Time Crystals

Krzysztof Sacha

Jagiellonian University in Kraków.

People involved in the research on time crystals:

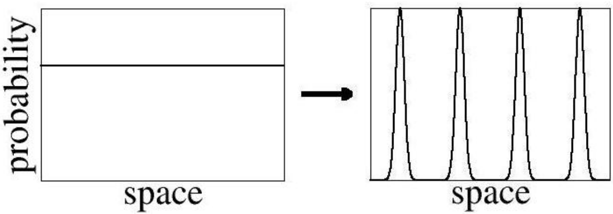
- **Alexandre Dauphin**, *Barcelona*
- **Dominique Delande**, *Paris*
- **Krzysztof Giergiel**, *Kraków*
- **Peter Hannaford**, *Melbourne*
- **Arkadiusz Kosior**, *Dresden*
- **Arkadiusz Kuroś**, *Kraków*
- **Maciej Lewenstein**, *Barcelona*
- **Paweł Matus**, *Kraków*
- **Marcin Mierzejewski**, *Wrocław*
- **Florian Mintert**, *London*
- **Artur Miroszewski**, *Warsaw*
- **Rick Mukherjee**, *London*
- **Luis Morales-Molina**, *Santiago*
- **Frederic Sauvage**, *London*
- **Andrzej Syrwid**, *Kraków*
- **Jakub Zakrzewski**, *Kraków*

Formation of space crystals

$$[\hat{H}, \hat{T}] = 0$$

$\hat{T}$ – translation operator of all particles by the same vector

t =const.

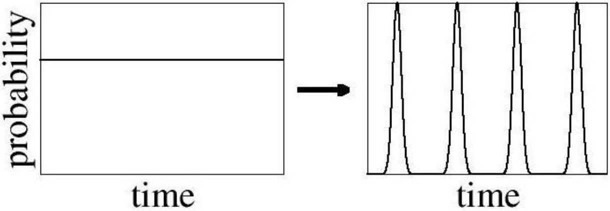


$$\langle \, \hat{\psi}^\dagger(x) \, \hat{\psi}^\dagger(x') \, \hat{\psi}(x') \, \hat{\psi}(x) \, \rangle$$

Formation of time crystals?

Eigenstates of a time-independent Hamiltonian H are also eigenstates of a time translation operator e^{-iHt}

$\vec{r}$ is fixed

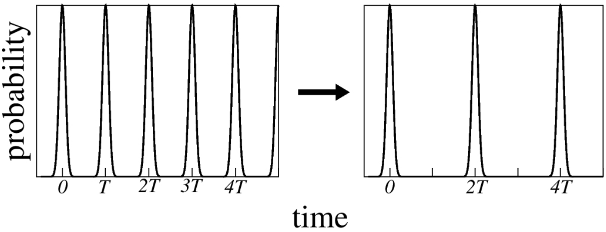


$$\langle\, \hat{\psi}^{\dagger}(t)\, \hat{\psi}^{\dagger}(t')\, \hat{\psi}(t')\, \hat{\psi}(t)\, \rangle$$

F. Wilczek, PRL **109**, 160401 (2012).
P. Bruno, PRL **111**, 070402 (2013).
H. Watanabe and M. Oshikawa, PRL **114**, 251603 (2015).
A. Syrwid, J. Zakrzewski, KS, PRL **119**, 250602 (2017).
V. K. Kozin and O. Kyriienko, *"Quantum Time Crystals from Hamiltonians with Long-Range Interactions"*, PRL **123**, 210602 (2019).

Discrete time crystals

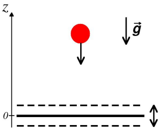
Spontaneous process



Discrete time crystals

Single particle bouncing on an oscillating mirror in 1D

Classically:



Floquet Hamiltonian:

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\ \psi_n(z,t) = E_n\ \psi_n(z,t)$$

E_n – quasi-energy

$\psi_n(z,t)$ – time periodic function

Discrete time crystals

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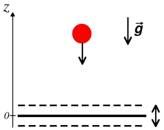

$2:1$ resonance

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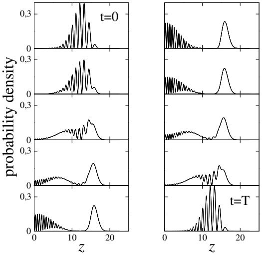


Two resonant Floquet states with the quasi energy difference $\frac{\omega}{2}$

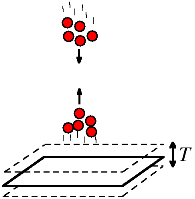
($\frac{\omega T}{2} = \pi$-difference)

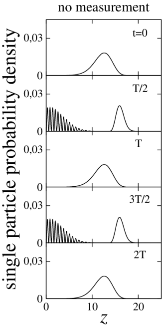
A. Buchleitner, D. Delande, J. Zakrzewski, Phys. Rep. **368**, 409 (2002).

