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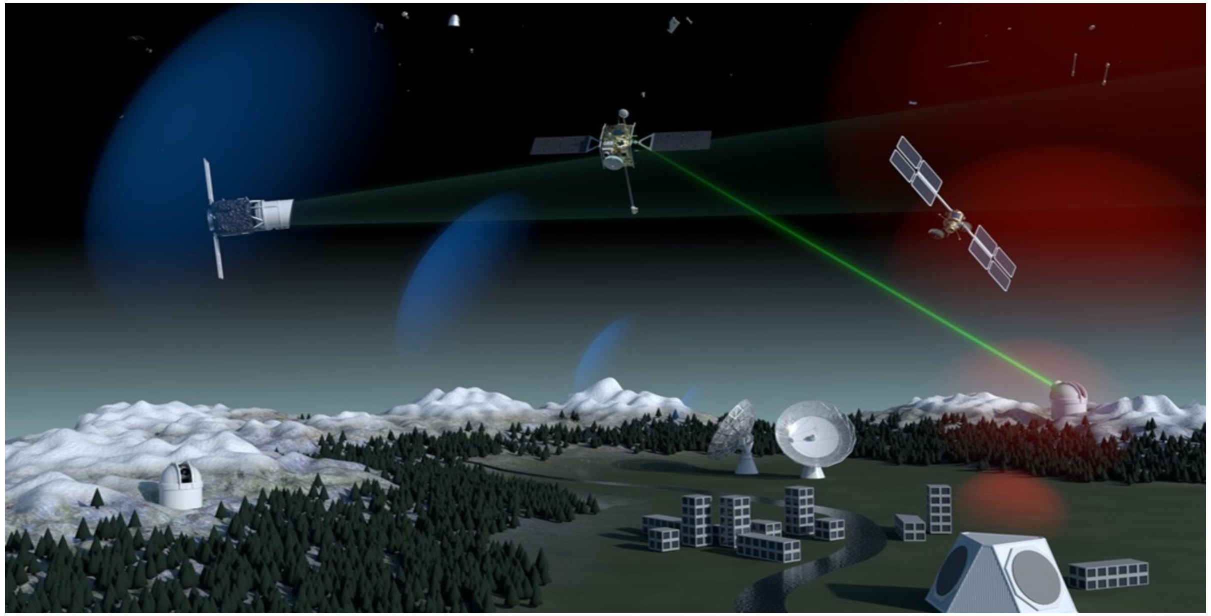
Seminar WFiIS/CTK
20.01.2023

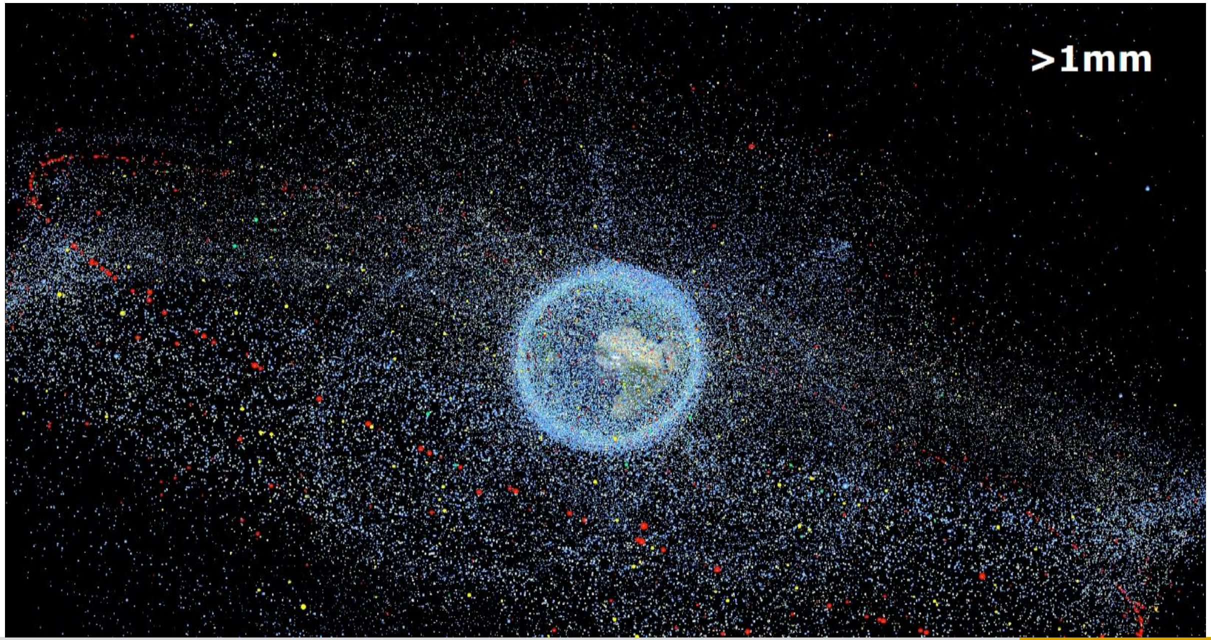
Space Surveillance and Tracking – SST

watching for:
***active and inactive satellites**

***discarded launch stages and**

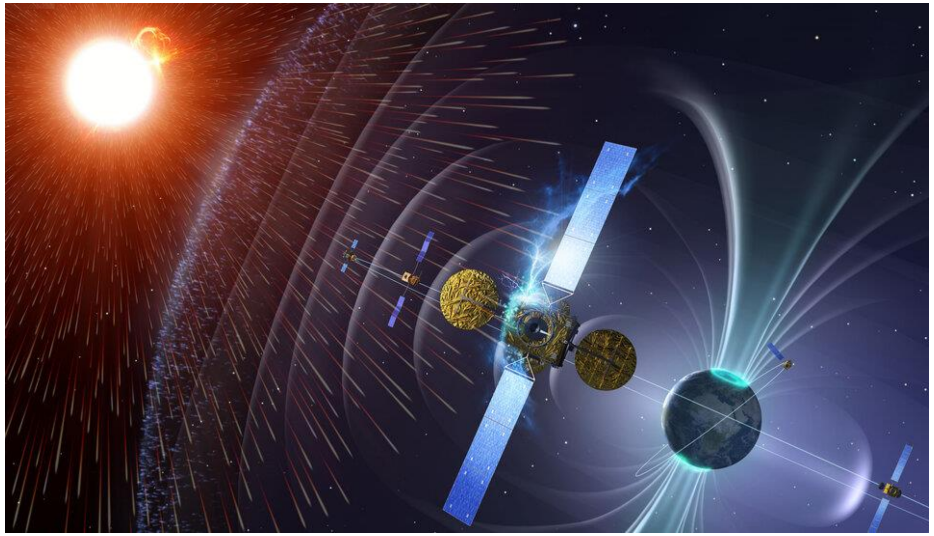
***fragmentation debris orbiting Earth**





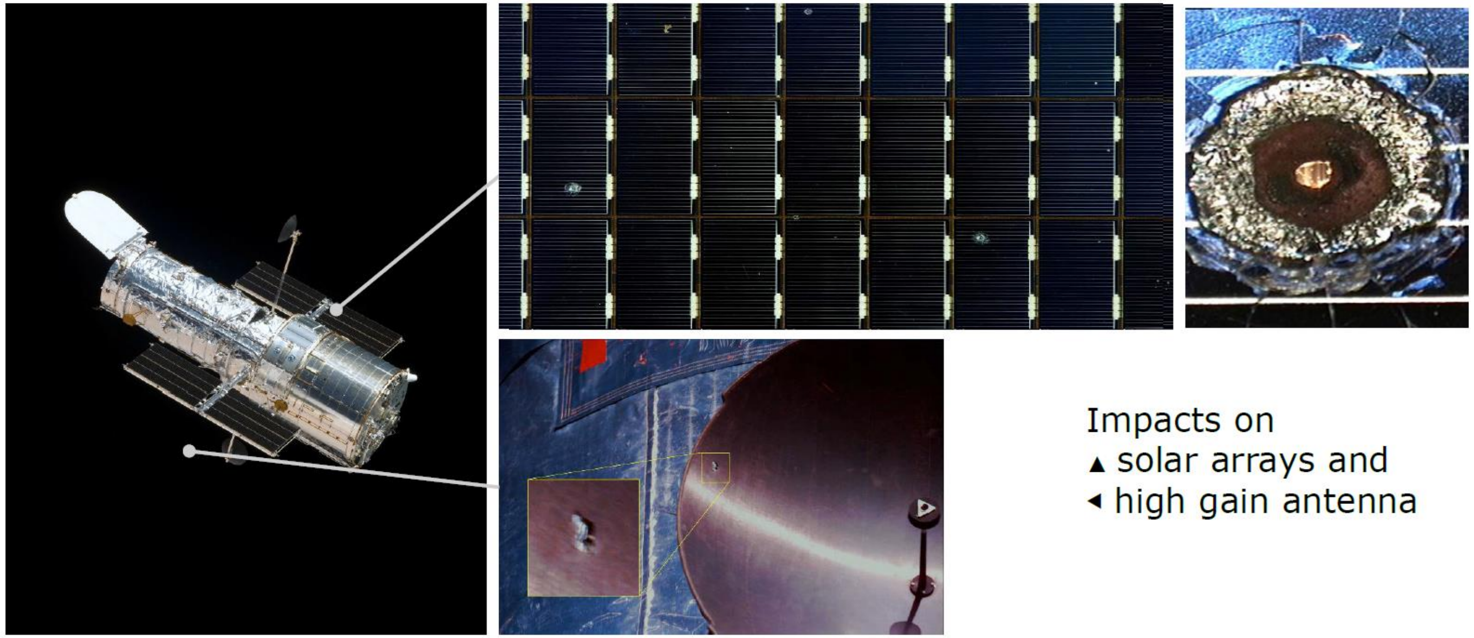
SST is a part of Space Situational Awareness **SSA**:

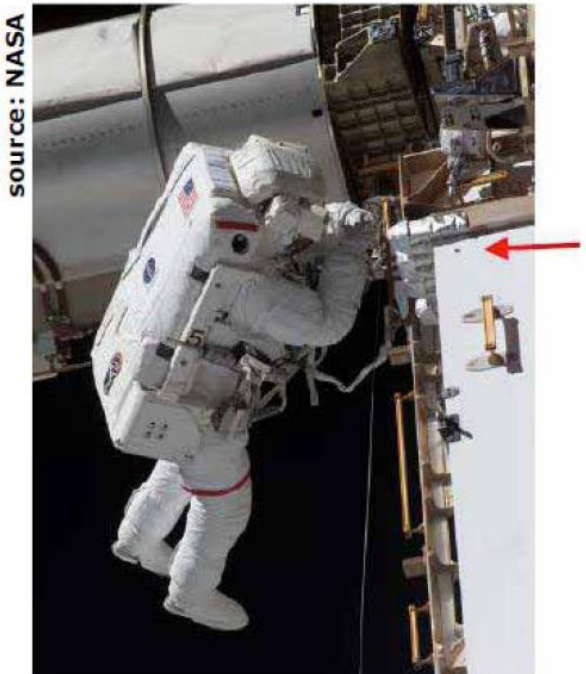
- **Near-Earth Objects (NEO)**: detecting natural objects such as asteroids that can potentially impact Earth and cause damage
- **Space Weather** (**SWE**): monitoring and predicting the state of the Sun and the interplanetary and planetary environments, including Earth's magnetosphere, ionosphere and thermosphere, which can affect spaceborne and ground-based infrastructure thereby endangering human health and safety

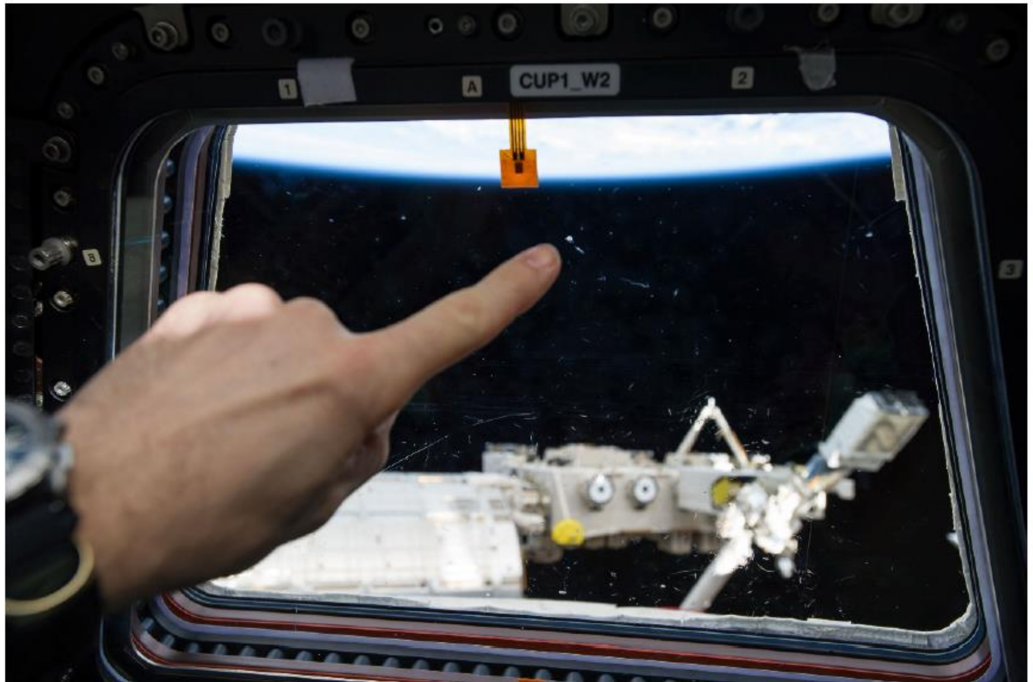


Hubble Space Telescope

Image credits: NASA







more than 30,000 space debris >10 cm,
more than 900 000 space debris >1 cm

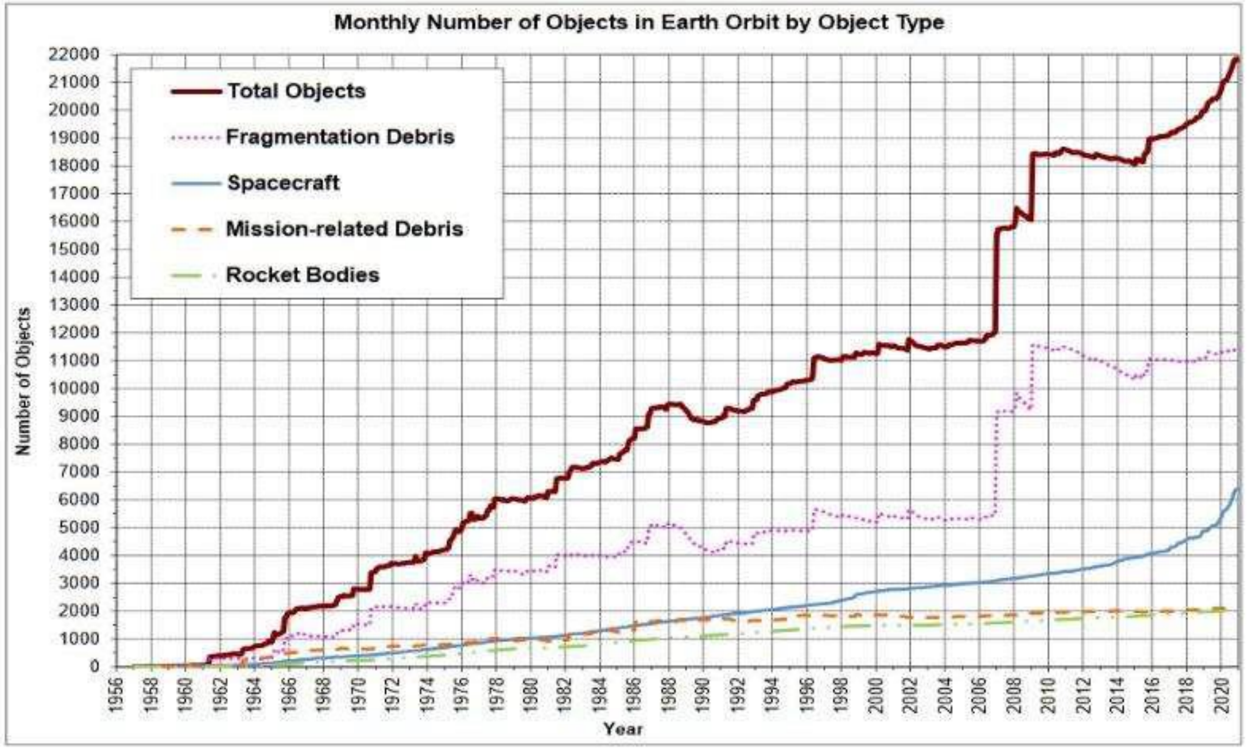
currently, more than 4,000 active satellites orbit the Earth; a number more than doubled since 2015

with the increasing number of space objects, increased risk of collisions, which often lead to the formation of new space debris

2007: Old Fengyun1C satellite is destroyed by a missile (ASAT) => Debris cloud of 2 000 cataloged objects

Contents

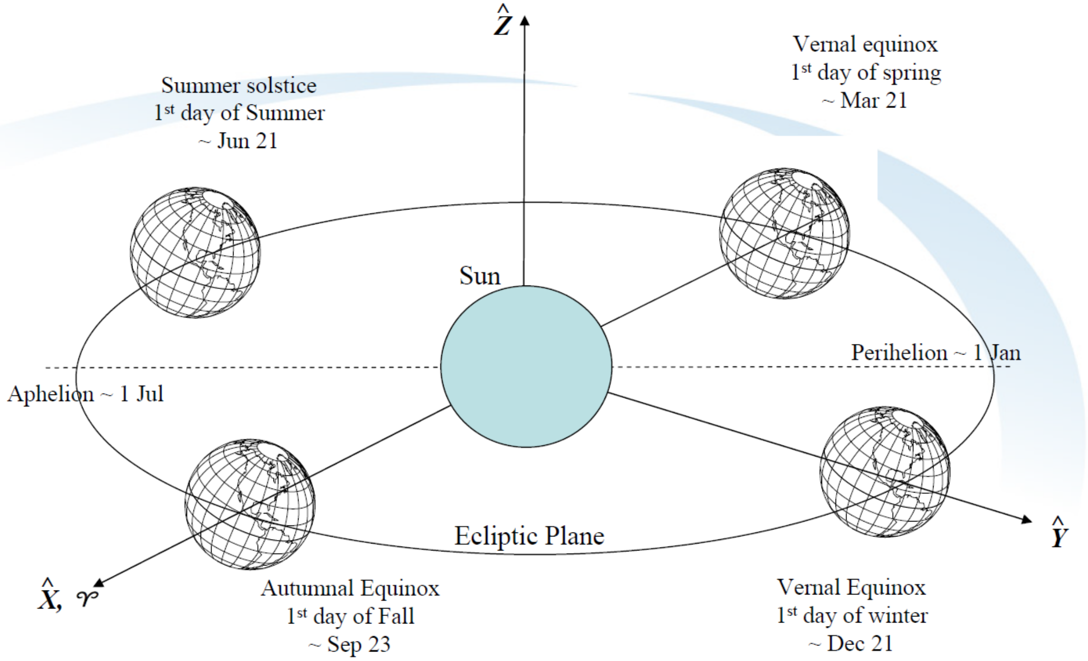
- Ideal Motion
- Non - Ideal Motion
- Orbit determination
- Observation techniques
- Data Analysis



