TAJEMNICE AMPLITUD ROZPRASZANIA W CHROMODYNAMICE KWANTOWEJ

PIOTR KOTKO

AGH

NCN GRANT DEC-2018/31/D/ST2/02731

Seminarium WFiIS

PLAN

1. Chromodynamika Kwantowa (QCD)
2. Amplitudy rozpraszania w QCD
3. Diagramy Feynmana i ich problemy

4. Amplitudy MHV, geometryczne niespodzianki i teoria twistorów
5. Linie Wilsona i równoważne teorie Yanga-Millsa
6. Metody on-shell i amplituhedron

7. Podsumowanie

QUANTUM CHROMODYNAMICS (QCD)

QCD — Quantum Field Theory with local SU(3) gauge symmetry

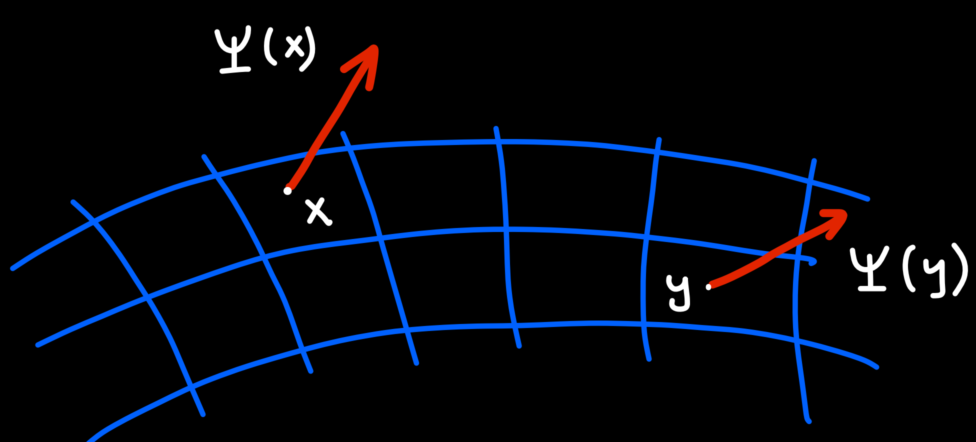
$$\Psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \end{pmatrix}$$

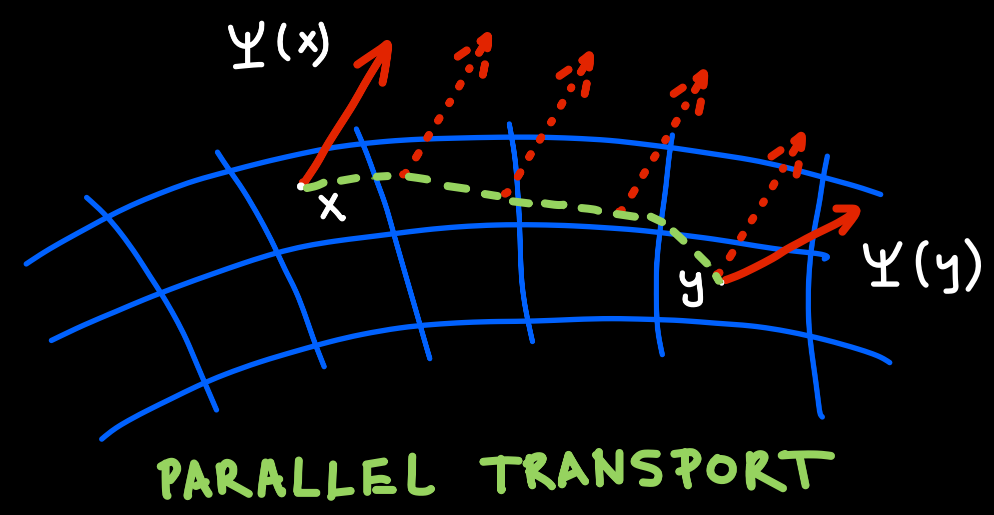
↑ QUARK FIELD

Local Gauge Symmetry: physics doesn't change when $\Psi(x)$ is rotated independently at every space-time point.

$$\Psi'(x) = U(x)\Psi(x)$$

Derivatives of fields (needed to define the evolution of fields) require subtracting fields at different points. But they transform differently…





Parallel transport is realized thanks to a connection — vector field $\hat{A}^\mu$

GLUON FIELD

QUANTUM CHROMODYNAMICS (QCD)

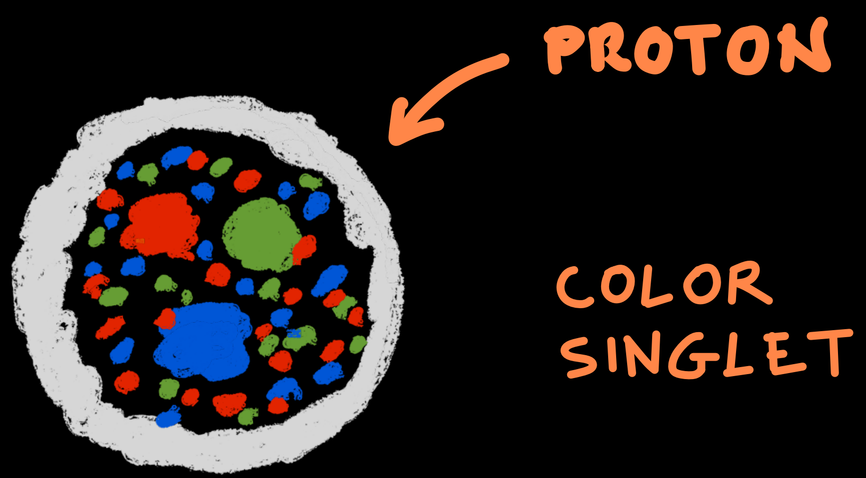
Asymptotic Freedom

Coupling constant gets weaker when the energy/resolution scale increases.

$$\alpha_s(\mu^2) = \frac{g^2(\mu^2)}{4\pi} \to 0 \quad \text{for} \quad \mu^2 \to \infty$$

Color Confinement

Free quarks and gluons are unobservable — hadrons are physical QCD states.



How do we study QCD?

Nonperturbative QCD

- Lattice QCD
- Effective models

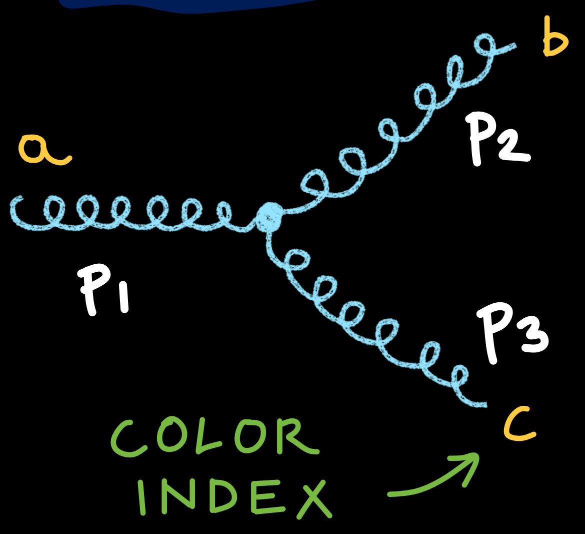
Perturbative QCD

Used when coupling const. is small, or/and a resummation to all orders can be done.

