semiconductor–metal crossovers

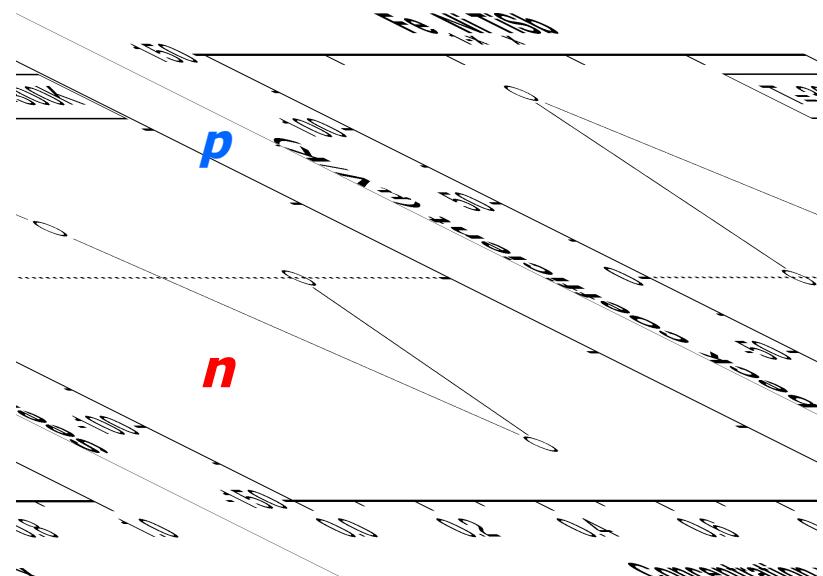


FeTiSb (VEC=17)  
Curie-Weiss PM ( $\sim 0.9 \mu_B$ )  
NiTiSb (VEC=19)  
Pauli PM