Discrete time crystals

Bosons with attractive interactions

$$\hat{H}_F \approx -\frac{J}{2}\left(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1\right) - \frac{|U|}{2}\left[(\hat{a}_1^\dagger)^2(\hat{a}_1)^2 + (\hat{a}_2^\dagger)^2(\hat{a}_2)^2\right] = -J\sum_{i=1}^{N} s_i^x - \frac{N|U|}{N}\sum_{i,j}^{N} s_i^z s_j^z, \quad \text{LMG model}$$





$$N = 10^4$$

$$|\psi\rangle \approx \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}}$$

KS, Phys. Rev. A **91**, 033617 (2015).

Discrete time crystals

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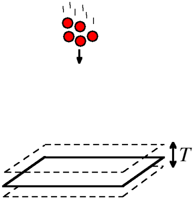
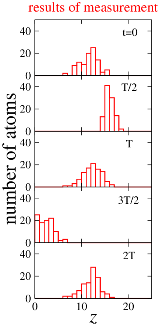




$N = 10^4$

$$|\psi\rangle \approx \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}} \longrightarrow |N-1,0\rangle \text{ or } |0,N-1\rangle$$

KS, Phys. Rev. A **91**, 033617 (2015).

Discrete time crystals

Spin systems

V. Khemani, A. Lazarides, R. Moessner, L. S. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

D. V. Else, B. Bauer, C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).

LETTER

doi:10.1038/nature21413

Observation of a discrete time crystal

J. Zhang[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], A. Lee[1], J. Smith[1], G. Pagano[1], I.-D. Potirniche[2], A. C. Potter[3], A. Vishwanath[2,4], N. Y. Yao[2] & C. Monroe[1,5]

LETTER

doi:10.1038/nature21426

Observation of discrete time-crystalline order in a disordered dipolar many-body system

Soonwon Choi[1*], Joonhee Choi[1,2*], Renate Landig[1*], Georg Kucsko[1], Hengyun Zhou[1], Junichi Isoya[3], Fedor Jelezko[4], Shinobu Onoda[5], Hitoshi Sumiya[6], Vedika Khemani[1], Curt von Keyserlingk[7], Norman Y. Yao[8], Eugene Demler[1] & Mikhail D. Lukin[1]

Discrete time crystals

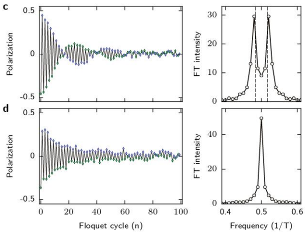
Spin systems

Chain of 10 ions: J. Zhang *et al.*, Nature (2017).

$$H = \begin{cases} H_1 = g(1-\varepsilon)\sum_i \sigma_i^y, & \text{time } t_1 \\ H_2 = \sum_i J_{ij}\sigma_i^x\sigma_j^x, & \text{time } t_2 \\ H_3 = \sum_i D_i\sigma_i^x & \text{time } t_3. \end{cases}$$

$$|\psi\rangle \approx \frac{|\uparrow\uparrow\ldots\uparrow\rangle_x \pm |\downarrow\downarrow\ldots\downarrow\rangle_x}{\sqrt{2}} \longrightarrow \quad |\uparrow\uparrow\ldots\uparrow\rangle_x \quad \text{or} \quad |\downarrow\downarrow\ldots\downarrow\rangle_x$$

10^6 impurities in diamond: S. Choi *et al.*, Nature (2017).



P. Matus, KS, "*Fractional Time Crystals*", Phys. Rev. A **99**, 033626 (2019)

Condensed matter physics in time crystals

Platform for time crystal research

Single particle systems

Integrable 1D system:

$$H_0(x,p) \longrightarrow H_0(I) \quad \Longrightarrow \quad I = const, \quad \theta = \Omega(I)\, t + \theta_0.$$

Time periodic perturbation:

$$H_1 = f(t)\, h(x) \longrightarrow H_1 = \left(\sum_k f_k e^{ik\omega t}\right)\left(\sum_n h_n e^{in\theta}\right).$$

Platform for time crystal research

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Assume s:1 resonance, $\omega = s\,\Omega(I)$. In the moving frame $\Theta = \theta - \frac{\omega}{s} t$

$$H \approx \frac{P^2}{2m_{eff}} + \sum_k f_{-k}\, h_{ks}\, e^{iks\Theta}.$$

Platform for time crystal research

Single particle systems

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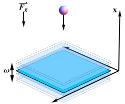
For example for $f(t) = \lambda \cos(\omega t)$, we get $H \approx \frac{P^2}{2m_{eff}} + V_0 \cos(s\Theta)$.

