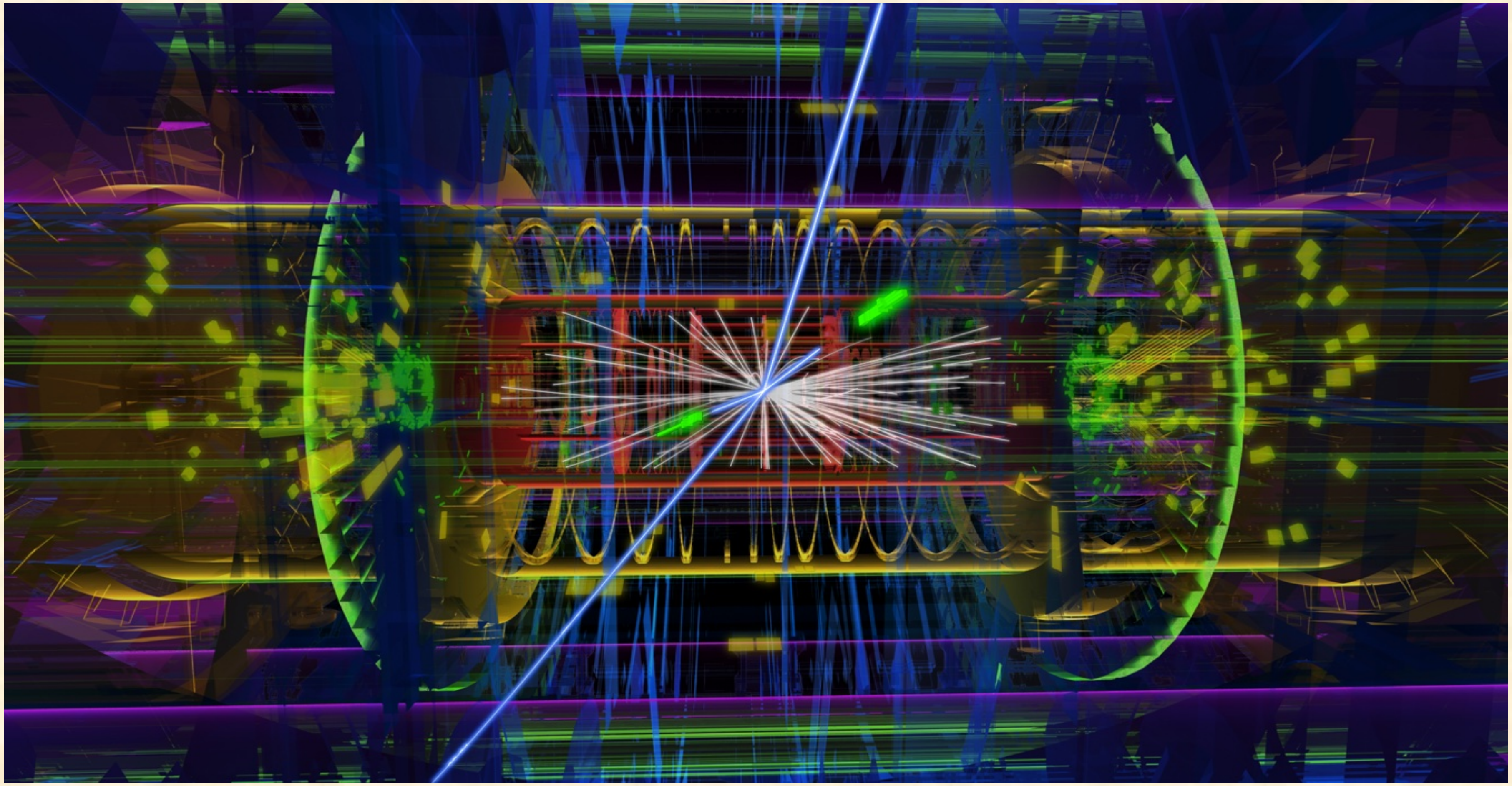
The Large Hadron-Electron Collider LHeC

Bernhard Holzer, CERN, for the LHeC and FCC-eh Study Group



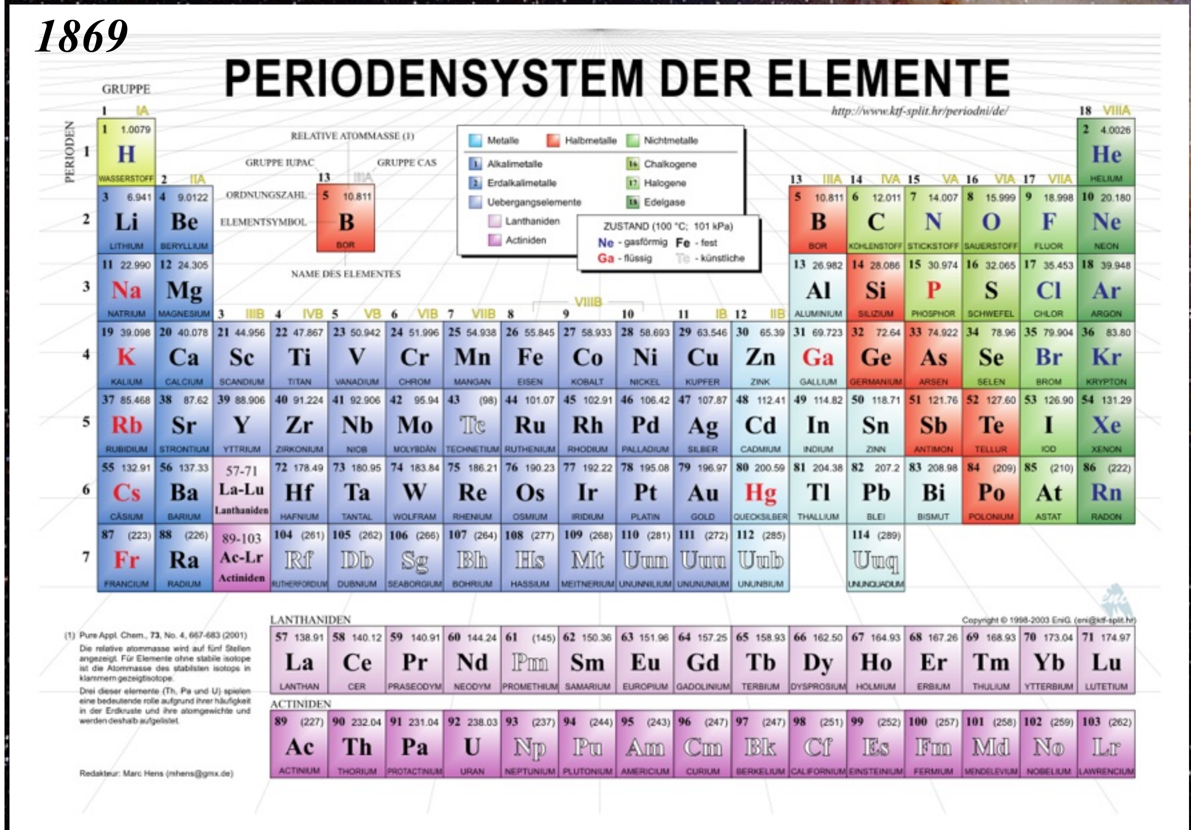
ATLAS event display: Higgs => two electrons & two muons

$$E = m_0 c^2 = m_{e1} + m_{e2} + m_{\mu 1} + m_{\mu 2} = 125.4\ GeV$$

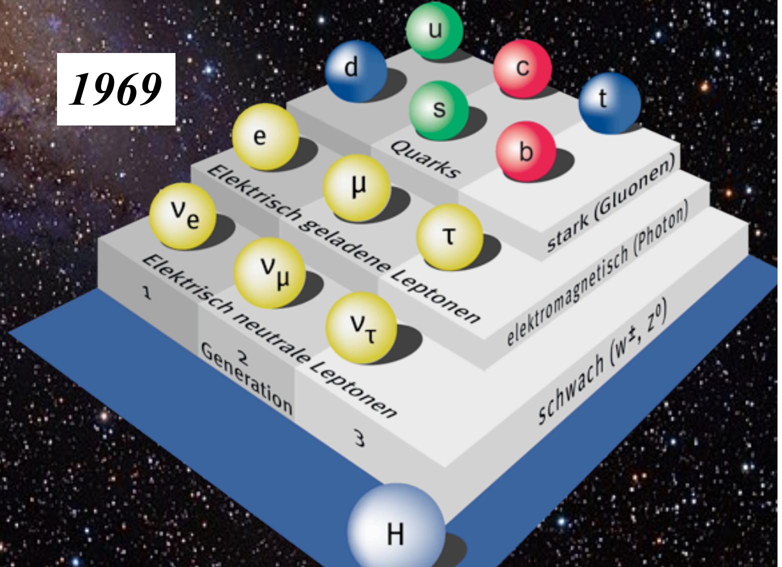
In the end and after all … : We try to explain the structure of "hadronic matter" in the universe.

In short words: "What is going on, up there ???"

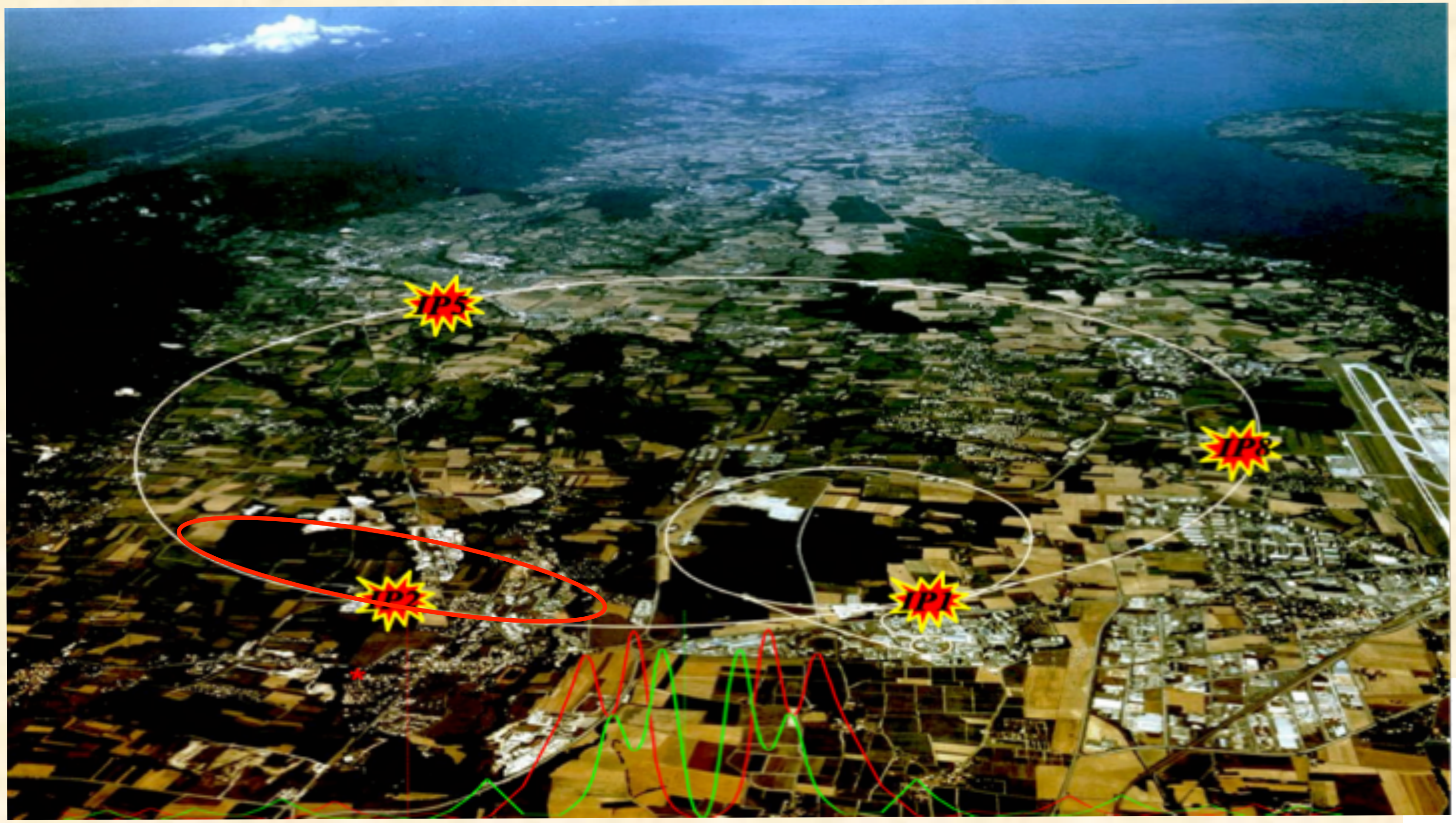
1869



1969



LHeC
Deep Inelastic Scattering of Electrons and LHC Protons



combine a compact ERL of 50 GeV with the 7 TeV protons

LHeC

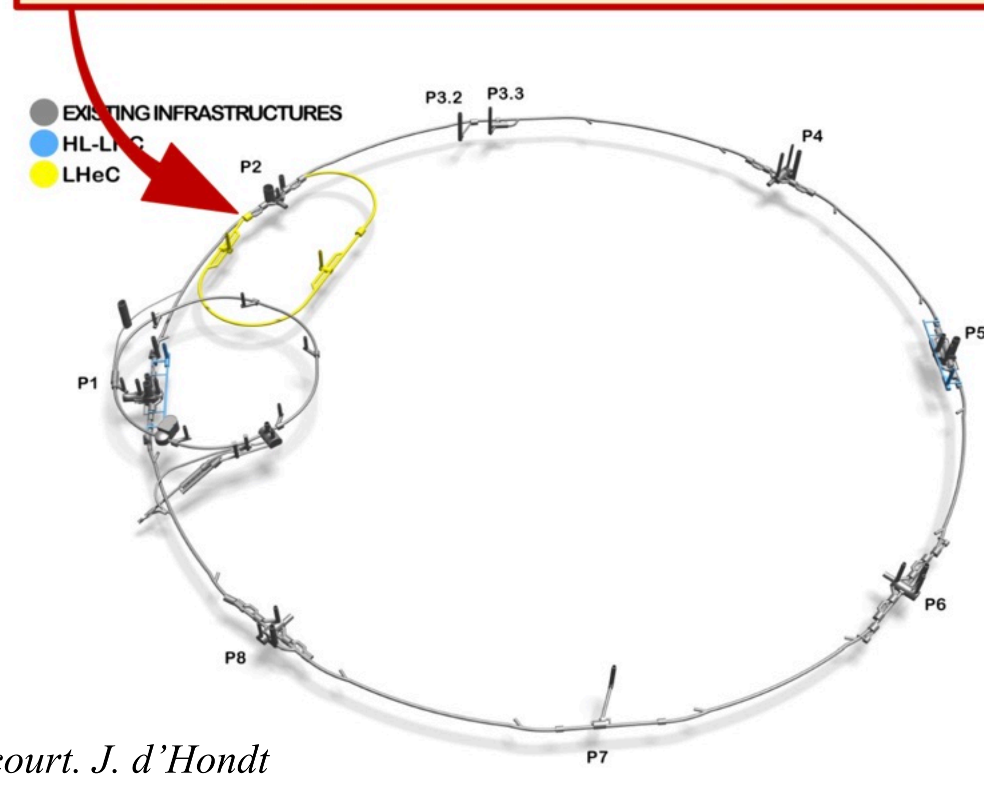
The Large Hadron-Electron Collider at the HL-LHC

LHeC Study Group

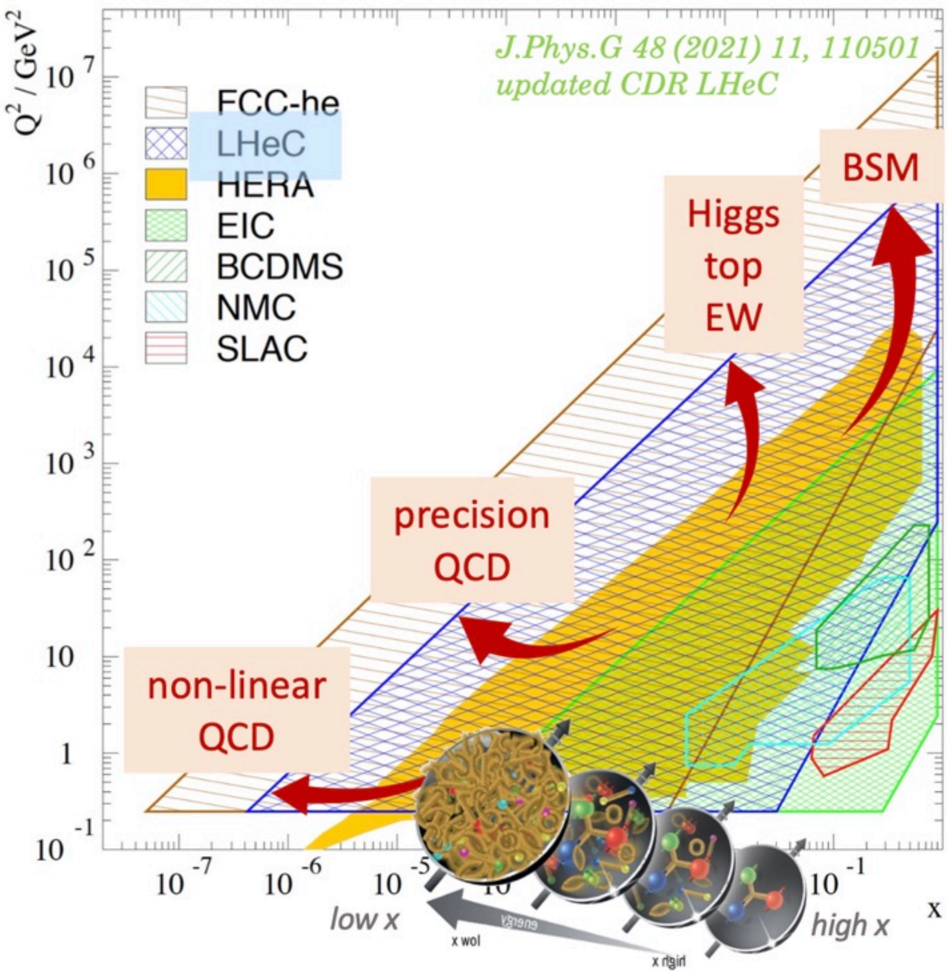
Published in J.Phys. G

The ultimate microscope in hadronic matter: a high-energy electron-hadron collider

LHeC (>50 GeV electron beams)
E_{cms} = 0.2 – 1.3 TeV, (Q^2,x) range far beyond HERA
run ep/pp together with the HL-LHC (≳ Run5)



court. J. d'Hondt



ERL: Energy Recovery Linac

The energy efficiency of present and future accelerators [...] is and should remain an area requiring constant attention.