*J. Tobola et al., PRB 64, 155103 (2001)*

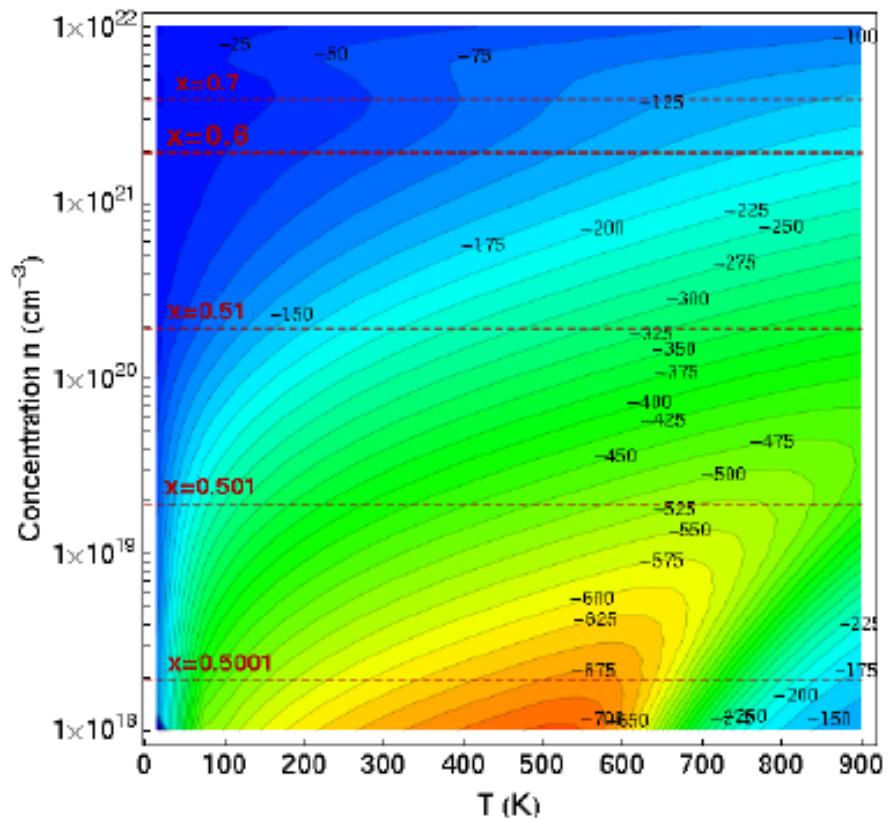


**Resistivity (experiment)**

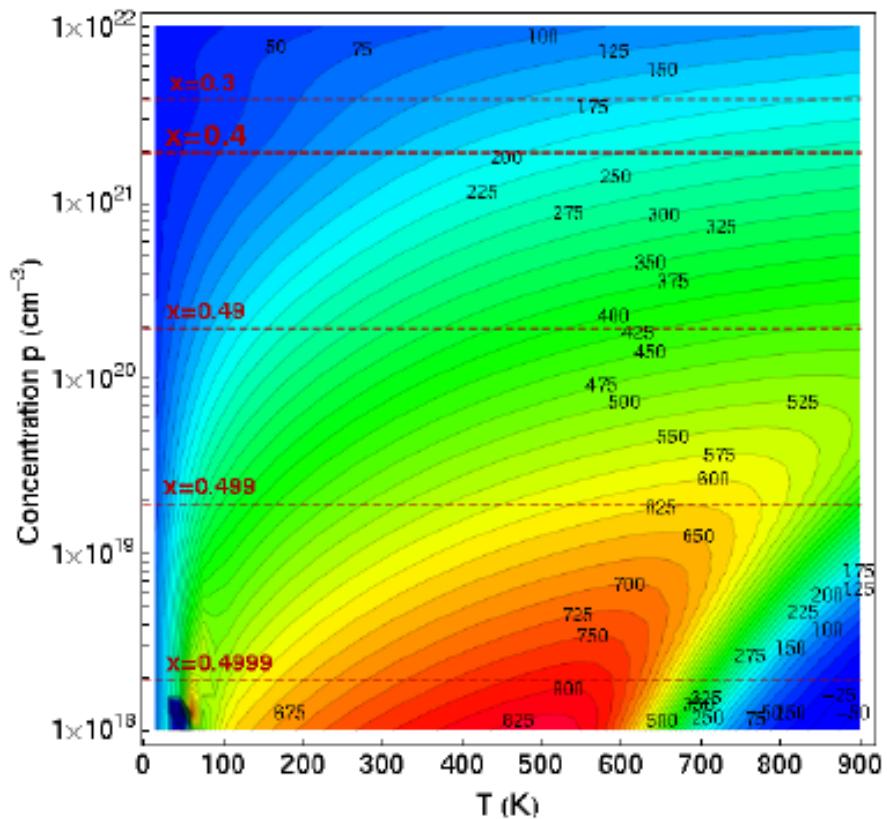


**Thermopower (experiment)**

# Seebeck coefficient vs. temperature & carrier concentration

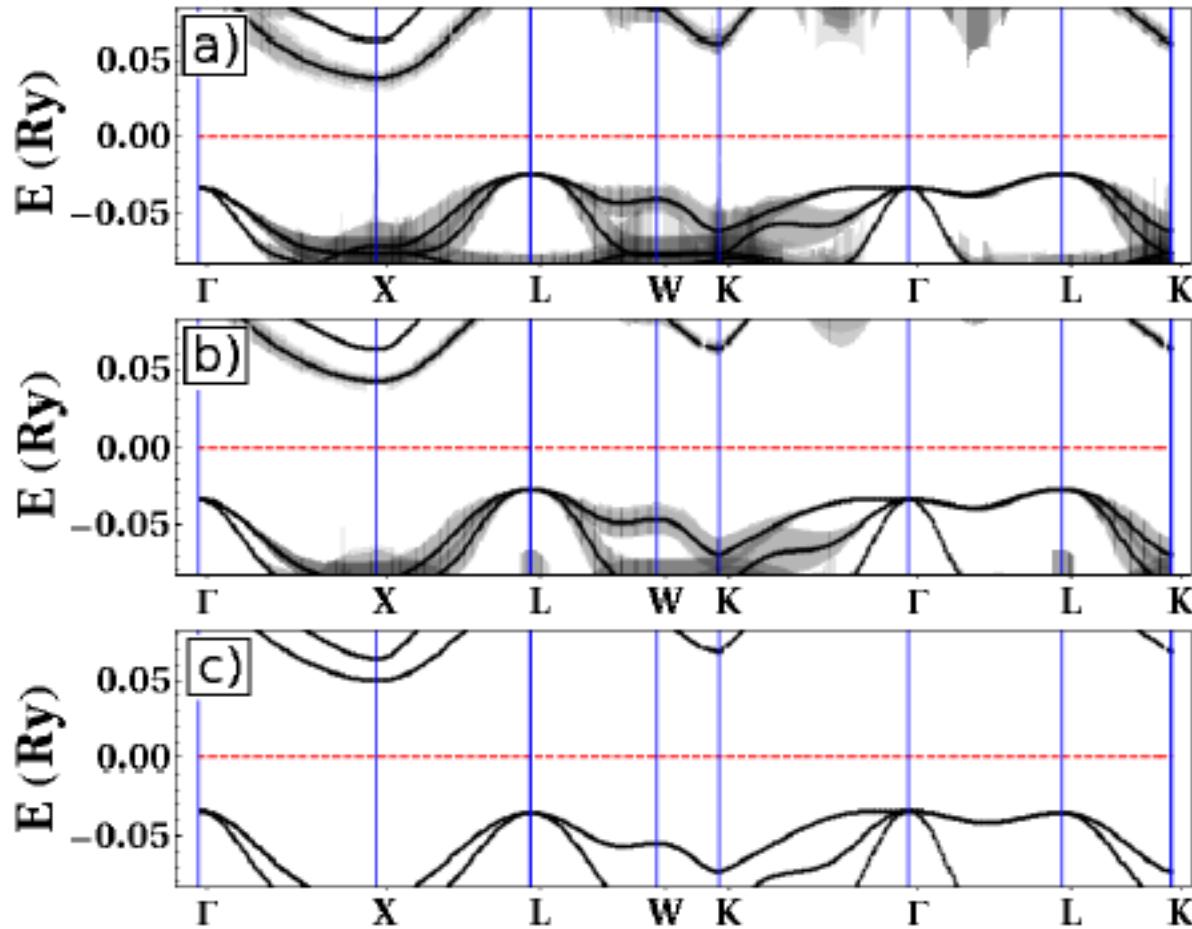


n-type



p-type

# Complex energy band „engineering”



$TiFe_{0.5}Ni_{0.5}Sb$

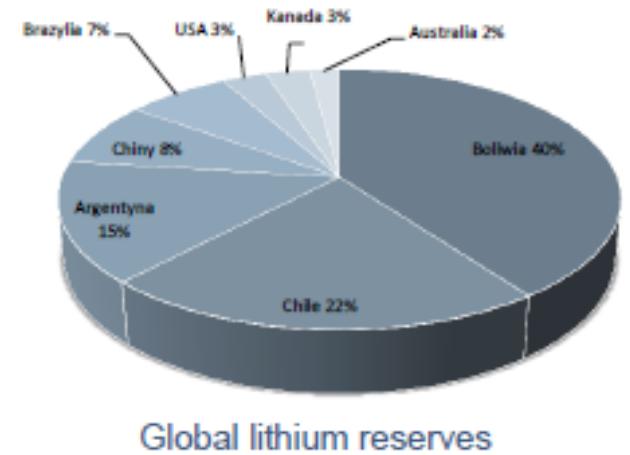
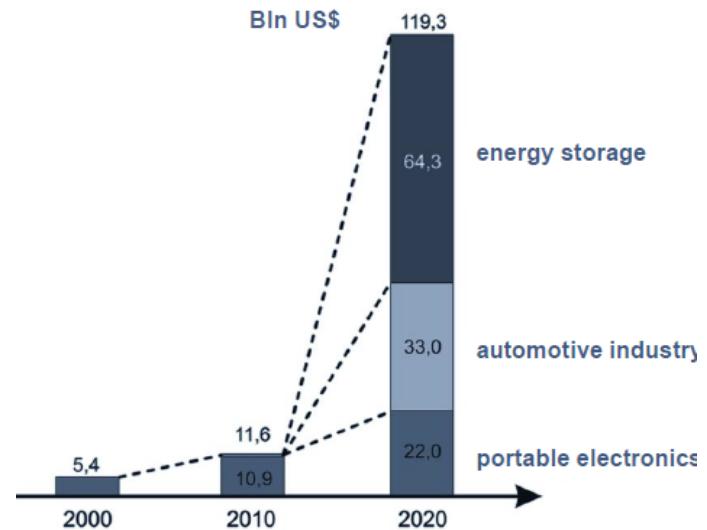
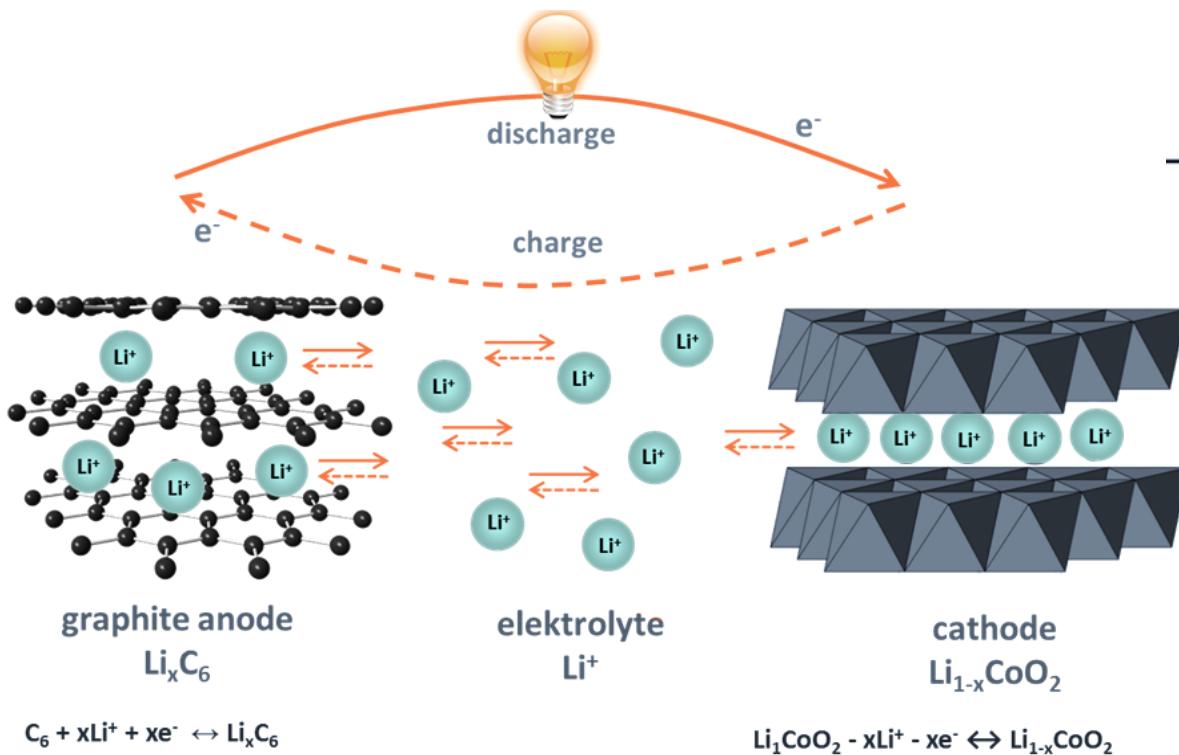
$TiFe_{0.3}Co_{0.4}Ni_{0.3}Sb$