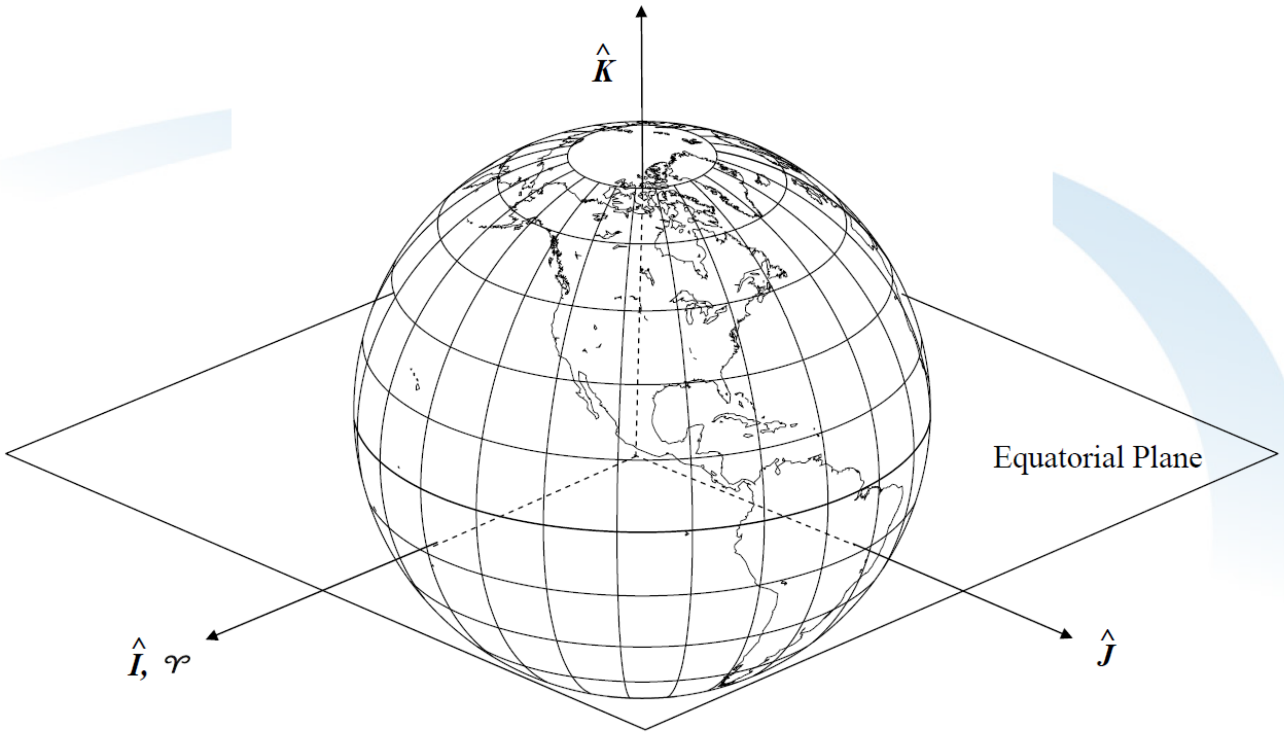
DESCRIPTION OF THE MOVEMENT OF THE EARTH'S ARTIFICIAL SATELLITES

ASTRODYNAMICS

Inertial Reference System: have origin at the Earth's center. The axes are not rotating with the Earth, but instead they are fixed respect to the fixed stars. The most commonly used inertial reference systems are J2000 and ICRF (International Celestial Reference Frame). **For orbit determination purposes we only refer to the inertial systems.**



Heliocentric

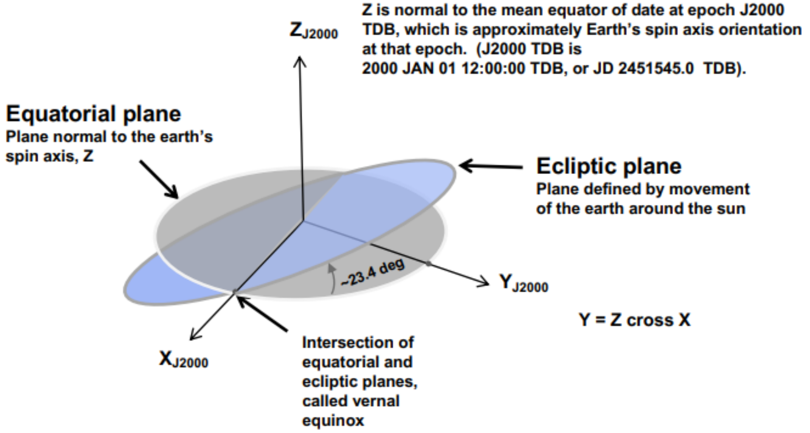


Geocentric

J2000 Frame

Navigation and Ancillary Information Facility

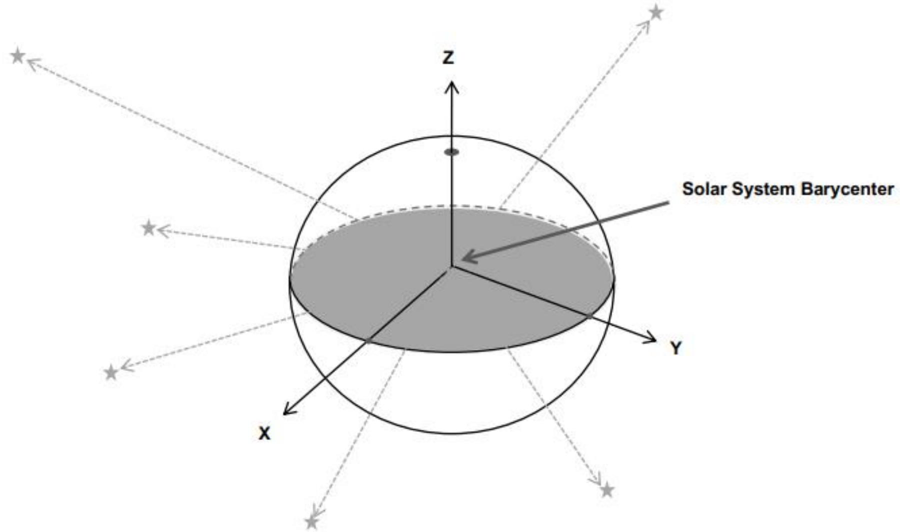
- **The J2000* (aka EME2000) frame definition is based on the earth's equator and equinox, determined from observations of planetary motions, plus other data.**



The ICRF Frame

Navigation and Ancillary Information Facility

- **The ICRF* frame is defined by the adopted locations of 295 extragalactic radio sources**



*ICRF = International Celestial Reference Frame

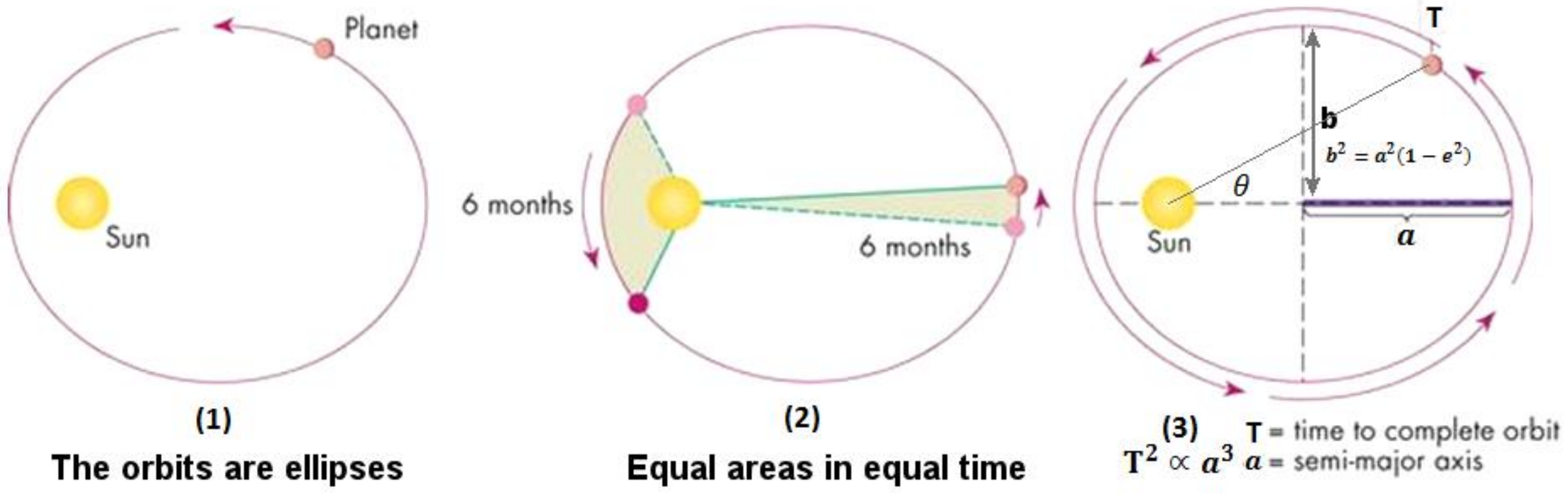
- Even though elaborate models have been developed to compute the motion of artificial Earth satellites to the high level of accuracy required for many applications today, **the main features of their orbits may still be described by a reasonably simple approximation**.

  This is due to the fact that the force resulting from the Earth's central mass outniles all other forces acting on the satellite by several orders of magnitude, **in much the same way as the attraction of the Sun governs the motion of the planets.**

  The laws of planetary motion, which were found empirically by Kepler about 400 years ago, may, therefore, equally well be applied to a satellite's orbit around the Earth.

Kepler Laws

- The orbit of a planet is an ellipse with the Sun at one focus
- The line joining a planet and the Sun sweeps out equal areas during equal intervals of time.
- The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.



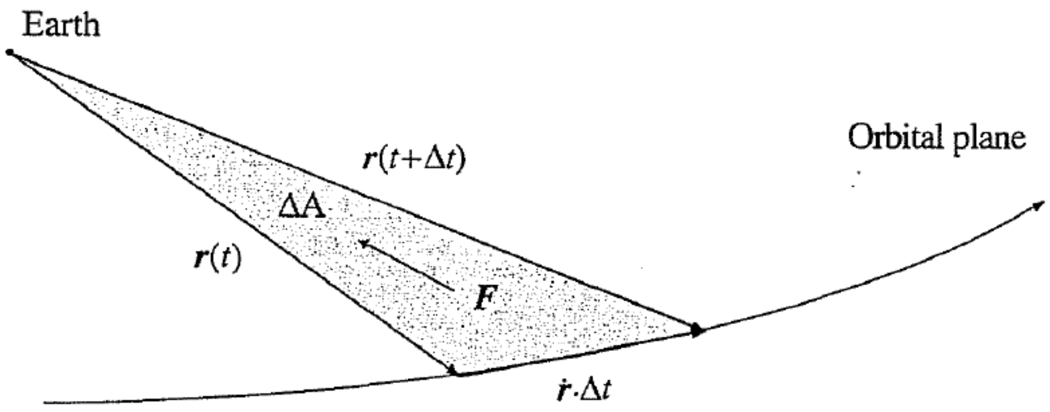
A zero-sum approximation of the problem of the motion of Earth's satellites is the issue of the two bodies with negligible mass of the satellite.

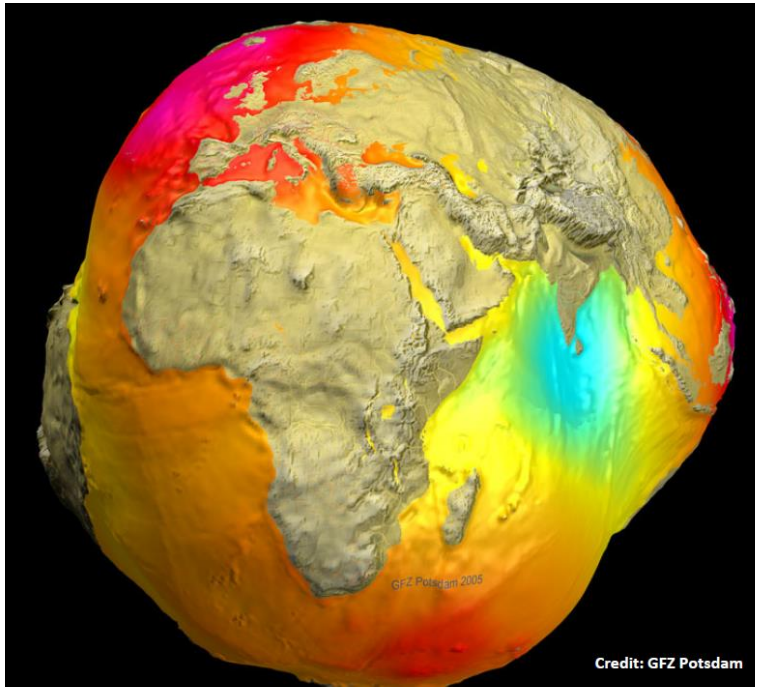
The equations of motion of a satellite in a geocentric system with a fixed axis orientation then have the form of a:

$$\ddot{\bar{r}} = -\frac{\mu}{r^3}\bar{r} \qquad \mu = G\, m_{\oplus} = 3.9860044 \times 10^{14}\, \mathrm{m}^3\mathrm{s}^{-2}$$

The Kepler approximation means that we treat the Earth as a spherical body with a radius [Earth's equatorial radius]: $a_{\oplus} = 6.37813766 \times 10^{6}\, \mathrm{m}.$

The satellite cannot leave the orbital plane, since the force is always anti-parallel to the position vector and, therefore, does not give rise to any acceleration perpendicular to the plane [$\vec{F} = \frac{\vec{r}}{r} F$ *the central force dose not alter the plane of the satellite's orbit*].





$$\vec{F} = -\,\mathrm{grad}\;V(r) \qquad V = \int_{r_0}^{r} \frac{a}{r^2}\,dr + V(r_0) = -\frac{a}{r} + \mathrm{const}$$

Non-spherical Earth –Geopotential

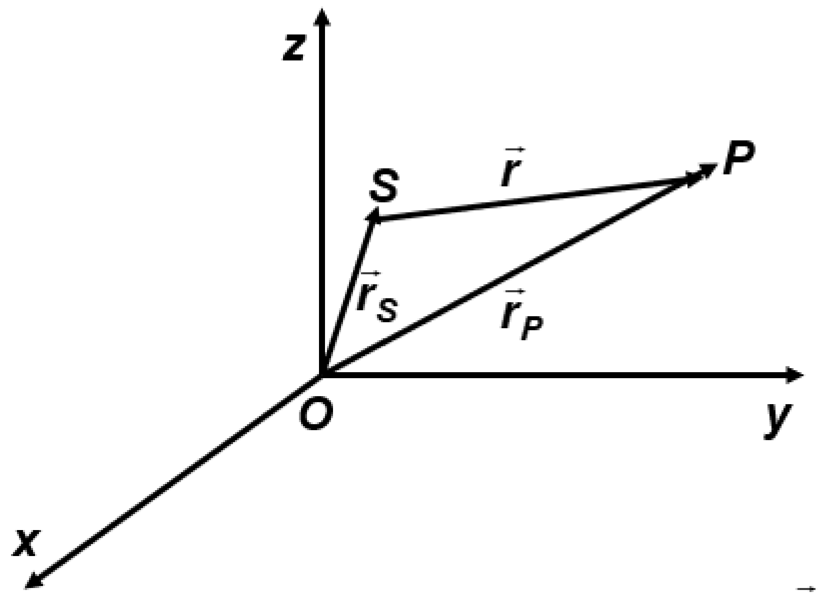
$$U = \frac{\mu}{r} \cdot \left(1 + \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left(\frac{R_{\oplus}}{r}\right)^{l} \cdot P_{l,m}(\sin\phi_g) \cdot \left(C_{l,m} \cdot \cos(m\cdot\lambda) + S_{l,m} \cdot \sin(m\cdot\lambda)\right) \right)$$

Ideal Motion: The Two Bodies Problem

$$\vec{r} = \vec{r}_P - \vec{r}_S$$
$$r = |\vec{r}|$$

$$\boldsymbol{F} = m \cdot \ddot{\boldsymbol{r}} = -G \cdot \frac{M \cdot m}{r^2} \cdot \frac{\boldsymbol{r}}{|r|}$$

$$\ddot{\boldsymbol{r}}_{2-body} = \nabla U_{2-body} = -\frac{\mu}{r^2} \cdot \frac{\boldsymbol{r}}{|r|}$$



equation of motion of the sun $$M\ddot{\vec{r}}_S = G\frac{Mm}{r^2}\frac{\vec{r}_P - \vec{r}_S}{r}$$

equation of motion of the planet $$M\ddot{\vec{r}}_P = -G\frac{Mm}{r^2}\frac{\vec{r}_P - \vec{r}_S}{r}$$

M –Sun's mass
m – planet's mass

$$\mu \frac{d^2\vec{r}}{dt^2} = -\frac{\vec{r}}{r}\frac{\alpha}{r^2}$$

- differential equation describing the relative motion of a planet [satellite] with respect to the Sun [Earth]

$$\vec{v}\,\mu\frac{d\vec{v}}{dt} = -\dot{\vec{r}}\,\frac{\vec{r}}{r}\frac{\alpha}{r^2} \quad \Rightarrow \quad \frac{\mu v^2}{2} - \frac{\alpha}{r} = \text{const}$$

- integral of the energy of two material points in motion relative to the center of mass

$$\mathcal{J}^{(S)} = \mu r^2 \dot{\varphi} = \text{const.}$$

- integral of angular momentum of a system of two material points in motion relative to the center-of-mass

$$v^2 = v_r^2 + v_\varphi^2 = \dot{r}^2 + r^2\dot{\varphi}^2 = \frac{\mathcal{J}^{(S)2}}{\mu^2}\left[\left(\frac{d}{d\varphi}\frac{1}{r}\right)^2 + \frac{1}{r^2}\right] \quad \Rightarrow \quad E^{(S)} = \frac{\mathcal{J}^{(S)2}}{2\mu}\left[\left(\frac{d}{d\varphi}\frac{1}{r}\right)^2 + \frac{1}{r^2}\right] - \frac{\alpha}{r} = \text{const.}$$

From the integral of Energy, we get **the track** of a material point [planet, satellite]: $r = r(\varphi)$

From the integral of angular momentum, the **time dependence** of relative motion: $\varphi = \varphi(t)$.

$$\frac{1}{r} = w = \frac{\alpha\mu}{\mathfrak{J}^{(S)^2}} + h\cos(\varphi - \gamma).$$

$$r = \frac{p}{1 + \varepsilon\cos(\varphi - \gamma)}$$

- trajectory of a material point [satellite, planet] in relative motion in a polar system with origin at a material point [Sun, Earth] in the plane of relative motion

$$p = \frac{\mathfrak{J}^{(S)^2}}{\alpha\mu}, \qquad \varepsilon = h\frac{\mathfrak{J}^{(S)^2}}{\alpha\mu}.$$

$$h = \frac{\alpha\mu}{\mathfrak{J}^{(S)^2}}\sqrt{1 + E^{(S)}\frac{2\mathfrak{J}^{(S)^2}}{\alpha^2\mu}}; \qquad \varepsilon = \sqrt{1 + E^{(S)}\frac{2\mathfrak{J}^{(S)^2}}{\alpha^2\mu}}.$$

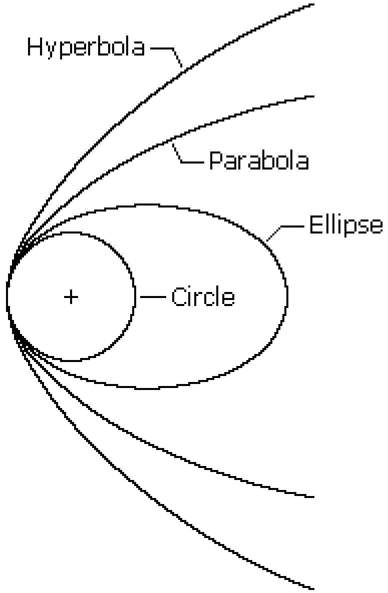
$E^{(S)} > 0,$		$\varepsilon > 1,$	track is a hyperbola
$E^{(S)} = 0,$	=>	$\varepsilon = 1,$	… is an ellipse
$E^{(S)} < 0,$		$\varepsilon < 1,$	… is a parabola