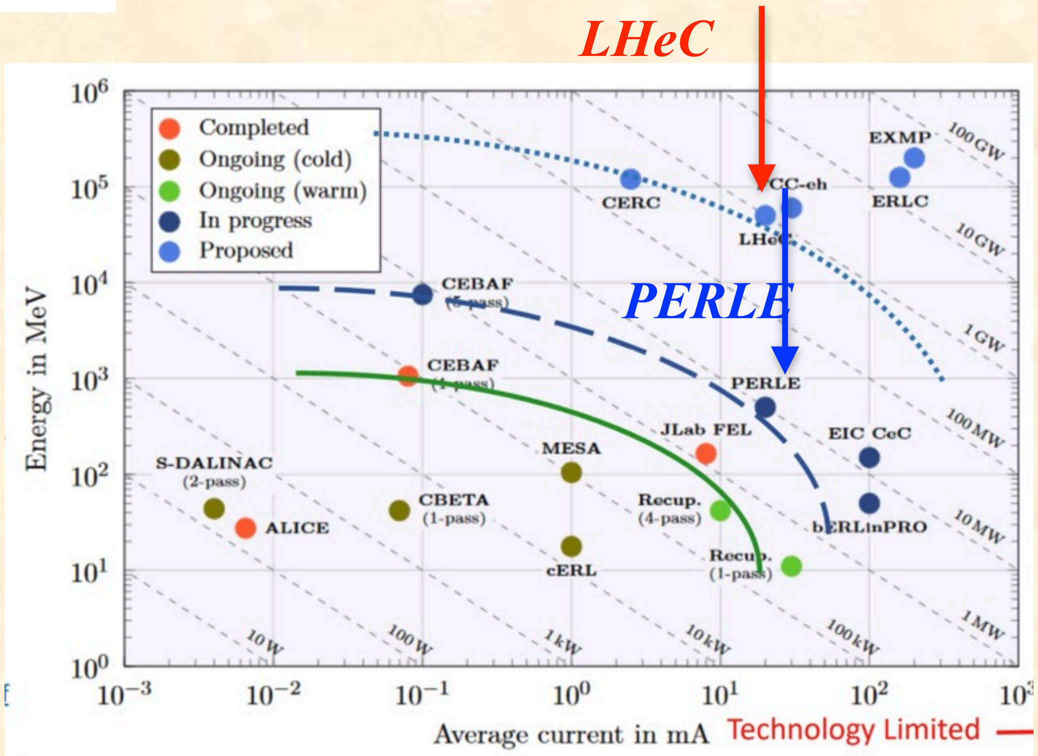
A detailed plan for the [...] saving and re-use of energy should be part of the approval process for any major project.

European Strategy for Particle Physics 2020

Instead of recirculating the electron beam (Storage Ring) (loosing brightness & energy) …

… recirculating the beam energy for new acceleration (high brightness, low radiation losses, high efficiency)

LHeC: Where are we ?



Main challenges,
… what is needed to provide e-p Collisions at LHC ??

Electron Acceleration:
compact, efficient, "green" — > ERL

IR-region: *Electron mini-beta*
Beam separation
Proton mini beta

Individual optics *for p-p collisions*
and e-p collisions

—> colliding p-beam
—> non-colliding proton beam

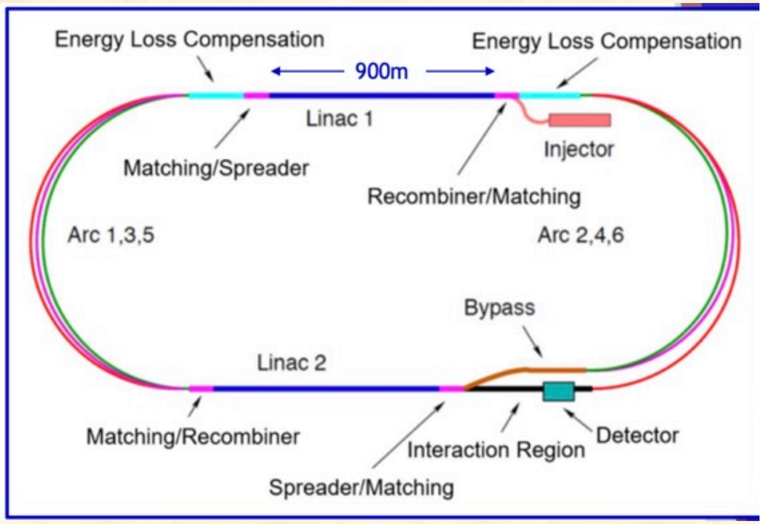
Concurrent e-p and p-p operation

Beam-Beam Effect & Luminosity: *limits & impact for p & e beam*

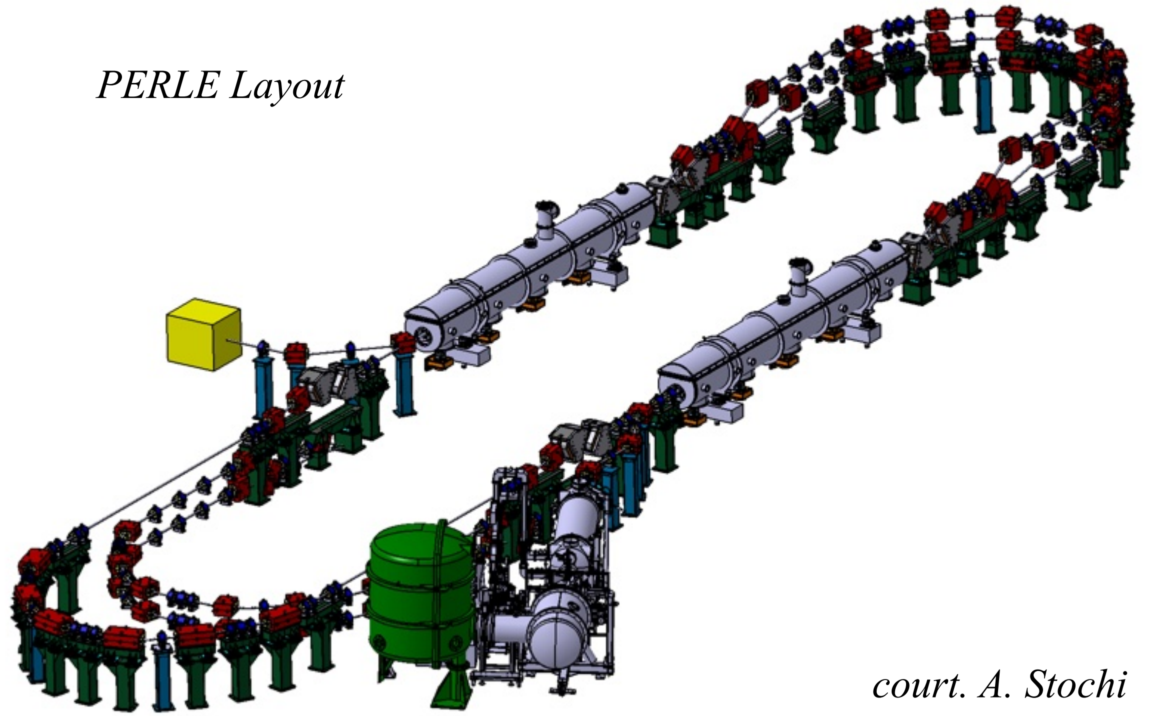
LHeC: The ERL

Design of an ERL based Linac to accelerate electrons and collide with one LHC proton beam

- * *limited size of the electron "ring"*
- * *beam-beam limit pushed far up*
- * *synchrotron radiation limited to "one turn"*
- * *beam energy recovered in the deceleration branch*



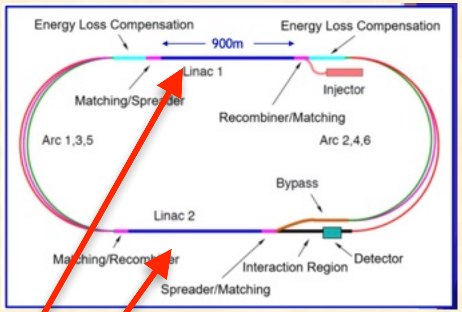
PERLE Layout

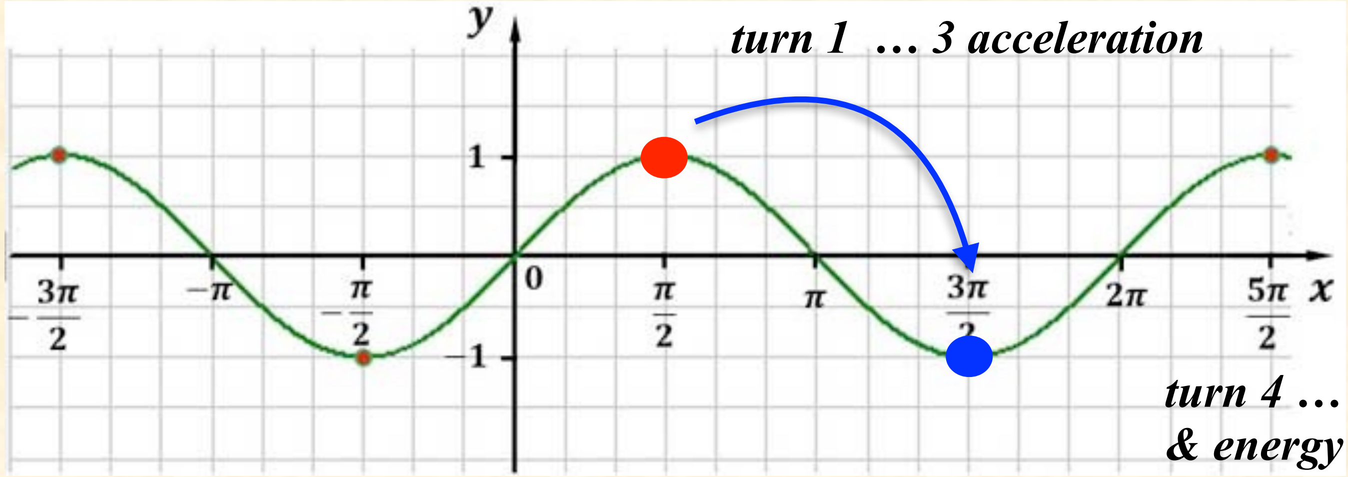


court. A. Stochi

Parameter	***Electrons***
Energy (GeV)	50
N_p /bunch (10^{11})	2.2
N_e /bunch (10^9)	3.1
bunch distance (ns)	25
I_e (mA)	20
Emittance (nm)	0.31
Beam size @ IP (μm)	6 / 6
Length (m)	6665
Luminosity (cm^{-2} s^{-1})	$10^{33} \ldots 10^{34}$
wall plug power	100 MW

LHeC Main Components: sc RF System



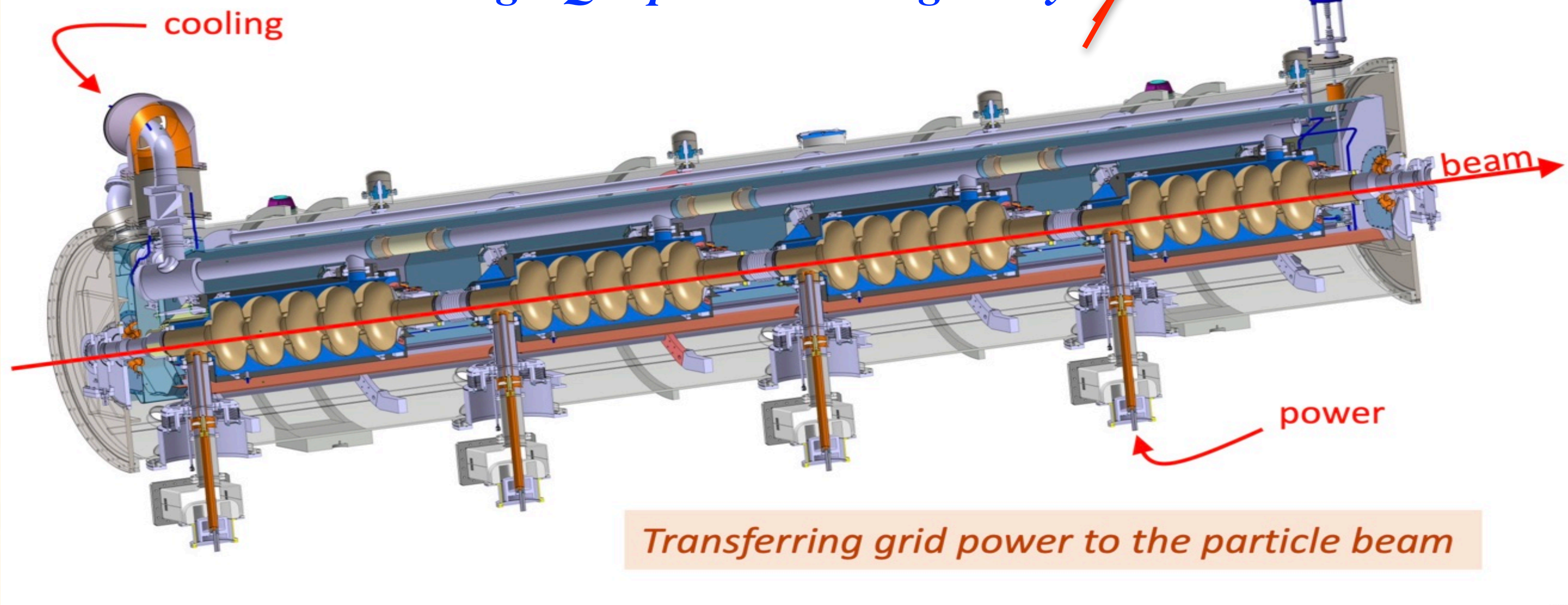


turn 1 … 3 acceleration

phase shift after collision
- via path length of arc 6 -

turn 4 … 6 de-acceleration
& energy recovery

High Q super conducting RF system

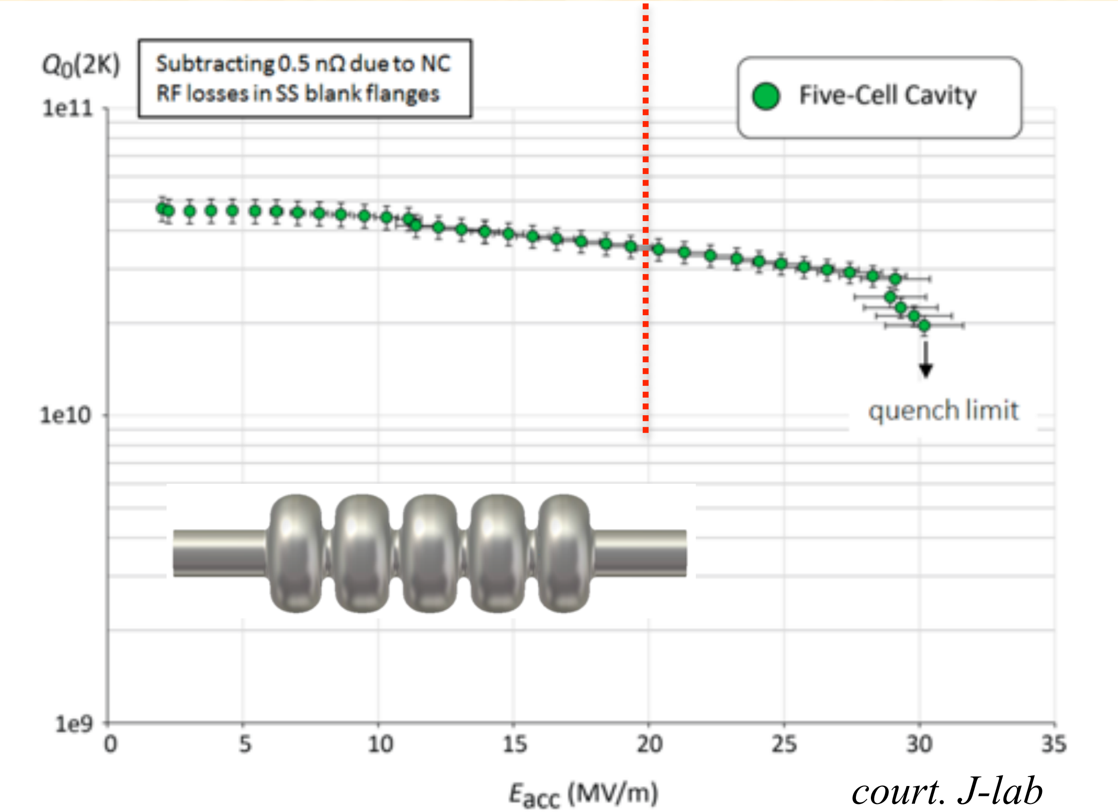


LHeC Main Components: sc RF System

I_e = 20 mA … at IP —> 3 turns for acceleration
—> 3 turns for deceleration

energy of decelerated bunches stored in RF field to accelerate the new bunches

careful balance of fields (and phases) — > high Q sc. RF system



court. J-lab

$$\text{quality factor } Q = \omega_0 \frac{W_{stored}}{P_{loss}}$$

Prototype design of 5 cell sc. cavity (J-lab)