Crystalline structure in time

s : 1 resonance

$$H \approx \frac{P^2}{2m_{eff}} + V_0 \cos(s\, \Theta)$$

$$\Theta = \theta - \frac{\omega}{s} t$$



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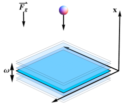
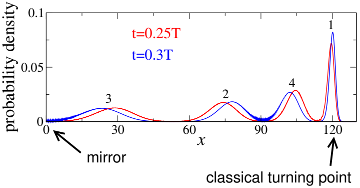


$s : 1$ resonance ($s = 4$):



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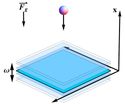

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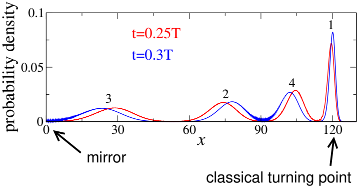
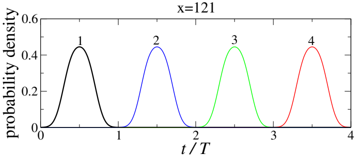




$$E_F = -\frac{J}{2} \sum_{j=1}^{s} (a_{j+1}^* a_j + \mathrm{c.c.})$$

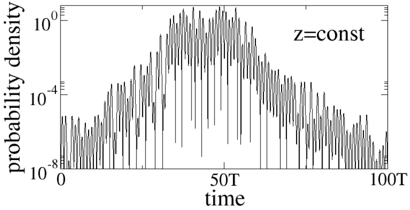
KS, Sci. Rep. **5**, 10787 (2015).
L. Guo, M. Marthaler, G. Schön, Phys. Rev. Lett. **111**, 205303 (2013).

Anderson localization in the time domain

$H'(t)$ is a perturbation that fluctuates in time but $H'(t+sT) = H'(t)$.

$$E_F = -\frac{J}{2}\sum_{j=1}^{s}(a_{j+1}^* a_j + \mathrm{c.c.}) + \sum_{j=1}^{s} \varepsilon_j |a_j|^2$$

Example for $s = 100$:



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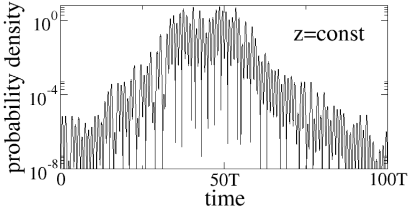
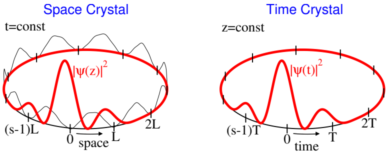


KS, Sci. Rep. **5**, 10787 (2015).



KS, D. Delande, PRA **94**, 023633 (2016).
K. Giergiel, KS, PRA **95**, 063402 (2017).
D. Delande, L. Morales-Molina, KS, PRL **119**, 230404 (2017).

Topological time crystals

A particle bouncing on an oscillating mirror

Mirror oscillations $\propto \lambda \cos(s\omega t) + \lambda_1 \cos(s\omega t/2)$

$$\textbf{SSH model:} \quad H \approx -\sum_{i=1}^{s/2} \left(J\, b_i^* a_i + J'\, a_{i+1}^* b_i \right)$$

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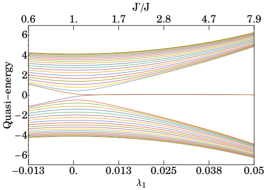
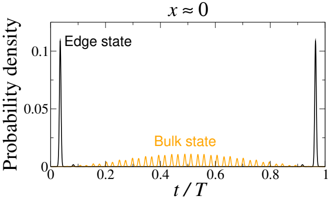
Mirror oscillations $\propto$ $\lambda\cos(s\omega t) + \lambda_1\cos(s\omega t/2) + f(t)$,

$f(t)$ creates an edge in time:





K. Giergiel, A. Dauphin, M. Lewenstein, J. Zakrzewski, KS, New J. Phys. **21**, 052003 (2019).
E. Lustig, Y. Sharabi, M. Segev, Optica **5**, 1390 (2018).