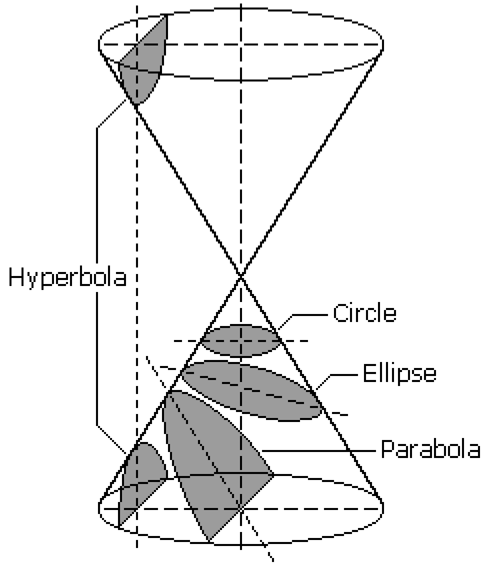
Ideal Motion (first Kepler's law): resulting orbits

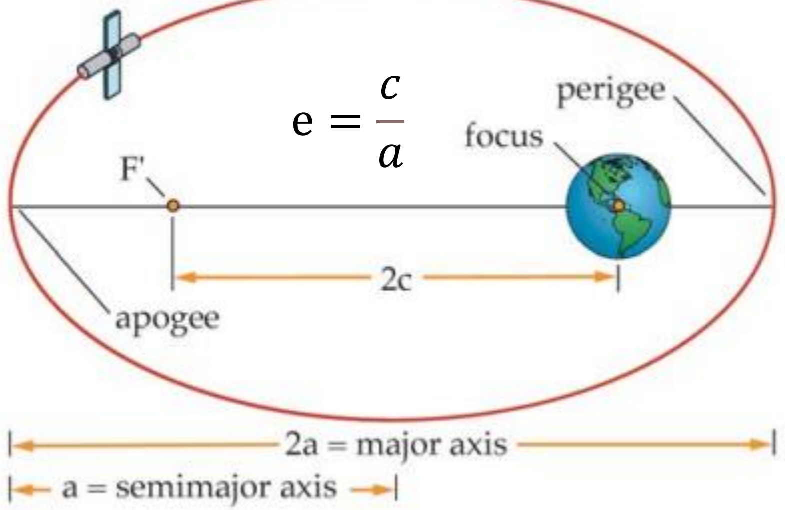
- solution of the equation of motion in polar coordinates is called Conic Equation:

$$r = \frac{p}{1 + e\cos(\vartheta)}$$

- object during motion assumes the shape of a cone, depending on the eccentricity and focus on the material point S (F')



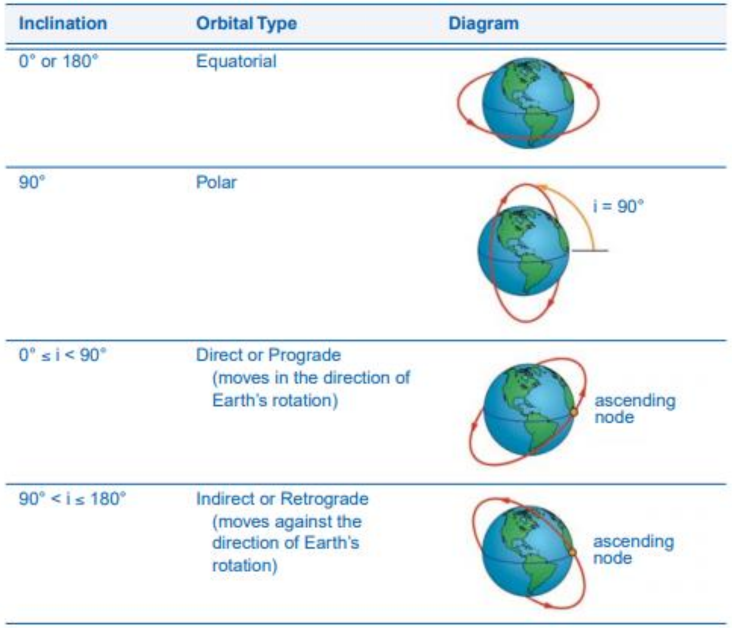


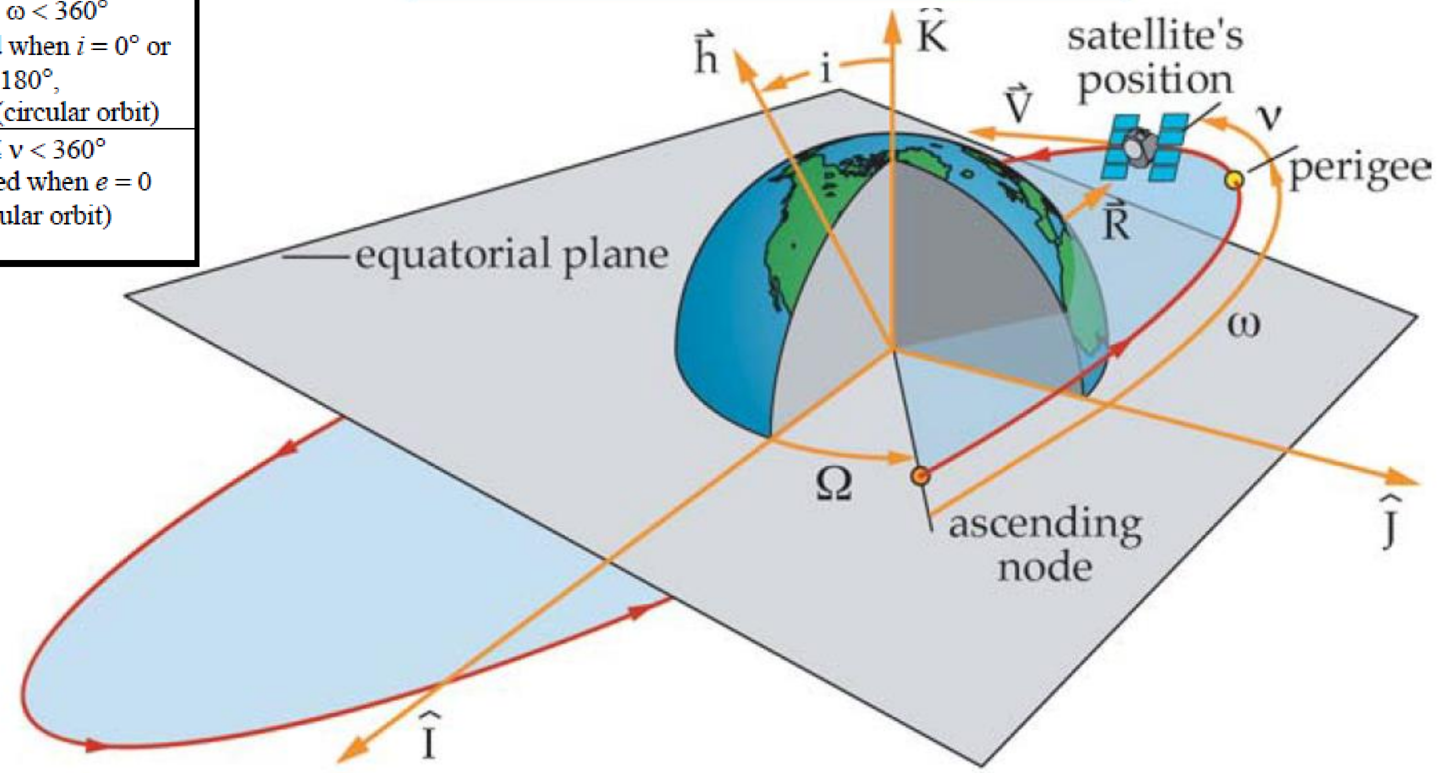


semi-major axis a gives information about the size of the orbit.

Classical Orbital Elements – Position Along the Orbit

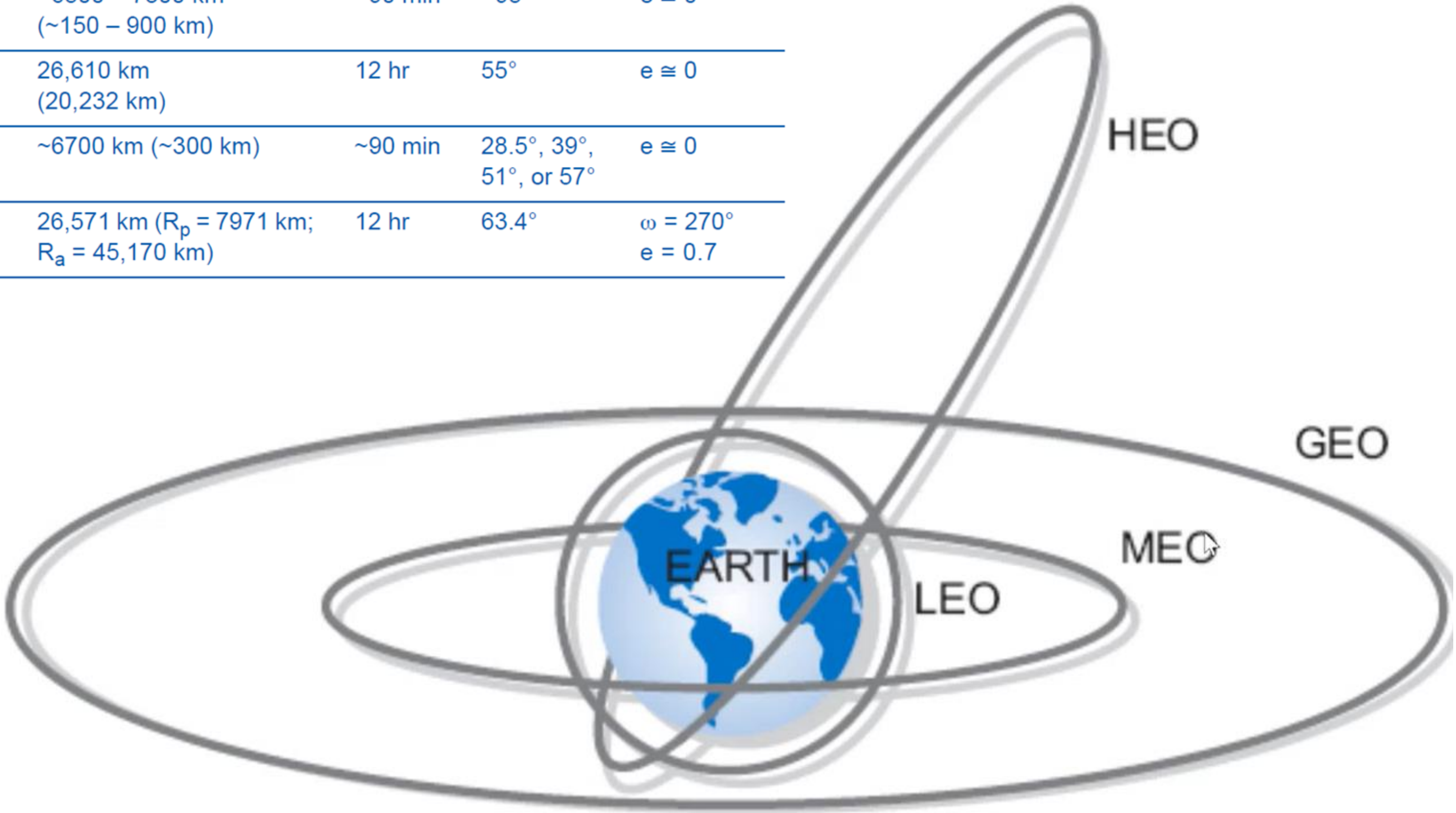
Element	Name	Description	Definition	Remarks
a	semi-major axis	orbit *size*	half of the long axis of the ellipse	orbital period and energy depend on orbit size
e	eccentricity	orbit *shape*	ratio of half the foci separation (c) to the semi-major axis	closed orbits: $0 \le e < 1$ open orbits: $1 \le e$
i	inclination	orbital plane's *tilt*	angle between the orbital plane and equatorial plane, measured counterclockwise at the ascending node	equatorial: $i = 0°$ or $180°$ prograde: $0° \le i < 90°$ polar: $i = 90°$ retrograde: $90° < i \le 180°$
Ω	right ascension of the ascending node	orbital plane's *rotation* about the Earth	angle, measured eastward, from the vernal equinox to the ascending node	$0° \le \Omega < 360°$ undefined when $i = 0°$ or $180°$ (equatorial orbit)
ω	argument of perigee	orbit's *orientation* in the orbital plane	angle, measured in the direction of satellite motion, from the ascending node to perigee	$0° \le \omega < 360°$ undefined when $i = 0°$ or $180°$, or $e = 0$ (circular orbit)
ν	true anomaly	satellite's *location* in its orbit	angle, measured in the direction of satellite motion, from perigee to the satellite's location	$0° \le \nu < 360°$ undefined when $e = 0$ (circular orbit)

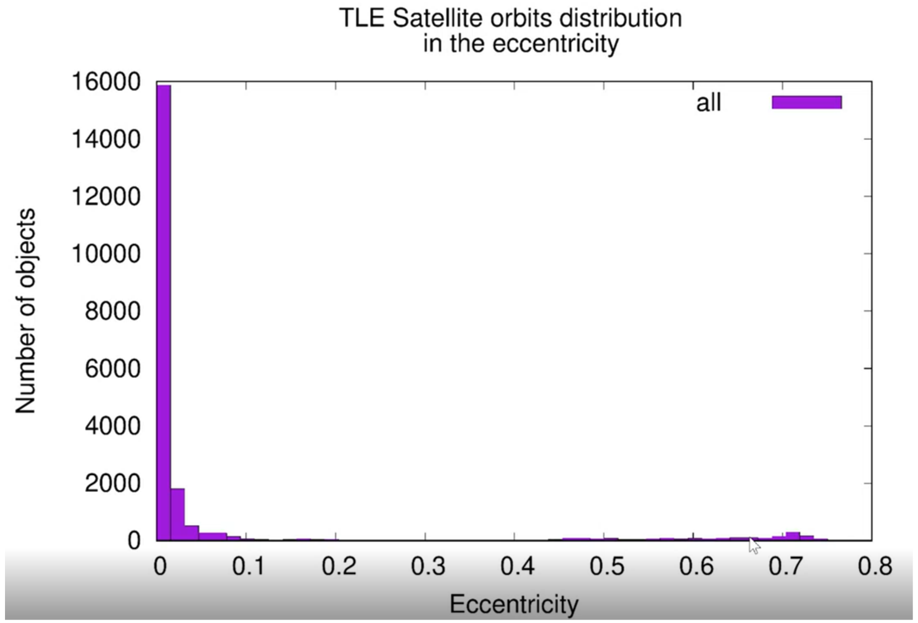


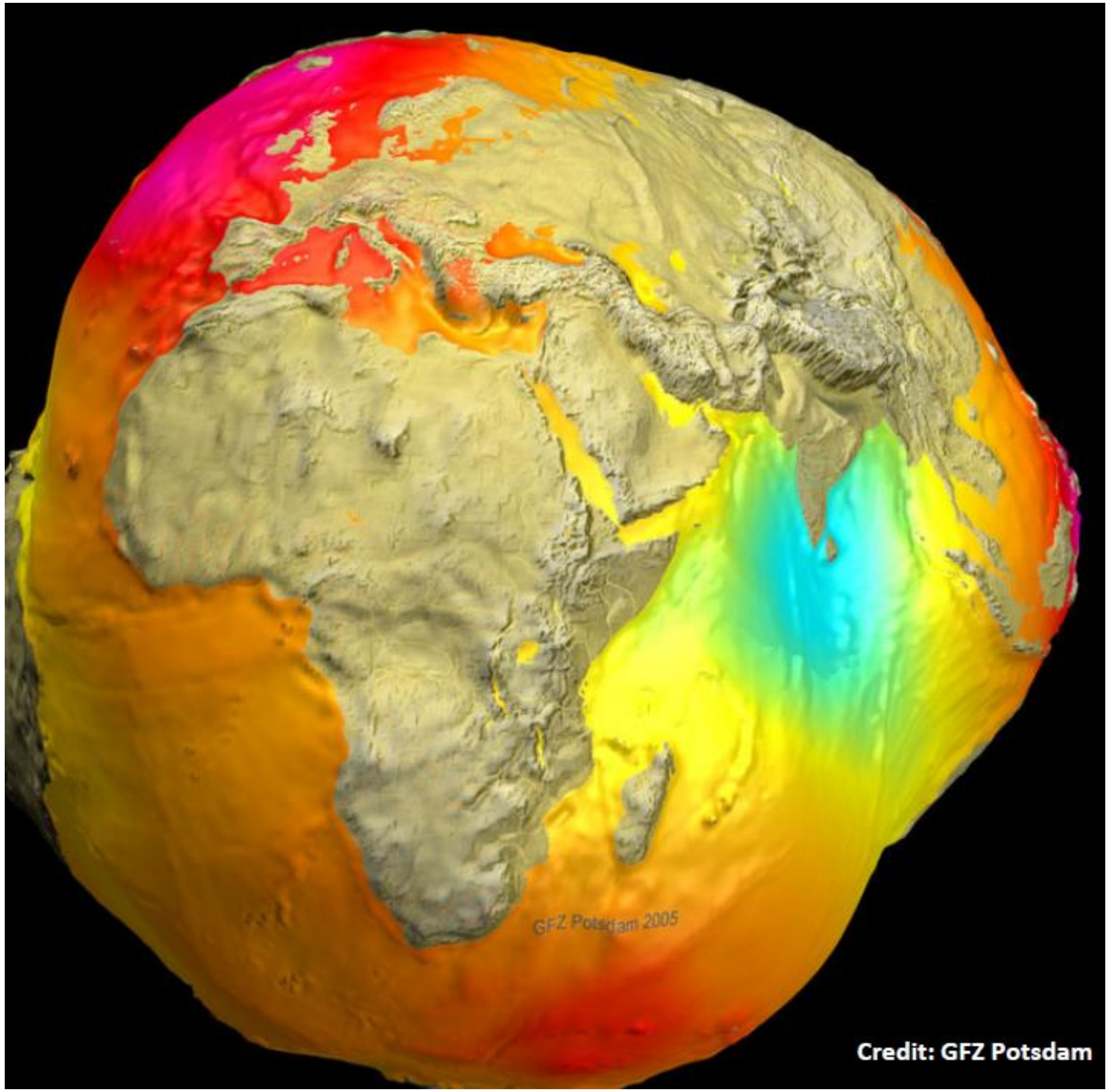


Orbital Elements for Various Missions.

Mission	Orbital Type	Semimajor Axis (Altitude)	Period	Inclination	Other
• Communication • Early warning • Nuclear detection	Geostationary	42,158 km (35,780 km)	~24 hr	~0°	$e \cong 0$
• Remote sensing	Sun-synchronous	~6500 – 7300 km (~150 – 900 km)	~90 min	~95°	$e \cong 0$
• Navigation – GPS	Semi-synchronous	26,610 km (20,232 km)	12 hr	55°	$e \cong 0$
• Space Shuttle	Low-Earth orbit	~6700 km (~300 km)	~90 min	28.5°, 39°, 51°, or 57°	$e \cong 0$
• Communication/ intelligence	Molniya	26,571 km (R_p = 7971 km; R_a = 45,170 km)	12 hr	63.4°	$\omega = 270°$ $e = 0.7$




TLE Satellite orbits distribution
in the eccentricity
all
Number of objects
16000
14000
12000
10000
8000
6000
4000
2000
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Eccentricity



Non - Ideal Motion

two groups of interactions disrupting Keplerian motion:

1. Gravitational forces:

a) constant in time gravitational field of the non-spherical Earth,

b) direct attraction of the satellite by the Sun and the Moon,

c) indirect influence of the Sun and the Moon through the flows of oceanic masses and the Earth's crust,

d) relativistic effects: *Earth's mass leads to a curvature of the four-dimensional space-time: correction by a factor of about* $\frac{v^2}{c^2}$.