$TiCoSb$

Tendency to alignment of bands near Fermi energy  
BUT it needs experimental proof whether TE properties are really improved

# Li-ion battery cathode materials

Must be gradually replaced by Na-ion battery?  
*world's resources of Li likely insufficient...*



Kim et al. Adv. Energy Mater. 2 (2012) 860.

from J. Molenda

# 'Electronic' model of Li-intercalation process $\text{Li}/\text{Li}^+/\text{Li}_x\text{M}_a\text{X}_b$

$$\mu_{\text{Li}}(\text{cathode}) - \mu_{\text{Li}}(\text{anode}) = -\Delta E$$



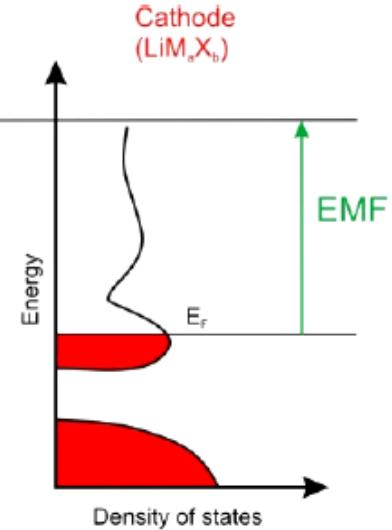
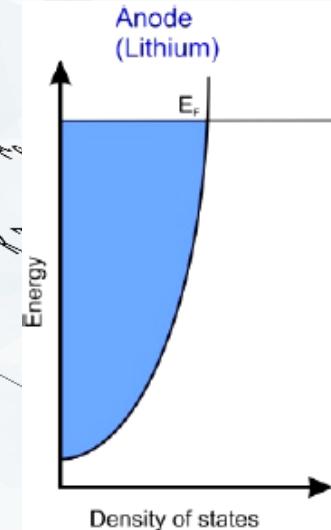
$$\mu_{\text{Li}}(\text{cathode}) = \mu_{\text{Li}}(\text{anode}) + \Delta E$$

Initial states

$\mu_{e^-}$  (initial Fermi level)

Li cathode

Li<sub>x</sub>M<sub>a</sub>X<sub>b</sub> cathode



$$\Delta V_{OC} \approx \Delta E_F^{cathode}$$

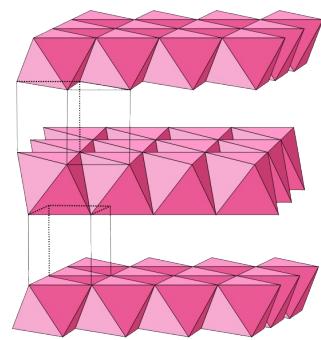
J. Molenda, Phys. Status Solidi B 165 (1991) 419

J. Molenda, Funct. Mater. Lett. 4 (2011) 107

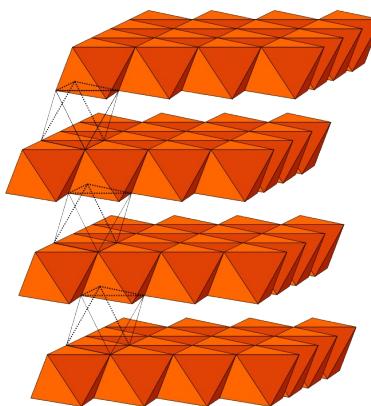
J. Molenda *et al.* Phys. Chem. Chem. Phys. 16 (2014) 14845