Exotic Interactions

Ultra-cold atoms bouncing on an oscillating mirror

Bosons:

$$\hat{H}_F = -\frac{J}{2}\sum_{j=1}^{s}(\hat{a}_{j+1}^{\dagger}\hat{a}_j + \mathrm{h.c.}) + \frac{1}{2}\sum_{i,j=1}^{s} U_{ij}\ \hat{a}_i^{\dagger}\hat{a}_j^{\dagger}\ \hat{a}_j\hat{a}_i$$

$$U_{ij} \propto \int_0^{sT} dt\ g_0(t)\ \int dx\ |\phi_i|^2\ |\phi_j|^2,$$

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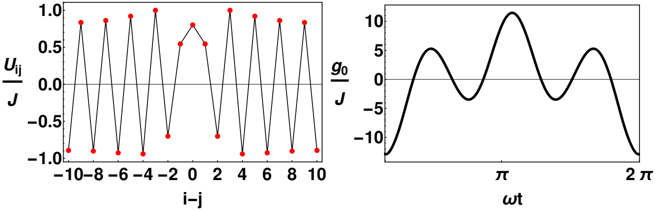
20:1 resonance



K. Giergiel, A. Miroszewski, KS, PRL **120**, 140401 (2018).

Many-body localization induced by temporal disorder

For example for bosons:

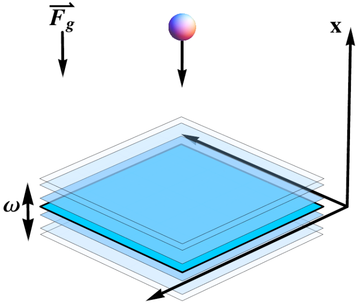
$$\hat{H}_F = -\frac{J}{2}\sum_{j=1}^{s}(\hat{a}_{j+1}^\dagger \hat{a}_j + \mathrm{h.c.}) + \sum_{j=1}^{s}\epsilon_j\ \hat{a}_j^\dagger \hat{a}_j + \frac{1}{2}\sum_{i,j=1}^{s} U_{ij}\hat{n}_i\hat{n}_j$$

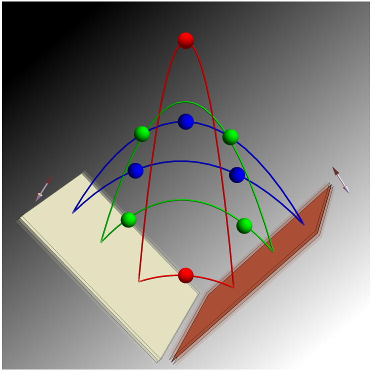
Many-body localization (MBL):

- vanishing of dc transport,
- absence of thermalization,
- logarithmic growth of the entanglement entropy,

M. Mierzejewski, K. Giergiel, KS, PRB **96**, 140201(R) (2017).

Time crystals with properties of 2D space crystals





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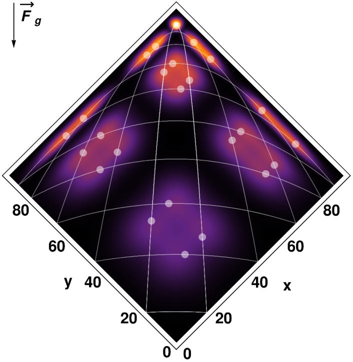
Time crystals with properties of 2D space crystals

5:1 resonances along x and y directions



$$\hat{H}_F = -\frac{J}{2}\sum_{\langle \mathbf{i},\mathbf{j}\rangle}(\hat{a}_{\mathbf{j}}^\dagger \hat{a}_{\mathbf{i}} + \mathrm{h.c.}) + \frac{1}{2}\sum_{\mathbf{i},\mathbf{j}} U_{\mathbf{ij}}\ \hat{a}_{\mathbf{i}}^\dagger \hat{a}_{\mathbf{j}}^\dagger\ \hat{a}_{\mathbf{j}}\hat{a}_{\mathbf{i}}$$

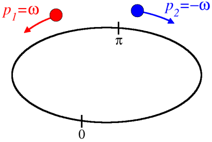
K. Giergiel, A. Miroszewski, KS, PRL **120**, 140401 (2018).