2. Non-gravitational forces:

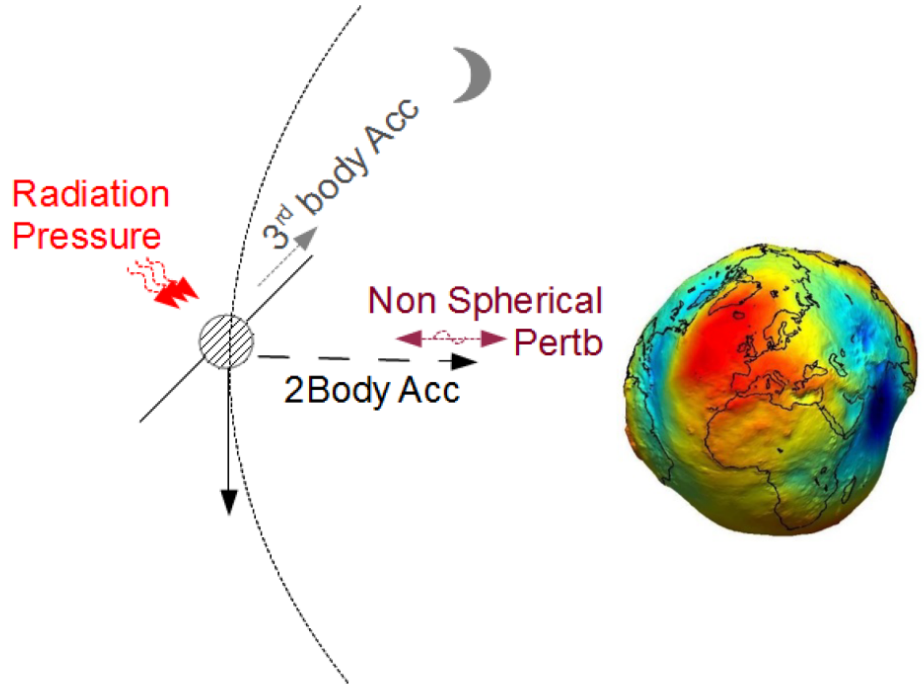
a) solar radiation pressure

b) atmospheric drag

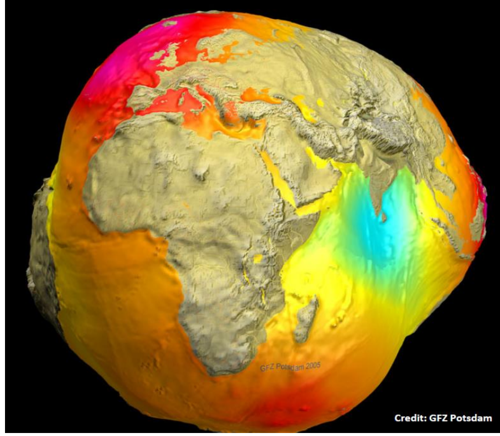
c) Earth radiation pressure: *radiation reflected by the Earth (albedo) generates a small amount of pressure on the satellite.*

• Non - Ideal Motion

Forces acting on a space-object



- Non - Ideal Motion ad 1. Gravitational forces: -



A common feature of gravitational forces is their potentiality:

- none of them can causa systematic changes in the semi-major axis or orbital eccentricity,
- satellite can be treated as a material point with negligible mass.

Main mathematical tool used to construction of geopotential models is the harmonic series of spherical functions [*the Earth's geopotential field as the sum of an infitite number of harmonics*]:

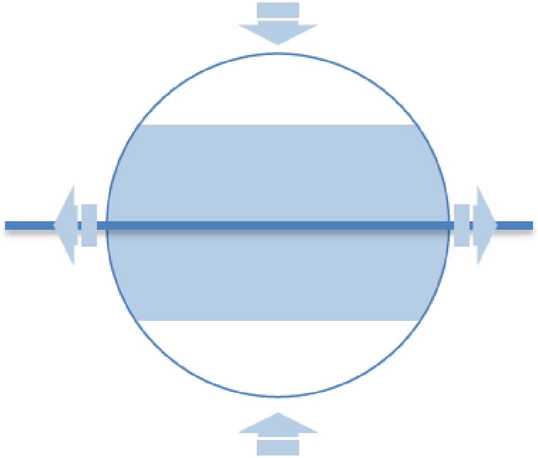
$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}+\lambda\right]f(\theta,\phi)=0$$

$$V_\oplus = \sum_{l=0}^{\infty}\sum_{m=0}^{l} V_{l,m},$$

harmonics of degree l and order m

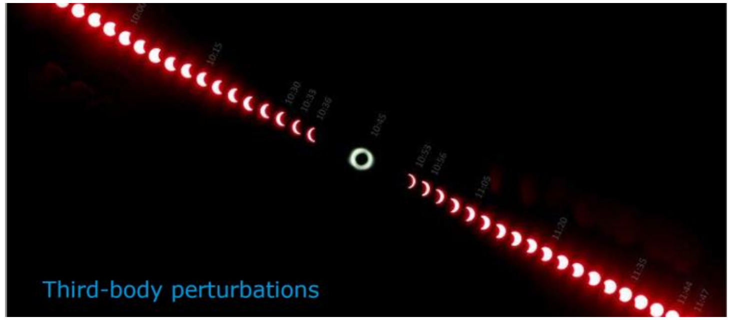
$$V_{l,m} = -\frac{\mu}{r}\left(\frac{a_\oplus}{r}\right)^l P_l^m(\sin\varphi)\,[C_{l,m}\cos m\lambda + S_{l,m}\sin m\lambda]\,.$$

geopotential coefficients



$C_{2,0} = -J_2$

$$\begin{aligned}
\bar{C}_{2,0} &= -4.84165371736\times10^{-4},\\
\bar{C}_{2,1} &= -1.86987635955\times10^{-10},\\
\bar{S}_{2,1} &= 1.19528012031\times10^{-9},\\
\bar{C}_{2,2} &= 2.43914352398\times10^{-6},\\
\bar{S}_{2,2} &= -1.40016683654\times10^{-6},\\
\bar{C}_{3,0} &= 9.57254173792\times10^{-7}.
\end{aligned}$$



- **Nature** of the **third-body perturbation**: namely, the Sun and Moon produce a secular variation in the satellite node, argument of perigee, and mean anomaly, similar to the Earth oblateness effect.

- **Earth tides** – gravitational force exerted by the Sun and the Moon lead to a time varying deformation of the Earth: has some small periodic variations that affect the motion of the satellite.

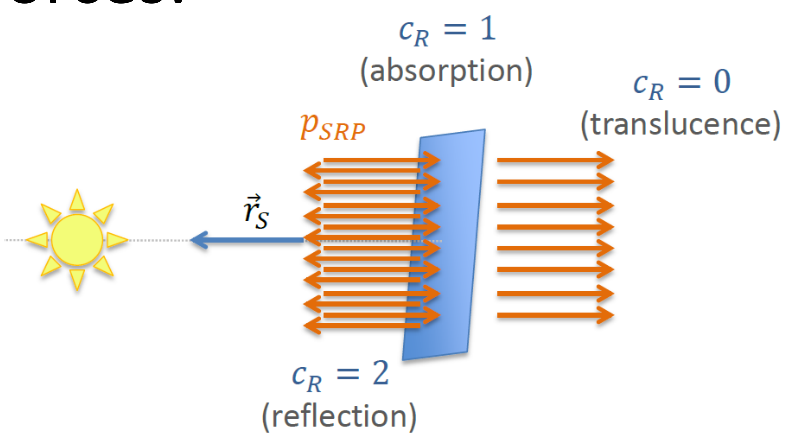
• Non - Ideal Motion ad 2. Non - gravitational forces:

a) solar radiation pressure

b) atmospheric drag

c) Earth radiation pressure: *radiation reflected by the Earth (albedo) generates a small amount of pressure on the satellite.*

• Non - Ideal Motion ad 2. Non - gravitational forces:



- cannot treat a satellite as a material point: mass, shape, size and the physical properties of its surface (even its color) become relevant.

- can induce a secular perturbation of semi-major axis and eccentricity and have a significant impact on the residence time of satellites in orbit.

Ad a) solar radiation pressure: $S_{RP} = \dfrac{\Phi}{c}$ $\quad \Phi = \dfrac{\Delta E}{A \Delta t}$

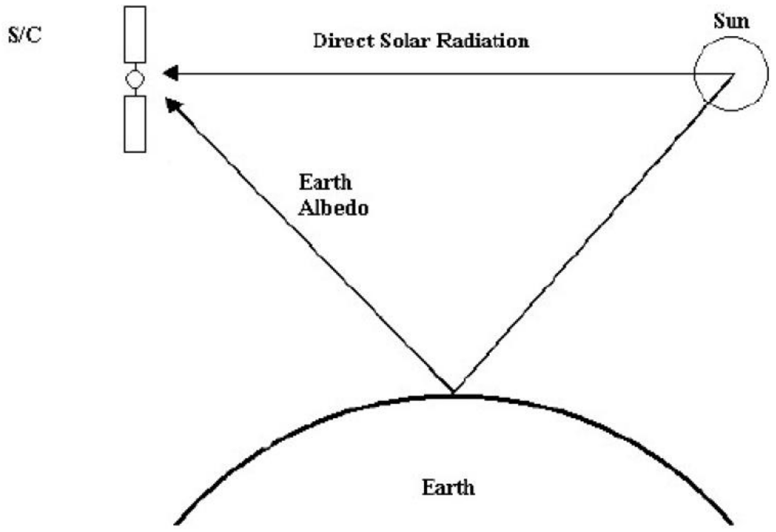
- S_{RP} creates the predominant perturbing acceleration above approximately 800 km altitude.

S_{RP} nearly constant for objects orbiting the Earth, about 1367 Wm^{-1}.

acceleration due to the Solar Radiation Pressure $\ddot{\mathrm{r}} = -S_{RP} C_R \dfrac{A}{m} \dfrac{1}{R_{Sun}^2}$

$C_R = 1 + \varepsilon$ solar radiation pressure coefficient

fraction of energy reflected by the satellite



Ad b) **atmospheric drag**: are really of high importance for the LEO satellites, being the biggest source of disturbance at that regime.

- is directed in the opposite velocity direction
- force is non-conservative and the overall effect is about to reduce the semimajor axis.

$$\ddot{r} = -\frac{1}{2} C_D \frac{A}{m} \rho v_r^2$$

drag coefficient: value depends on the interaction between the atmosphere and the satellite surface; in general: 2.0÷2.3

$$BC = C_D \frac{A}{m}$$

satellite cross sectional area,

ballistic coefficient

$$\rho = \rho_0 e^{-\beta(h-h_0)}$$

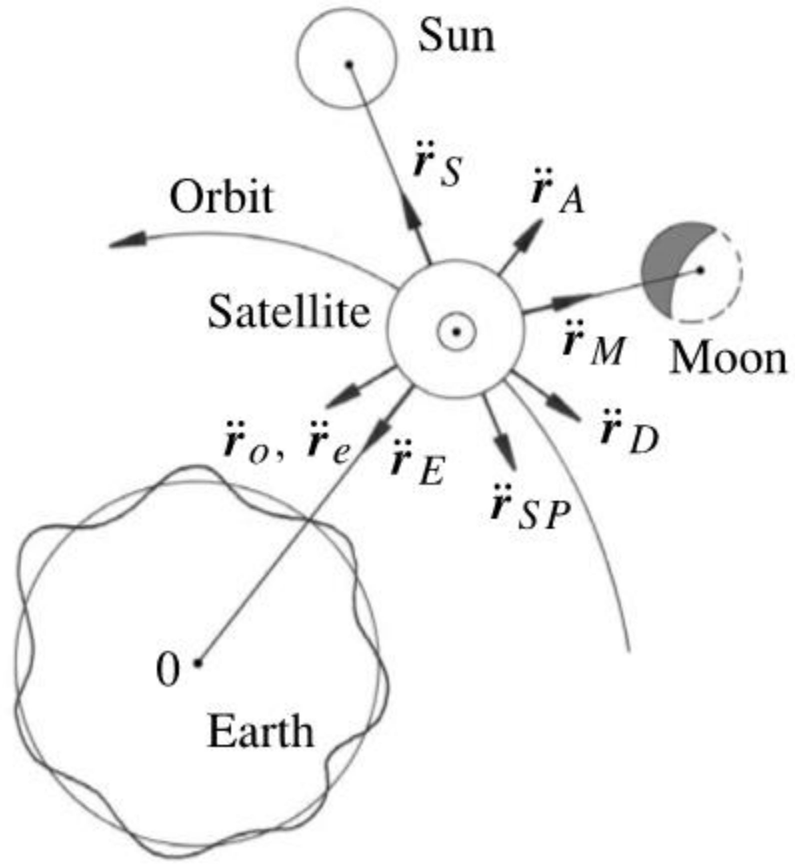
atmospheric density: function of the altitude

- β scale factor: function of the temperature

- the actual temperature in the higher atmosphere is a complex function of the air's molecular components,
- effects of the solar radiation have a great impact in the overall computation.

• Perturbed Satellite Motion

$$\ddot{\boldsymbol{r}} = -\frac{GM}{r^3}\boldsymbol{r} + \boldsymbol{k}_s$$
$$\boldsymbol{k}_s = \ddot{\boldsymbol{r}}_E + \ddot{\boldsymbol{r}}_S + \ddot{\boldsymbol{r}}_M + \ddot{\boldsymbol{r}}_e + \ddot{\boldsymbol{r}}_o + \ddot{\boldsymbol{r}}_D + \ddot{\boldsymbol{r}}_{SP} + \ddot{\boldsymbol{r}}_A.$$



Perturbing forces acting on a satellite

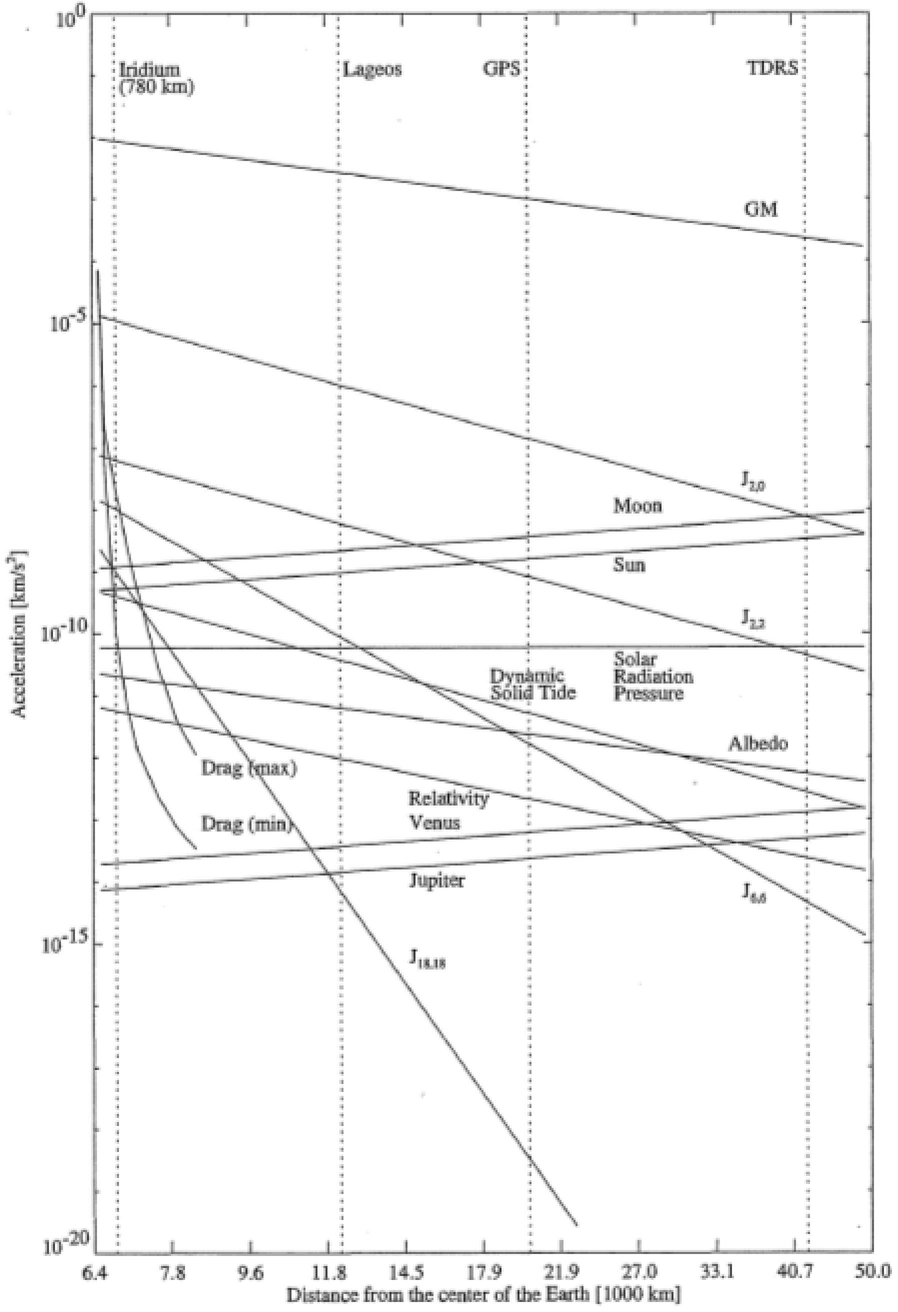
1. Accelerations due to the non-spherically and inhomogeneous mass distribution within Earth (central body), $\ddot{r}_E$.

2. Accelerations due to other celestial bodies (Sun, Moon and planets), mainly $\ddot{r}_S$, $\ddot{r}_M$.

3. Accelerations due to Earth and oceanic tides, $\ddot{r}_e$, $\ddot{r}_o$.

4. Accelerations due to atmospheric drag, $\ddot{r}_D$.

5. Accelerations due to direct and Earth-reflected solar radiation pressure, $\ddot{r}_{SP}$, $\ddot{r}_A$ **.**

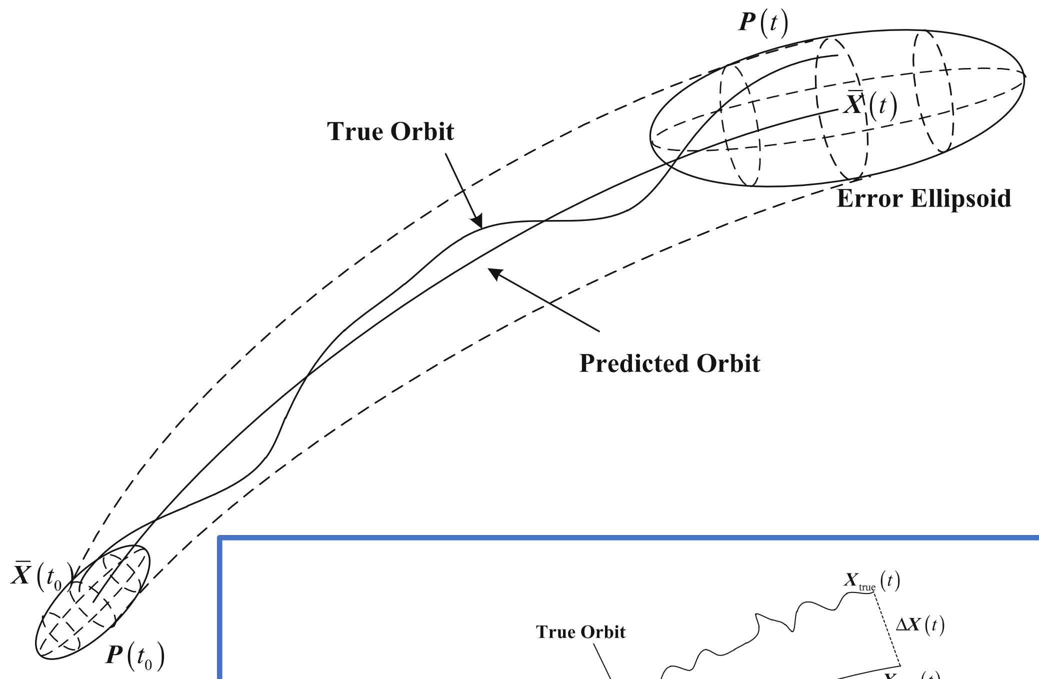


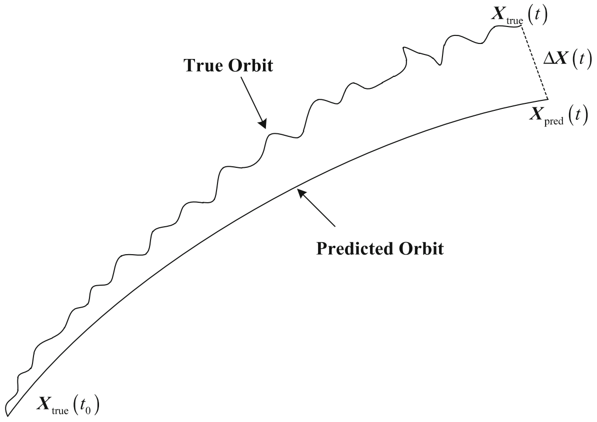
Order of magnitude of various perturbations of a satellite orbit.