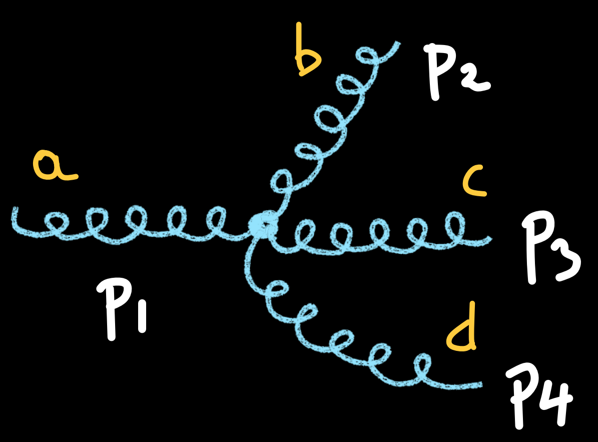
QUANTUM CHROMODYNAMICS (QCD)

Basic Interactions

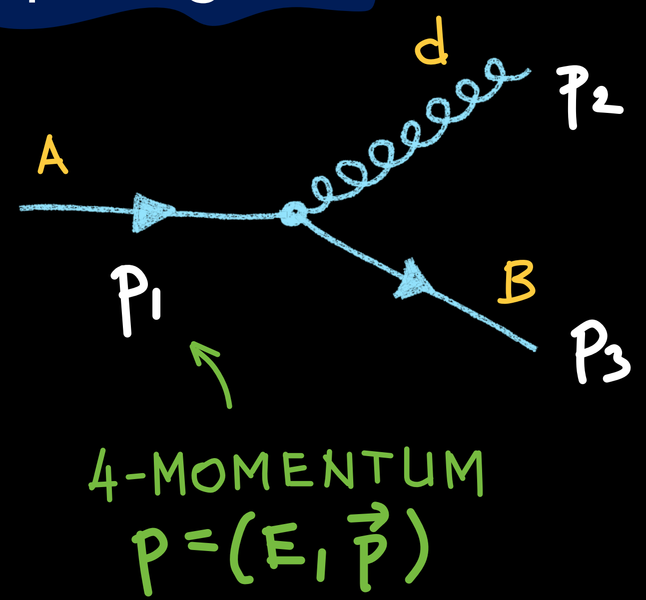
3-gluon



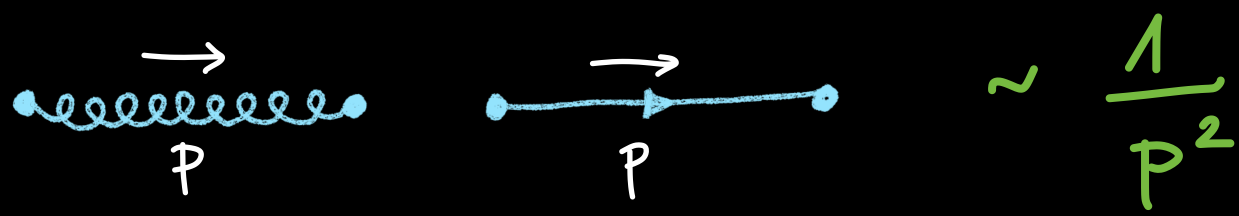
4-gluon



quark-gluon



Propagators:



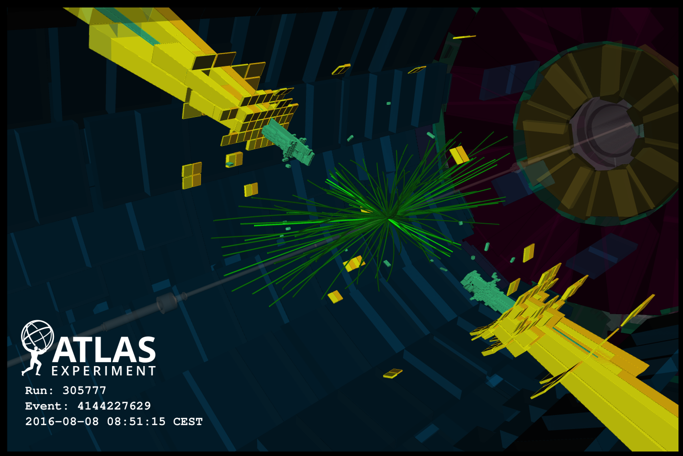
Each graphic symbol has corresponding analytic expression (Feynman rules).

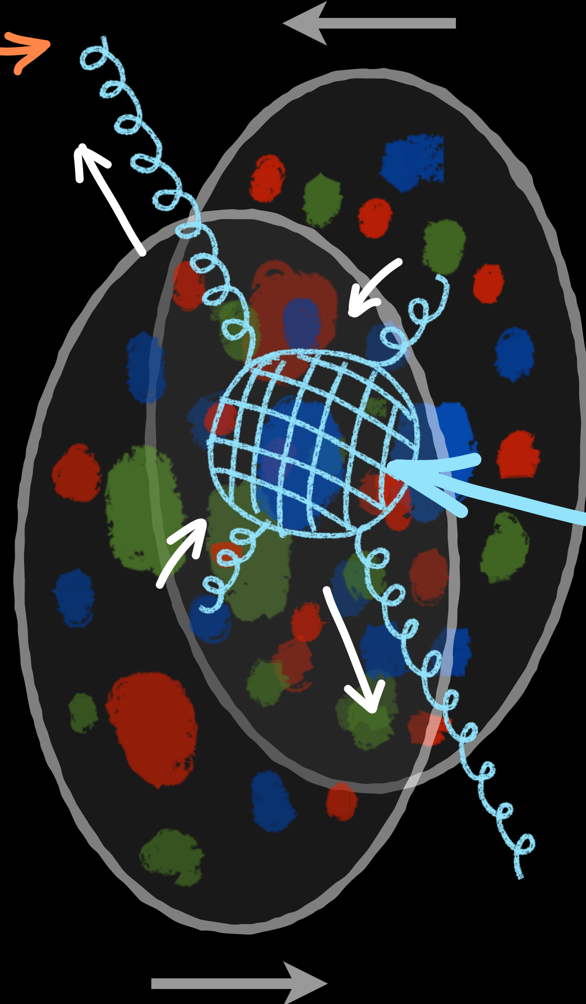
These are the building blocks of the (perturbative) theory*.

QUANTUM CHROMODYNAMICS (QCD)

High energy collisions

Proton-proton collision at Large Hadron Collider (LHC)





Eventually produce JETS due to color confinement.

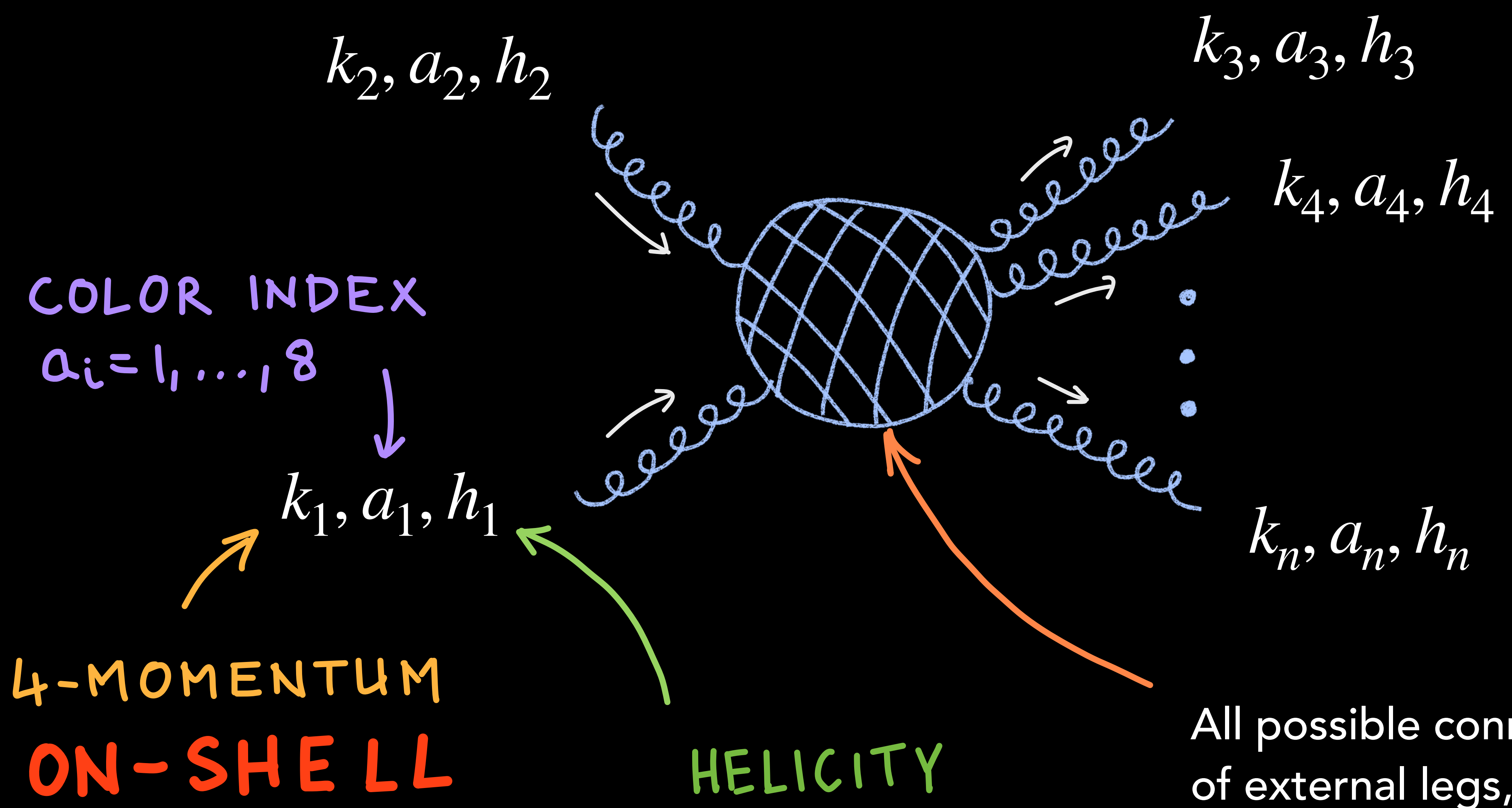
Jets provide large energy scale, so that coupling constant is small for part of the process.