Time engineering

Anderson molecule

Two atoms bound together not due to attractive interaction but due to destructive interference

$$H = \frac{p_1^2 + p_2^2}{2} + \delta(\theta_1 - \theta_2)\, f(t) \quad \longrightarrow \quad H_{eff} = \frac{P_1^2 + P_2^2}{2} + \sum_k f_{-2k} e^{ik(\Theta_1 - \Theta_2)}$$



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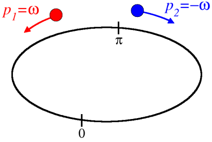

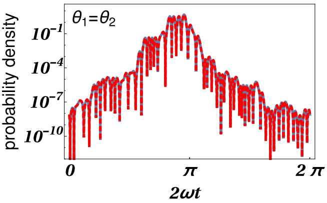

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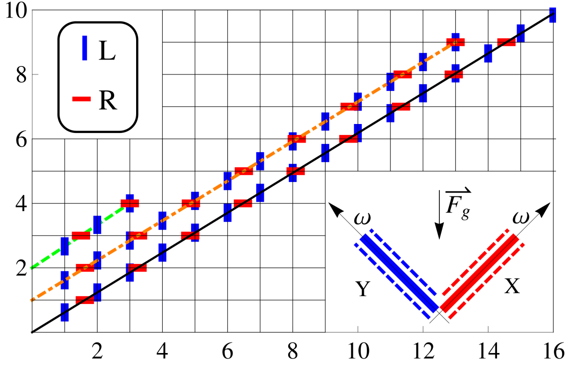
Spontaneous formation of quasi-crystals in time

Spontaneous formation of time quasi-crystals

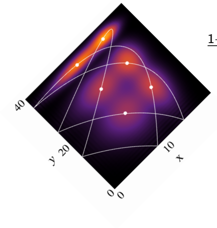
Fibonacci quasi-crystal: *LRLLRLRLLR*...



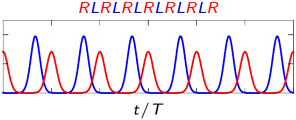
Spontaneous formation of time quasi-crystals

$$H(t+T) = H(t)$$

$s_x : 1$ and $s_y : 1$ resonances



$\frac{1+\sqrt{5}}{2} \approx \frac{s_y}{s_x} = \frac{3}{2}$



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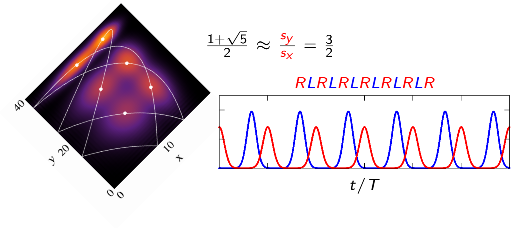
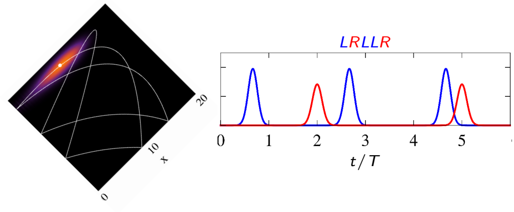
$s_x : 1$ and $s_y : 1$ resonances

$\frac{1+\sqrt{5}}{2} \approx \frac{s_y}{s_x} = \frac{3}{2}$



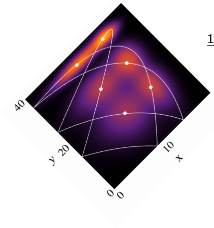


K. Giergiel, A. Kuroś, KS, "*Discrete Time Quasi-Crystals*", PRB **99**, 220303(R) (2019)

Spontaneous formation of time quasi-crystals

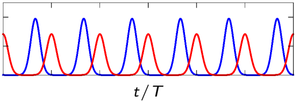
$$H(t+T) = H(t)$$

$s_x : 1$ and $s_y : 1$ resonances



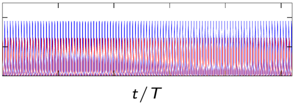
$\frac{1+\sqrt{5}}{2} \approx \frac{s_y}{s_x} = \frac{3}{2}$

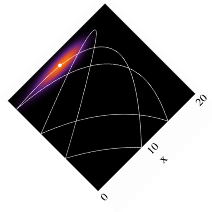
RLRLRLRLRLRLR



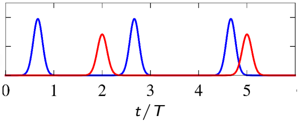
$\frac{s_y}{s_x} = \frac{13}{8}$

RLRLRLRLRLRLRLRLRL . . .