# Types of crystal structure capable for alkaline-ion intercalation

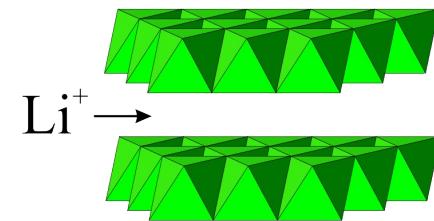
$\text{Na}_x\text{CoO}_2$



$\text{LiCoO}_2$



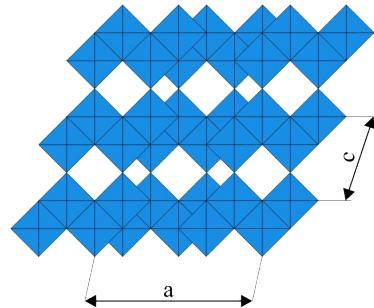
$\text{TiS}_2$



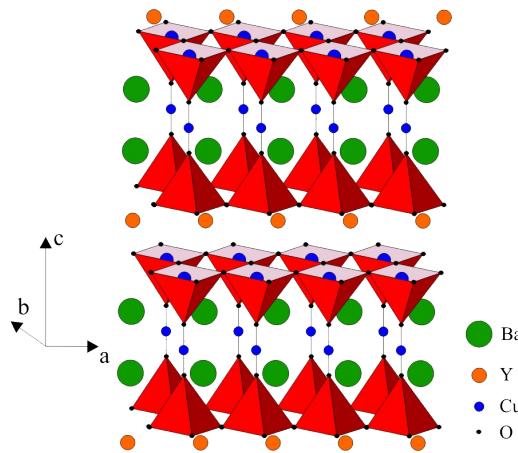
$\text{Li}^+ \rightarrow$



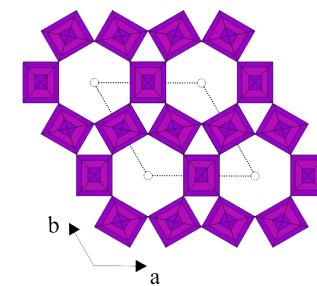
$\text{VO}_2$



$\text{YBa}_2\text{Cu}_3\text{O}_7$



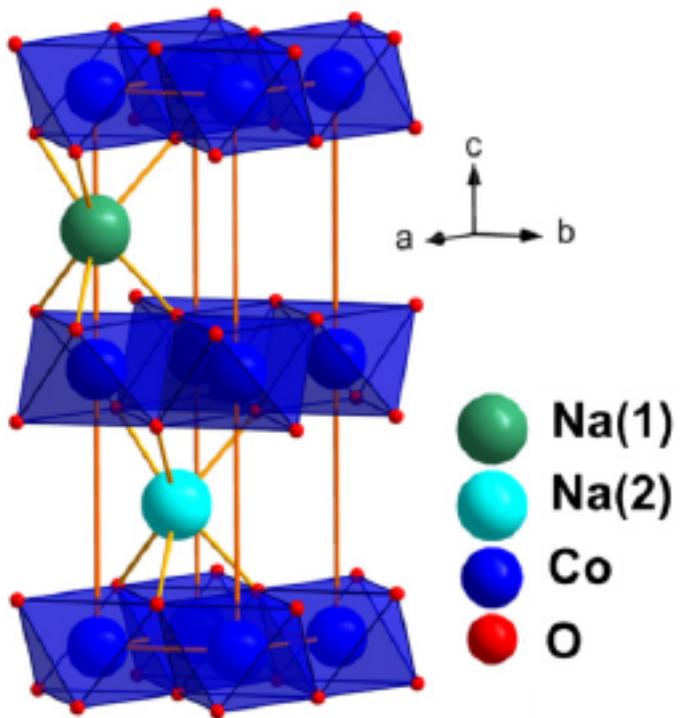
$\text{WO}_3$



● Ba  
● Y  
● Cu  
● O

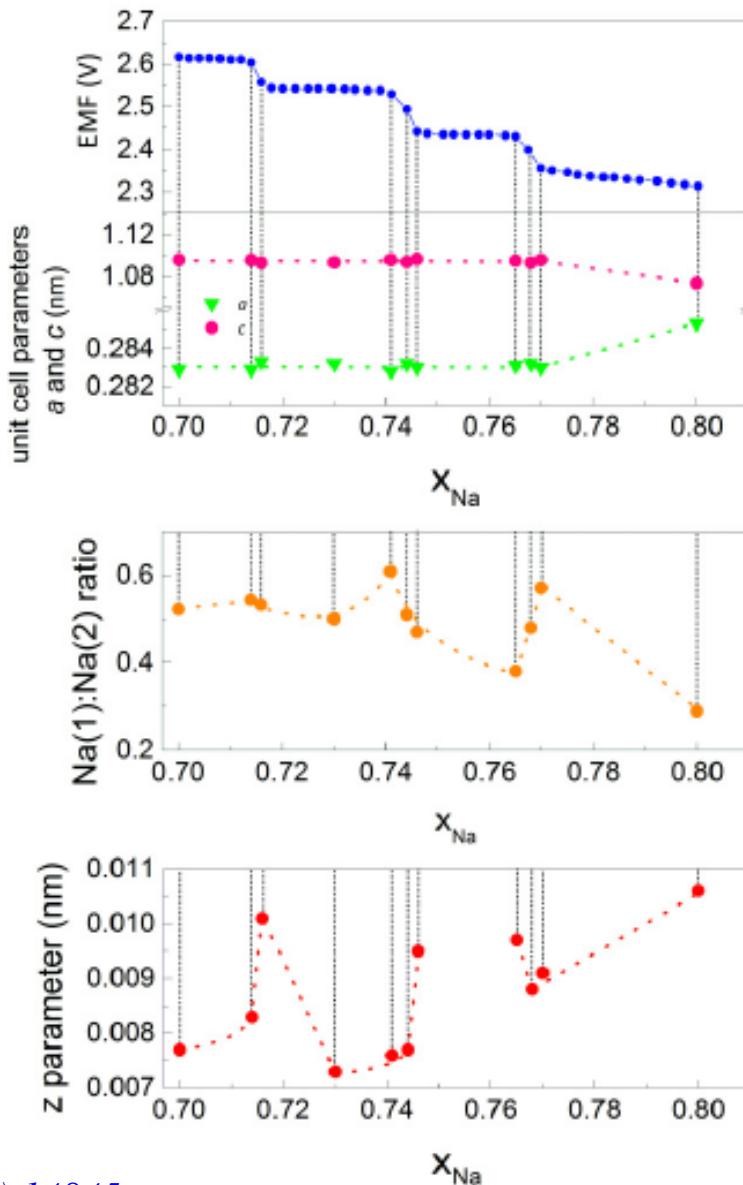
From J. Molenda

# Important structural aspects in $\text{Na}_x\text{CoO}_2$



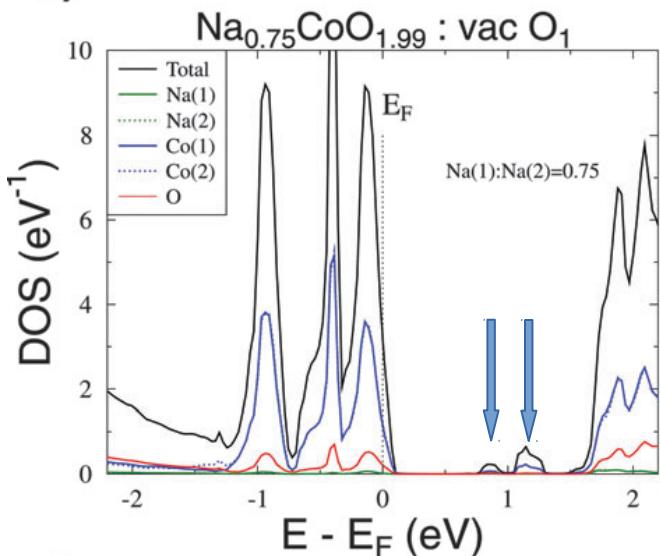
## S.G. P6<sub>3</sub>/mcm

- partial occupancy of Na(1) & Na(2) sites
- O atoms on 4f sites no more equivalent
- lattice constants change regularly, but z parameter defining O octahedron, irregularly.

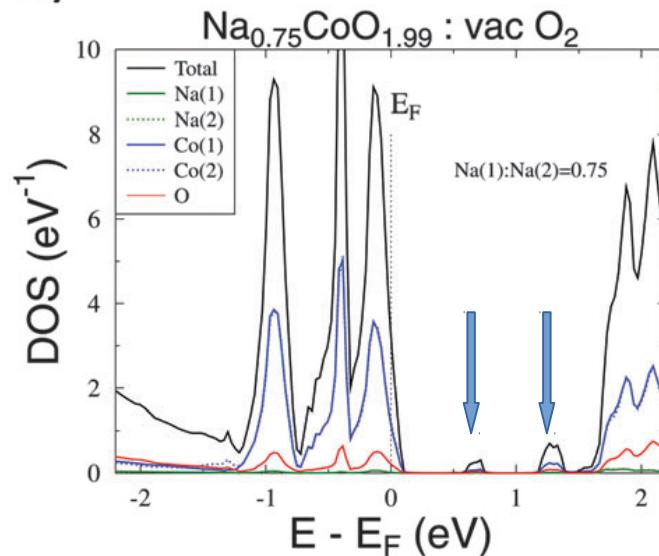


# O vacancy in $\text{Na}_x\text{CoO}_2 \rightarrow$ extra DOS peaks

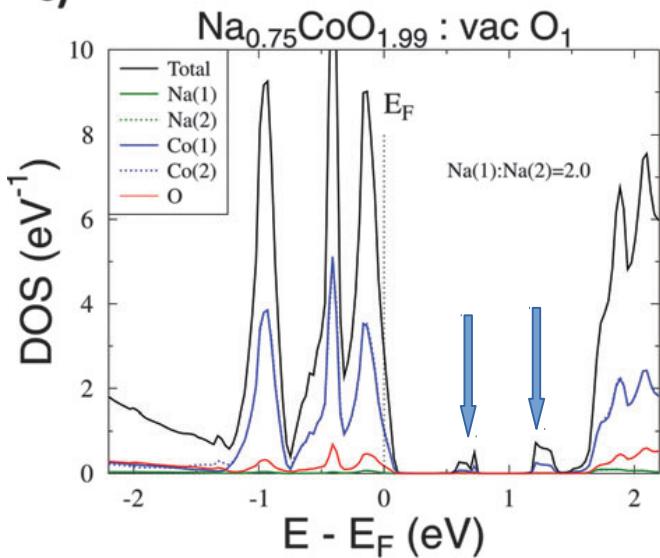
a)



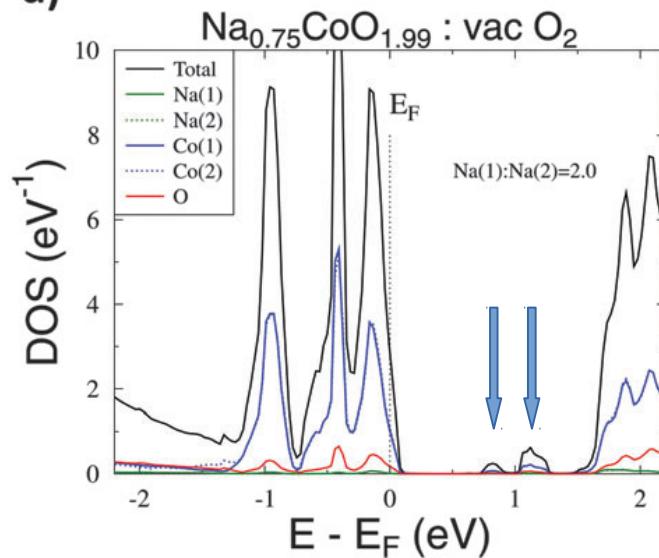
b)



c)