Object built from basic vertices and propagators, calculable order-by-order.

QCD Factorization Theorem

ON-SHELL AMPLITUDES

Pure gluon amplitudes



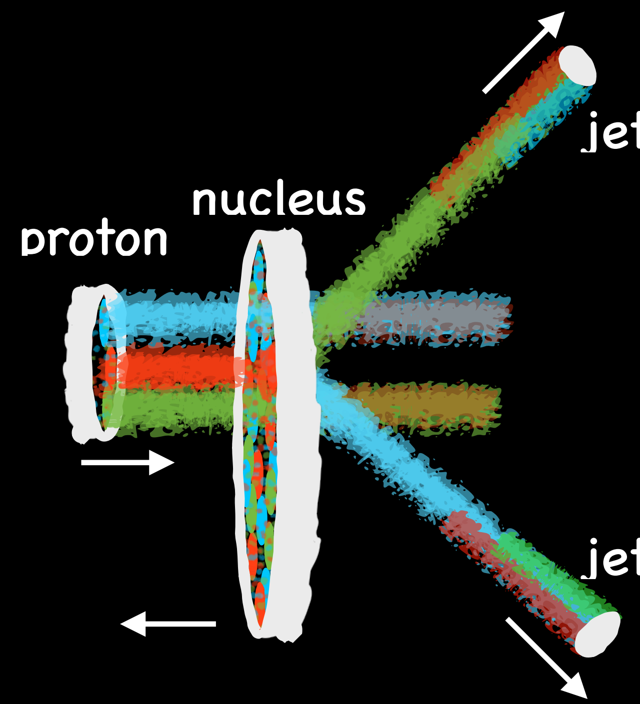
$$k_1^2 = E^2 - \vec{p}^{\,2} = 0$$

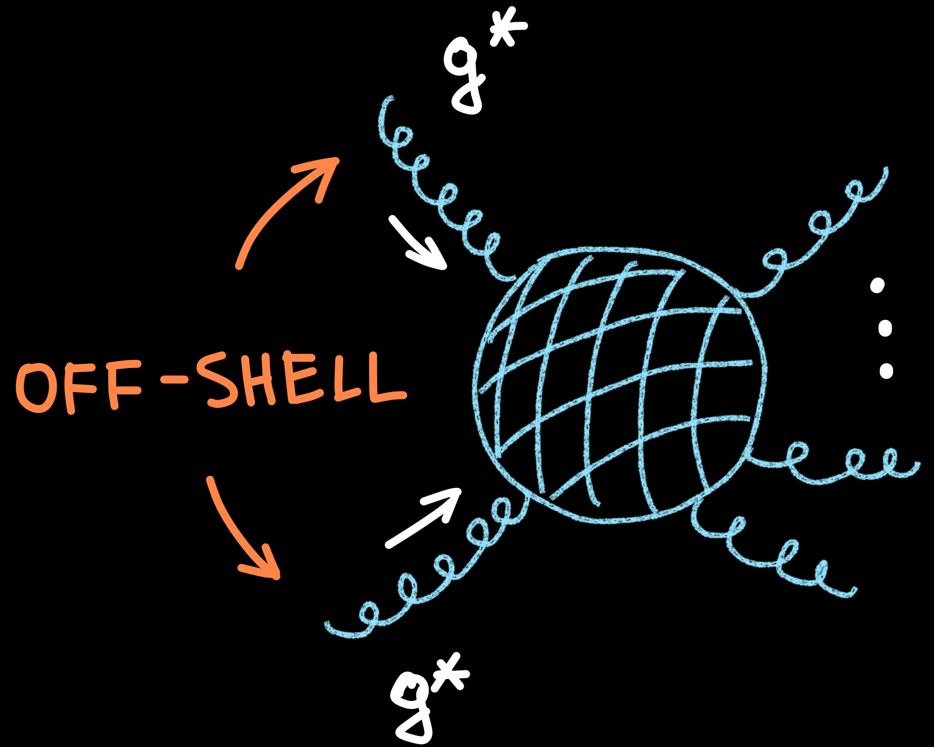
All possible connections of external legs, using interaction vertices.

OFF-SHELL AMPLITUDES

Scattering at very high energies and forward jets

Processes for which on-shell amplitudes are not adequate.





NOT GAUGE INVARIANT

We need additional contributions to restore the symmetry.

Methods of calculating:

[A. van Hameren, P. Kotko, K. Kutak, 2012]
[A. van Hameren, P. Kotko, K. Kutak, 2013]
[P. Kotko, 2014]
[E. Blanco, A. van Hameren, P. Kotko, K. Kutak, 2020]

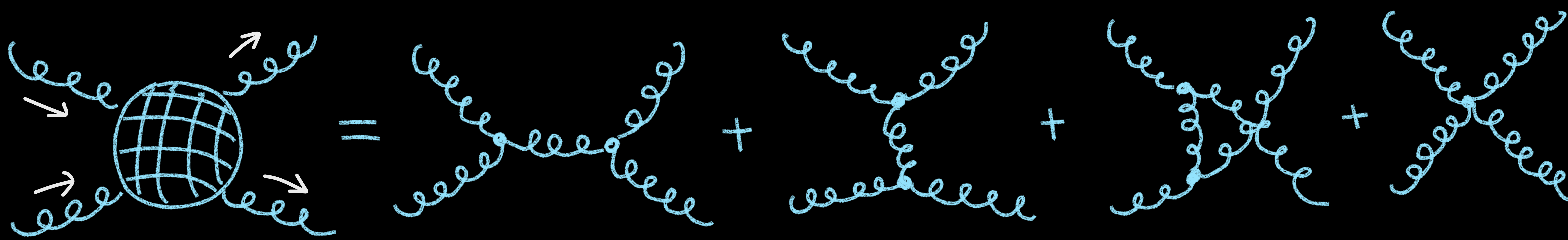
Recent theory and phenomenology:

[M. Bury, A. van Hameren, P. Kotko, K. Kutak, 2020]
[H. van Haevermat, A. van Hameren, P. Kotko, K. Kutak, P. van Mechelen, 2020]
[A. van Hameren, P. Kotko, K. Kutak, S. Sapeta, 2019]
[T. Altinoluk, R. Boussarie, P. Kotko, 2019]
[M. Bury, P. Kotko, K. Kutak, 2019]

SCATTERING AMPLITUDES

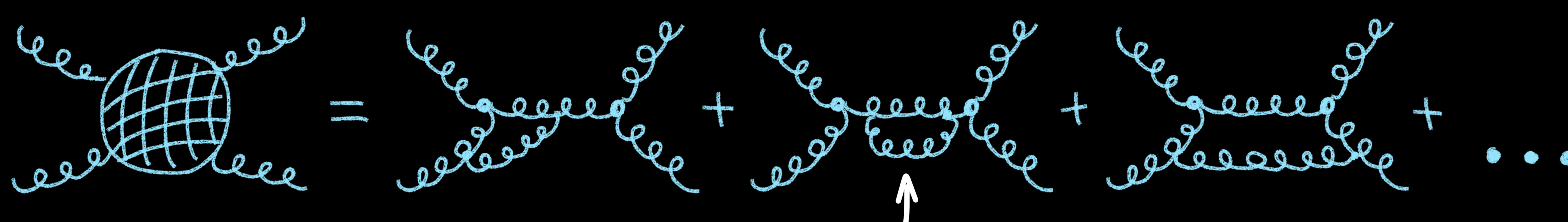
Example: 4-leg amplitude

TREE AMPLITUDE $\mathcal{O}(\alpha_s)$



4-momenta of all internal lines are fixed by the external data.