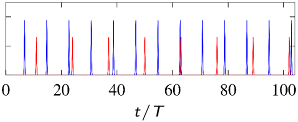




LRLLR



LRLLRLRLLR . . .



K. Giergiel, A. Kuroś, KS, "*Discrete Time Quasi-Crystals*", PRB **99**, 220303(R) (2019)

Summary and outlook:

- Condensed matter physics in the time dimension:
  - Spontaneous breaking of time translation symmetry in periodically driven systems (discrete time crystals, time quasi-crystals, fractional time crystals, DQPT)
  - Condensed matter phenomena in the time dimension (Anderson localization, many-body localization, topological time crystals, exotic interactions, time lattices with properties of 2D and 3D space crystals).
- Experiments in progress — Peter Hannaford (Melbourne).
- Novel phenomena with the help of time engineering (e.g. Anderson molecule).

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Time crystals enter the real world of condensed matter

Time crystals break time-translational symmetry rather than spatial symmetry, as ordinary crystals do. **Peter Hannaford** and **Krzysztof Sacha** look at how this exotic state could have similar applications to condensed-matter devices

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Look at a computer processor or a superconducting device and imagine what's inside – countless electrons flying between the ions that form a solid-state crystal. Now try to imagine it's all happening not in space but in the fourth dimension of time. Is it possible that condensed-matter devices and conventional electronics can enter the time dimension?

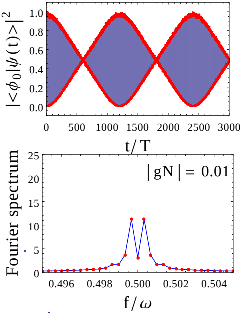
In 2012 the Nobel-prize-winning physicist Frank Wilczek published his seminal article on "quantum time crystals", in which he posed the provocative question of whether time-translational symmetry – where one instant in time is equivalent to any other – can be spontaneously broken in the lowest-energy state of a quantum-mechanical system (*Phys. Rev. Lett.* **109** 160401). Such symmetry breaking would long as the period of the driving force, to create what is known as a "discrete" time crystal (*Phys. Rev. A* **91** 033617). Classically, such "period-doubling" of a driven oscillatory system is well known. However, in the quantum world, stationary (that is, time-independent) solutions of the Schrödinger equation must follow the period of the force. If a system spontaneously chooses stationary motion with a different period, the discrete time-translational symmetry is broken. We call the symmetry discrete because not every point in time is equivalent to any other for the periodically changing force. The only points that are equivalent are those that correspond to a discrete jump in time by the period of the force.

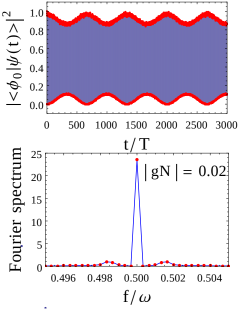
Similar ideas were later proposed that involve peri-

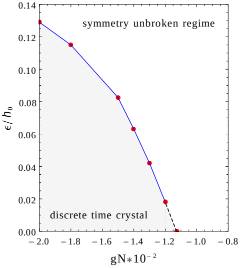
Phase diagram for discrete time crystals

$h = h_0 + \epsilon$

h_0 – turning point of the resonant orbit







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Phase transition in Anderson localization in the time domain

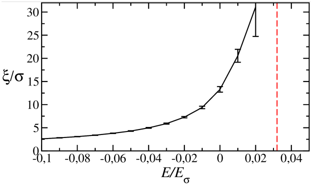
$$H = \frac{p_\theta^2 + p_\psi^2 + p_\phi^2}{2} + V_0 g(\theta) g(\psi) g(\phi) f_1(t) f_2(t) f_3(t),$$

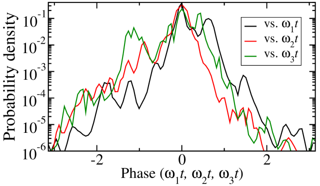
where $f_i(t + 2\pi/\omega_i) = f_i(t) = \sum_k f_k^{(i)} e^{ik\omega_i t}$.

In the moving frame, $\Theta = \theta - \omega_1 t, \ \Psi = \psi - \omega_2 t, \ \Phi = \phi - \omega_3 t$,

$$H_{\text{eff}} = \frac{P_\Theta^2 + P_\Psi^2 + P_\Phi^2}{2} + V_0 h_1(\Theta) h_2(\Psi) h_3(\Phi),$$

where $h_i(x) = \sum_k g_k f_{-k}^{(i)} e^{ikx}$ are disordered potentials.





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