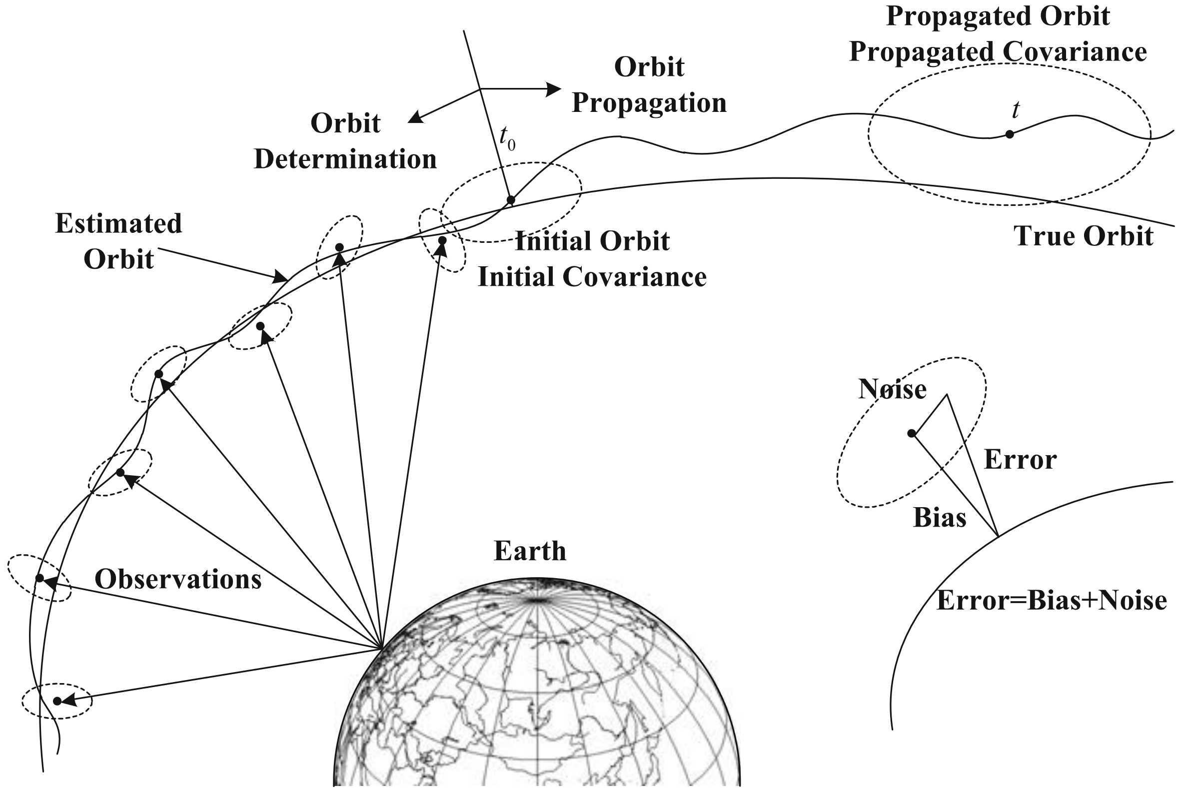
Orbit determination

„Satellite Orbit Determination: method of determining the position and velocity, i.e., the state vector of an orbiting object such as an interplanetary spacecraft or an Earth – orbiting satellite"

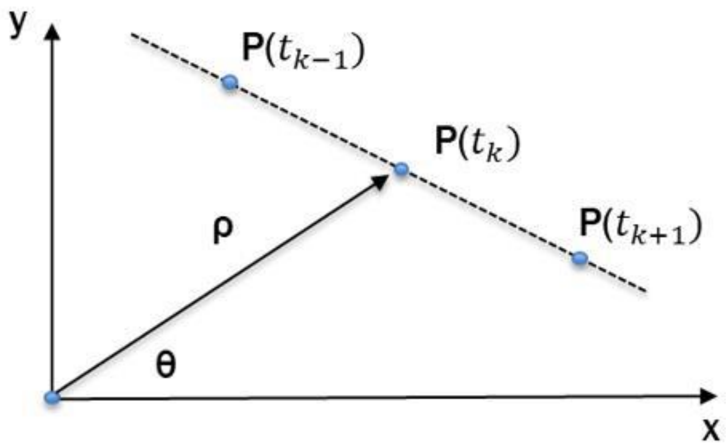
Mathematical Description: Goddard Trajectory Determination System







- the minimal set of parameters is the position and velocity vectors at some given epoch
- dynamic and measurement model parameters: i.e., tracking equipment biases and environmental forces affecting satellite motion.



$$x(t) = \begin{pmatrix} r(t) \\ v(t) \\ p \\ q \end{pmatrix}$$

p and q affects the force and the measurement models

the estimate of the state will differ from the true state because of :

- Mathematical formulation and parameter errors embedded in the equations of motion;
- Mathematical formulation and parameter errors in the observation-state relationship;
- Random or systematic errors in the observations;
- Numerical errors in the computational procedures used in the estimation process.

➢ estimation of the trajectory will never be exact since the observations will be always subject to both random and systematic errors.

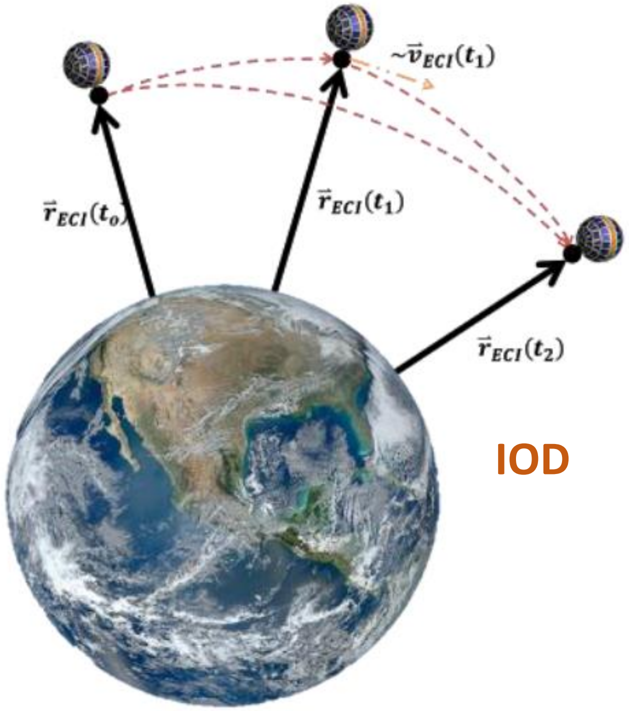
Orbit determination :==: state estimation

- The problem of determining the best estimate of the state over time of a spacecraft, whose initial state is unknown, from observations influenced by random and systematic errors, using a mathematical model that is not exact, is referred to as the problem of *state estimation*

$$x_{k+1} = f(t_k, x_k, u_k)$$

1. initial orbit determination **IOD** used for the direct computation of the 6 orbital elements from at least three observations with no a priori knowledge of the spacecraft's trajectory:

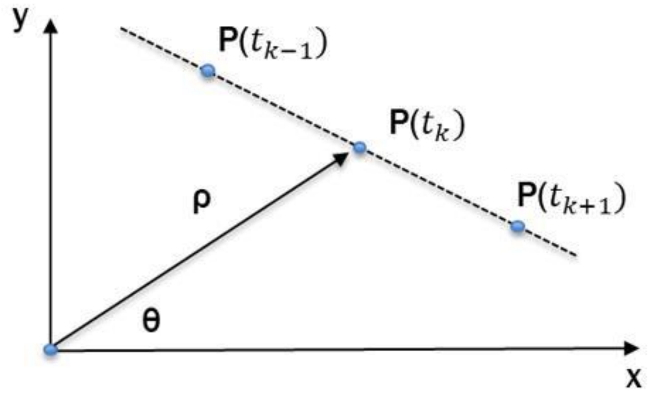
2. orbit estimation (differential correction) used for the improvement of the a priori orbital elements from a large set of tracking data



IOD

- *non-linear systems:* describe the problem by introducing **the state transition matrix** composed by the partial derivatives of the state at time t_{k+1} with respect to the state at time t_k:

$$\mathbf{\Phi}(t_{k+1}, t_k) = \frac{\partial x_{k+1}}{\partial x_k}, \mathbf{\Phi} \in \mathbb{R}^{n*n}$$



For a simplified case of unperturbed Keplerian orbits the state vector is $x = (a, e, i, \Omega, \omega, M)$ with M = mean anomaly. In this case the first five elements remain unchanged over time, while M varies according with the following law: $M(t_{k+1}) - M(t_k) = n(t_{k+1} - t_k)$. Since n (mean motion) is a function of a (semimajor axis) we have a variation in M when a variation in a occurs. The state transition matrix is:

$$\mathbf{\Phi}(t_{k+1}, t_k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{\partial M}{\partial a} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{3}{2}\frac{n}{a}(t_{k+1} - t_k) & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

process noise: account the effects of the non-modeled forces.

$$\boldsymbol{x}_{k+1} = \mathbf{\Phi}(t_{k+1}, t_k)\boldsymbol{x}_k + \boldsymbol{G}(t_k)\boldsymbol{u}_k + \boldsymbol{w}_{k+1}$$

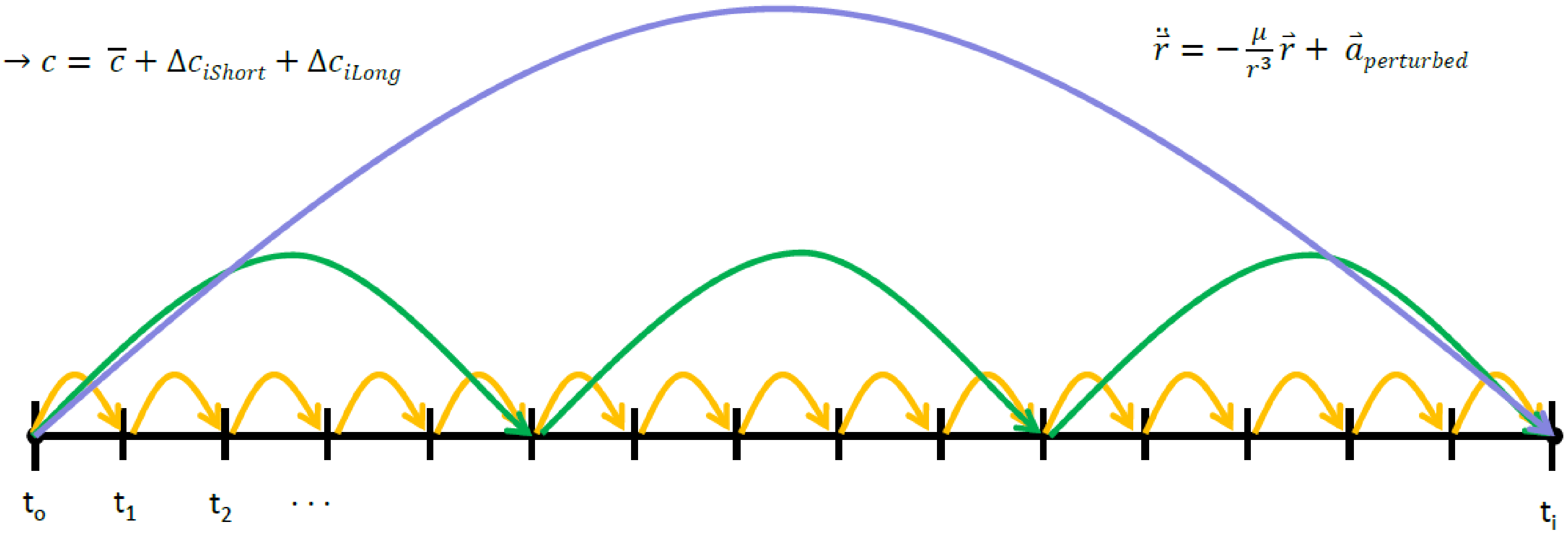
Methods of propagation

Analytical Propagator

$$\frac{d\vec{c}}{dt} = f(\vec{c}, t) \rightarrow c = \bar{c} + \Delta c_{iShort} + \Delta c_{iLong}$$

Numerical Propagator

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_{perturbed}$$



Semi-analytical Propagator

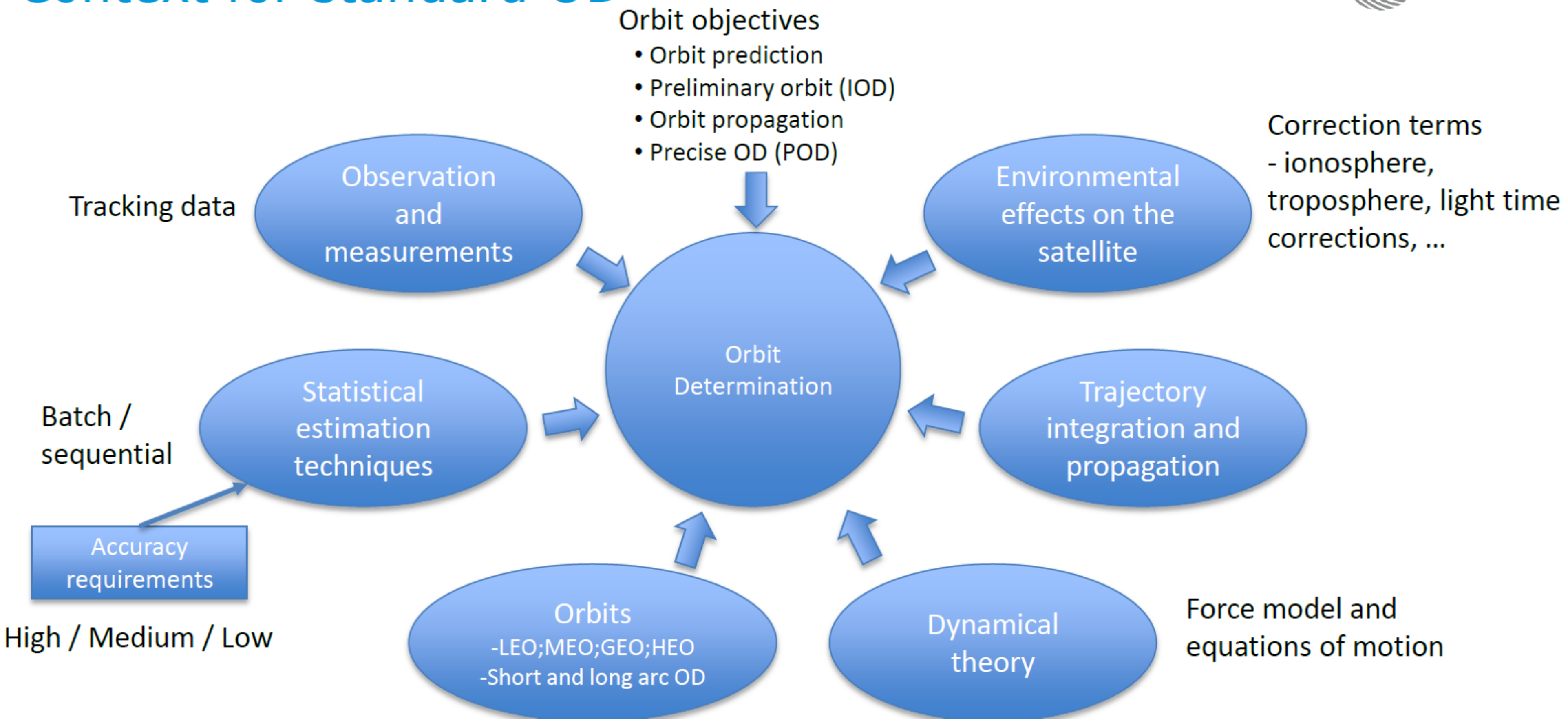
$$\frac{d\bar{c}_i}{dt} = \sum f(ci, \vec{c})_{long}$$

Step 2. To integrate over large time steps

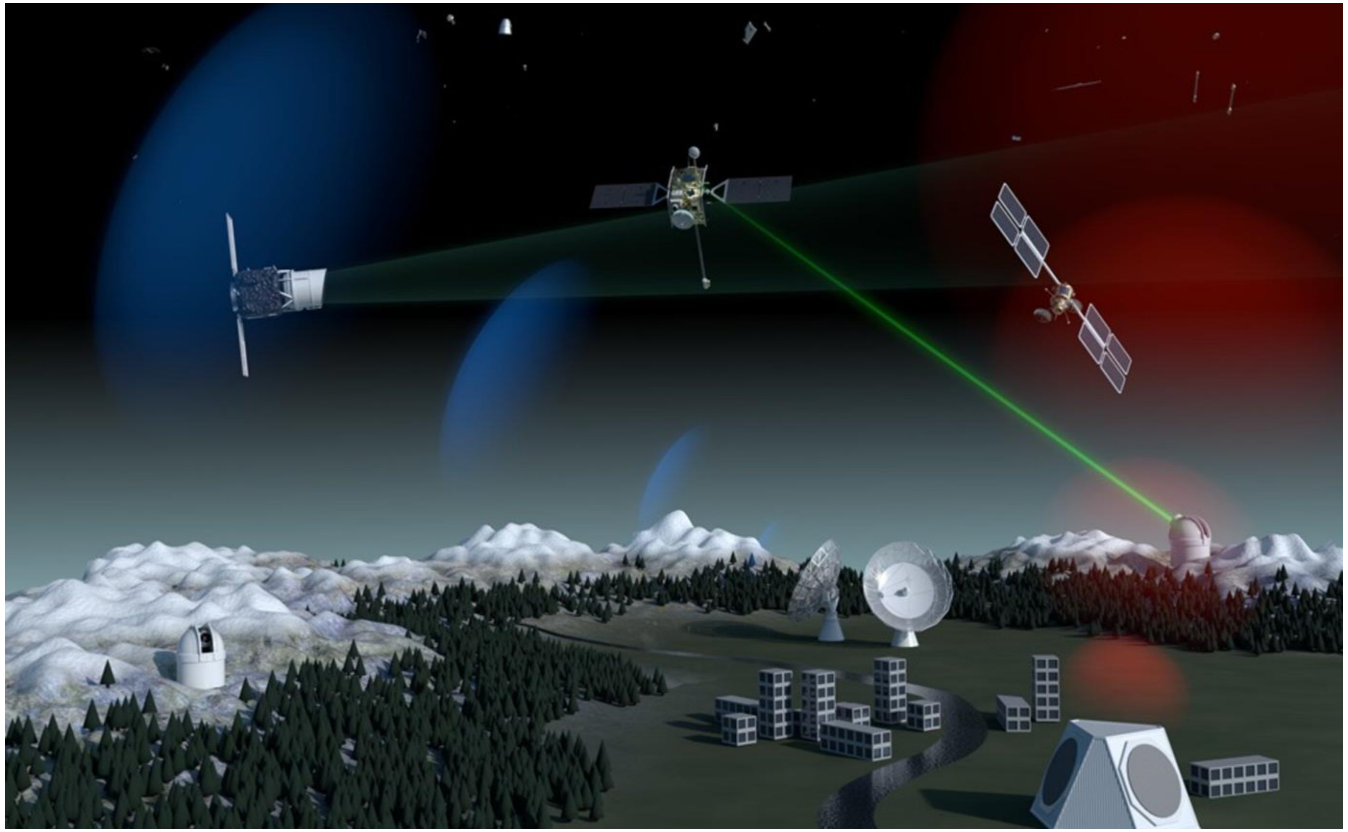
$$c_i = \bar{c}_i + \sum f(ci, \vec{c})_{short}$$