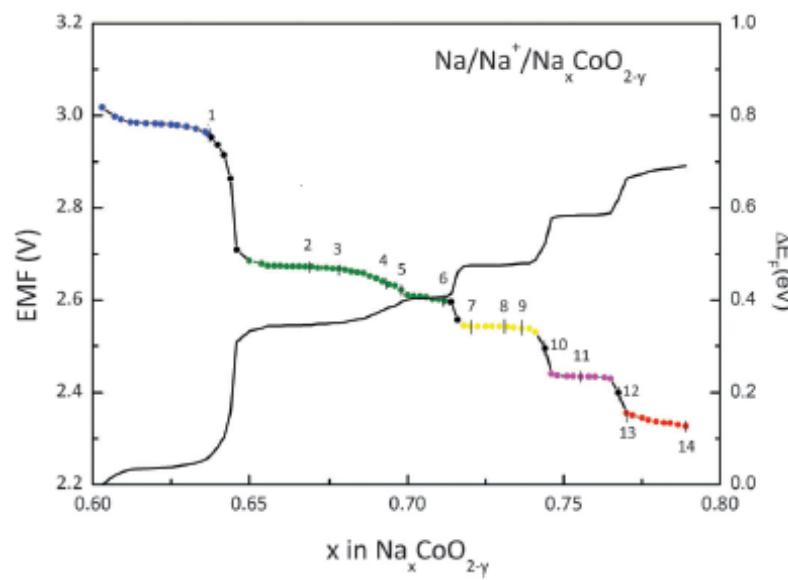
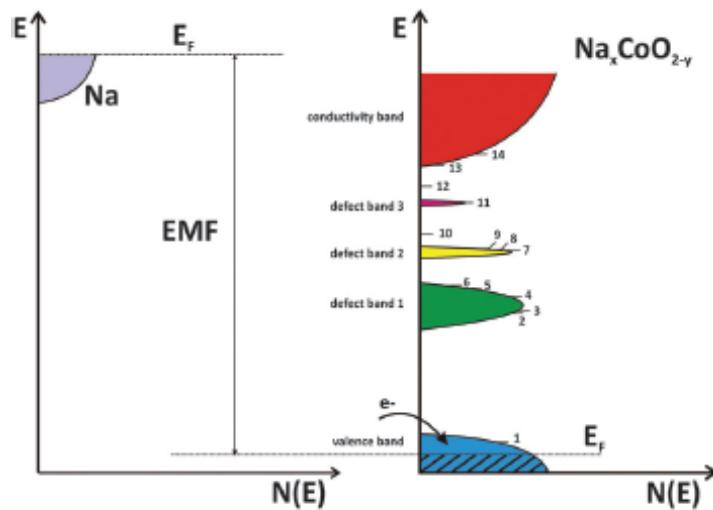
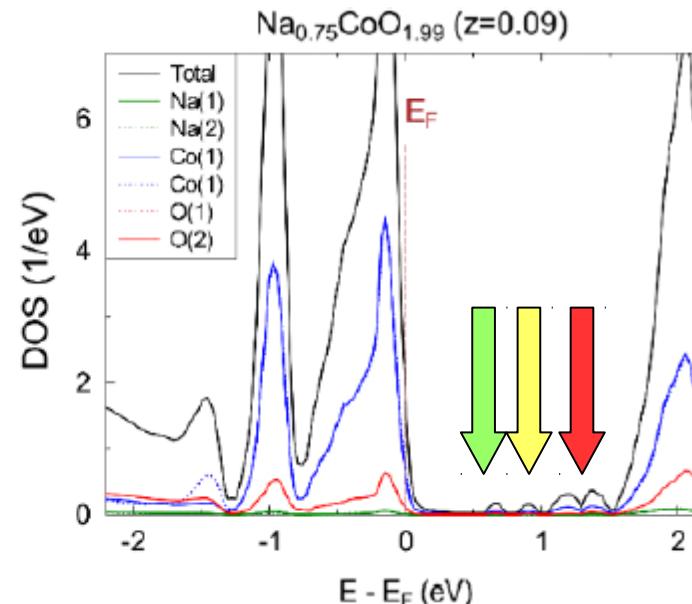
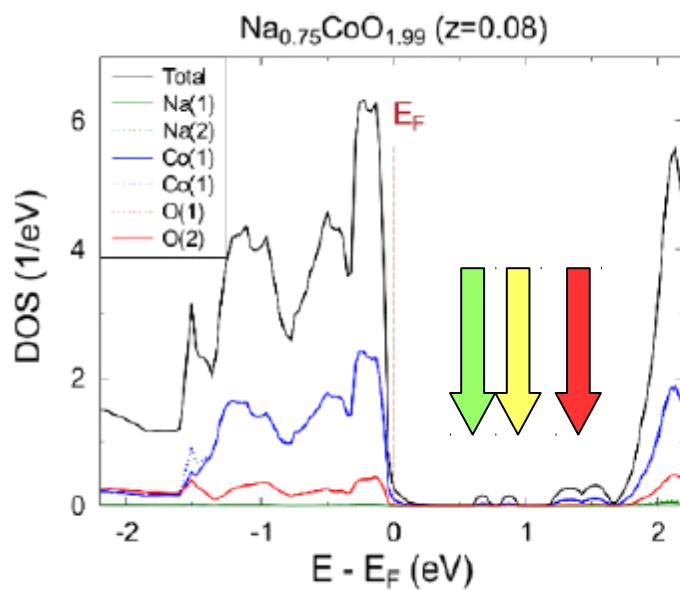
d)