Required Acc Gradient: 20 MV/m
Frequency: 801.58MHz

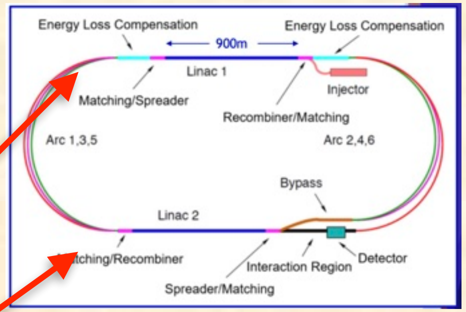
crucial test: PERLE

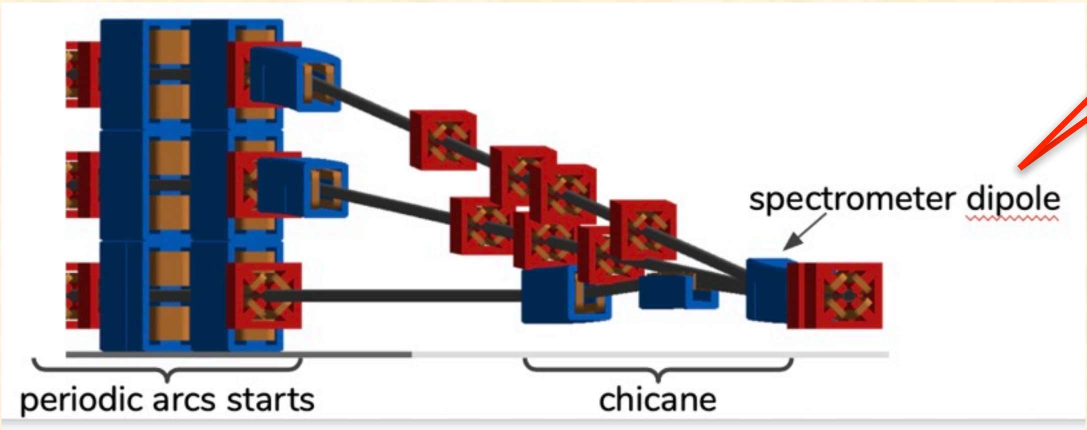
LHeC Main Components: Beam Spreader / Re-Combiner

Distribute / re-combine the beam before / after each linac to the corresponding arc structure

***Challenge:** minimise emittance dilution … in the vertical plane*

- ***Non-dispersive (i.e. "achromatic") vertical deflection system***
- ***Gently matched beam optics between Linacs and Arcs***
- ***Optimised for smallest impact on ε_y***

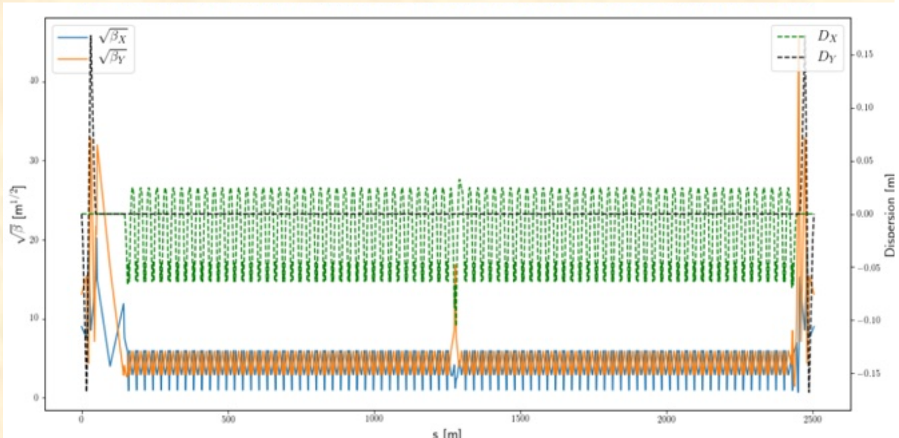




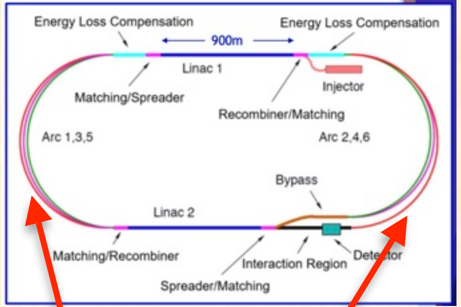
$$\mathscr{H}_x = \gamma_x(\eta_x)^2 + 2\alpha_x\eta_x\eta_x' + \beta_x(\eta_x')^2$$

ERL beam optics: spreader,
dispersion suppressor
arc structure & re-combiner

court. A. Bogacz, K. André

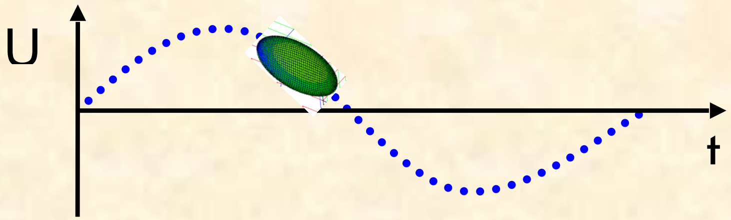


Return Arcs: … a piece of Art *court. A. Bogacz*

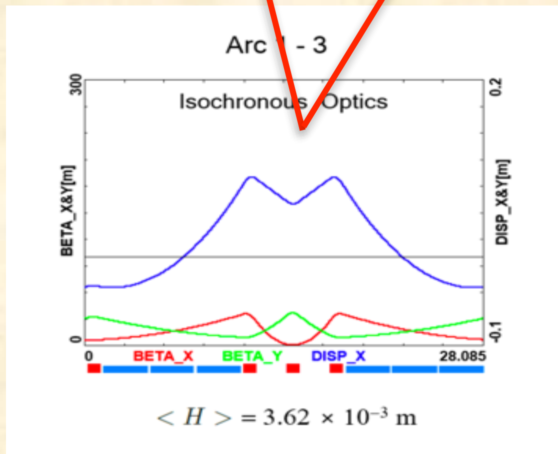


FMC cell to optimise for low / high energy arcs:

**** low energy: keep bunches short for optimum phase "spread"***



isochronous
optics arc 1,2,3

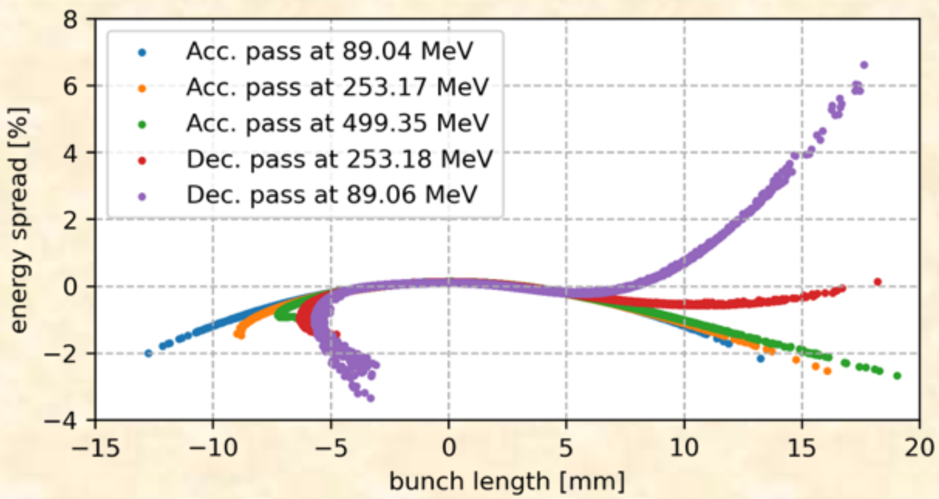


Develop a focusing structure for constant revolution time, independent of the particle energy

—> keep the bunch length short

Simulation for the PERLE-ERL

long bunches "see" the non-linear rf field and distort in phase space



Return Arcs: … a piece of Art

High Energy Arcs: ***Arc 4 … 6***

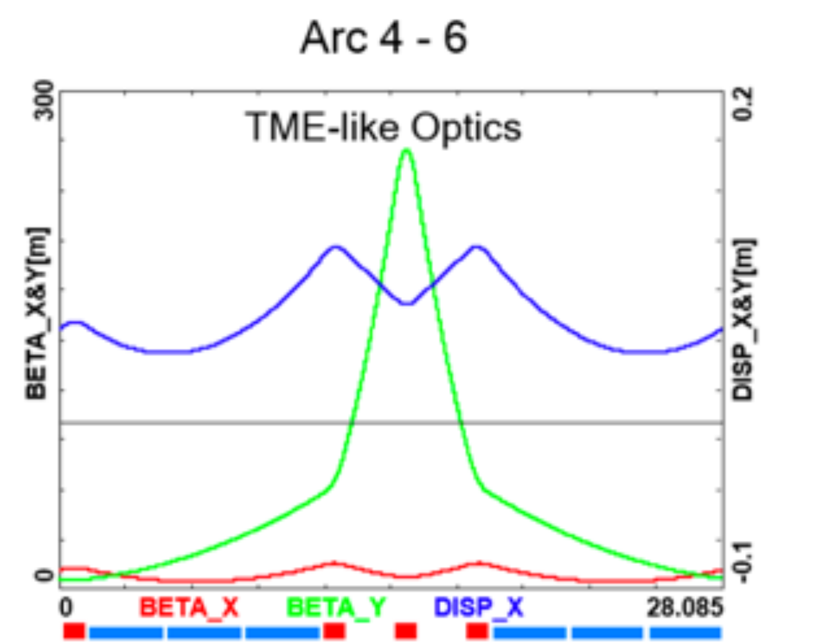
v = const, keep radiation effects small
well known problem in synchrotron light sources

$$\varepsilon_0 = C_q \frac{\gamma^2}{j_x} \frac{I_5}{I_2}$$

$$I_2 = \oint \frac{1}{\rho^2} ds \approx \frac{2\pi}{\rho}$$

$$I_5 = \oint \frac{\mathcal{H}_x}{\rho^3} ds$$

$$\mathcal{H}_x = \gamma_x (\eta_x)^2 + 2\alpha_x \eta_x \eta_x' + \beta_x (\eta_x')^2$$



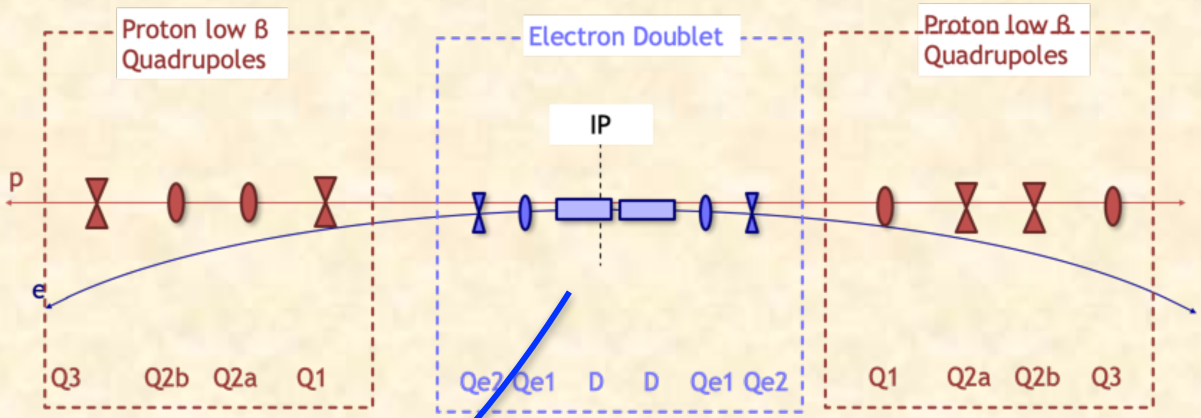
$< H > = 0.59 \times 10^{-3}$ m

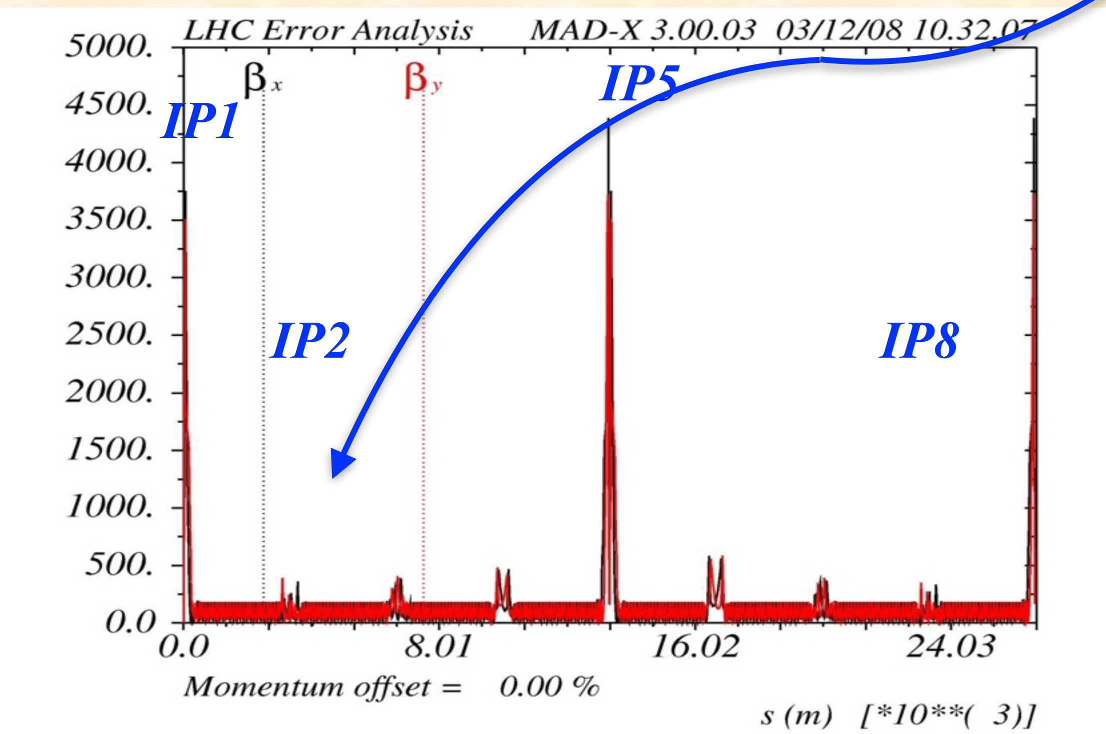
low emittance
optics arc 4,5,6

Main Systems: Interaction Region

court. T. vonWitzleben

Double Mini-Beta Insertion
imbedded e-p collisions in LHC standard structure





proton optics "modular" within the LHC periodic arc structure

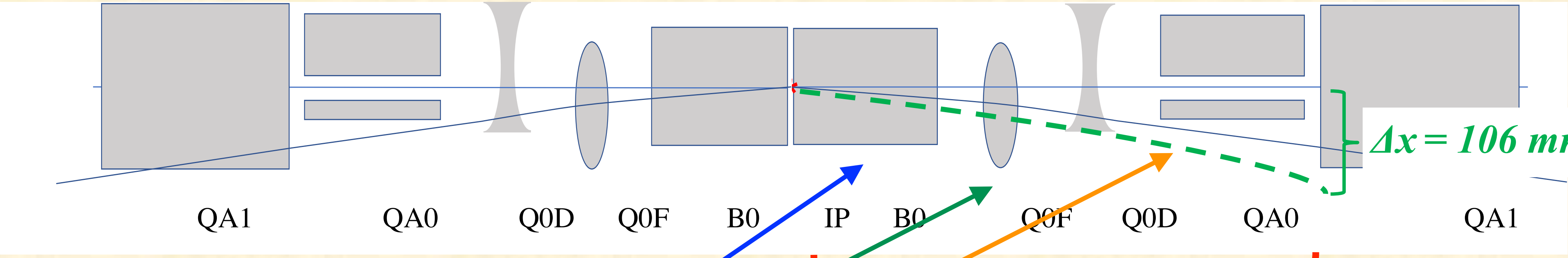
electron optics insertion within the p-final focusing

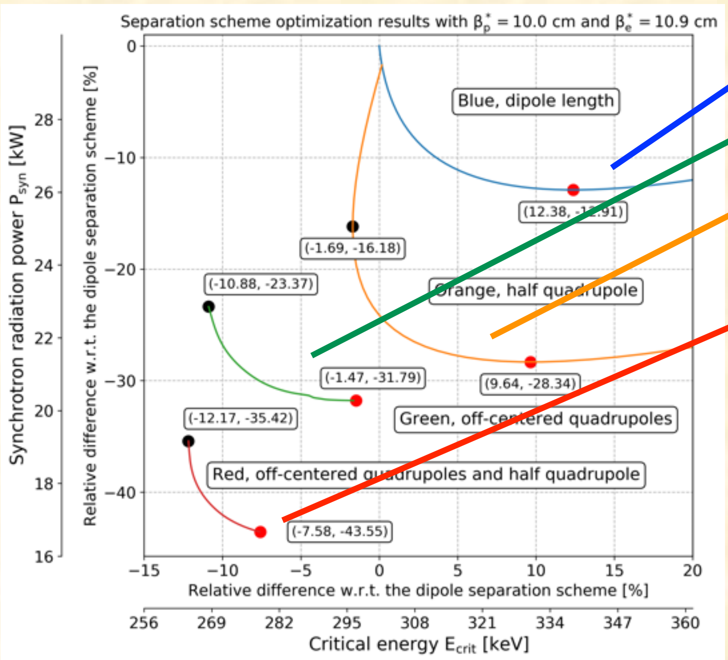
early beam separation scheme

The Interaction Region

Separation Scheme of the Electrons

court. Kevin André





emitted synchrotron radiation power

$$P_\gamma = \frac{e^2\, c}{6\pi\epsilon_0} \cdot \frac{\gamma^4}{\rho^2}$$

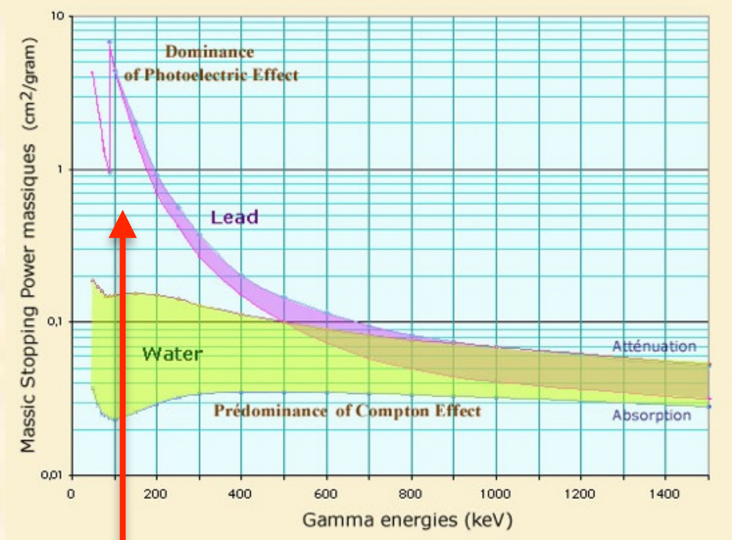
the complete magnet structure is used to provide soft bending of the electron beam for minimum emitted synchrotron light.