LOOP AMPLITUDE $\mathcal{O}(\alpha_s^2)$



Internal virtual 4-momenta are not fixed by external data and must be integrated over.

SCATTERING AMPLITUDES

Issue with the Feynman diagram method

$n = 5$ 25 tree diagrams

$n = 6$ 220 tree diagrams

$n = 7$ 2485 tree diagrams

$n = 8$ 34300 tree diagrams

$n = 9$ 559405 tree diagrams

$n = 10$ 10525900 tree diagrams

HUGE PROLIFERATION OF DIAGRAMS

10^7 TERMS

10^6 LOOP DIAGRAMS

($\sim 10^{10}$ TERMS)

The actual results turn out to be actually simple!

$\Longrightarrow$

- gauge invariance
- hidden geometry

Amplitudes are functions of 4-momenta and helicity (ignore color)

$$k_i = \left(E_i, \vec{k}_i\right) \equiv \left(k_i^0, k_i^1, k_i^2, k_i^3\right)$$

$$k_i^2 = 0$$

polarization vectors

$$\varepsilon_i^{h_i} \equiv \varepsilon^{h_i}(k_i, q), \quad h_i = \pm$$

REFERENCE MOMENTUM

$$\mathscr{M}\left(k_1, \varepsilon_1^{h_1}, \ldots, k_n, \varepsilon_n^{h_n}\right)$$

$$\varepsilon^{h_i}(k_i, q') = \varepsilon^{h_i}(k_i, q) + \beta\, k_i$$

Ward identity: $$\mathscr{M}\left(k_1, \varepsilon_1^{h_1}, \ldots, k_i, k_i, \ldots, k_n, \varepsilon_n^{h_n}\right) = 0$$

REPLACE $\varepsilon_i^{h_i} \to k_i$

GAUGE INVARIANCE

Spinor helicity formalism

Express all external data in terms of helicity spinors:

4-MOMENTUM

2-COMPONENT SPINORS

$$k_{\alpha\dot{\alpha}} = (k_\mu \sigma^\mu)_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$$

PAULI MATRICES

2×2 MATRIX

$$\alpha, \dot{\alpha} = 1,2$$

$$\begin{pmatrix} -k^0 + k^3 & k^1 - ik^2 \\ k^1 + ik^2 & -k^0 - k^3 \end{pmatrix}$$

$$v_+ = \begin{pmatrix} \lambda_\alpha \\ 0 \\ 0 \end{pmatrix}, \quad v_- = \begin{pmatrix} 0 \\ 0 \\ \tilde{\lambda}_{\dot{\alpha}} \end{pmatrix}$$

where $v_\pm$ are solution to Dirac equation:

$$\gamma_\mu k^\mu v_\pm(k) = 0$$

Similarly, we express the polarization vectors.

$$\Longrightarrow \quad \mathcal{M}\left(\lambda_1, \tilde{\lambda}_1, h_1, \ldots, \lambda_n, \tilde{\lambda}_n, h_n\right)$$

Color decomposition

The dependence of color quantum numbers can be made explicit

$$\mathcal{M}^{a_1,\ldots,a_n} = \sum \mathrm{Tr}(t^{a_1},\ldots,t^{a_n})\,\mathcal{A}(1,\ldots,n)$$

non-cyclic permutations

COLOR-ORDERED AMPLITUDES

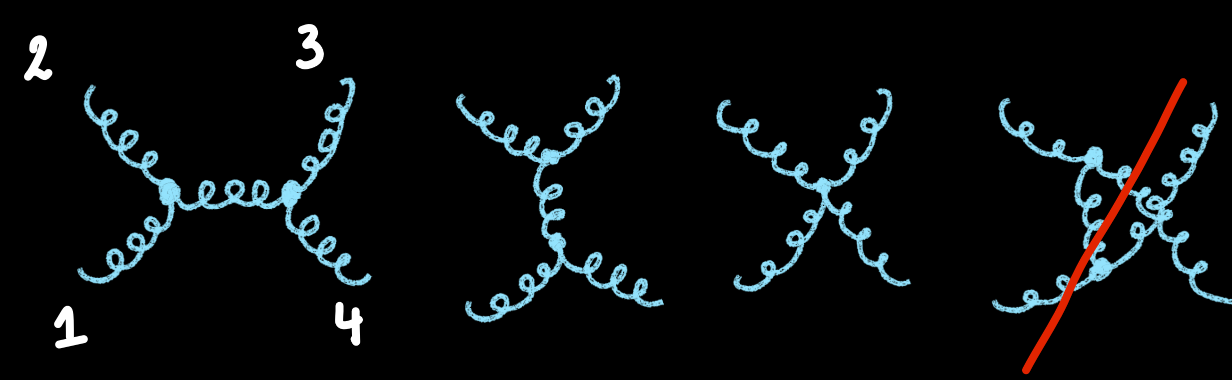
SU(3) GROUP GENERATORS

$$\mathrm{Tr}(t^a t^b) = \delta^{ab}$$

$$[t^a, t^b] = i f^{abc} t^c$$

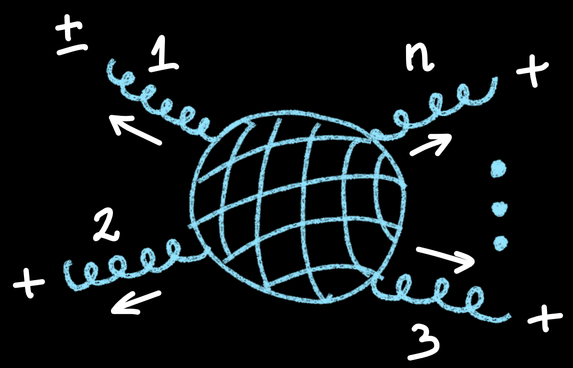
GROUP STRUCTURE CONSTANTS

Only 'planar' diagrams (no crossed lines)



Results for any number of gluons (tree level)

Two simplest (color-ordered) helicity amplitudes:



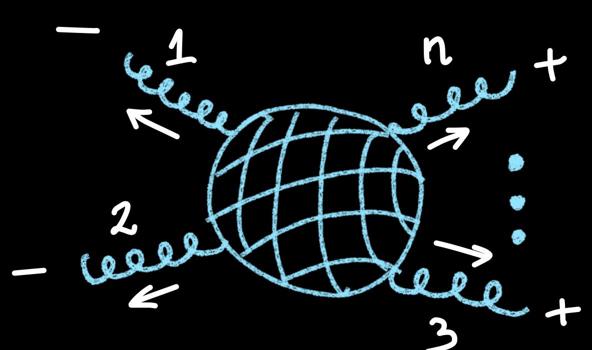
HELICITY

$$\mathscr{A}(1^+,2^+,\ldots,n^+) = 0$$

$$\mathscr{A}(1^-,2^+,\ldots,n^+) = 0$$

Maximally helicity violating (MHV) amplitudes:

[S.J. Parke, T.R Taylor, 1986]



$$\mathscr{A}(1^-,2^-,3^+,\ldots,n^+) \sim \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle (n-1)n\rangle\langle n1\rangle}$$

WHY SO SIMPLE ??