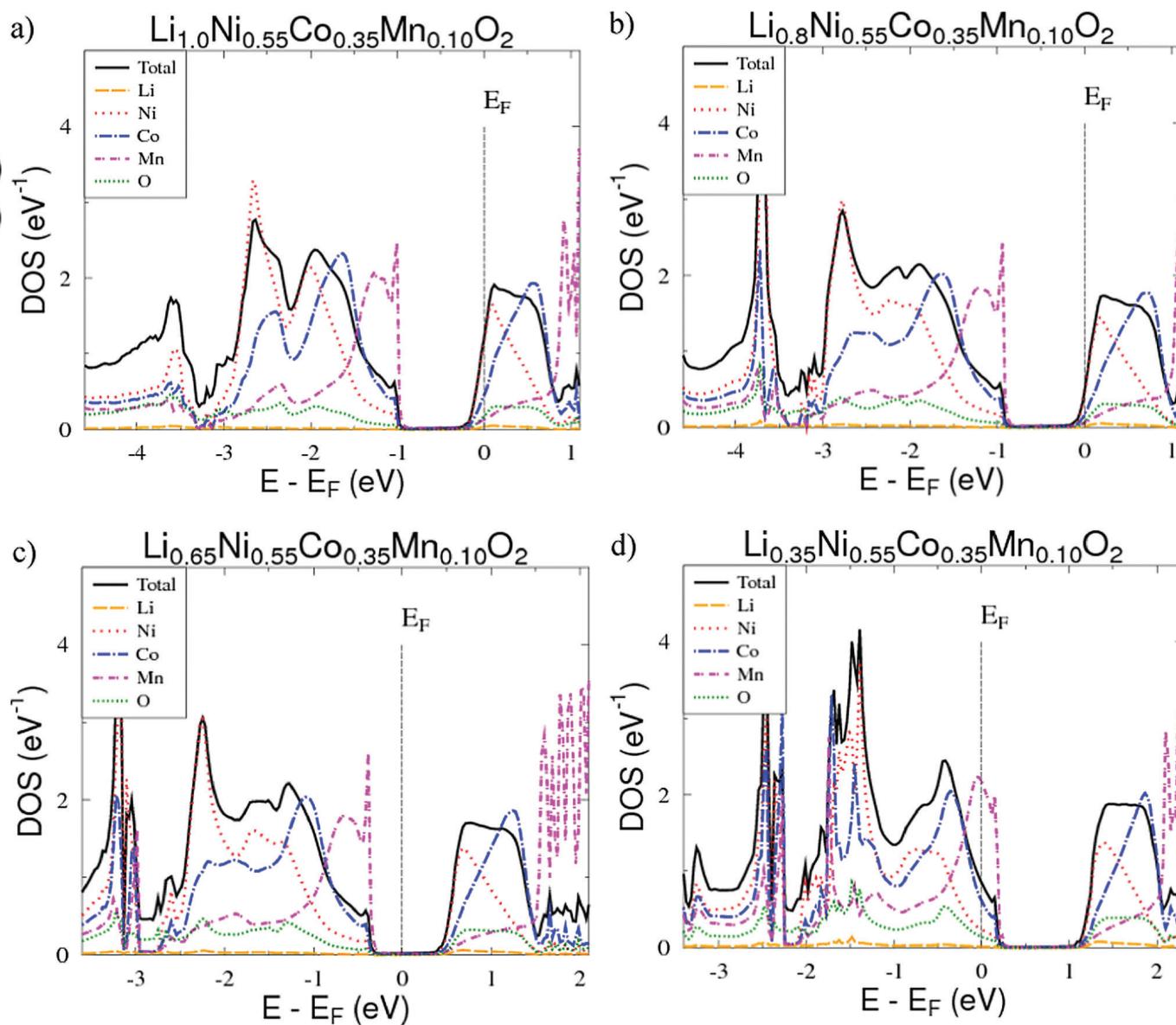
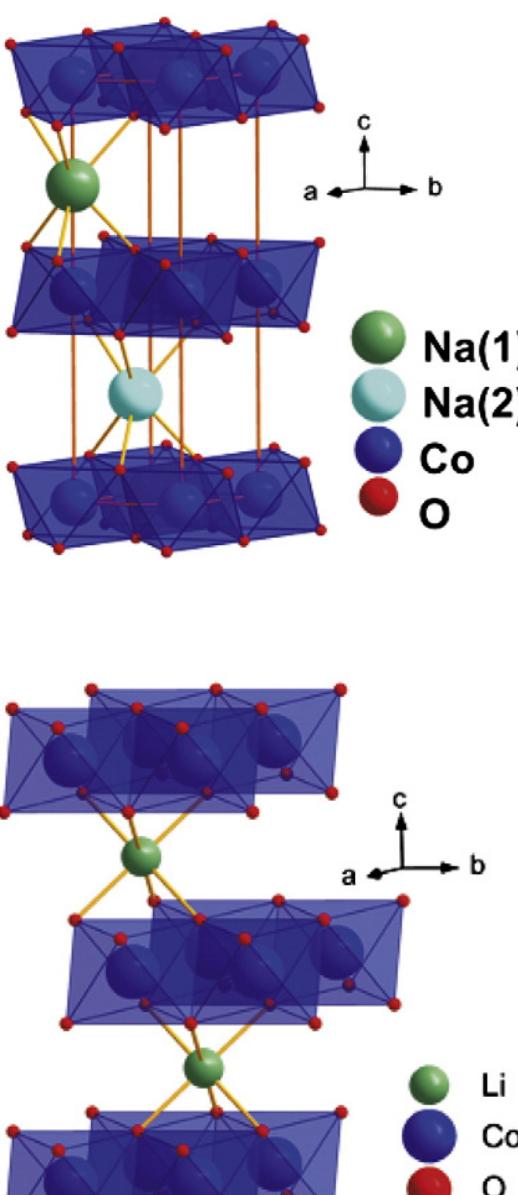
Vacancies on two pairs of O atoms generate extra DOS peaks inside bandgap in slightly different places, which comes from various occupancy of Na(1) & Na(2) sites.

"Defect" peaks contain mostly  $d$ -states from the closest Co atoms, with some admixture of  $p$ -states of O.

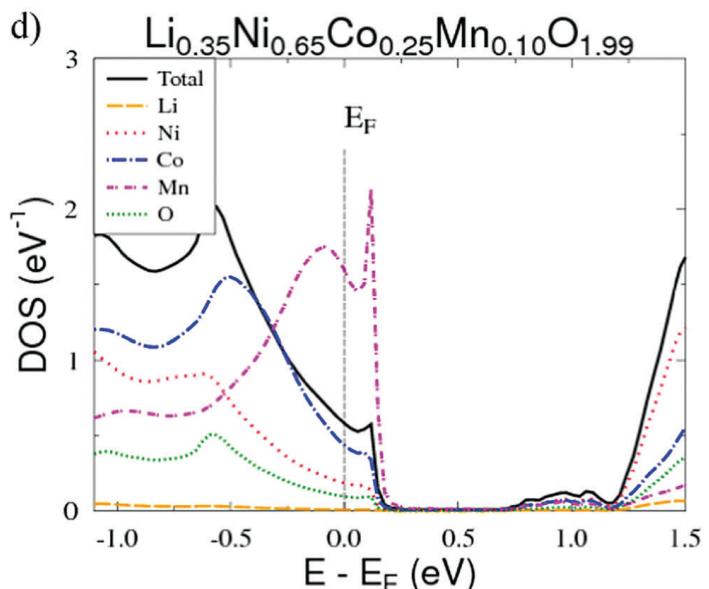
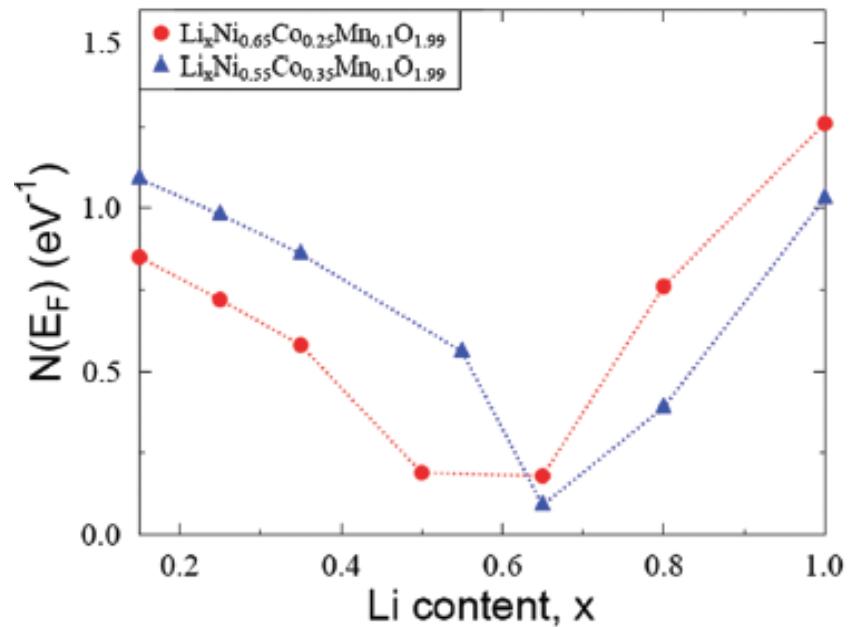
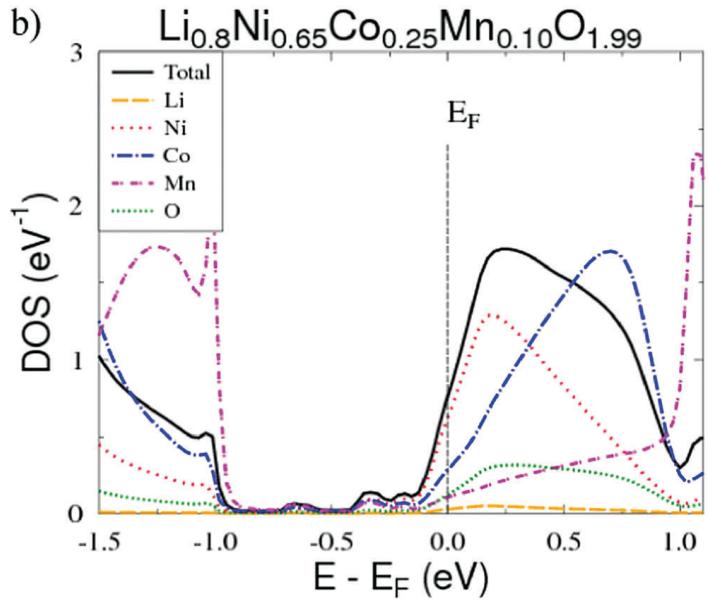
# 'Defects bands' versus of EMF character



# Electronic structure of $\text{Li}_x(\text{Ni-Co-Mn})\text{O}_2$



# Electronic structure of $\text{Li}_x(\text{Ni-Co-Mn})\text{O}_{2-y}$ : + O vacancy



Strong effect of O vacancy which produces extra states inside the gap leading to its vanishing in view of KKR-CPA

For lower Li content Fermi level falls into high DOS peak of d-states on Mn.