The Interaction Region:

Synchrotron Light & Emittances

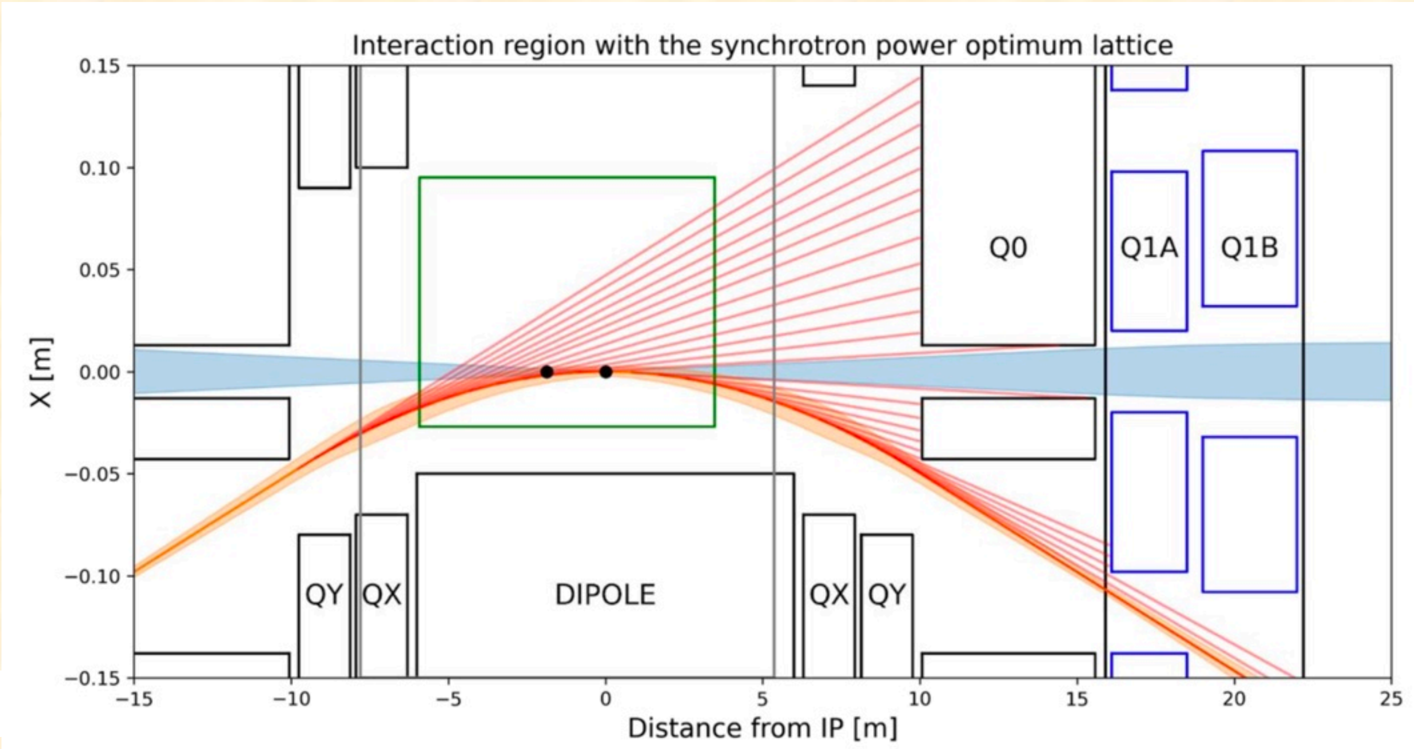
critical energy $$E_{crit} = \frac{3hc}{2}\frac{\gamma^3}{\rho}$$

radiated power $$P_{syn} = \frac{e^2c}{6\pi\varepsilon_0}\frac{\gamma^4}{\rho^2}$$



emitted synchrotron radiation during beam separation

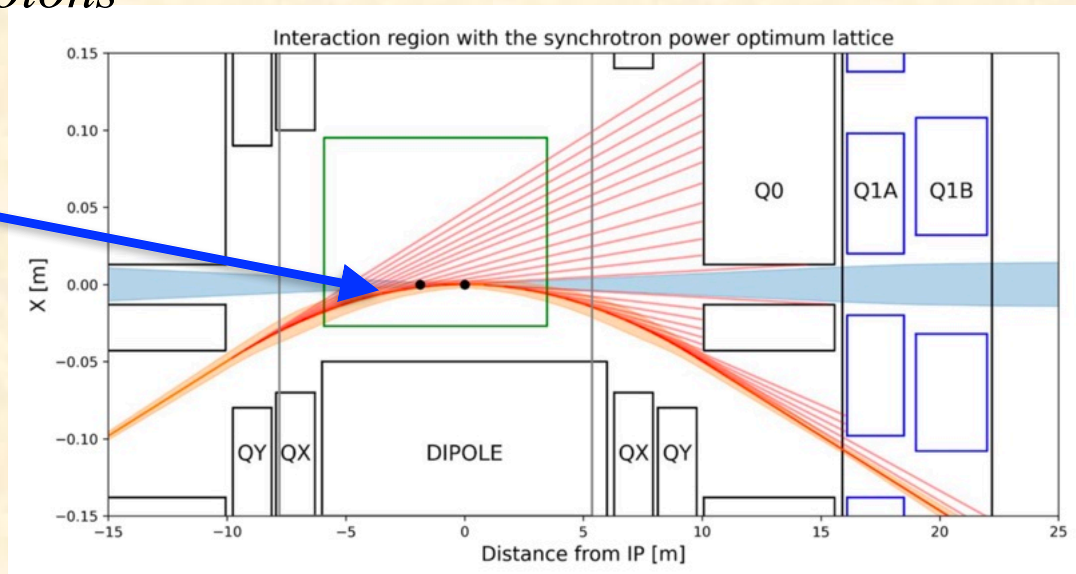
Shielding of Detector & sc Magnets

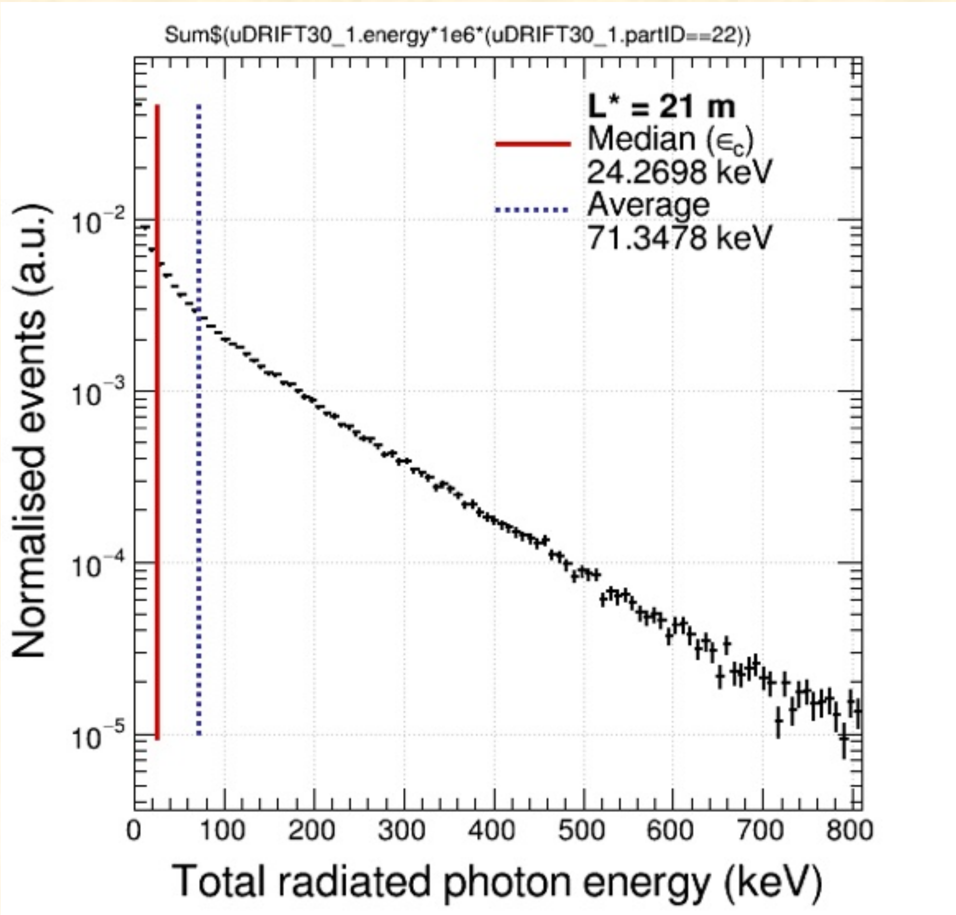


court. Dan Hanstock

The Interaction Region:
Synchrotron Light & Emittances

Detector beam pipe
optimised for vertex reconstruction & photons





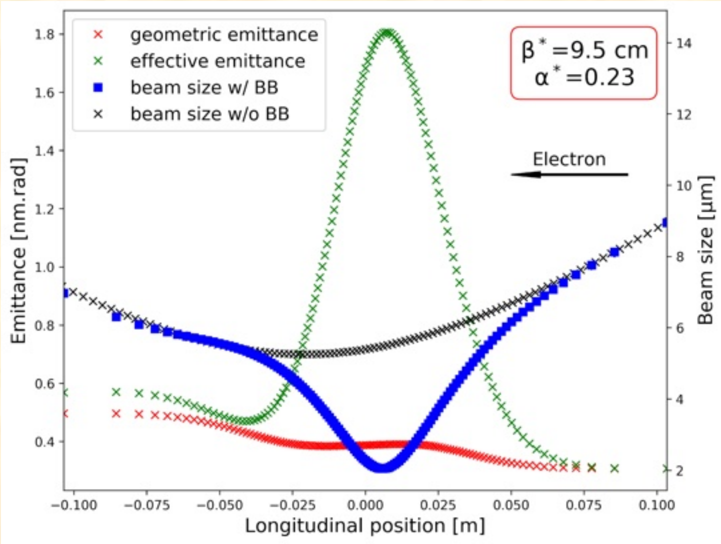
Critical energy ≈ 120 keV
challenging but possible

court. Laurent Forthomme, AGH

Electron Emittance & Beam Beam Effect

court. K. André

Optimise optics: Rematch including the beam-beam focusing

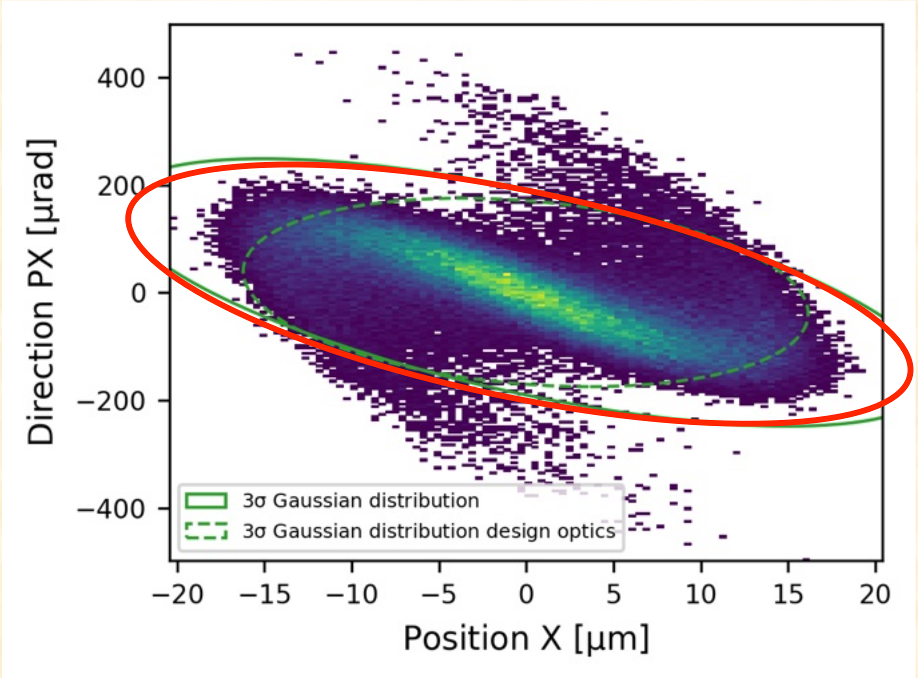


IR Optics for minimum
Optics mismatch

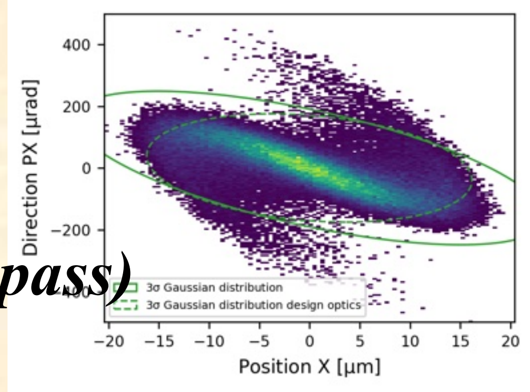
Performance Limit: Beam disruption

development of tails due to
non-linear beam beam force

$$D_{x,y} = \frac{2\ N\ r_0\ \sigma_z}{\gamma\ \sigma_{x,y}\ (\sigma_x + \sigma_y)}$$



ERL Performance:



front-to-end tracking, including

… emittance blow up (radiation in arcs, spreader, bypass)
beam separation scheme
energy gain in linacs
energy loss in arcs
beam-beam effects

particle distribution after IP
as starting conditions for the deceleration & energy recovery

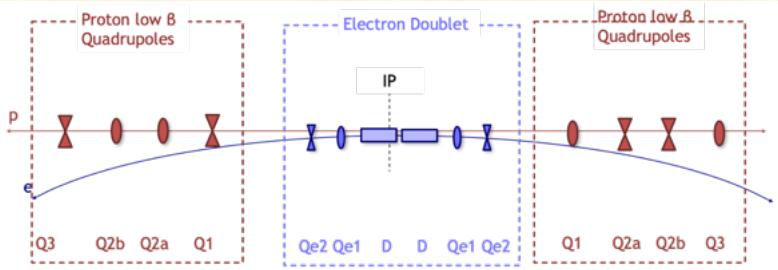
ERL performance: ≈ 98 %

1/3	unit	Injection	Until IP	Post IP	Dump	Energy recovery
εx, εy	um.rad	25.4, 29.4	30.0, 30.0	47.7, 45.2	89.6, 202.6	
dpp	%	0.02	0.0210	0.0210	4.174	97.9 %
Transmission	%	-	100	100	99.93	