SPINOR PRODUCT

$$\langle ij\rangle = \lambda_i^\alpha \lambda_{j\alpha}$$

Hidden geometry

In Minkowski space it is hard to see that MHV amplitudes are special…

MHV amplitude in Twistor space

[E. Witten, 2004]

MINKOWSKI SPACE

$$\frac{\delta^{2\times 2}(\sum_i \lambda_i \tilde{\lambda}_i)\,\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle (n-1)n\rangle\langle n1\rangle}$$

MOMENTUM CONSERVATION

AMPLITUDE IS NON-LOCAL

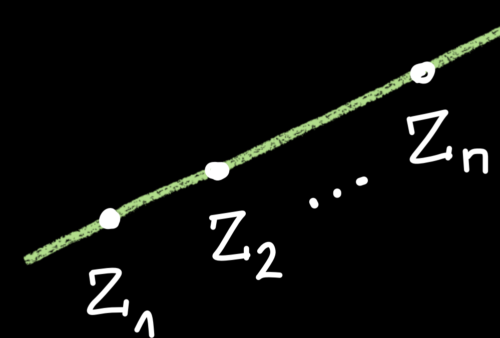
TWISTOR SPACE

$$\mathbb{A}(Z_1, Z_2, \ldots, Z_n)$$

TWISTOR VARIABLES

COMPLEX

$$Z_i = \left(Z_i^0, Z_i^1, Z_i^2, Z_i^3\right)$$

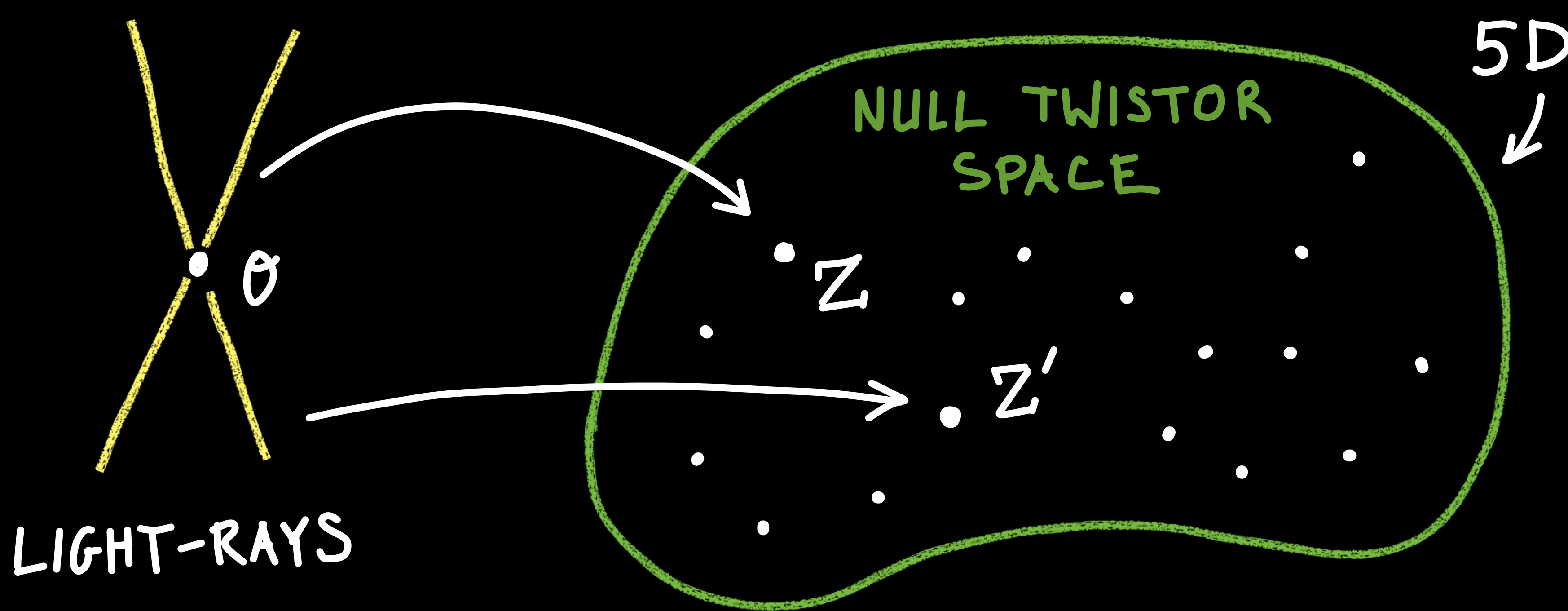


AMPLITUDE LIVES ON A LINE

Basic idea

[R. Penrose, 1967]

One can imagine Twistor space as a space of light-rays



Full Twistor space is a complex projective space $\mathbb{CP}^3$ (6D).

DUALISM:

MINKOWSKI		TWISTOR
LINES	⟷	POINTS
POINTS	⟷	LINES

projective nature: $Z \sim tZ$

$$Z \equiv \left(1, Z^1, Z^2, Z^3\right)$$

Secret of MHV amplitudes

MHV amplitudes live on a line in Twistor space, so they should behave like a local object (a vertex) in Minkowski space!

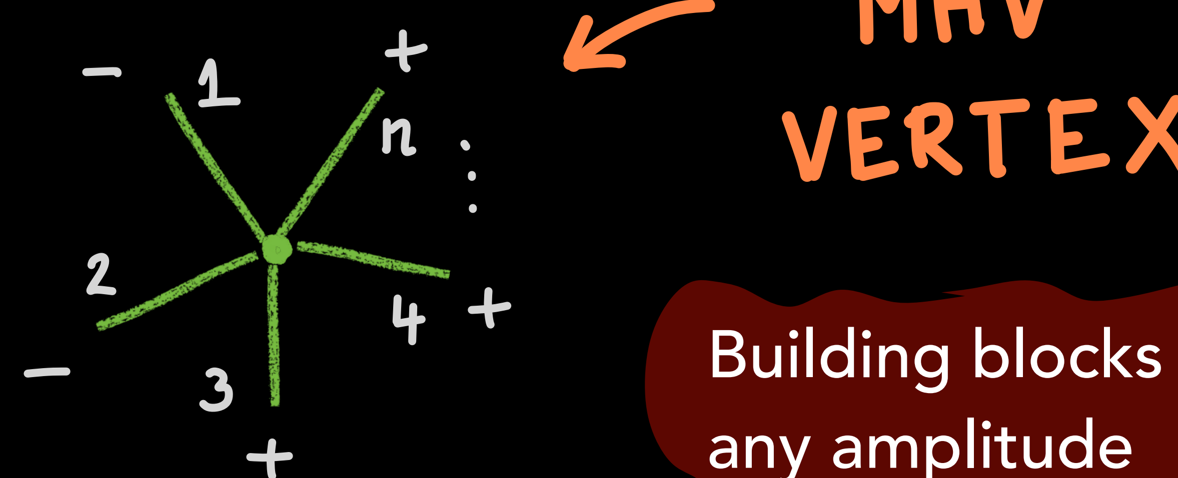
Cachazo-Svrcek-Witten (CSW) discovery

[F. Cachazo, P. Svrcek, E. Witten, 2004]