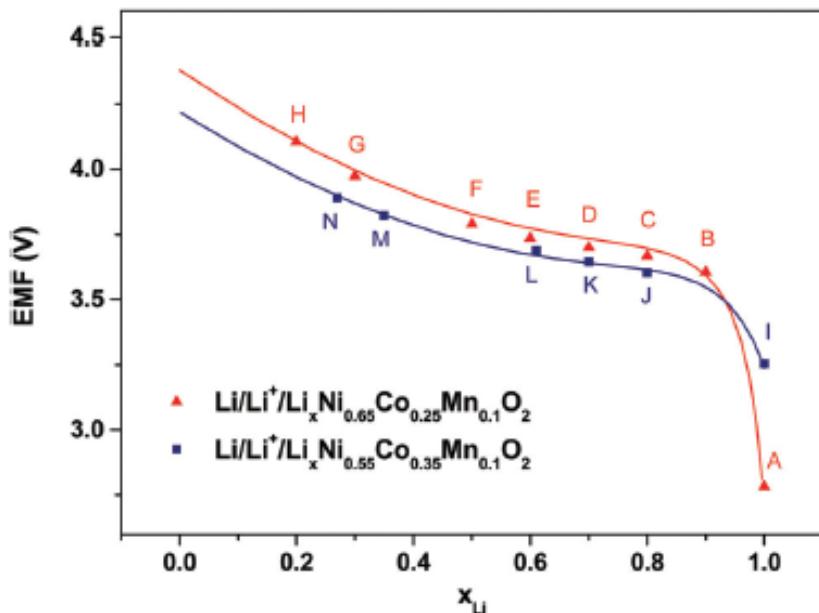
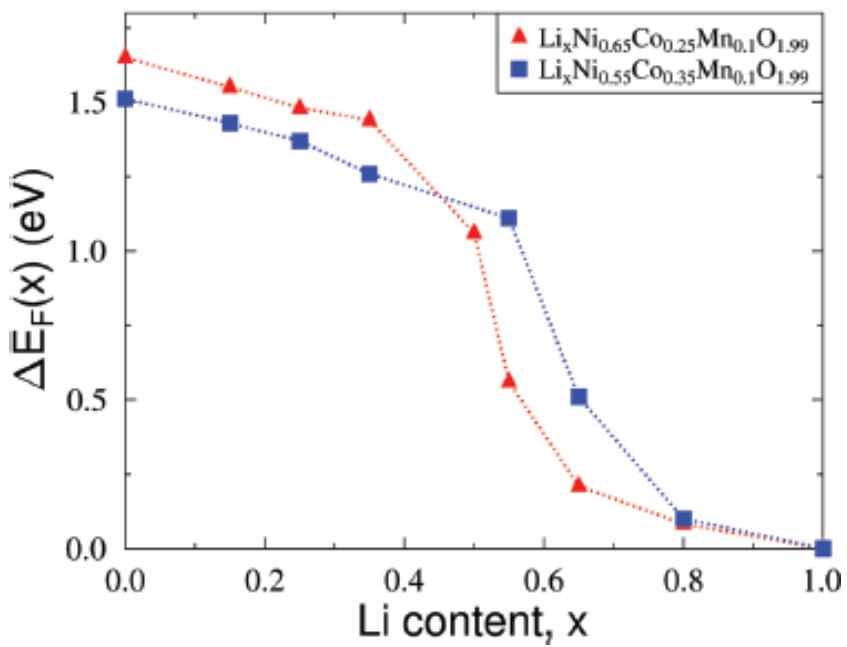
Without O-vacancies transport properties can not be interpreted coherently.

# $\text{Li}_x(\text{Ni-Co-Mn})\text{O}_{2-y}$ : correlation of EMF & $\Delta E_F$

Experimental EMF curve shows the voltage change of about **1.2-1.5 V**



More detailed computations would require analysis of **formation energy at finite T**.

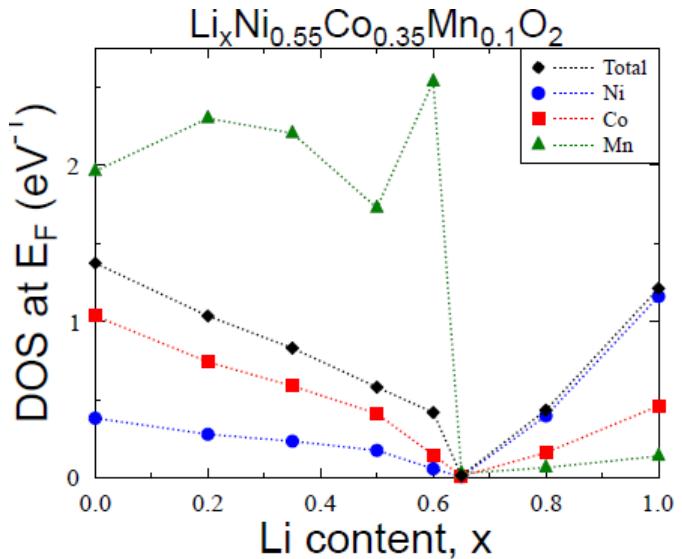


Variation of Fermi level from KKR-CPA calculations of about **1.5-1.6 eV** in reasonable agreement with EMF