Proton Beam Performance

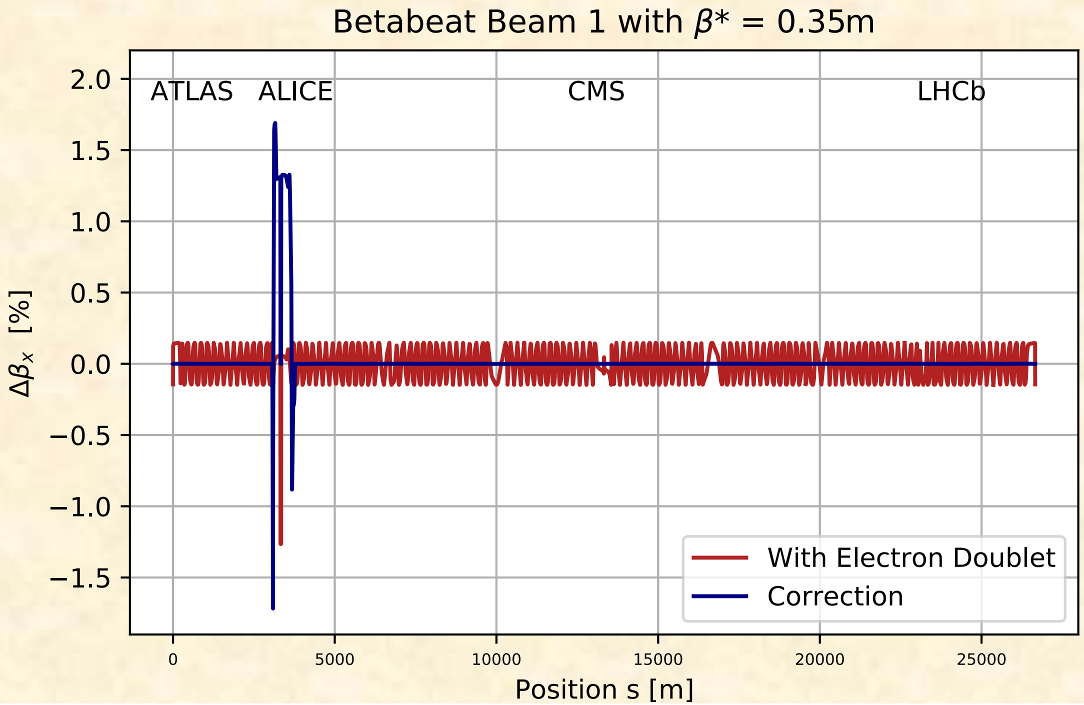
p-Optics & e-Quadrupoles

court. T. vonWitzleben



local orbit bump
local optics distortion
—> on colliding proton beam
—> non-colliding proton beam

corrected locally via LHC matching quadrupoles



beam-beam effect: —> negligible

in linear approximation —> tune shift

$$\Delta Q_{x,y} = \frac{N\, r_0\, \beta^*_{x,y}}{2\pi\gamma\, \sigma_{x,y}\, (\sigma_x + \sigma_y)}$$

$\Delta Q_{p,p} \approx -3.1 \cdot 10^{-3}$ *per IP* *for* $N_p = 1.5 \cdot 10^{11}$

for $N_e = 3.1*10^{9}$ *—> negligible* $\Delta Q_{e,p} \approx +6.4 \cdot 10^{-5}$ *for* $N_e = 3.1 \cdot 10^{9}$

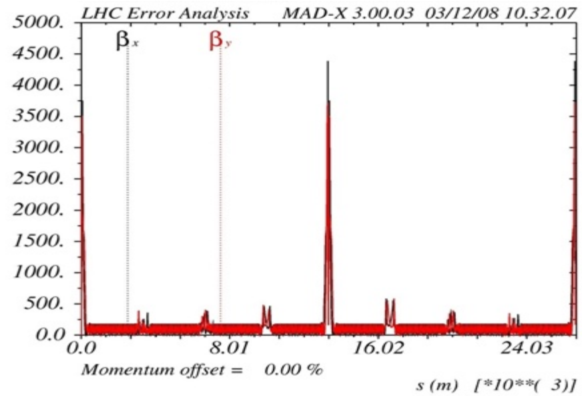
Proton Beam Performance: *two fold operation mode*

Create a three beam optics, with e-p collisions and one - relaxed - proton beam passing by

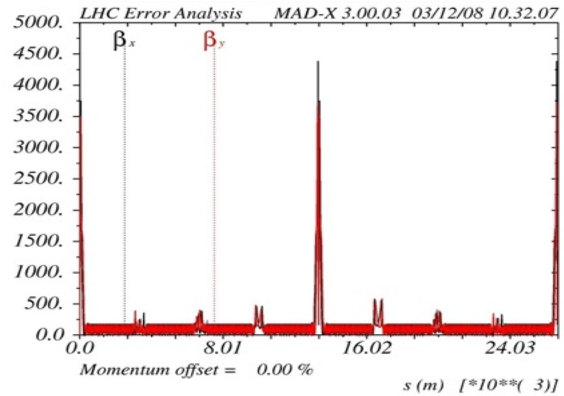
h-h operation:

standard LHC collision optics

p-beam 1

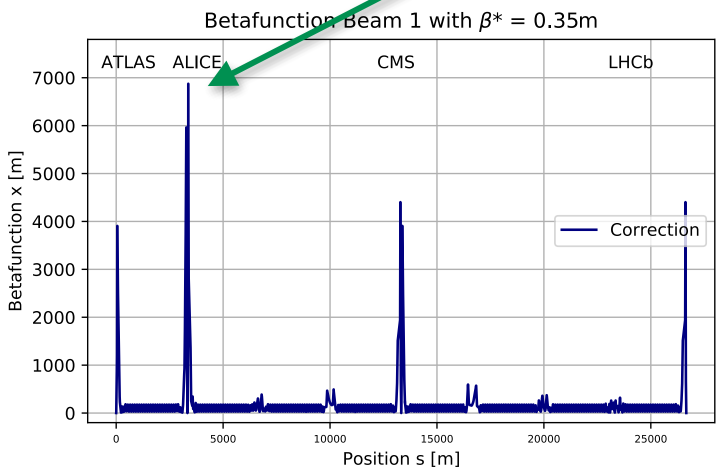


p-beam 2

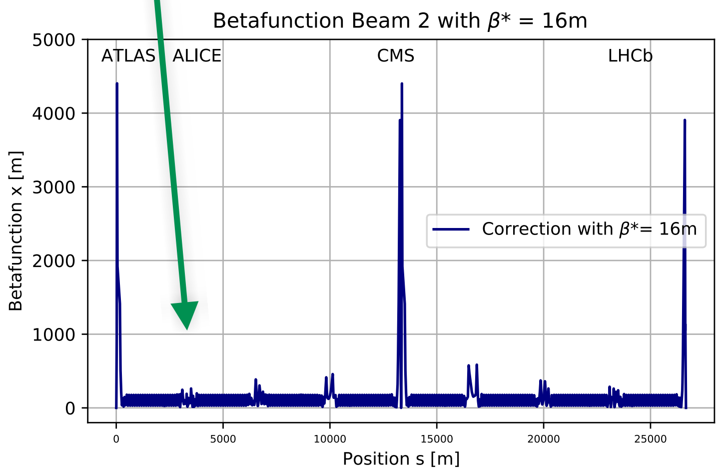


e-p operation:

p-beam 1
—> high luminosity optics



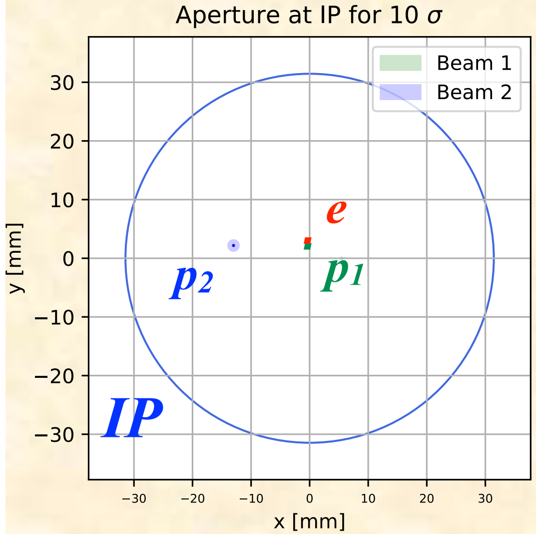
p-beam 2
—> relaxed optics
max aperture margin



court. T. vonWitzleben

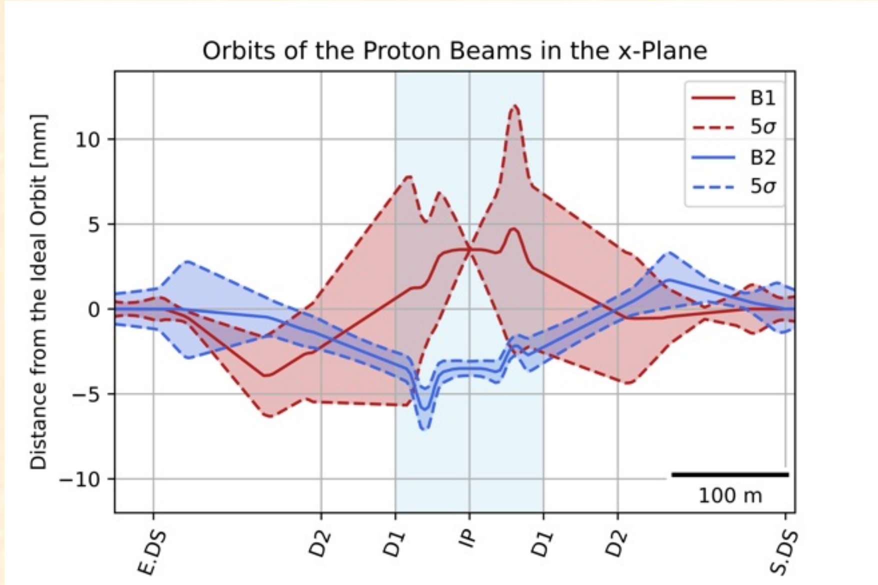
Proton Beam Performance

Design Orbits & Aperture Need

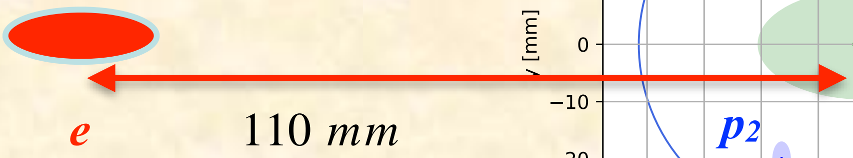


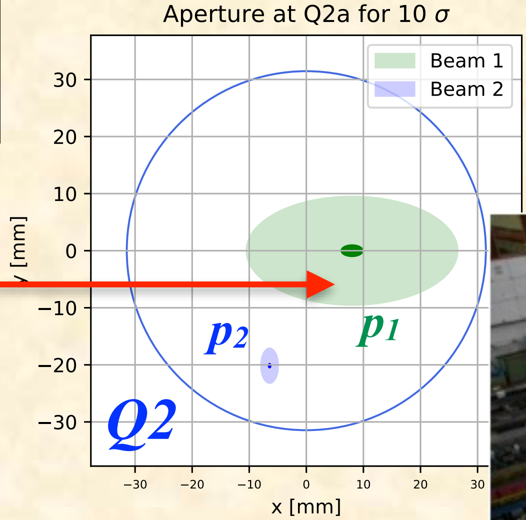
$$\sigma_1 = \sqrt{\varepsilon\beta} = 10\mu m$$

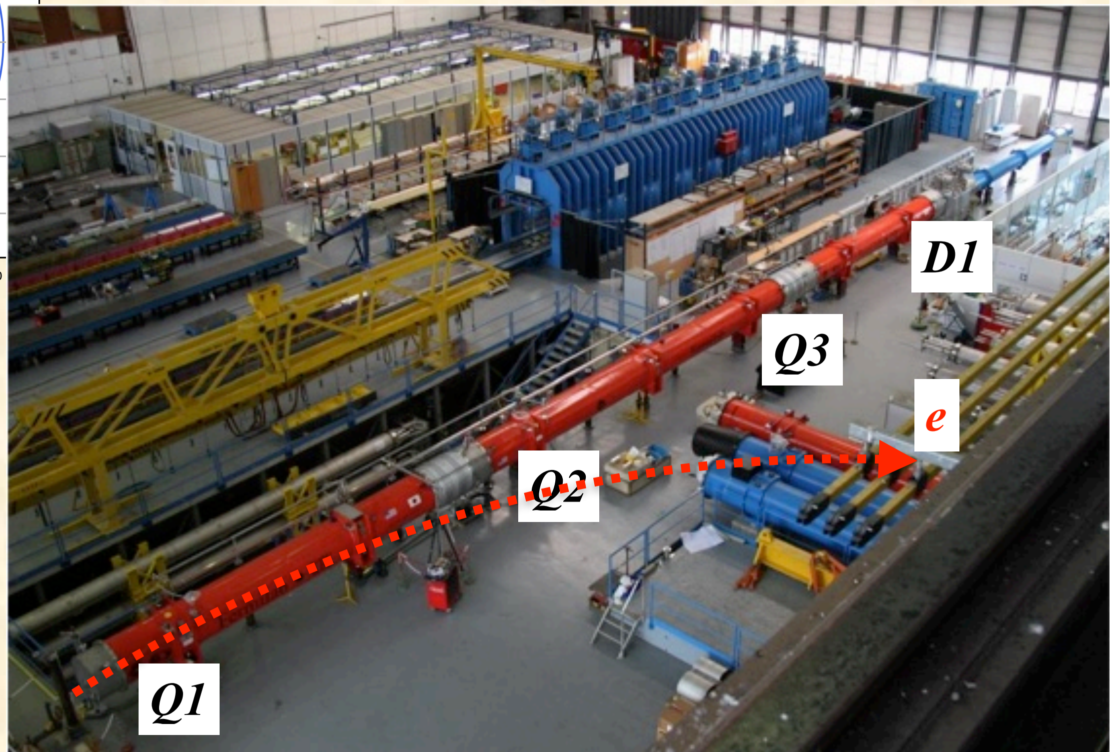
$$\sigma_2 = \sqrt{\varepsilon\beta} = 73\mu m$$



court. M. Smith



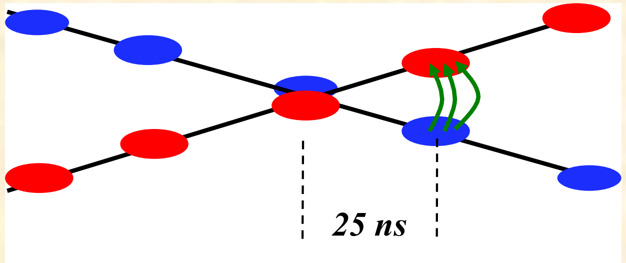




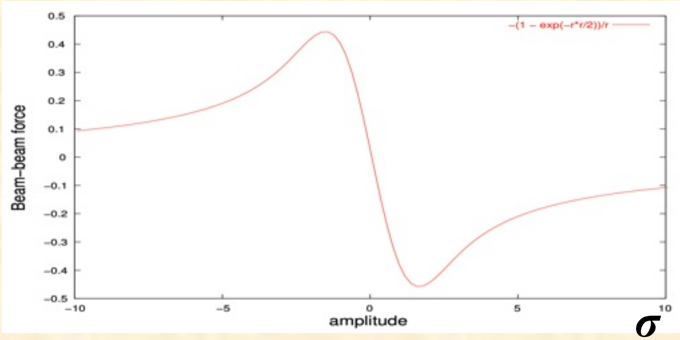
LHC Mini-Beta Insertion

Proton Emittance & Beam Beam Effect

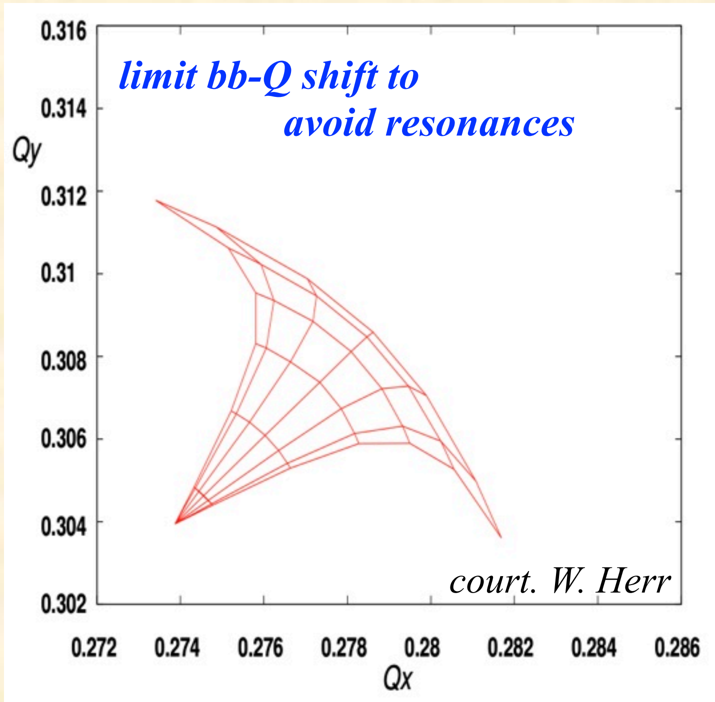
… a non-problem



… the ultimate limit of any collider
space charge of the colliding bunch has a detrimental effect on the opposing bunch



Beam Beam Force (round beams)



in linear approximation:
tune shift (LHC)

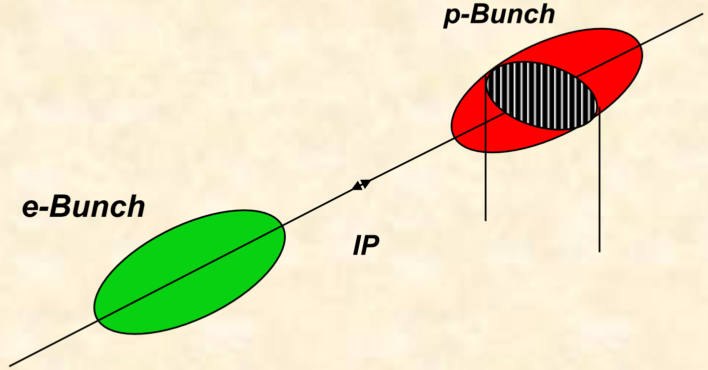
$$\Delta Q_{x,y} = \frac{N_e \, r_0 \, \beta^*_{x,y}}{2\pi\gamma \, \sigma_{x,y} \, (\sigma_x + \sigma_y)}$$

$N_e \approx 3 \cdot 10^9 \quad \leftarrow \rightarrow \quad N_p \approx 2.2 \cdot 10^{11}$

LHeC adds to the tune shift in the percent level

Finally the Luminosity:

$$L = \frac{N_e \cdot N_p \cdot n_b \cdot f_{rev}}{2\pi\sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \cdot \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}} \cdot \Sigma_i H_i$$



matched beam sizes

$$\sigma_{px} = \sigma_{ex} \quad = \quad \sigma_{py} = \sigma_{ey}$$

Correction factors

$$\Sigma_i H_i \approx 1$$

$$L = \frac{N_e \cdot N_p \cdot n_b \cdot f_{rev}}{4\pi\ \varepsilon_p\ \beta_p^*}$$