MHV amplitudes can be treated as local interaction vertices

$$\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle (n-1)n\rangle\langle n1\rangle}$$

with suitable off-shell continuation of spinor products $\langle ij\rangle$

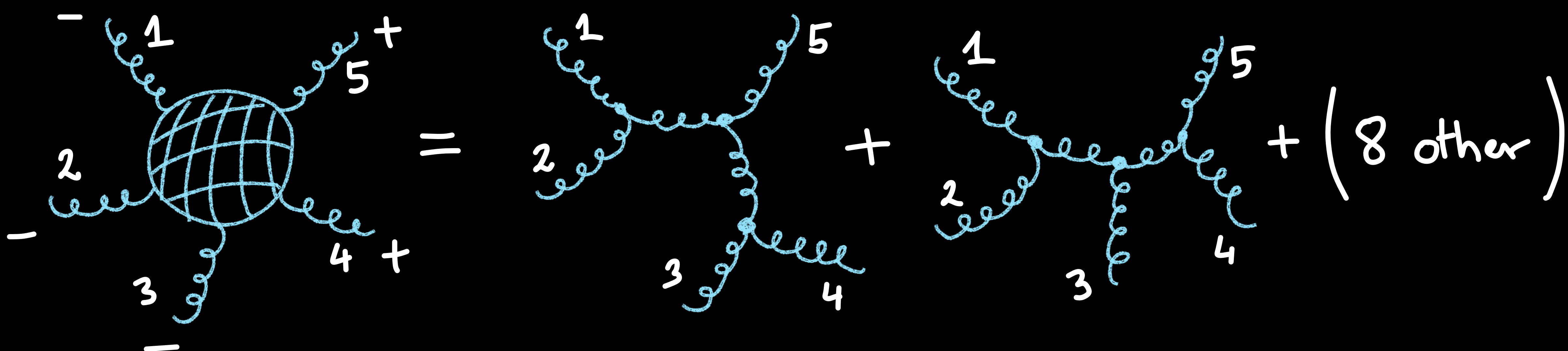


Building blocks for any amplitude

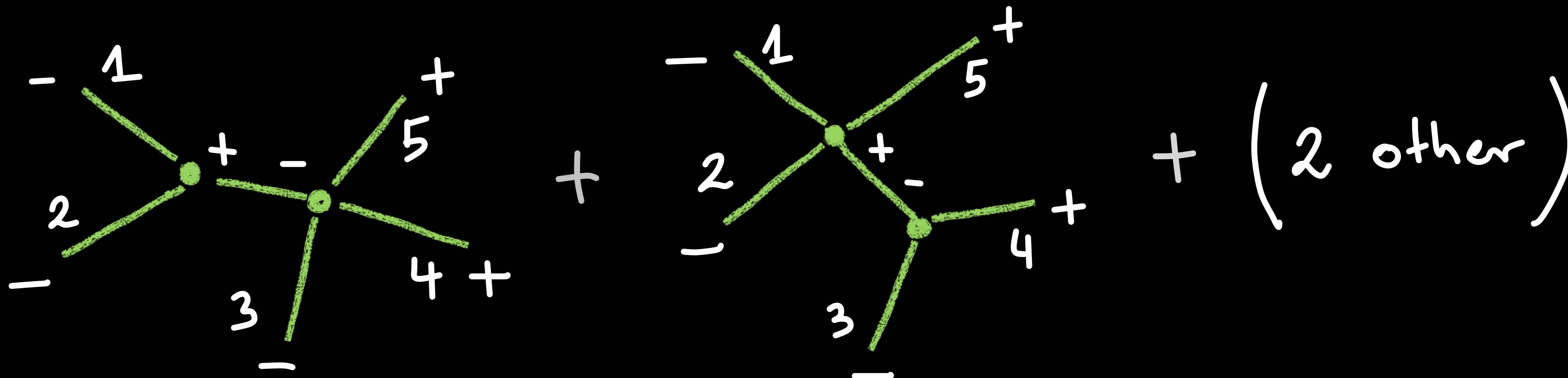
CSW METHOD

Example: $\mathscr{A}(1^-,2^-,3^-,4^+,5^+)$

Feynman diagram method:



CSW method:



What is the QFT of the MHV vertices in Minkowski space?

YANG-MILLS ACTION → $S_{\mathrm{Y-M}}[A_+, A_-]$ ← POSITIVE AND NEGATIVE HELICITY GLUON FIELDS

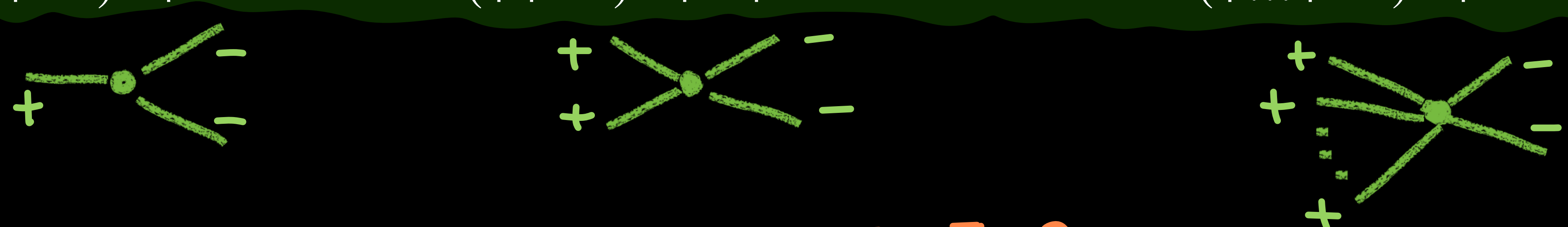
interaction part: $V_{(++-)}A_+A_+A_- + V_{(+--)}A_+A_-A_- + V_{(++--)}A_+A_+A_-A_-$

There exists transformation of fields that gives equivalent theory with MHV vertices.

$\{A_+, A_-\} \rightarrow \{B_+, B_-\}$

$S_{\mathrm{MHV}}[B_+, B_-]$

$$\mathscr{V}_{(+--)}B_+B_-B_- + \mathscr{V}_{(++--)}B_+B_+B_-B_- + \ldots + \mathscr{V}_{(+\ldots+--)}B_+\ldots B_+B_-B_-$$



BUT WHAT ARE $B_\pm[A_\pm]$?

[P. Mansfield, 2006]

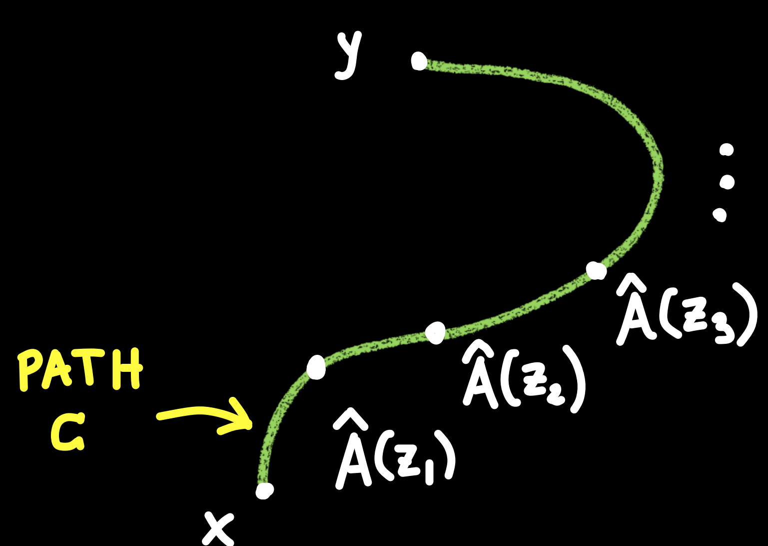
Parallel-transporter

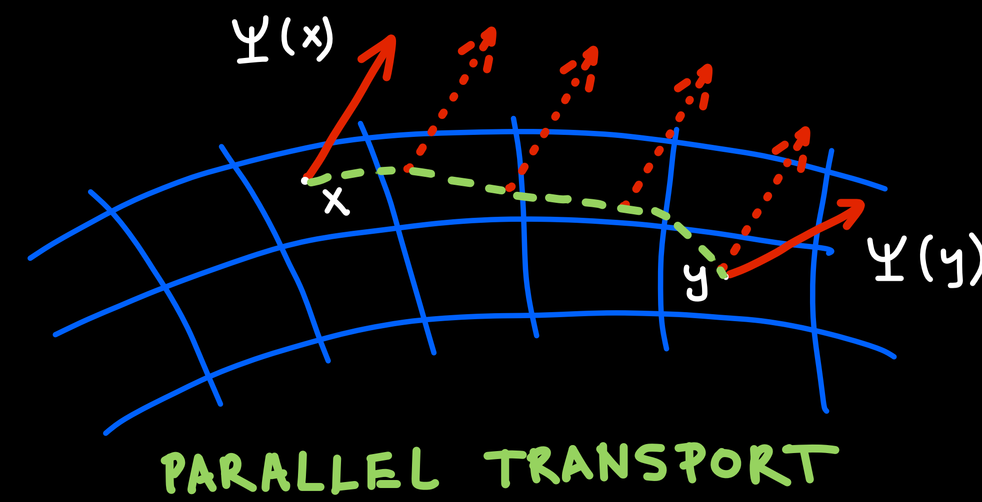
Parallel transport realized thanks to
parallel-transporter
(or gauge link, or Wilson line)

$$\mathscr{W}_C[A](x,y) = \mathbb{P}\exp\left\{ ig\int_C \hat{A}^\mu(z)\, dz_\mu \right\}$$

PATH ORDERING

GLUON FIELD
$\hat{A} = A_a t^a$





Wilson lines can be considered even more basic than gluon fields.