Step 1. To compute the short periodic variations analytically

Context for standard OD



Observational Techniques

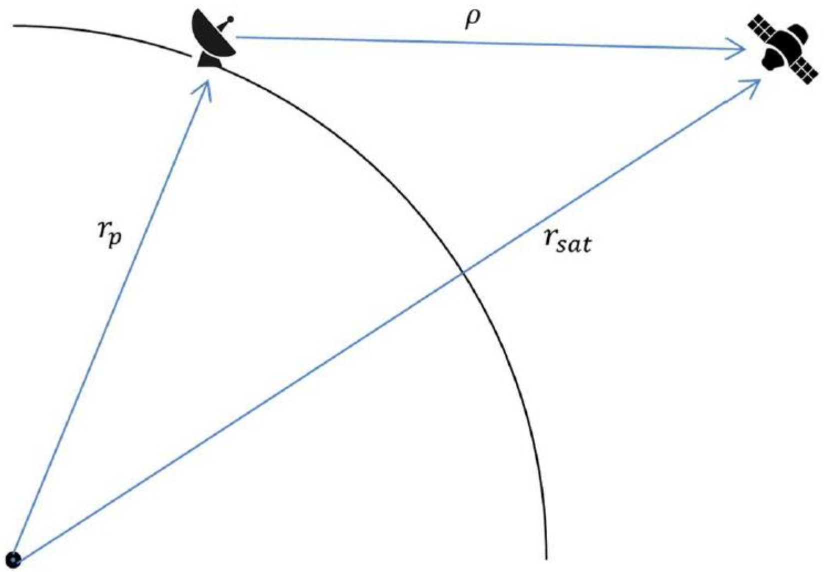


Radars:

- used to determine position and/or velocity of satellites since 30. XX century
- use the effect of reflection of electromagnetic waves and the Doppler effect
- a satellite is observable at least as it rises above the horizon (no weather condition and seasonal constrains)
- Two types of radio observations: *active – satellite transmitting the tracking signal, e.i. one-way*
  pasive - using a reflected signal from a ground-based transmitter, e.i. two-way

$$\boldsymbol{r}_{sat} = \boldsymbol{r}_p + \boldsymbol{\rho}$$

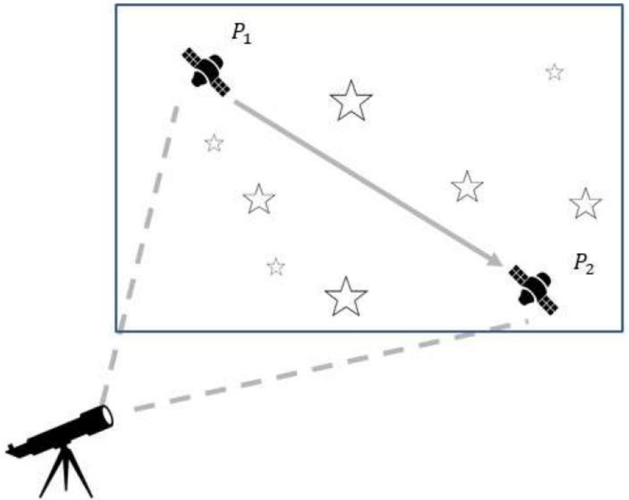
$$\rho = \frac{1}{2} c \Delta t$$



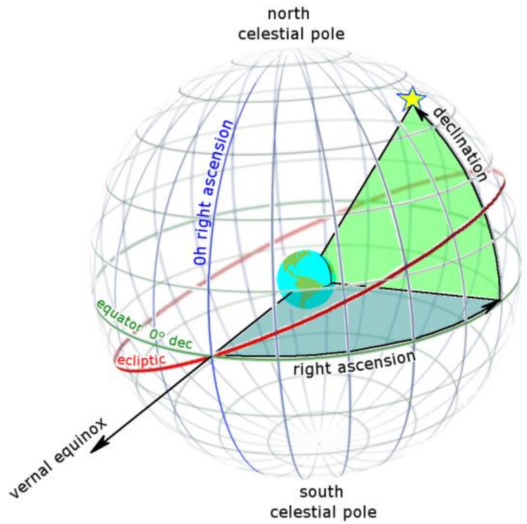
- Observation data
  - Azimuth - angular distance, measured along the horizon
    $(0 < \mathrm{Az} < 360°)$
  - Elevation – angular distance above the horizon
    $(0 < \mathrm{El} < 90°)$
- Affected by:
  - atmospheric delay
  - transponder delay (~1000-3000nsec: 150-450m)
  - satellite motion

Telescops:

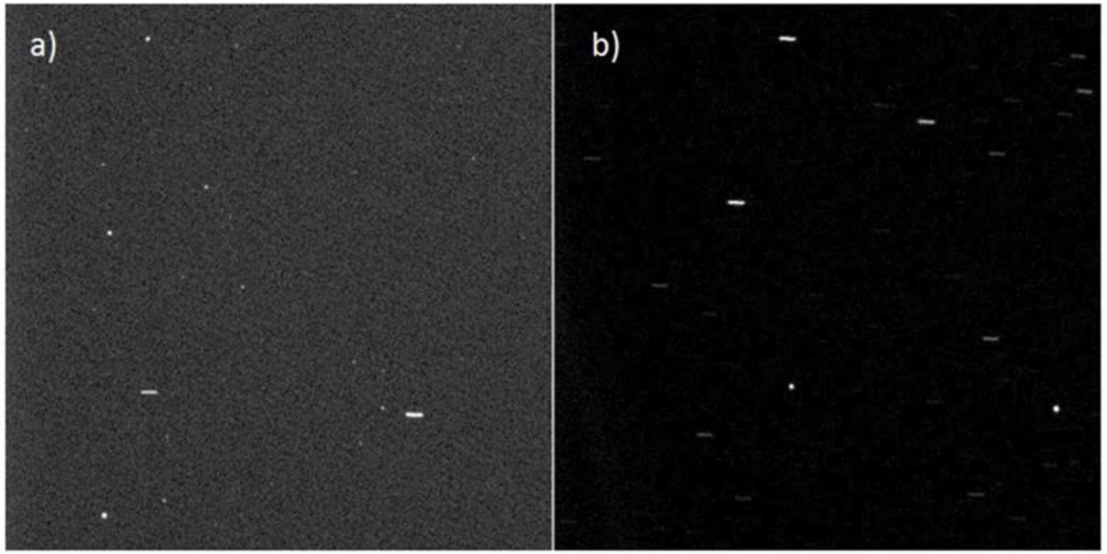
- electromagnetic wave detectors in the visible range
- for SST:
    - field of view
    - speed of assembly
    - camera readout speed



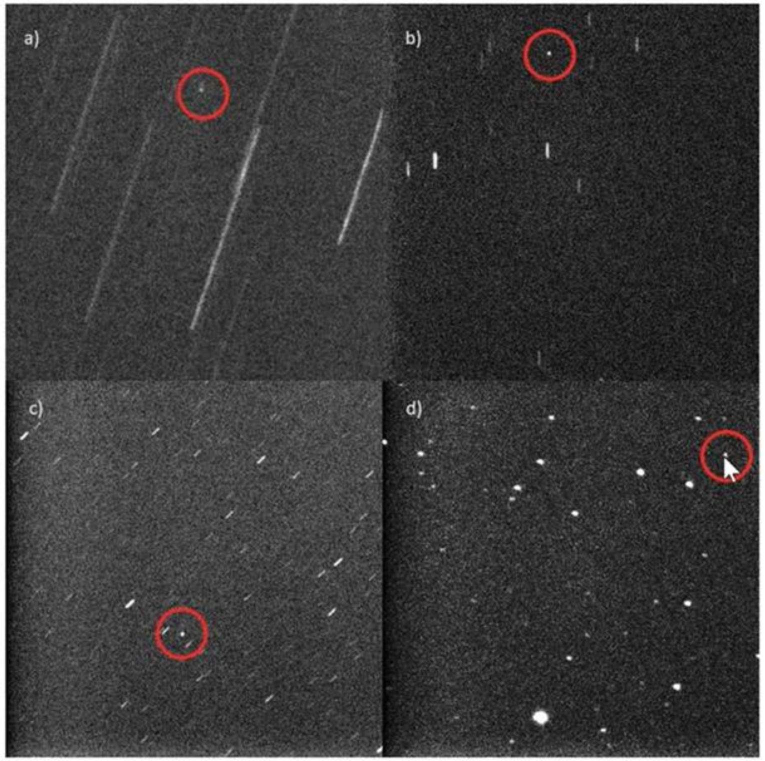
- optical measurements: right ascension and declination
- the satellite position is measured against the fixed star background by using image technigues
- shutter times are used as a time reference: t_1- *time of shutter opening*, t_2 - *time of shutter closing*
- two main errors:
    - aberration
    - light time delay- *satellite position is relevant to an instant in time before the time of shooting*



characterization of objects



Geostationary satellite Directv 8 (28659) observed in different modes a) star-tracking, b) satellite-tracking.



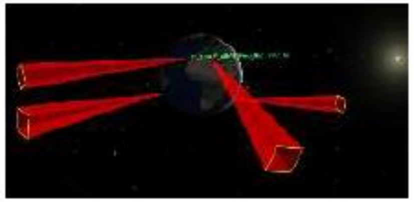
Sample images of satellites and space debris:
a) Fengyun-1C (29782) - LEO, b) Lageos-I (08820) - MEO,
c) Galaxy 12 (27715) - GEO, d) CXO (25867) - HEO

LASERS: *light amplification by stimulated emission of radiation*

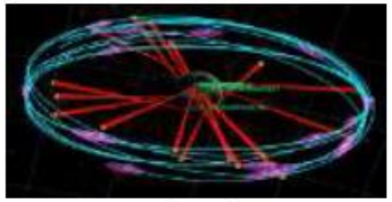
- for SST
  - satellite laser ranging, SLR
  - highest measurement precision

- active satellites with retroreflectors, up to 36 000 km
- space debris, up to 3 000 km

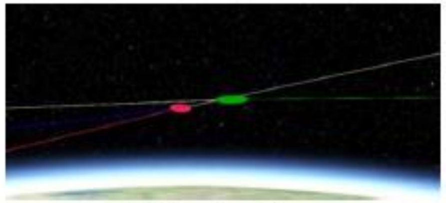
Data Analysis SST



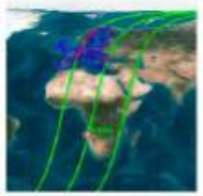
Coverage



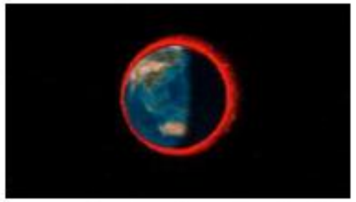
Cataloguing



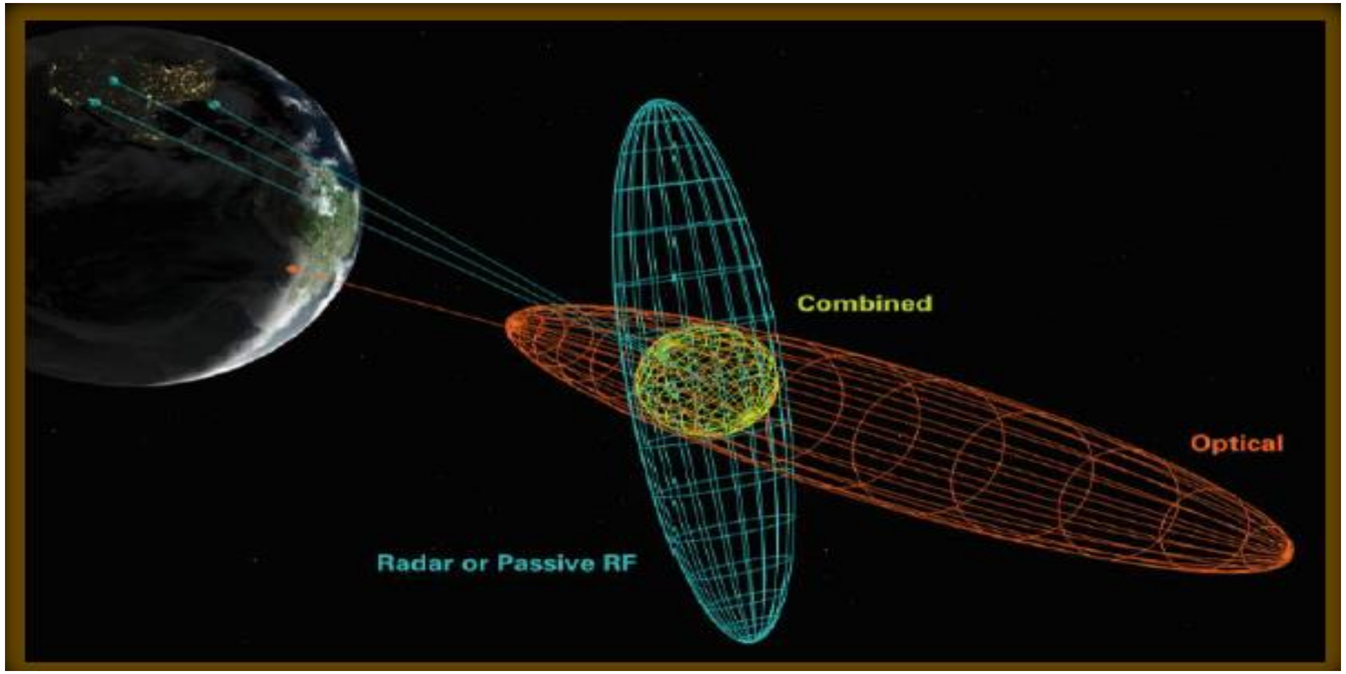
Collision Avoidance



Re-entry Analysis



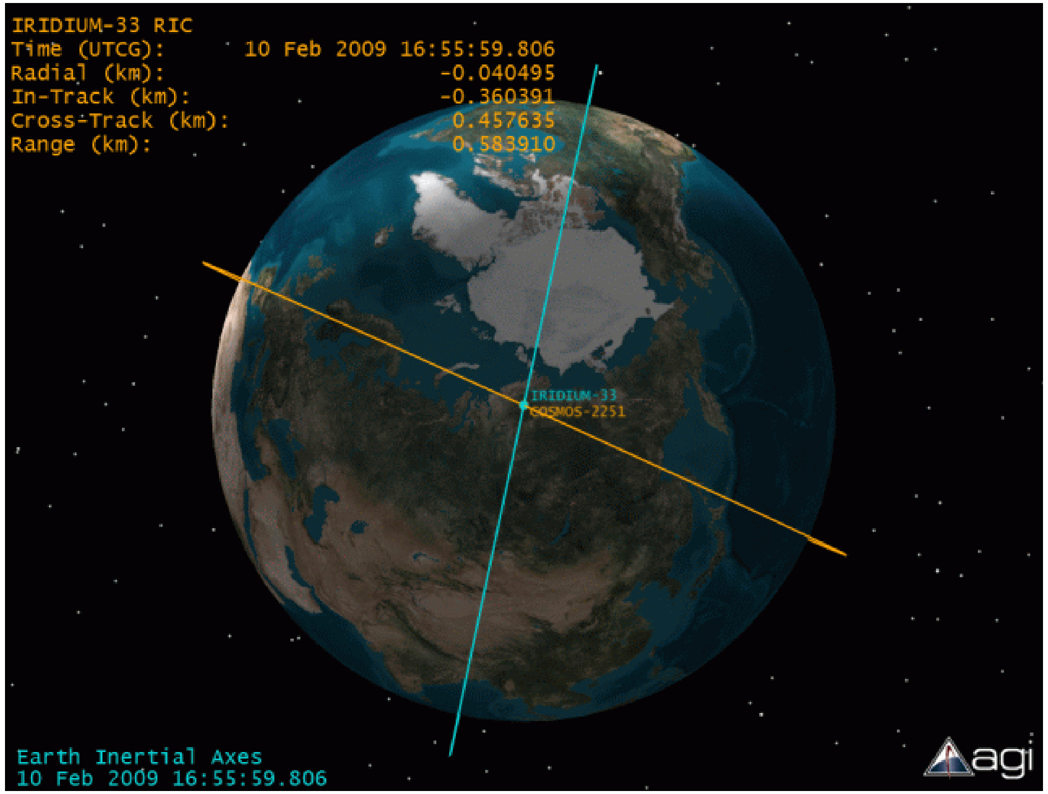
Fragmentation Analysis

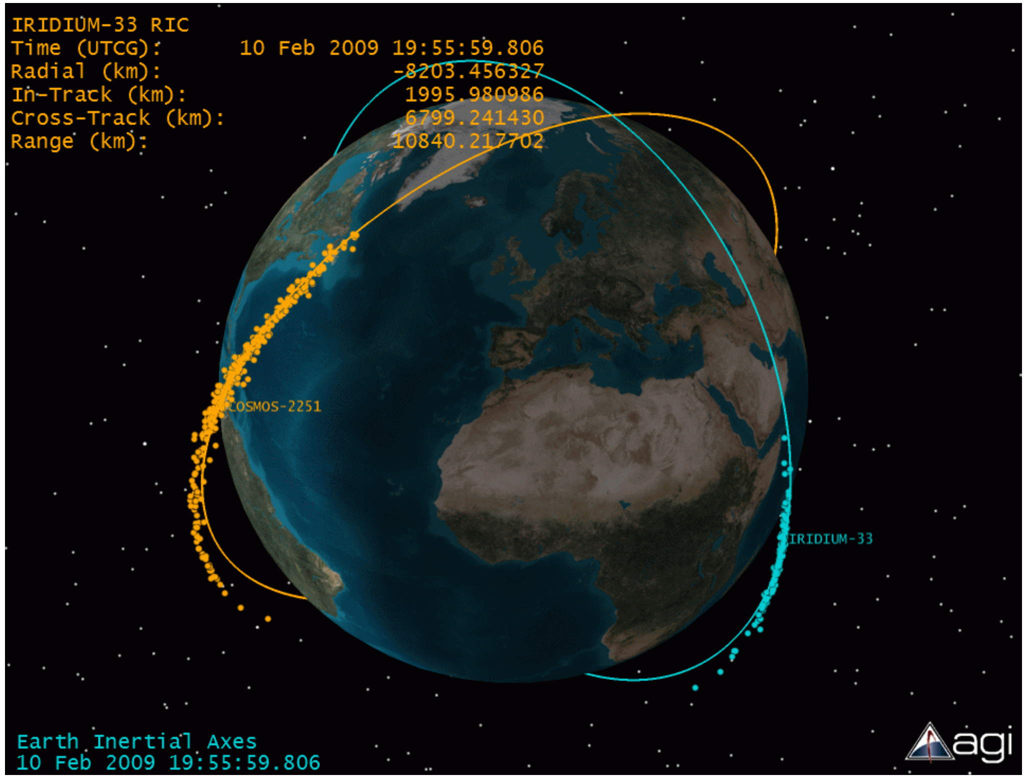


$$P_c = \frac{1}{\sqrt{(2\pi)^3 |\mathbf{C}|}} \iiint_V e^{-\frac{1}{2}\vec{r}^T \mathbf{C}^{-1} \vec{r}} \, dX dY dZ$$

Combined covariance C which is the sum of the two objects' covariance

- **2009 February 10th: Operational Iridium 33 satellite collides with Cosmos 2251 satellite** (not operational since 1995) **=> 2 debris clouds of respectively 600 and 1 600 cataloged objects**





[CelesTrak: Iridium 33/Cosmos 2251 Collision]