	Electrons	Protons
Energy (GeV)	50	7000
N /bunch	$3.1\ 10^{9}$	$2.2\ 10^{11}$
bunch distance (ns)	25	
I (mA)	20	1100
Emittance (nm)	0.31	0.33
Beam size @ IP (µm)	6 / 6	
Luminosity ($cm^{-2}\ s^{-1}$)	$9*10^{33}$	

$$\beta_p^* \approx 35\ cm \ldots 10\ cm$$

wall plug power: 100 MW


The Next Generation Ring Collider
FCC-ee / FCC-hh / FCC-eh
FCC
LHC
PS
SPS
FCC
hh ee he

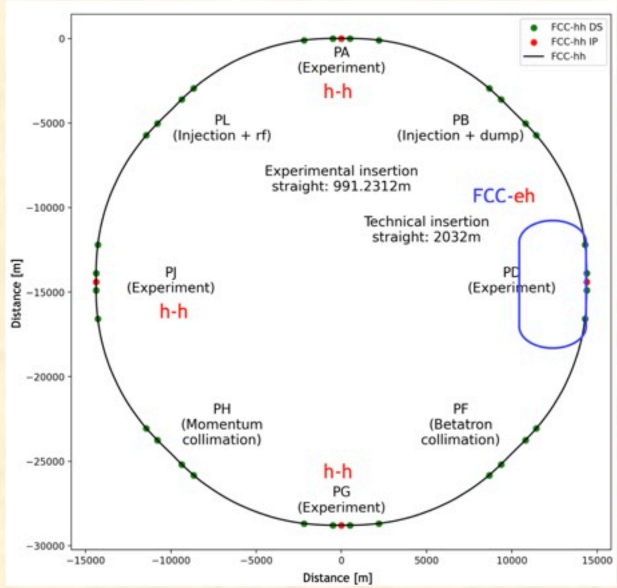
FCC-eh

Condition for an ideal circular orbit:

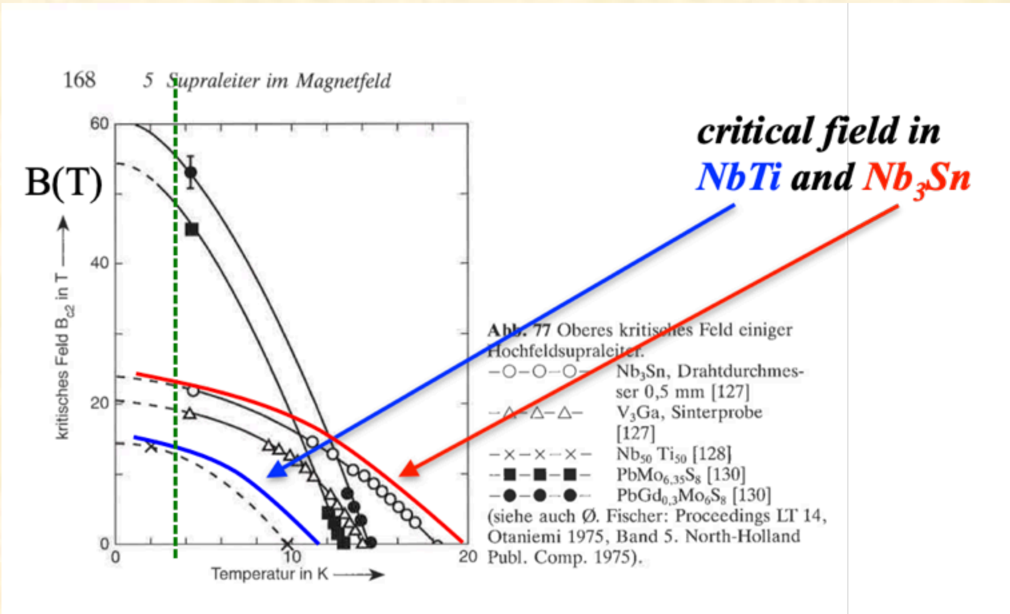
Lorentz force $$F_L = e\,v\,B$$

centrifugal force $$F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

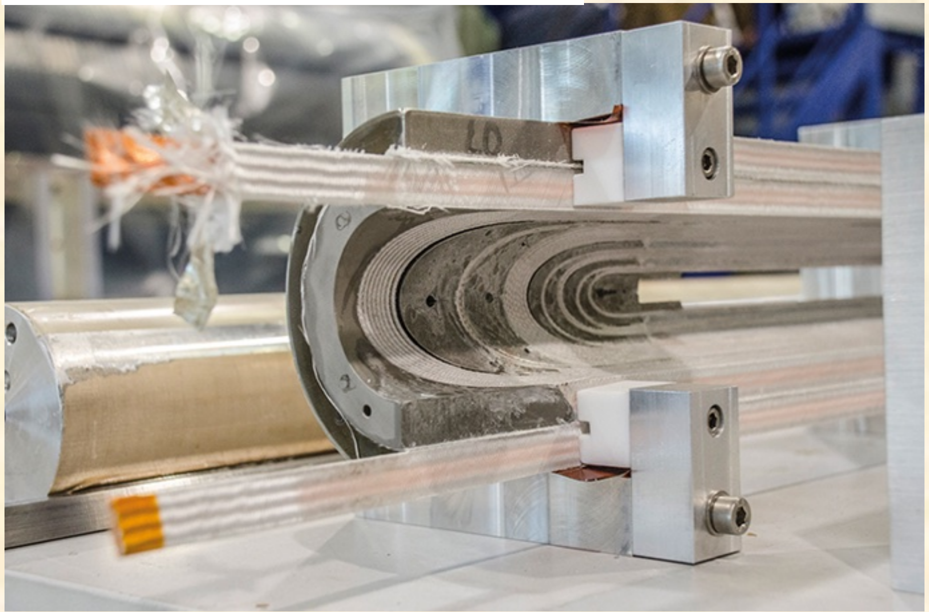
momentum $$\frac{\gamma\, m_0\, v^2}{\rho} = e\,v\,B$$



B ρ = "beam rigidity" $$\frac{p}{e} = B\rho$$



Nb_3Sn FCC type dipole coils,
11 T – 16 T

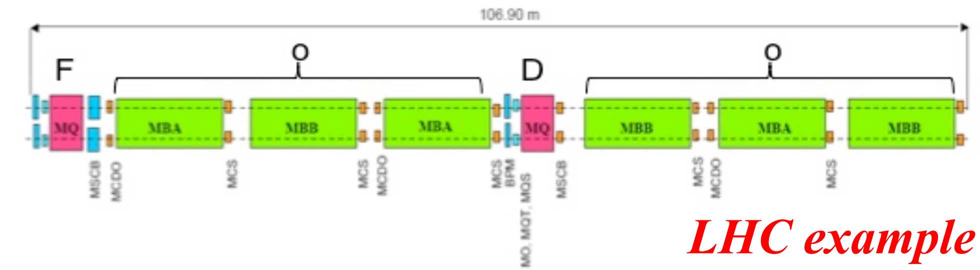


FCC-hh Parameter List

Pushing the limit (Dipole Fill Factor):

12 dipoles per cell, l_{dipole}=14.2m
34 cells per arc
12 arcs
dipole field = 16T <—> 50TeV

} *5016 dipoles*



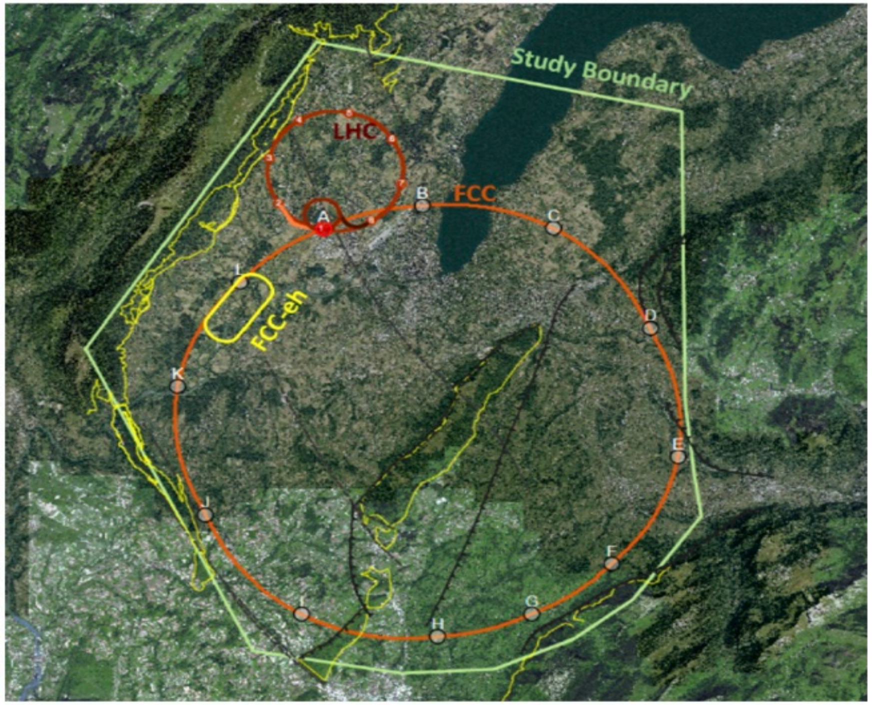
LHC example

	LHC	HL-LHC	FCC-hh	
			Initial	Nominal
Main parameters and geometrical aspects				
c.m. Energy (TeV)	14		100	
Circumference C (km)	26.7		97.75	
Dipole field (T)	8.33		<16	
Physics performance and beam parameters				
Peak luminosity[1] (10^{34} cm^{-2}s^{-1})	1.0	5.0	5.0	<30.0
Beam parameters				
Number of bunches n	2808		10 400	
Bunch spacing (ns)	25	25	25	
Bunch population $N(10^{11})$	1.15	2.2	1.0	
RMS bunch length[2] (cm)	7.55		8	
IP beta function (m)	0.55	0.15 (min)	1.1	0.3
RMS IP spot size (μm)	16.7	7.1 (min)	6.8	3.5
Full crossing angle (μrad)	285	590	104	200[3]
Other beam and machine parameters				
Stored energy per beam (GJ)	0.392	0.694	8.3	
SR power per ring (MW)	0.0036	0.0073	2.4	

FCC-eh

e-p IR-Design modular

—> ERL & IR can be imbedded at any straight section e.g. point "L"



FCC-CDR: Eur.Phys.J.ST 228 (2019, 4.755) ***FCC-hh/eh***

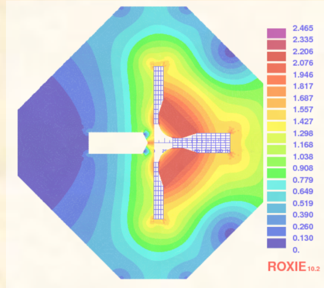
	Electrons	Protons
Energy	60 GeV	50 TeV
N /bunch	3.1 10^{9}	2.2 10^{11}
bunch distance (ns)	25	
I (mA)	20	1100
Emittance (nm)	0.31	0.05
Beam size @ IP (µm)	2.5 / 2.5	
Luminosity (cm^{-2} s^{-1})	$1.5*10^{34}$	

60 GeV × 50 TeV

—> 1.5 TeV collider

Operation: 2050 +

Challenges & Next Steps:

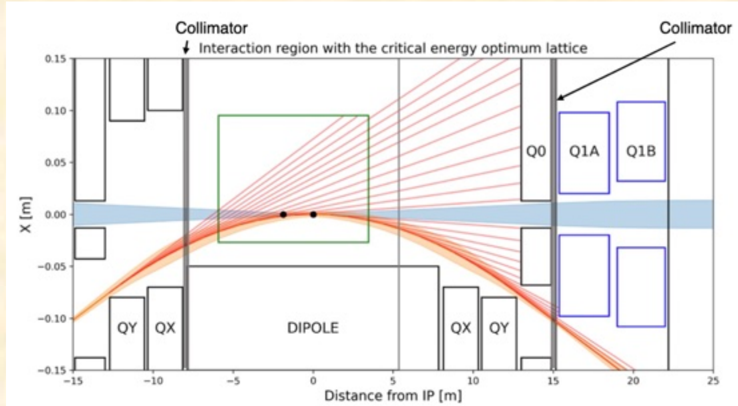


Design of ERL, Proton Optics ✓

- *beam separation scheme*

Design for prototypes of special magnets Q0/Q1: ✓

- *half-quadrupole in IR*
- *sc. "field free" quadrupole*

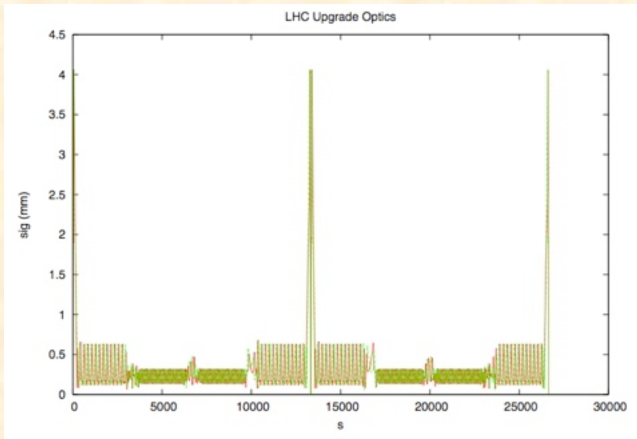


Synchrotron Radiation & Shielding ✓

- *MDI - inner detector*
- *sc. Quadrupoles down-stream IP*

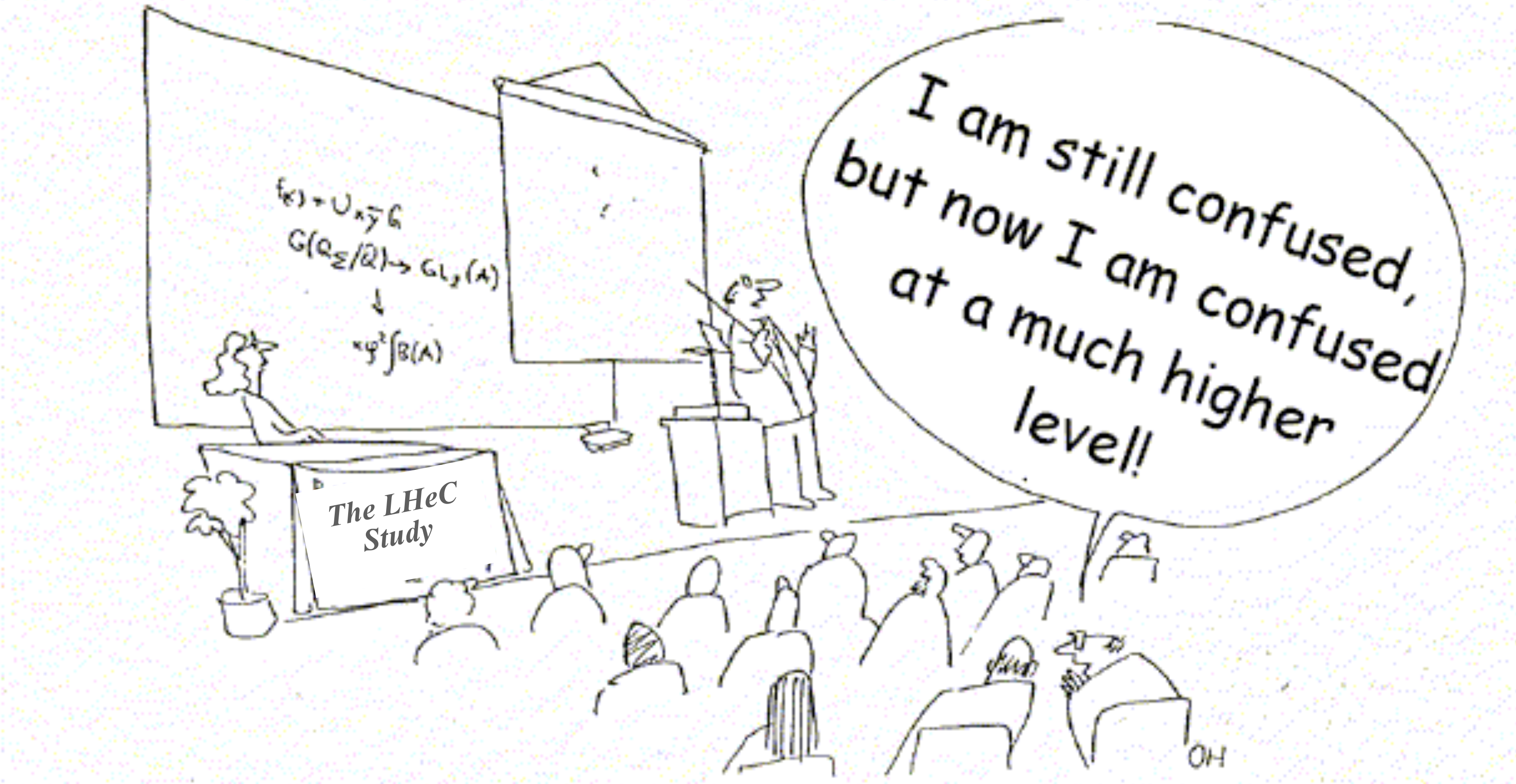
Optimise e-p Performance ✓

- *HL-LHC optics*
  —> ATS for extreme p-optics



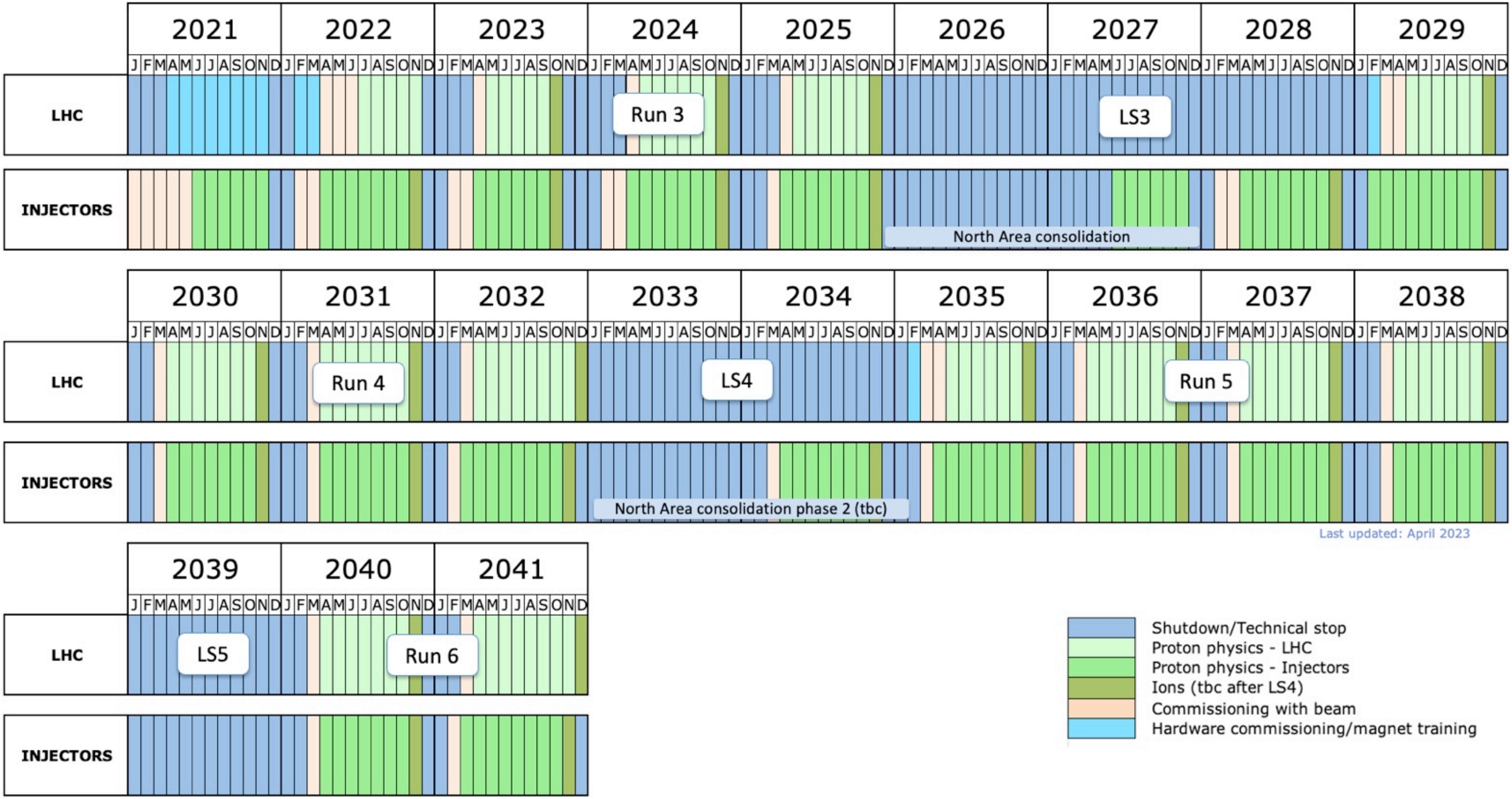
Long Term stability ✓

- *tracking calculations, bb effect*


I am still confused, but now I am confused at a much higher level!
The LHeC Study
OH

… the timing

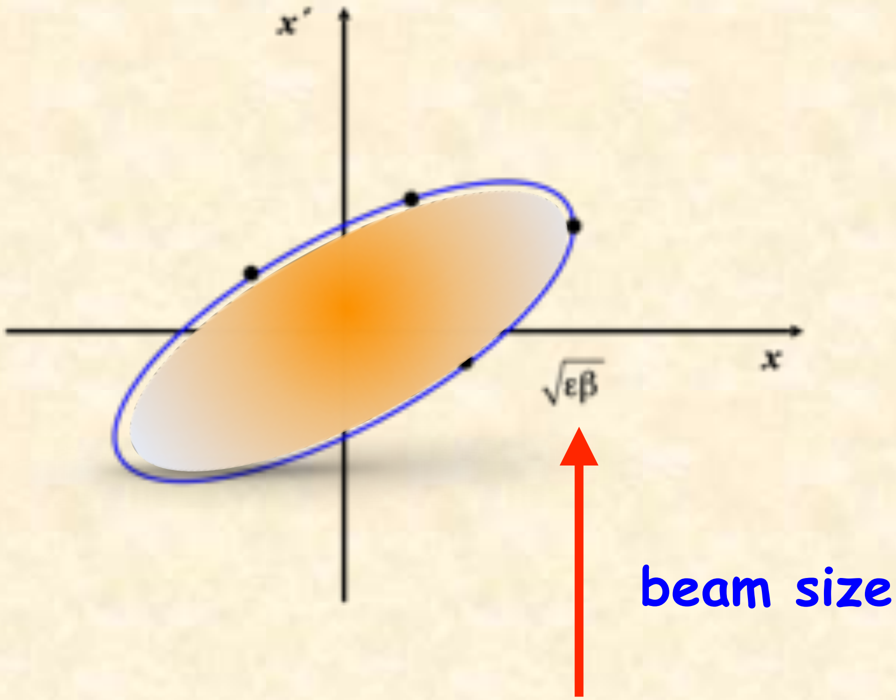
- *assuming ≈10 years of construction time for the ERL*
- *ideal scenario —> LS4 for IR modification & connection to the LHC*
- *commisioning & physics run: Run 5*

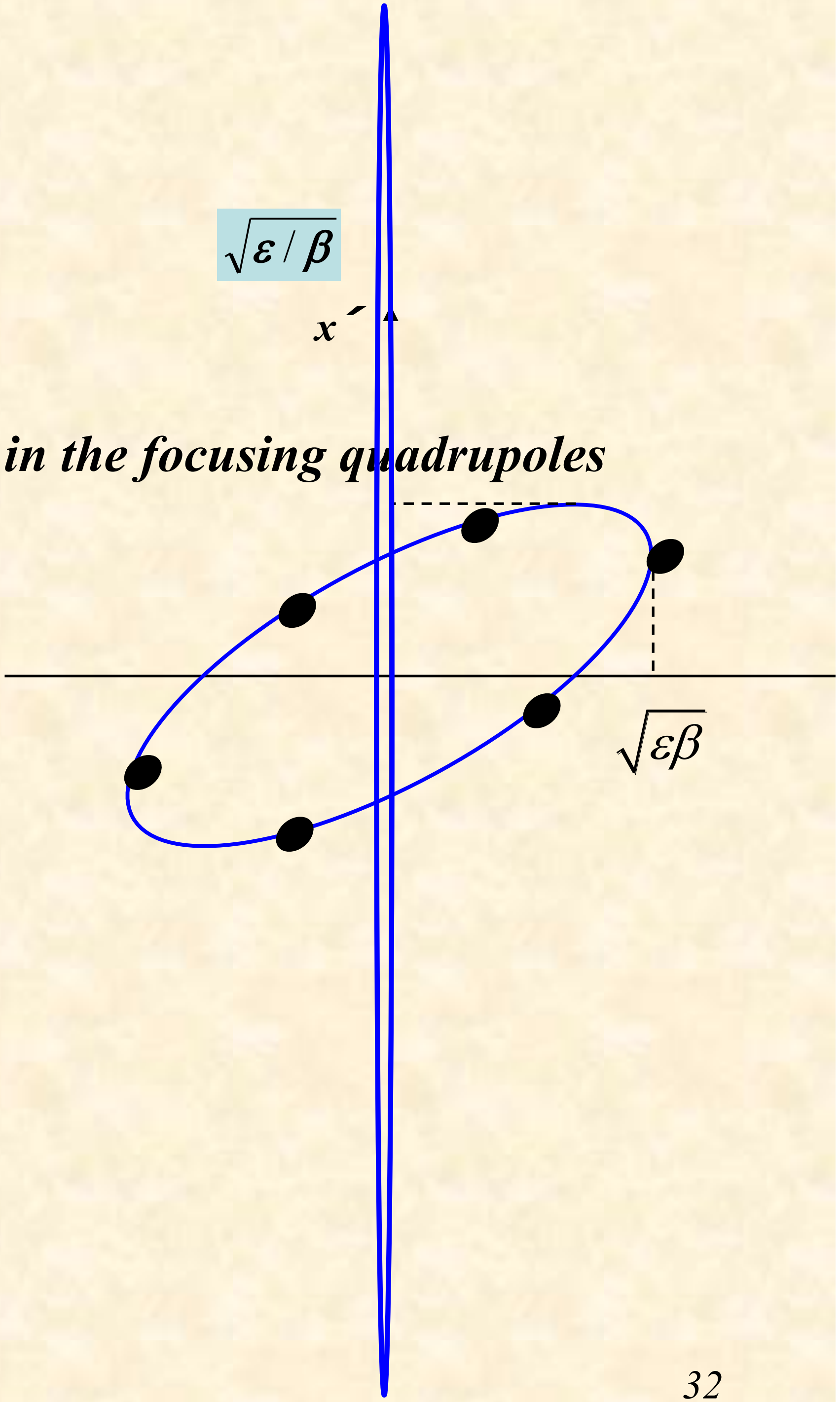


Proton Beam Performance Liouville

under the influence of conservative forces, the particle kinematics will always follow an ellipse in phase space;
x, x'phase space area = constant

—> strong focusing to smallest beam size
leads to large beam divergence and aperture limitations in the focusing quadrupoles





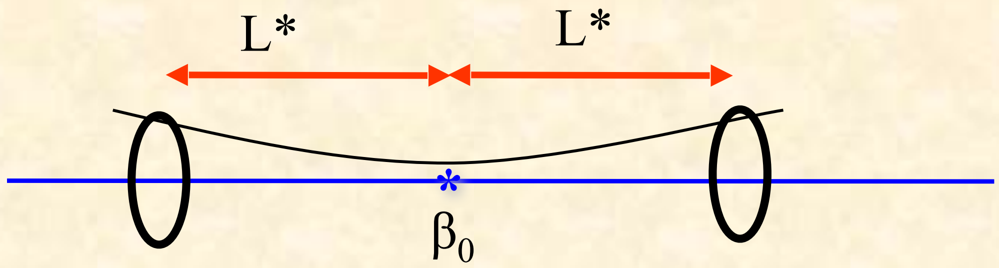
β-Function in a Drift:

A direct consequence of "Liouville",
i.e. phase space conservation, is that … if we make the beam size smaller, the divergence increases.

in our β-language:

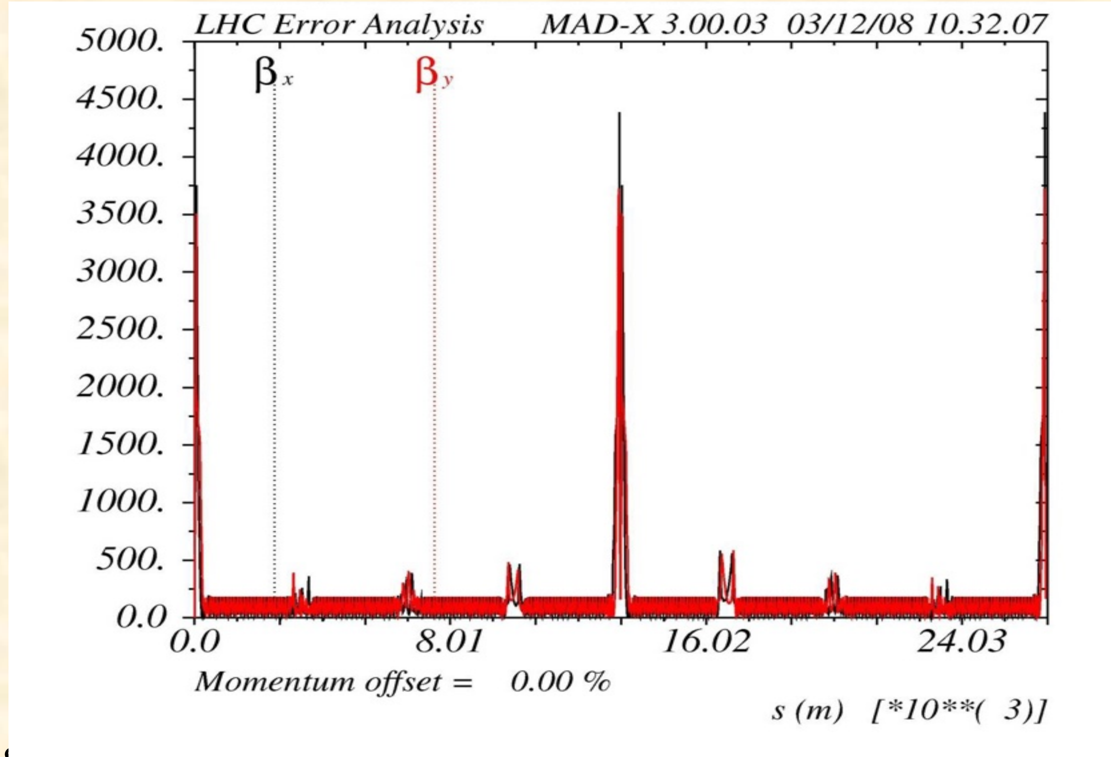
$$\beta(L) = \beta_0 + \frac{L^2}{\beta_0}$$

! ! !



At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.
-> here we get the largest beam dimension.