It is possible to expresses the Yang-Mills theory in terms of Wilson loops alone.

$$\to \sum_\gamma \mathrm{Tr}\, \mathscr{W}_\gamma$$

Wilson lines & MHV action

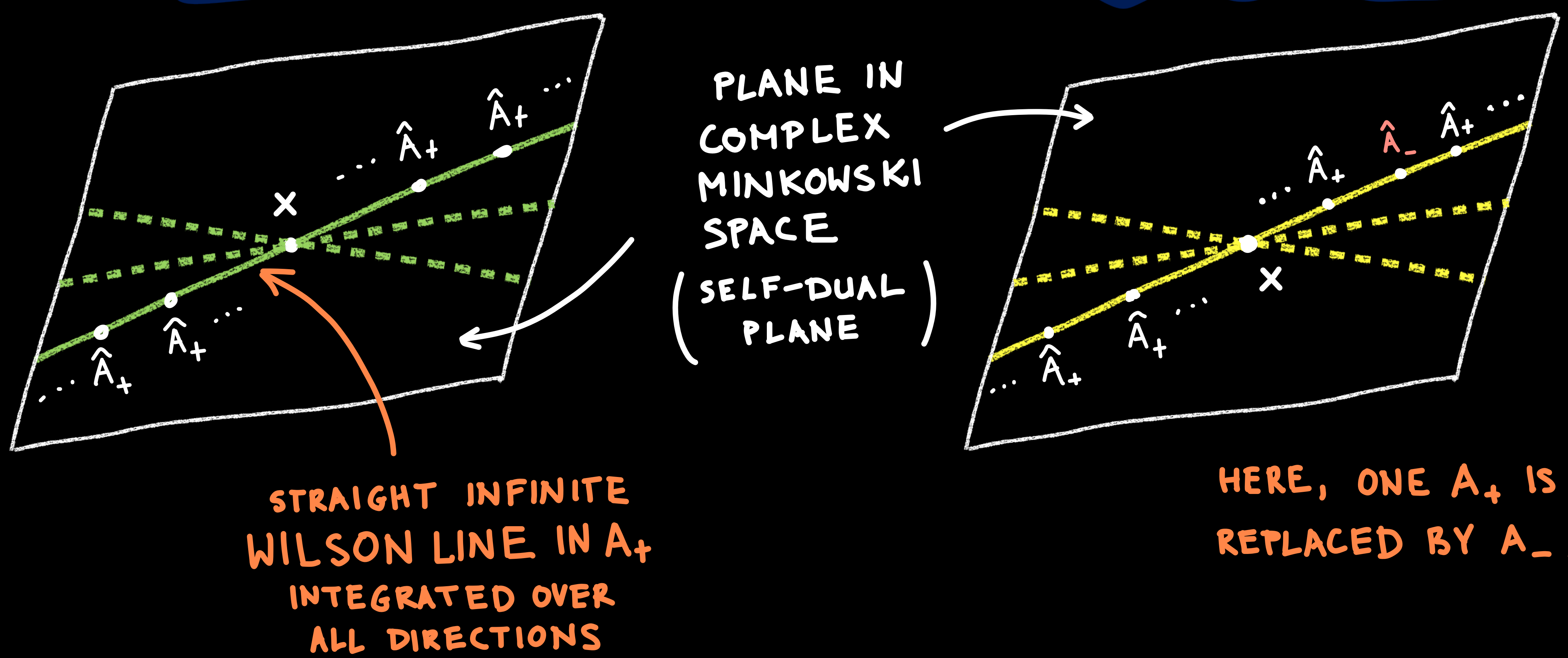
The field transformations $\{A_+, A_-\} \rightarrow \{B_+, B_-\}$ have geometric interpretation too…

[P. Kotko, 2014] [P. Kotko, A. Stasto, 2017]

[H. Kakkad, P. Kotko, A. Stasto, 2020]

$$B_+ = B_+[A_+]$$

$$B_- = B_-[A_+, A_-]$$



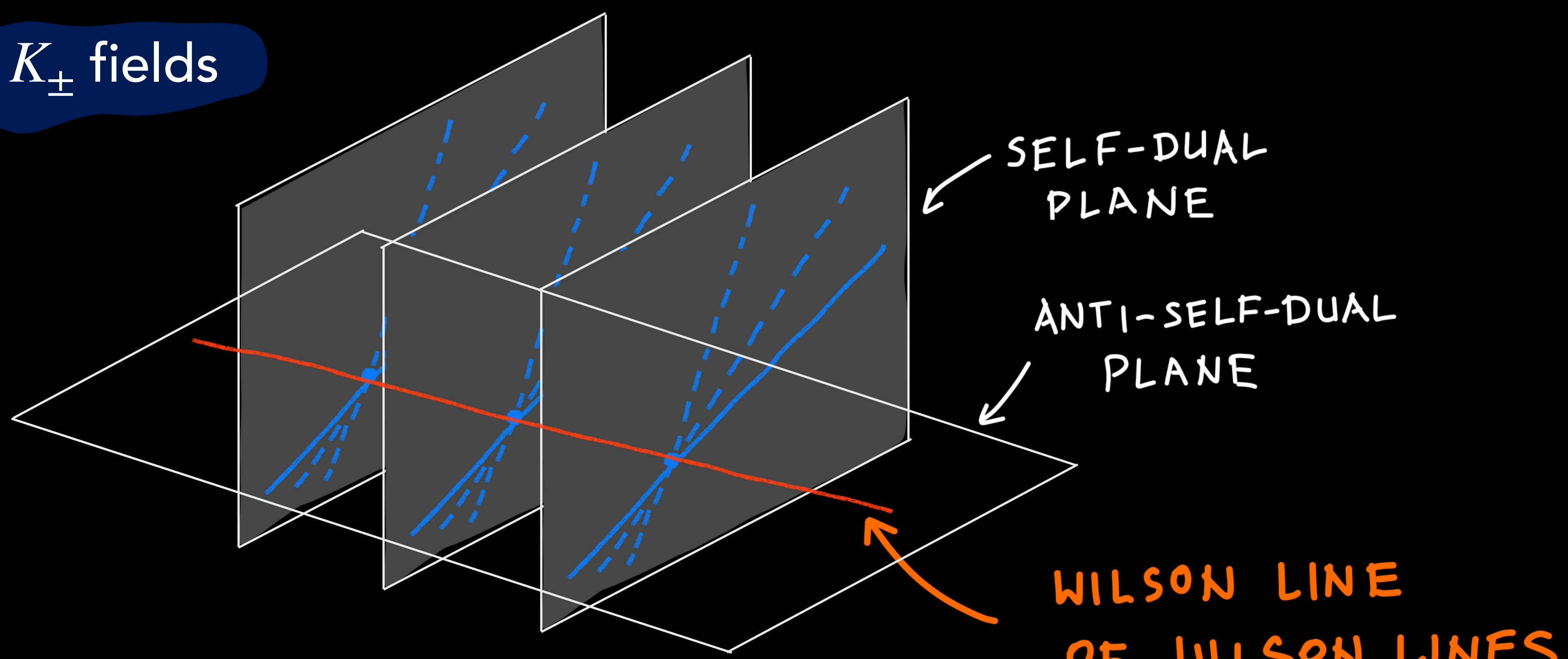
We can find even better Lagrangian then the MHV…

There exists a transformation of fields $\{A_+, A_-\} \to \{K_+, K_-\}$ that gives the Yang-Mills Lagrangian with vertices starting from 4-point vertex, and containing vertices beyond the MHV.