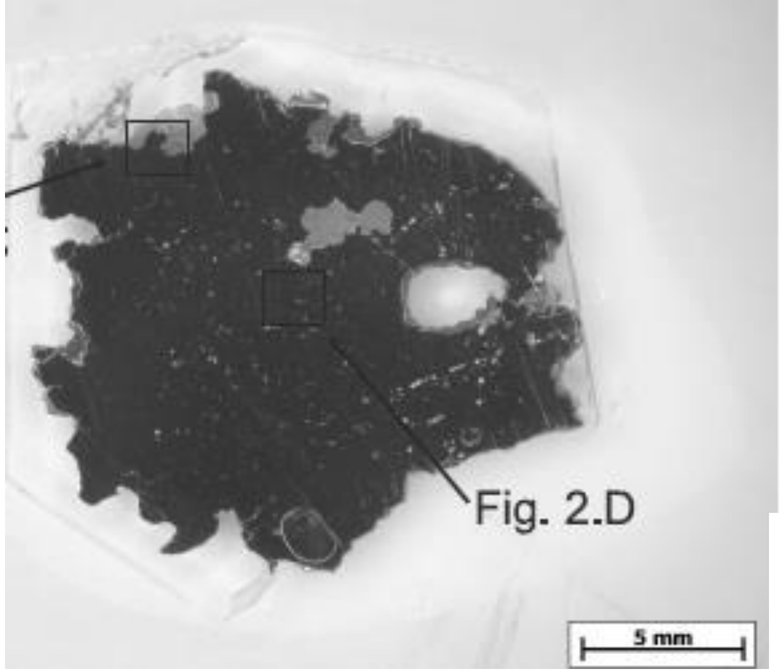
Re-entry on-ground risk analysis

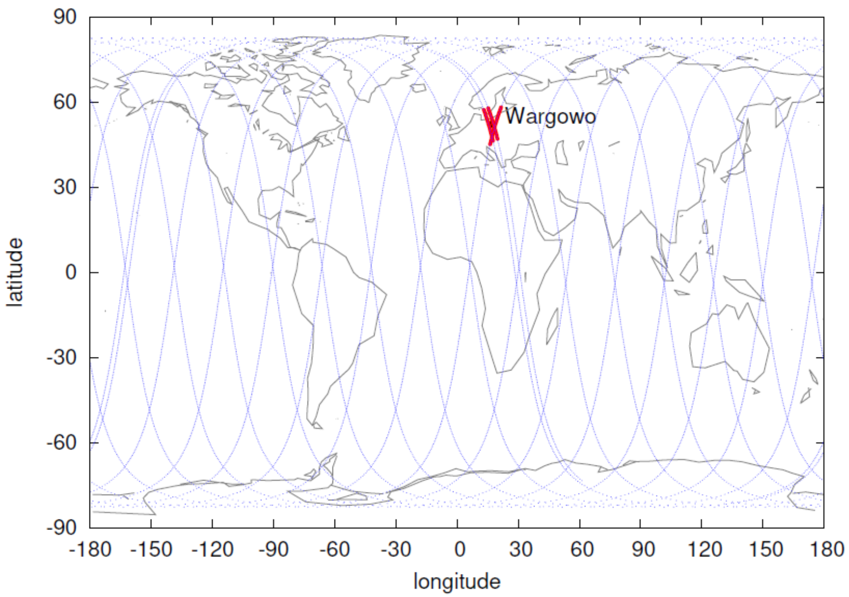
▲ 1997 (Texas, USA)
Delta 2 second stage
main propellant tank

▲ 2001 (Saudi Arabia)
Delta 2 third stage
motor casing

▲ 2004 (Brazil)
Ariane third stage
oxidiser tank

Large uncertainty in mass (in this case depending on how much fuel still on board or if fuel was dumped) →large uncertainty in drag (function of Area/mass ratio) →shifts in passes usual
In general for RE, depending on re-entry angle/attitude/rotational state/solar activity uncertainty in the predictions can be large and the object can “skip” a revolution, or even “bounce” on the atmosphere like skipping stones.





Possible Fengyun-1C debris fall

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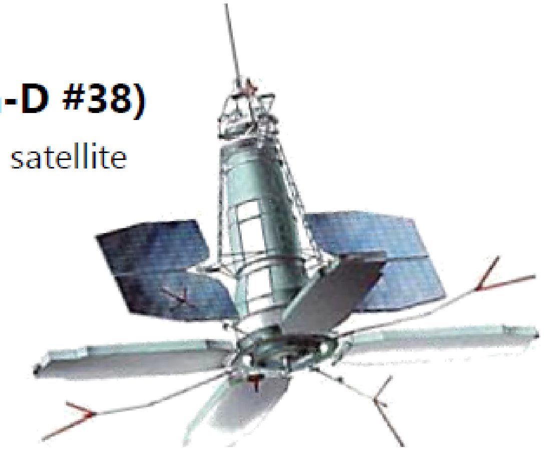
Abstract

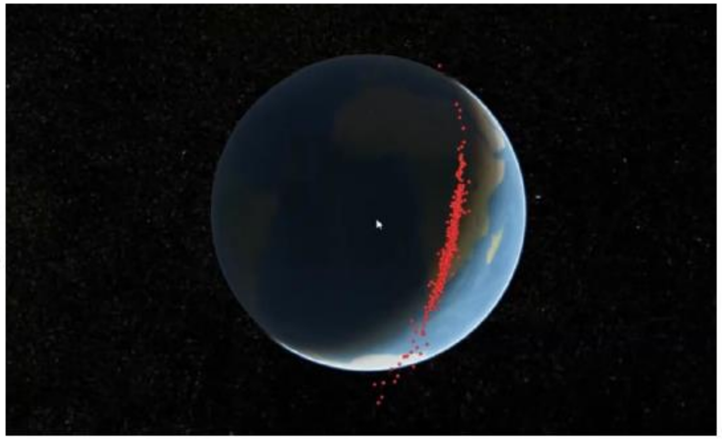
A fall of small objects took place on 27th April 2012 in Wargowo village near Oborniki, about 25 km NW from Poznań (Poland). There was only one eye-witness of the fall, who found two separate pieces, with several small additional fragments. After microscopic observations and chemical analysis a meteoritic origin of these objects was excluded. They are identified as space debris, therefore man-made. The most probable source of the observed fall was space debris 35127 Fengyun 1C DEB, created during destruction of the Chinese weather satellite Fengyun-1C (FY-1C).

Keywords: space debris; meteorwrong; re-entry

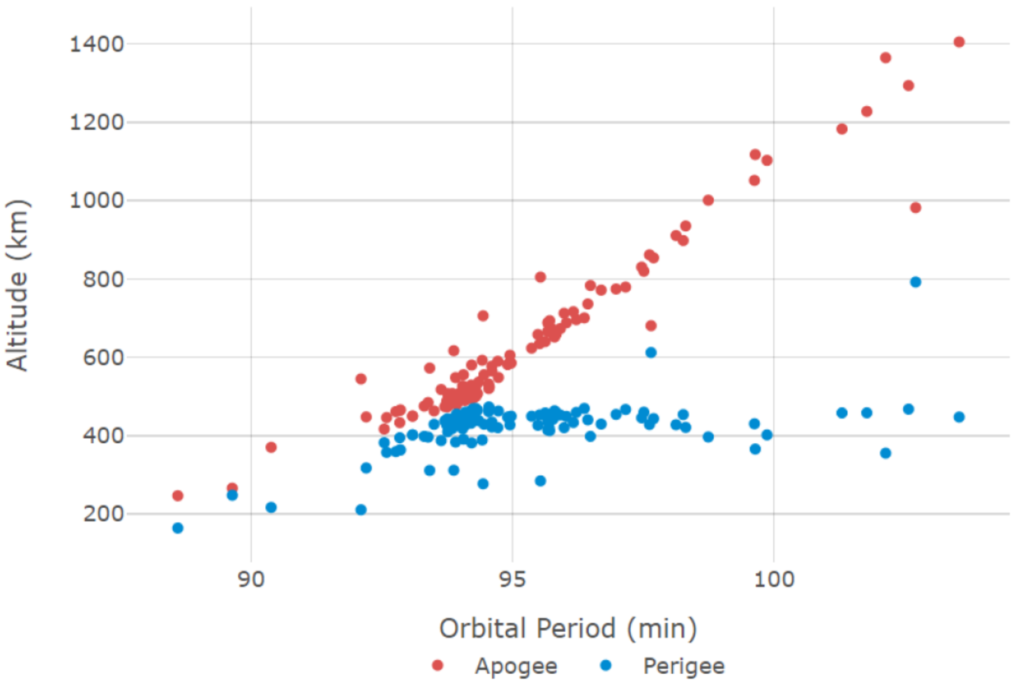
ASAT-related fragmentation

- Initially alleged ASAT, now confirmed
  - 1982-092A #801 **Kosmos 1408 (Tselina-D #38)**
    - Russian electronics and signals intelligence (ELINT) satellite
    - 465 x 490 km orbit, 82.5° inclination
    - > 2000 kg, inactive for decades
- **ASAT** took place on **15/11** likely **~2:47am UTC**
  - Launch from **Plesetsk**

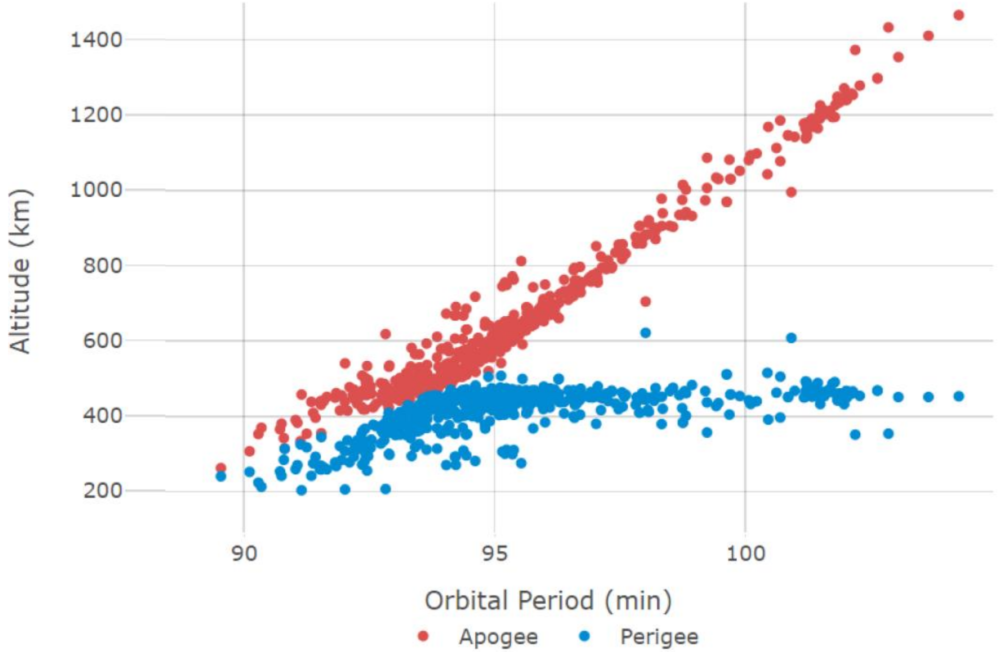




Gabbard diagram representing the monitored fragments as of 18 November



Gabbard diagram representing the monitored fragments as of 24 November



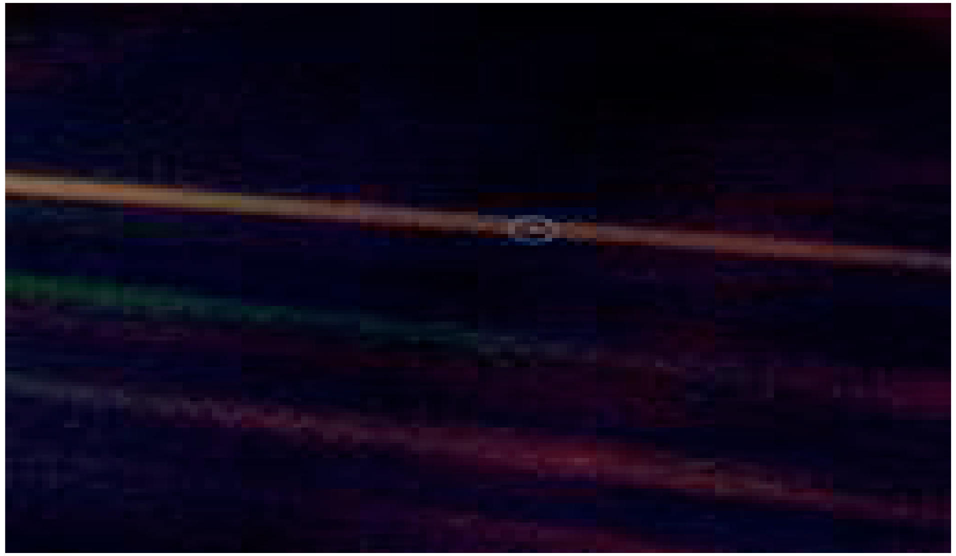
Current opportunities

Date of activity*	Programme	Deadline to apply	Type	Status
17-20 January 2023	Robotics Workshop	5 September 2022, 23:59 CET	Training Session	Closed
February 2023	FYS Test Opportunities	01 October 2022, 23:59 CEST	Hands-on	Closed
November 2022-April 2024	FYS Design Booster	2 October 2022, 23:59 CEST	Hands-on	Closed
31 January – 3 February	Space Systems Engineering Training Course	5 December 2022, 23:59 CEST	Training Session	Closed
14-17 February 2023	CubeSat Concurrent Engineering Workshop 2023	18 January 2023, 23:59 CET	Training Session	Open
13-17 March 2023	Ladybird Guide to Spacecraft Communications 2023	18 January 2023 23:59 CET	Training Session	Open
28-31 March 2023	Clean Space Training Course 2023	23 January 2023 23:59 CET	Training Session	Open

DZIĘKUJĘ ZA UWAGĘ

"It has been said that astronomy is a humbling and character-building experience. There is perhaps no better demonstration of the folly of human conceits than this distant image of our tiny world. To me, it underscores our responsibility to deal more kindly with one another, and to preserve and cherish the pale blue dot, the only home we have ever known."

Carl Sagan
" Pale Blue Dot: A Vision of the Human Future in Space"



***Pale Blue Dot:* from a distance of about 6. 4 bln km**

BACKUP

$$\Delta y = \text{Observed} - \text{computed}$$

- Residual Root Variance

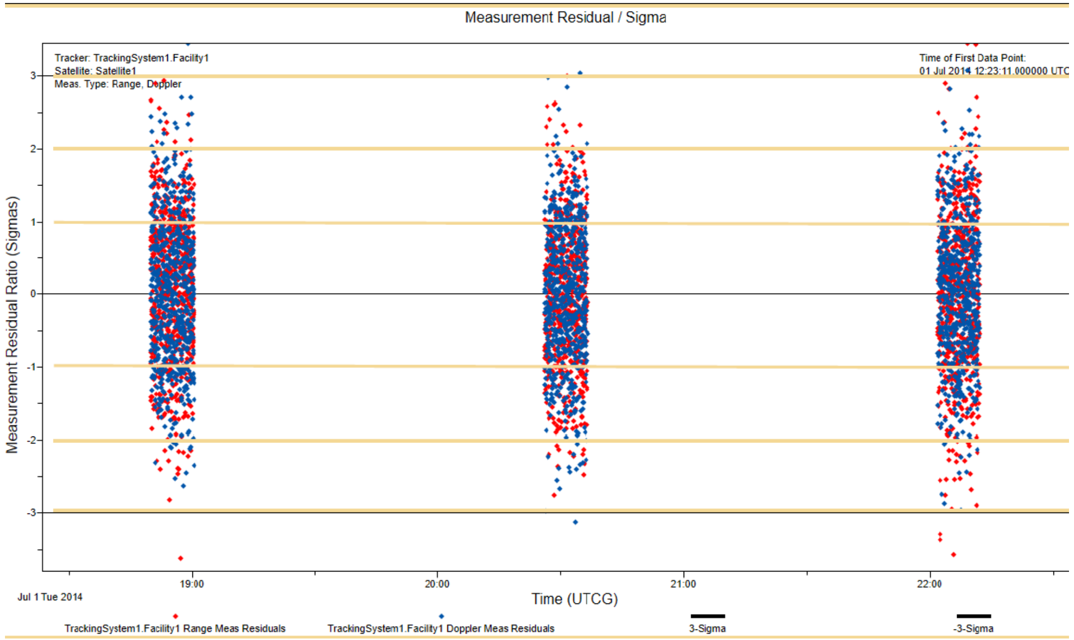
$$\sigma^*_{\Delta y} = \sqrt{HPH^T + \sigma^2_{\Delta y}}$$

$$P = \text{filter covariance}$$

$$H = \text{measurement partial w.r.t. filter state}$$

$$\sigma_{\Delta y} = \text{measurement noise sigma}$$

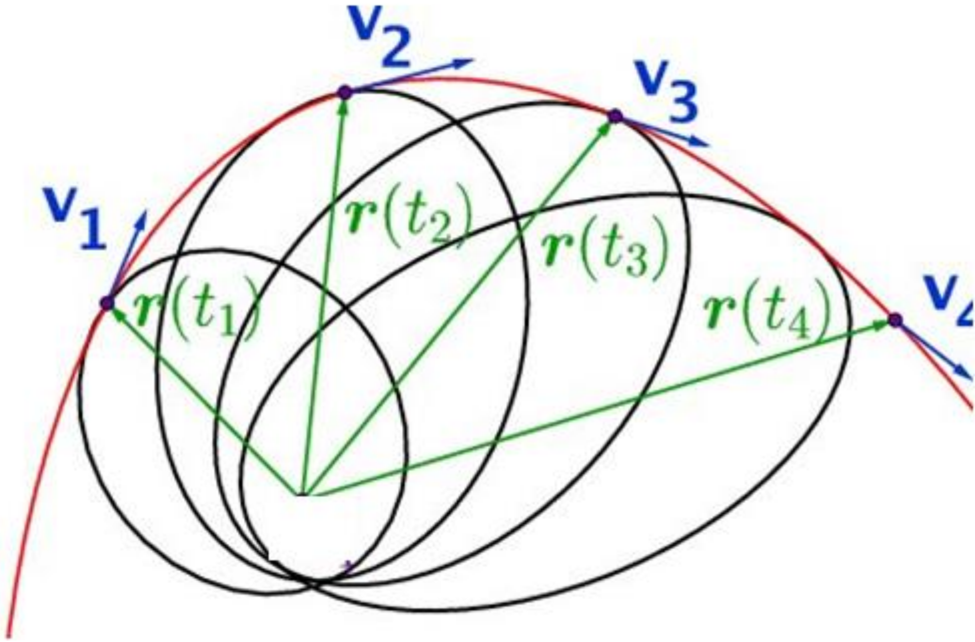
- Residual ratio $R = \Delta y / \sigma^*_{\Delta y}$
  - $|R| \leq 3$ accept measurement y
  - $|R| > 3$ reject measurement y
- R should be Gaussian (everything's modeled except the white noise)



- e.g. methods:
- Herrick-Gibbs: measurement type set {*range, azimuth ,elevation*},
- Gooding: measurement type set {*azimuth, elevation*},

Osculating and Mean Orbital Elements

Fig. Definition of osculating elements. The instantaneous conics ale always tangential to the perturbed actual physical orbit of the body.



- the satellite is located on a different osculating orbit for each particular epoch,
- The true satellite orbit can be regarded to be the envelope of all successive osculating orbits with the osculating elements $a(t_k)$, $e(t_k)$, . . . $M(t_k)$; t_k a time parameter, continuously increasing,
- the perturbed satellite motion can be interpreted to be a Keplerian motion with time-variable elements:

$$a(t),\ e(t),\ i(t),\ \omega(t),\ \Omega(t),\ \overline{M}(t).$$

- "history" of an osculating element $a_i(t)$ is represented as the sum of long- and short-periodic terms:

$$a_i(t) = \overline{a}_i(t) + \Delta a_i(t)$$

- $\overline{a}_i(t)$ contains the sum of low frequency, secular and constant parts - called *mean elements*.
- $\Delta a_i(t)$ represents the high-frequency oscillations.
- Thus, mean elements can be considered as osculating elements with vanishing periodic terms.