To some extent it reflects change in chemical potential of cathode material,

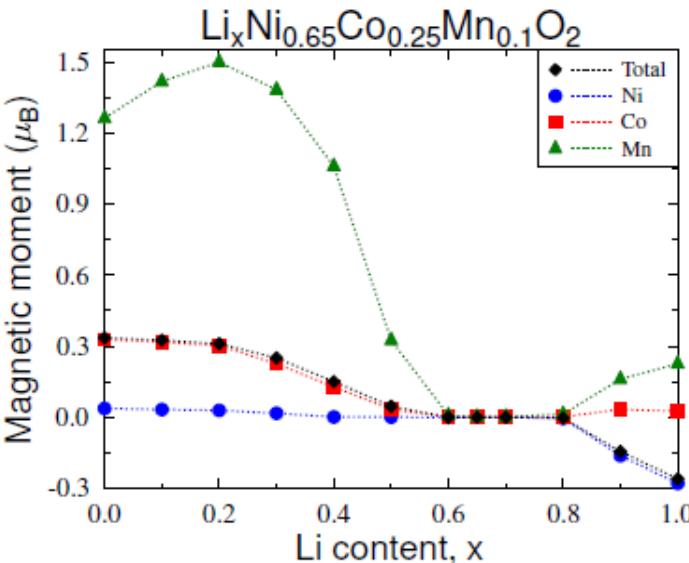
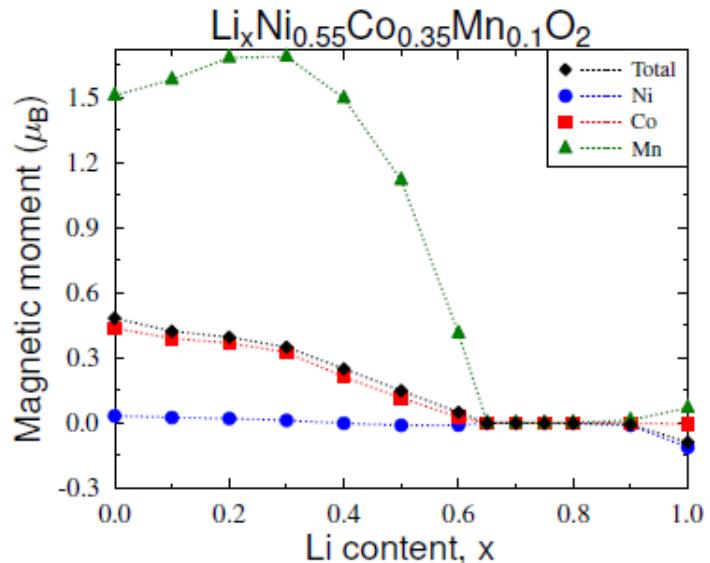
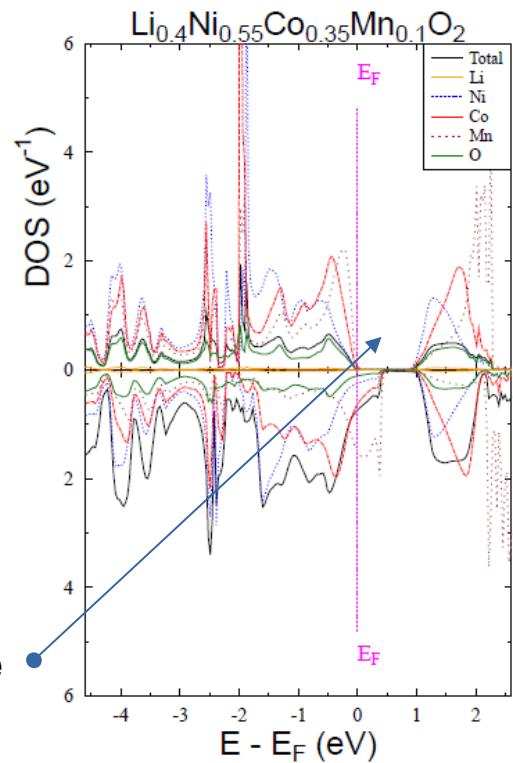
# Prediction of magnetism in $\text{Li}_x(\text{Ni-Co-Mn})\text{O}_{2-y}$



With decreasing Li content  
Magnetic moments can be  
born on TM atoms (Mn, Co).

It should have an effect on  
discharge curve if magnetic  
order present at battery  
operating T.

half-metallic state

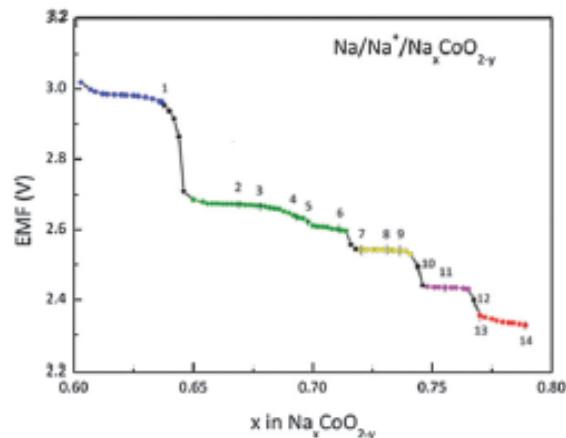
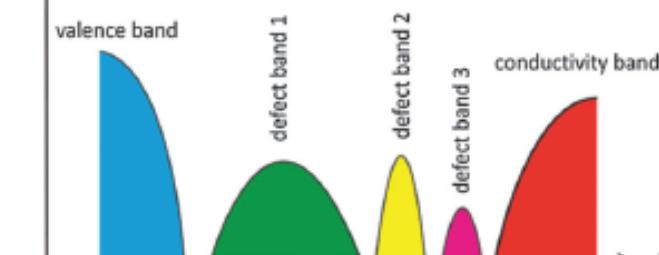


# Summary

- I Boltzmann approach combined with *ab initio* electronic structure calculations in disordered materials give reliable predictions of  $T$ -dependent & carrier concentration (n/p).
- II Electronic structure calculations are helpful in electronic structure searching for band degeneracy or band alignment (complex energy bands) in alloys improving thermoelectric properties e.g. Mg<sub>2</sub>(Si-Sn) or of strongly anisotropic, non-parabolic bands & Fermi surfaces etc.
- III KKR-CPA calculations accounting for realistic defects (alloying, vacancy defects) in battery systems as Li<sub>x</sub>(Co-Ni-Mn)O<sub>2</sub> vs. Na<sub>x</sub>CoO<sub>2</sub> show close correlation of variation of DOS near EF and character of EMF (more quantitative comparison needed to support this concept).
- IV Relativistic effects (S-O coupling) and electron correlations play a important role in materials converting energy and should be accounted for calculations.

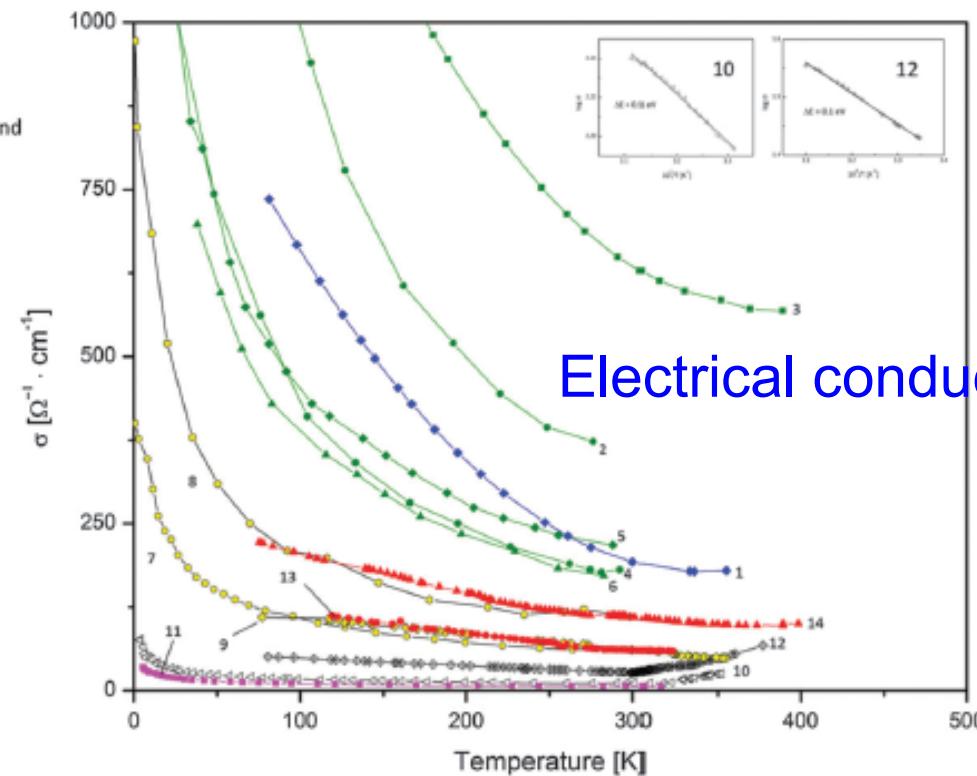
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# $\text{Na}_x\text{CoO}_{2-\delta}$



Electronic  
specific heat

$$\gamma = \frac{1}{3}\pi^2 k_B^2 N(E_F)$$



Electrical conductivity