Amplitudes can be calculated using fewer vertices then within the MHV theory…

[H. Kakkad, P. Kotko, A. Stasto, in preparation]

Structure of $K_\pm$ fields

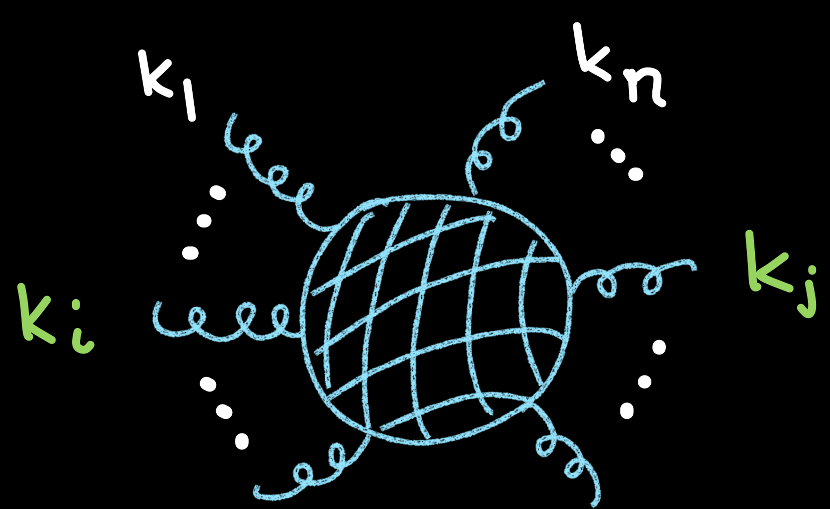


Britto-Cachazo-Feng-Witten (BCFW) recursion

[R. Britto, F. Cachazo, B. Feng, E. Witten, 2005]

CSW method allows to construct any (tree) amplitude from MHV ones.

Natural question: can any amplitude be a ‘building block’?



COMPLEX DEFORMATION OF MOMENTA

$$k_i^\mu \to k_i^\mu + z\varepsilon^\mu, \quad k_j^\mu \to k_j^\mu - z\varepsilon^\mu$$

$z \in \mathbb{C}$

$$k_1 + k_2 + \ldots + k_n = 0$$

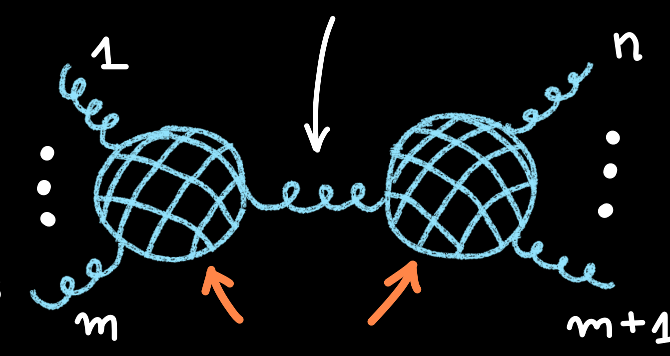
$$k_i^2 = k_j^2 = 0$$

$$f(z) = \frac{1}{z}\mathscr{A}(z)$$

ONLY SIMPLE POLES (GIVEN BY PROPAGATORS)

RESIDUE THEOREM

$$\mathscr{A}(0) = \sum_{\text{partitions}}$$



SHIFTED PROPAGATOR (AT THE RESIDUE)

ON-SHELL

ON-SHELL DIAGRAMS

New geometric formulation of QFT

[N. Arkani-Hamed, J. Bourjaily, F. Cachazo, A. Goncharov, A. Postnikov, J. Trnka, 2012]

No unphysical virtual particles
No fields
No Lagrangian

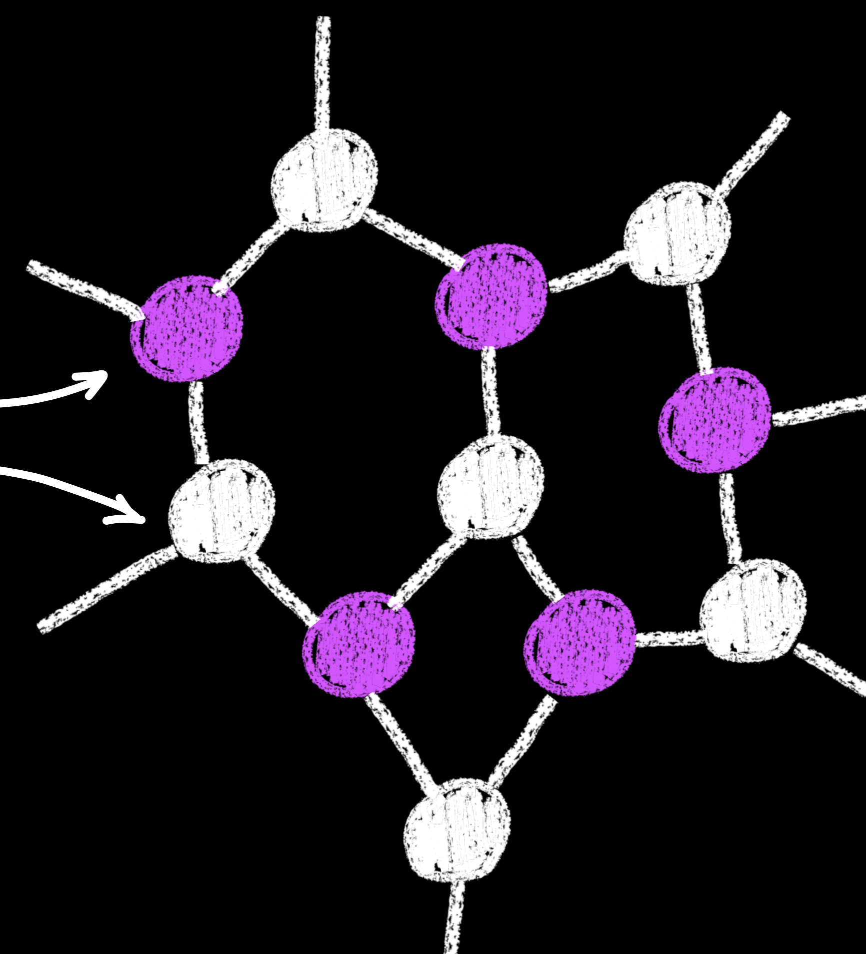
(mostly developed for Maximally Supersymmetric Yang-Mills theory)

ONLY ON-SHELL DIAGRAMS

On-shell diagrams live on Grassmannians

ALL k-PLANES IN n-DIMENSIONS

THREE-POINT ON-SHELL DIAGRAMS



Calculating amplitudes is reduced to calculating a volume of Amplituhedron

GENERALIZATION OF POSITIVE GRASSMANNIAN

PODSUMOWANIE

Amplitudy rozpraszania stanowią kluczowy element teorii

- Jest to „zawartość" teorii.
- Są konieczne do precyzyjnych testów teorii w zderzaczach.
- Struktura amplitud odzwierciedla „głęboką" strukturę teorii.

Krótka historia amplitud rozpraszania on-shell:

- Diagramy Feynmana są intuicyjne, ale nie efektywne. Współczesne obliczenia nie są wykonywane przy ich użyciu.
- Niespodziewana prostota amplitud MHV i związek z przestrzenią twistorów.
- Transformacje pól gluonowych i linie Wilsona, jako sposób na równoważne ale bardziej efektywne teorie oddziaływań kolorowych.
- Czysto geometryczny opis amplitud rozpraszania.
- State of the art: 2 pętlowa amplituda z 5 gluonami o takich samych helicity używając metod on-shell.