- second Kepler's law := constancy of areal velocity

$$\frac{\mathcal{J}}{2m}(t-t_0)=\frac{1}{2}\int_{t_0}^{t}|\vec{r}\times\vec{v}|\,dt, \qquad \frac{1}{2}\,r^2\dot{\varphi}=\frac{\mathcal{J}^{(S)}}{2\,\mu}=\text{const}$$

=>

$$E^{(S)}=-\frac{\alpha}{2a}$$

$$a=\frac{p}{1-\varepsilon^2}=-\frac{\alpha}{2E^{(S)}}$$

$$b=\frac{p}{\sqrt{1-\varepsilon^2}}=\sqrt{\frac{-\mathcal{J}^{(S)2}}{2E^{(S)}\mu}}$$

- third Kepler's law

$$\frac{\mathcal{J}^{(S)}}{2\mu}\,T=\pi ab. \quad => \quad T^2=-\frac{2\pi^2\,\mu\,a^2}{E^{(S)}}=\frac{4\pi^2\,\mu\,a^3}{\alpha}$$

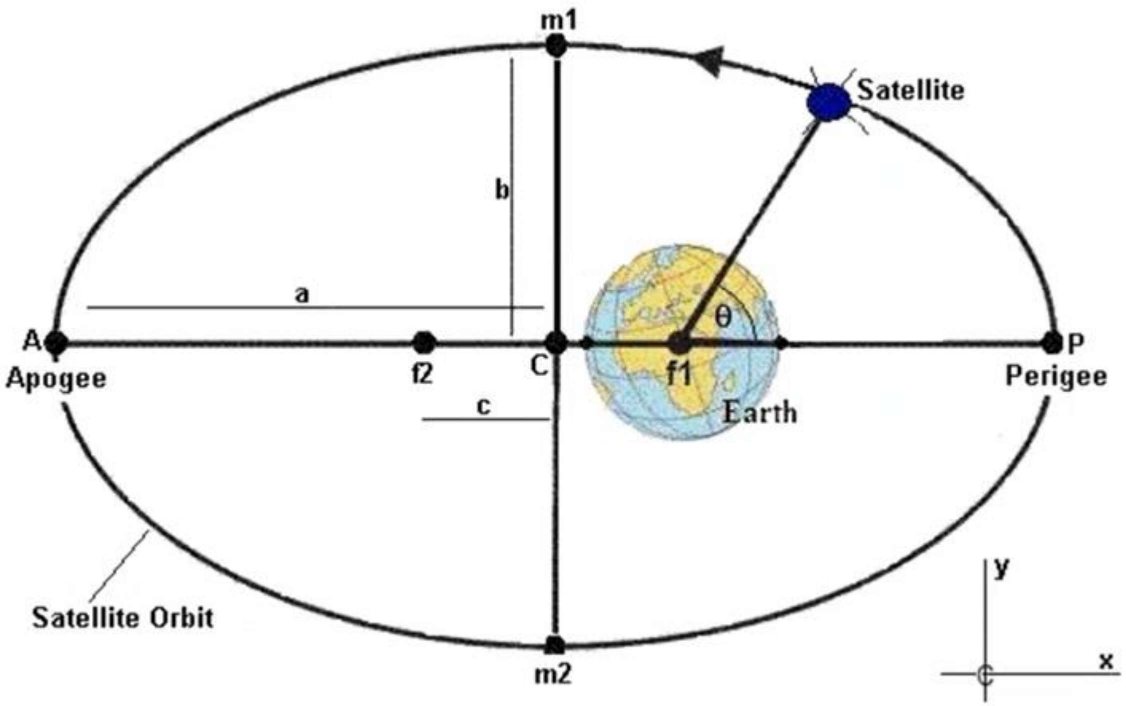
distance to perycenter $r_p=a(1-e)$

closest point to the focus F (center of attraction)

distance to apocenter $r_a=a(1+e)$

height of perigee $h_p=r_p-r_\oplus$

apogee height $h_a=r_a-r_\oplus$



Satellite in an Elliptical Orbit Around the Earth

Since 2009 JSpOC (18 SpCS) distributes CM to O/O

What is a *18 SpCS*'sCDM

The best available data to avoid collision in space: Takes benefit of the US SPcatalog,
Is distributed to all O/O

A description of a forecasted conjunction :TCA: Time of Closest Approach;

Orbit information of the 2 objects: Position / Velocity at TCA,
Covariance,
Orbit Determination characteristics

Information on the size of the object;

Generated with geometric criteria (*Miss distance & Radial separation*):Emergency criteria, up to 3 days before TCA:LEO: 1 km / 200 m,
GEO: 10 km / 5 km

Large criteria (95% capture screening):LEO: 50 km / 2 km (maximum value), up to 7 days before TCA,
GEO: 360 km / 12 km, up to 14 days before TCA

CDM Analysis implies to evaluate :

I.**Position & Velocityof the two objects at the TCA**

II.**Covarianceof the two objects at the TCA**

III.**Radius of the englobing sphere of each object at the TCA**

Measurement Errors

When we perform observations, we generally have a set of different observational data types for each observation time t_k :

$$\boldsymbol{y}_k = [y_{1,k} \quad y_{2,k} \quad \cdots \quad y_{q,k}]^T$$

We can also correlate the state with the observation vector at any observation time (this is needed since the state cannot be observed directly):

$$\boldsymbol{y}_k = \boldsymbol{h}(\boldsymbol{x}_k, t_k) + \boldsymbol{\varepsilon}_k$$

Measurements errors

The function $\boldsymbol{h}(\boldsymbol{x}, t)$ analytically describes the measurement model as a (generally nonlinear) function of the state at time t_k. By linearizing we get:

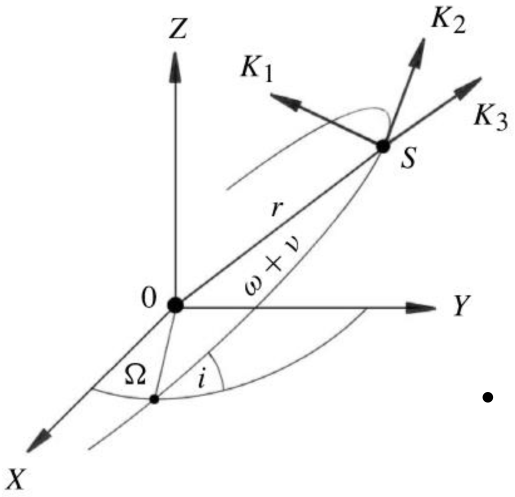
$$\boldsymbol{y}_k = \widetilde{\boldsymbol{H}}_k \boldsymbol{x}_k + \boldsymbol{\varepsilon}_k \quad \text{, with} \quad \widetilde{\boldsymbol{H}}_k = \frac{\partial \boldsymbol{h}(\boldsymbol{x}_k, t_k)}{\partial \boldsymbol{x}_k}$$

Lagrange's Perturbation Equations

In a non-central force field

$$V = \frac{GM}{r} + R,$$

$$F = \frac{GM}{r} + R - T = \frac{GM}{2a} + R.$$



- function ***R*** contains all components of V excluding the central term GM/r : ***disturbing potential***.

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial R}{\partial \overline{M}},$$
$$\frac{de}{dt} = \frac{1-e^2}{na^2e}\frac{\partial R}{\partial \overline{M}} - \frac{\sqrt{1-e^2}}{na^2e}\frac{\partial R}{\partial \omega},$$
$$\frac{d\omega}{dt} = -\frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} + \frac{\sqrt{1-e^2}}{na^2e}\frac{\partial R}{\partial e},$$
$$\frac{di}{dt} = \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \Omega},$$
$$\frac{d\Omega}{dt} = \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i},$$
$$\frac{d\overline{M}}{dt} = n - \frac{1-e^2}{na^2e}\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial a}.$$

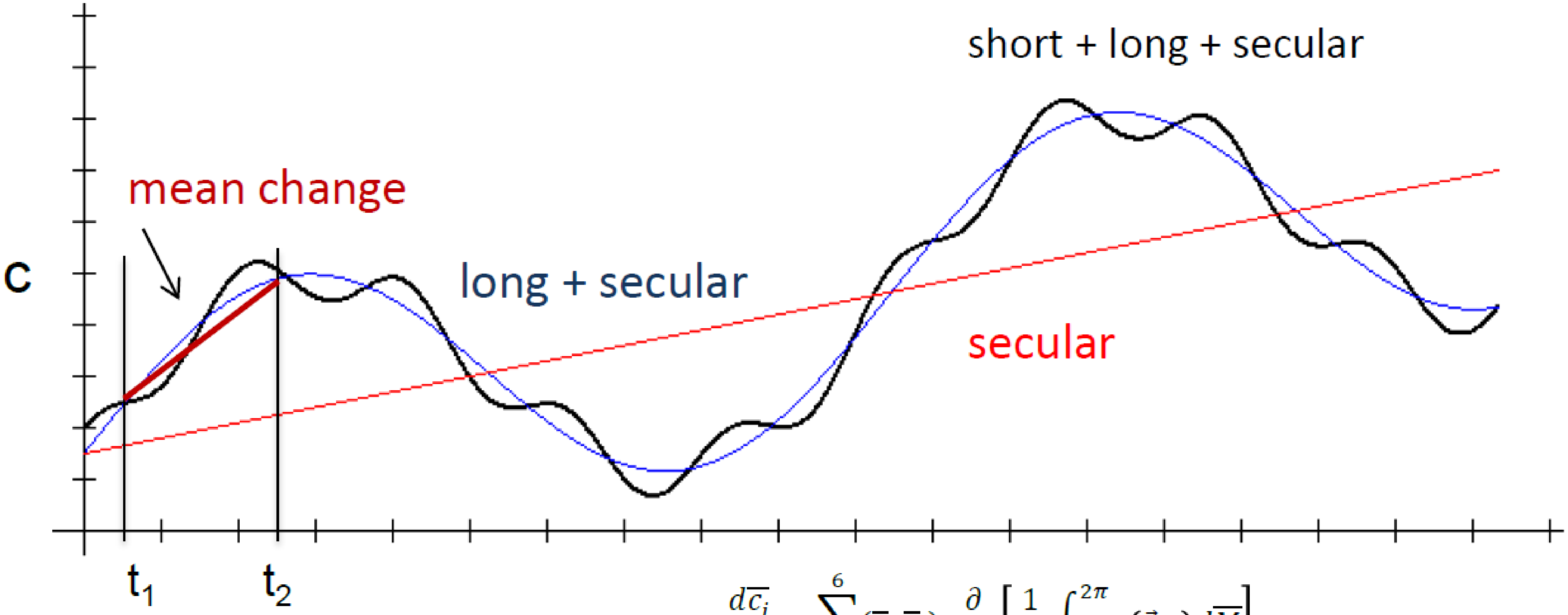
Gaussian Form of Perturbation Equation

$$\text{grad } R = \nabla R = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix}$$

- K_1 is capable of changing the orientation of the orbital plane (elements Ω and i).
- K_2 change of the semi-major axisfor $e \ll 1$, i.e. by the component in the direction of satellite motion.

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left(e\sin\nu K_3 + \frac{p}{r}K_2\right),$$
$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}(\sin\nu K_3 + (\cos E + \cos\nu)K_2),$$
$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nae}\left(-\cos\nu K_3 + \left(\frac{r}{p}+1\right)\sin\nu K_2\right) - \cos i\frac{d\Omega}{dt},$$
$$\frac{di}{dt} = \frac{1}{na\sqrt{1-e^2}}\frac{r}{a}\cos(\omega+\nu)K_1,$$
$$\frac{d\Omega}{dt} = \frac{1}{na\sqrt{1-e^2}}\frac{r}{a}\frac{\sin(\omega+\nu)}{\sin i}K_1,$$
$$\frac{d\overline{M}}{dt} = n - \frac{1}{na}\left(\frac{2r}{a} - \frac{1-e^2}{e}\cos\nu\right)K_3 - \frac{1-e^2}{nae}\left(1+\frac{r}{p}\right)\sin\nu K_2.$$

Method of averaging

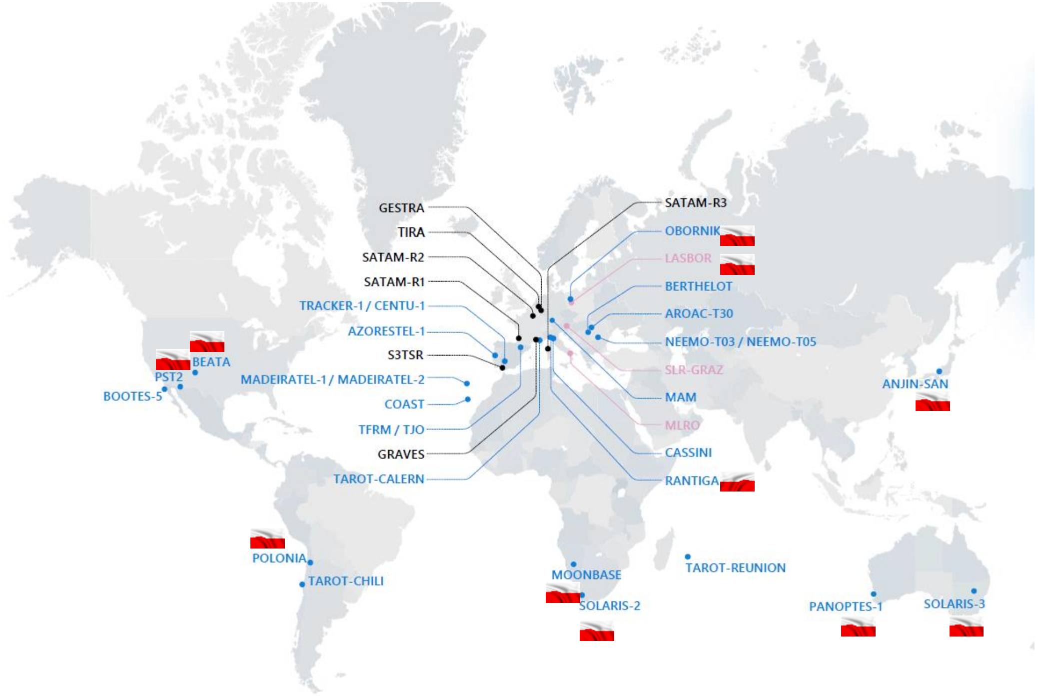


Analytical averaging: $$\frac{d\bar{c}_i}{dt} = \sum_{j=1}^{6} (\bar{c}_i, \bar{c}_j) \frac{\partial}{\partial \bar{c}_j} \left[\frac{1}{2\pi} \int_0^{2\pi} g(\vec{c}, t) d\bar{M} \right]$$

Period of averaging

Numerical averaging: $$\frac{d\bar{c}_i}{dt} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\partial \bar{c}_j}{\partial x} \cdot g\,(\vec{c}, t) d\bar{M}$$

3 lata POLSKI w EU SST



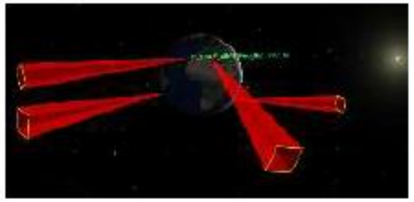
- **Działania operacyjne**: nadzór przesyłania danych do EUSATCEN oraz nad działaniem sensorów PAK, odpowiedzi na żądania wykonania pomiarów specyficznych obiektów np. ponownego wejścia w atmosferę – zachowanie reżimu czasowego maks. 24h, prowadzenie okresowej kalibracji sensorów

Działania organizacyjne: czynności wynikające z pracy komitetów roboczych EU SST oraz projektów wynikających z działalności konsorcjum 1SST2018-20 (grant operacyjny: 299/G/GRO/COPE/19/11109) i 23SST2018-20 (**EU project 952852** R&D H2020).

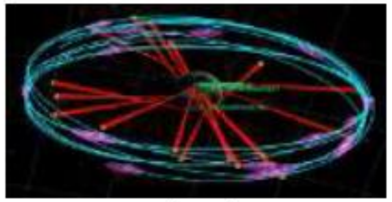
- **Integracja istniejącego oprogramowania Ansys AGI**: ODTK oraz STK na cele operacyjne poprzez moduł GSTT GMSPAZIO Satellite Tracking Toolkit

- **Budowa OPTICAL FENCE -** budowa systemu sensorów o bardzo szerokim polu widzenia do triangulacji optycznej

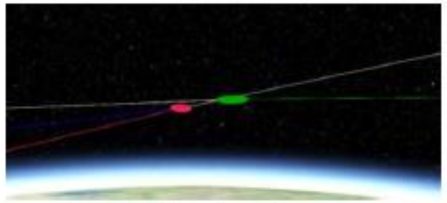
- **Upgrade krajowych sensorów** wykorzystywanych na potrzeby EUSST – grant 23SST.



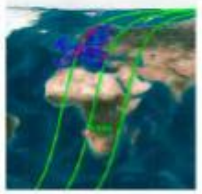
Coverage



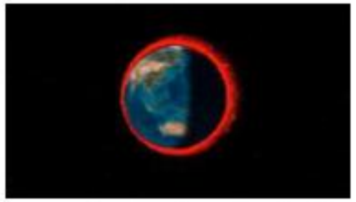
Cataloguing



Collision Avoidance



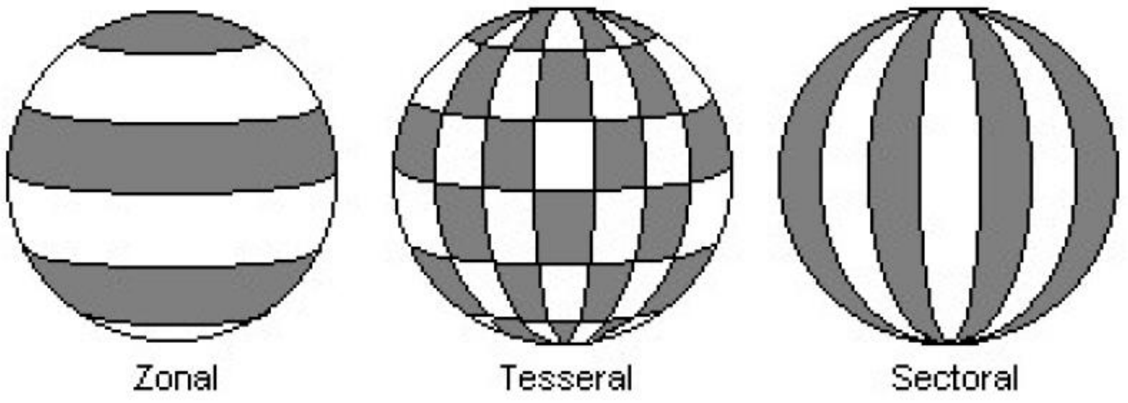
Re-entry Analysis



Fragmentation Analysis

- three types of harmonics:

1. zonal – only depend from the latitude
2. sectorial – only depend from the longitude
3. teseral – depend from both



- first zonal harmonics, **J2,** takes into account that the Earth is an oblate spheroid (the equatorial radius exceeds the polar radius by about 10 km).
- disturbing force that pulls the satellite towards the equatorial plane. This motion is westward for posigrade orbits (inclination <90 degrees) and eastward for retrograde orbits (inclination > 90 degrees). Approximation of the nodal precession:

$$\dot{\Omega} = -\frac{3}{2} J_2 n \frac{R_E^2}{[a(1-e^2)]^2} \cos i$$

- another effect of **J2** is the variation over time of the argument of perigee. Orbit rotates in its orbital plane and the argument of perigee value change over time:

$$\dot{\omega} = \frac{3}{4} J_2 n \frac{R_E^2}{[a(1-e^2)]^2} (5cos^2 i - 1)$$

  - **Those two effects** are called **secular:** variation of the orbital parameter occurs always in the same direction (just depends from the inclination) and have a long term effect.
  - **Other effects** whose net result in an orbit is null: they are called **periodic perturbation.**