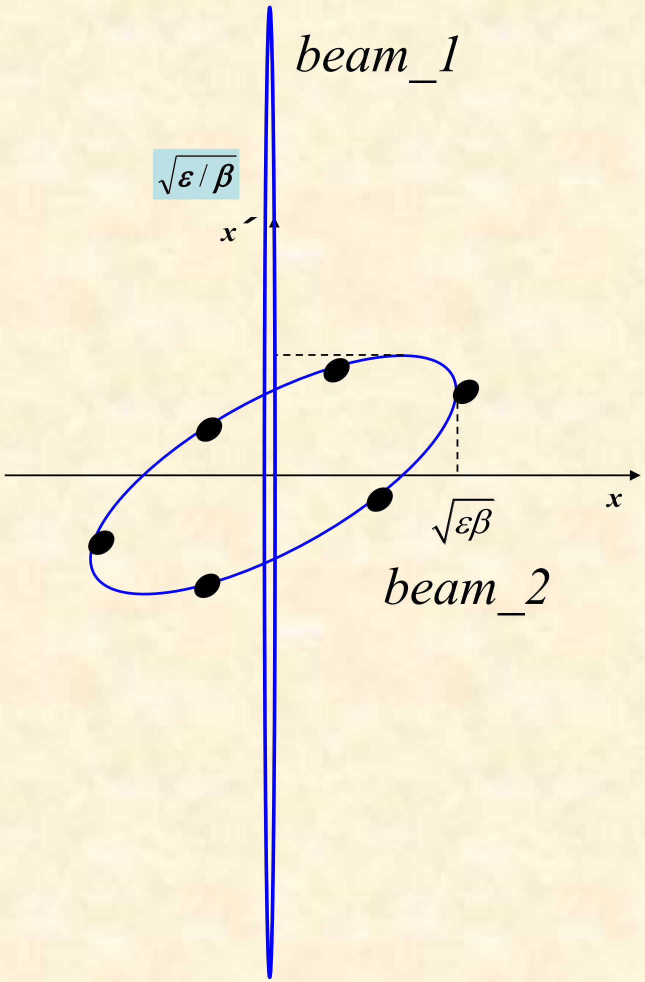
-> keep L* as small as possible

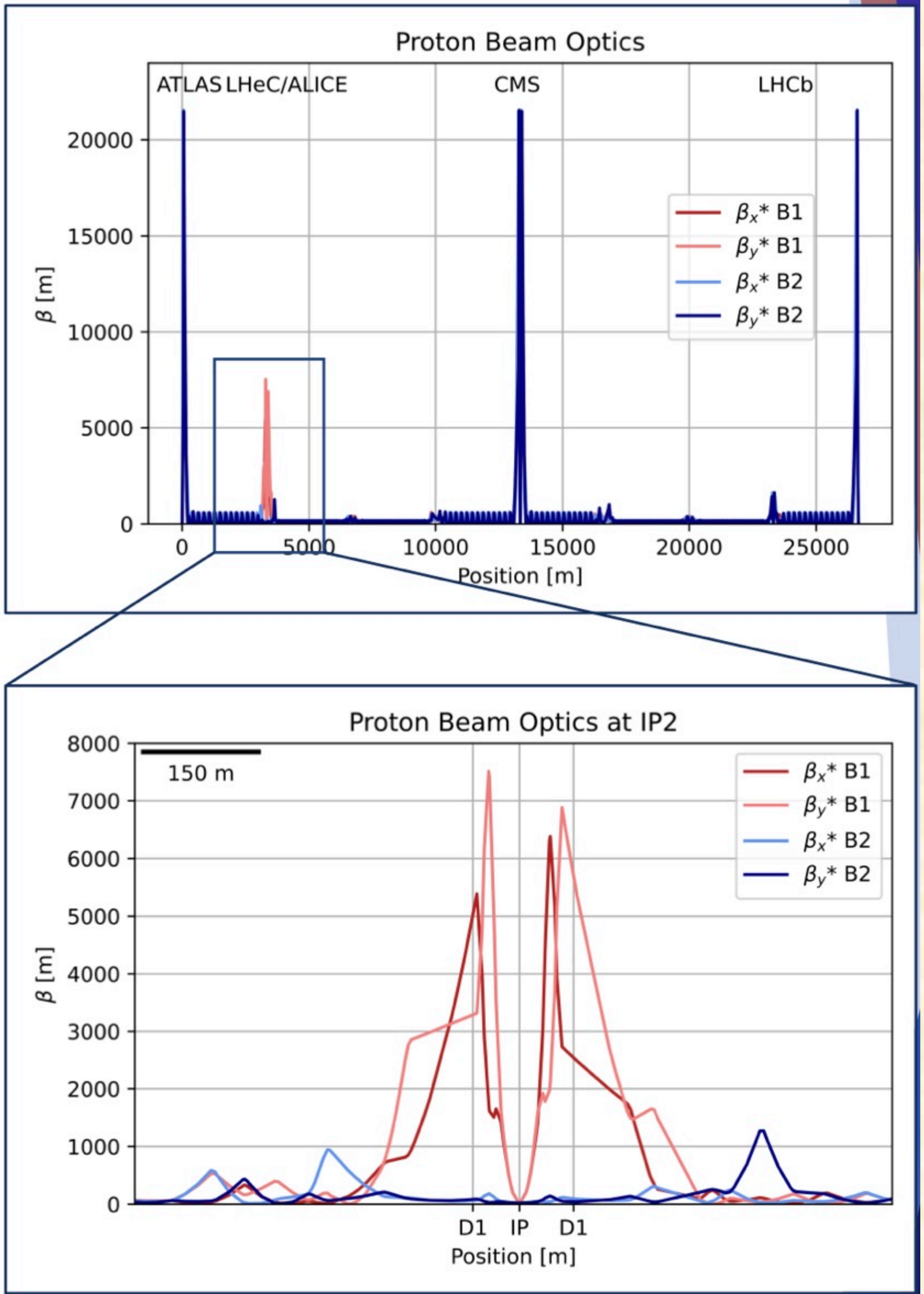


Proton Beam Performance

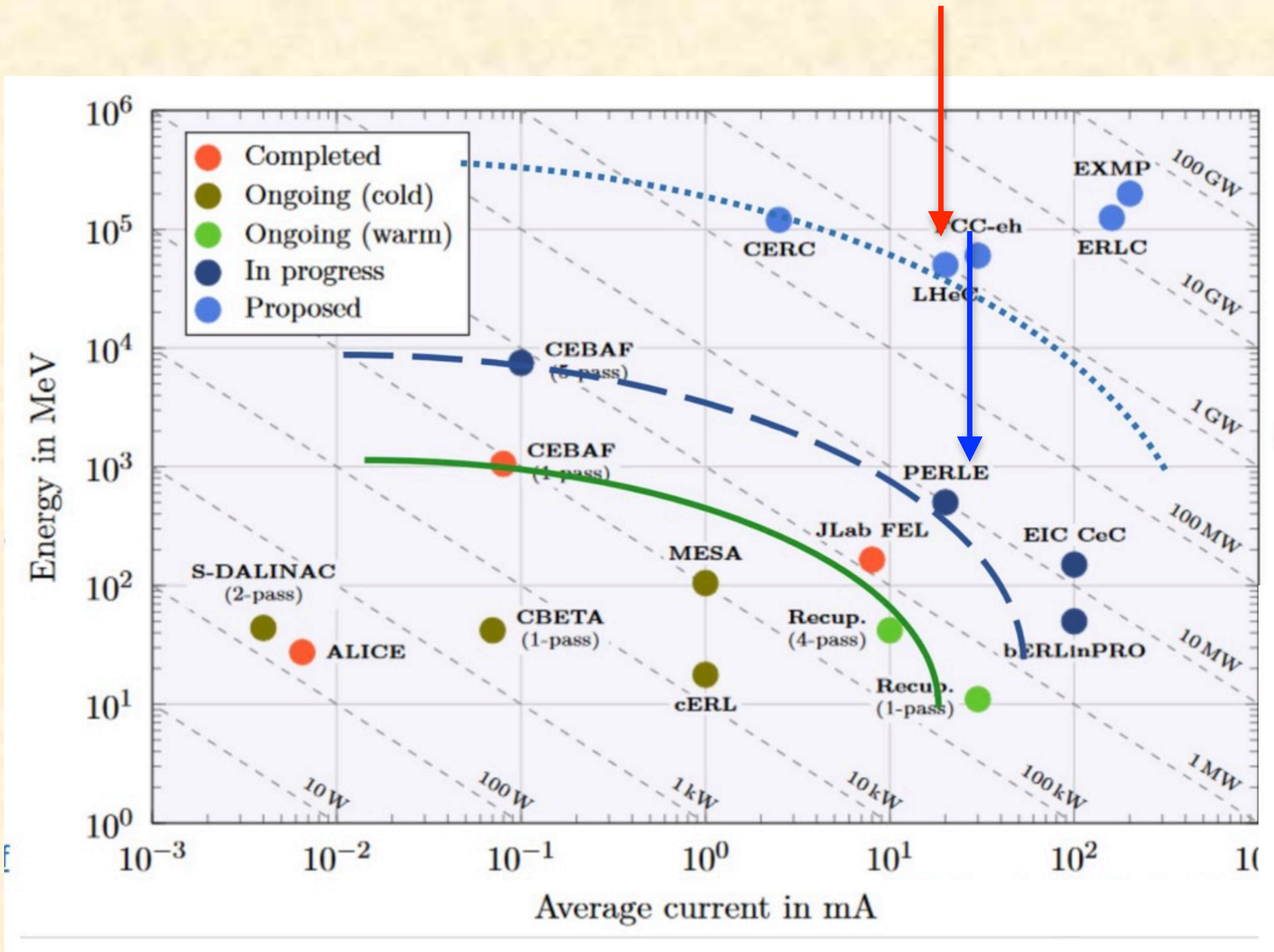
two different beam optics in IP2
for LHC beam_1 and beam_2

—> relax aperture need for beam_2
—> leave space for Liouville in beam_1





LHeC ERL: Where are we ?



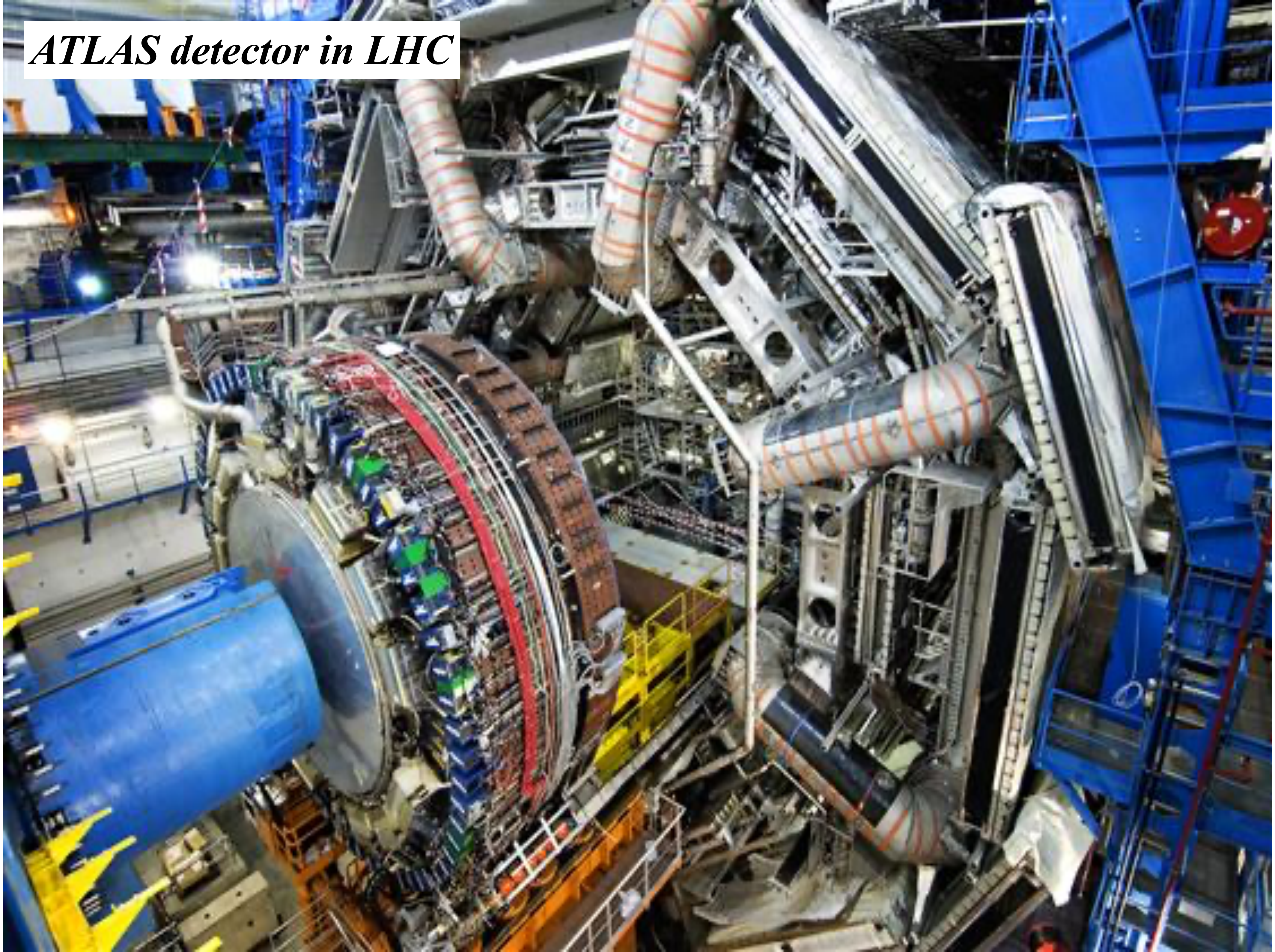
Parameter	***Electrons***
Energy (GeV)	50
N_p /bunch (10^{11})	2.2
N_e /bunch (10^{9})	3.1
bunch distance (ns)	25
I_e (mA)	20
Emittance (nm)	0.31
Beam size @ IP	6 / 6
Length (m)	6665
Luminosity (cm^{-2}	10^{33} … 10^{33}
wall plug power	

3.) The HL-LHC

* increasing the luminosity of LHC
* higher bunch intensities
* smaller β^*

	LHC	HL-LHC
Energy	7 TeV	7 TeV
Particles / bunch	$1.2*10^{11}$	$2.2*10^{11}$
number of bunches	2808	2748
β^*	55 cm	15 cm
ε	$5.0*10^{-10}$ m rad	$3.3*10^{-10}$ m rad
σ	16 μm	7 μm
Luminosity	$1.0*10^{34}\ cm^{-2}\ s^{-1}$	$7.0*10^{34}\ cm^{-2}\ s^{-1}$

ATLAS detector in LHC

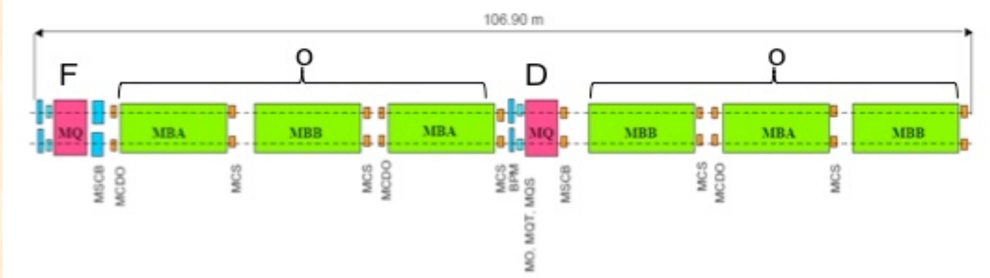


Scaling for FCCpp: Dipole Fill Factor for present Version V3:

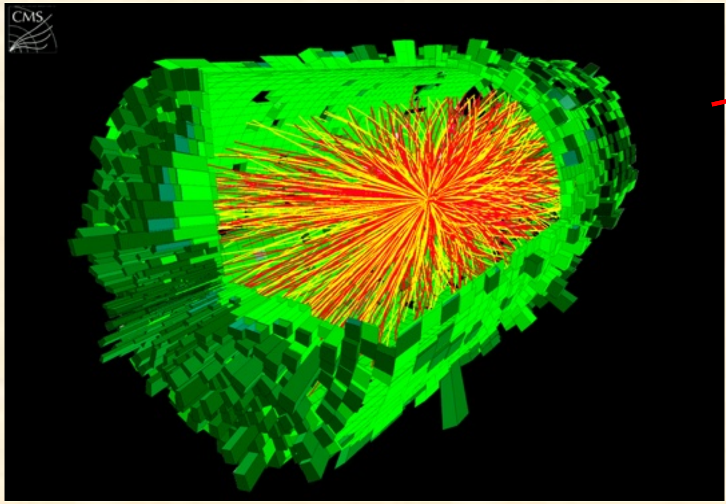
Pushing the limit (Dipole Fill Factor):

12 dipoles per cell, l_{dipole}=14.2m
34 cells per arc
12 arcs
dipole field = 15T <--> 50TeV
or 16T

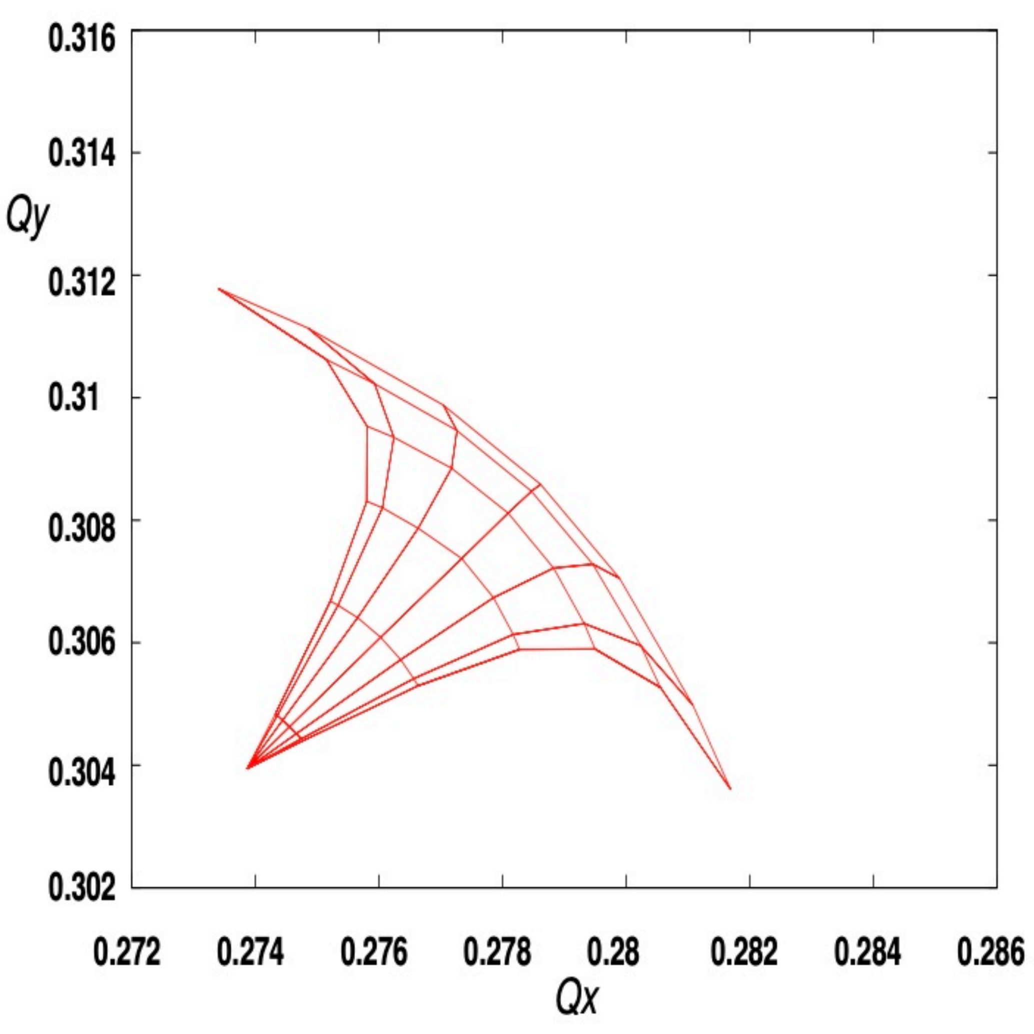
LHC example



FCC: 5016 dipoles
drifts a la LHC: dipole-quad=3.6m
dipole-dipole=1.3m
Double cell length = 200m



	FCC-hh Baseline	FCC-hh Ultimate
Luminosity L [$10^{34}cm^{-2}s^{-1}$]	5	20-30
Background events/bx	170 (34)	<1020 (204)
Bunch distance Δt [ns]	25 (5)	
Bunch charge N [10^{11}]	1 (0.2)	
Fract. of ring filled η_{fill} [%]	80	
Norm. emitt. [μm]	2.2(0.44)	
Max ξ for 2 IPs	0.01 (0.02)	0.03
IP beta-function β [m]	1.1	0.3
IP beam size σ [μm]	6.8 (3)	3.5 (1.6)
RMS bunch length σ_z [cm]	8	
Crossing angle [σ']	12	Crab. Cav.
Turn-around time [h]	5	4



Combined head-on and long-range interactions

, one horizontal and one vertical crossing (right).

$$S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x} \tan \frac{\phi}{2})^2}} \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x} \frac{\phi}{2})^2}} \, .$$

Crossing angle loss factor

on of this result is to consider it a correction to th

:

$$\sigma_{eff} = \sigma \cdot \sqrt{1 + (\frac{\sigma_s}{\sigma_x} \frac{\phi}{2})^2} \, .$$

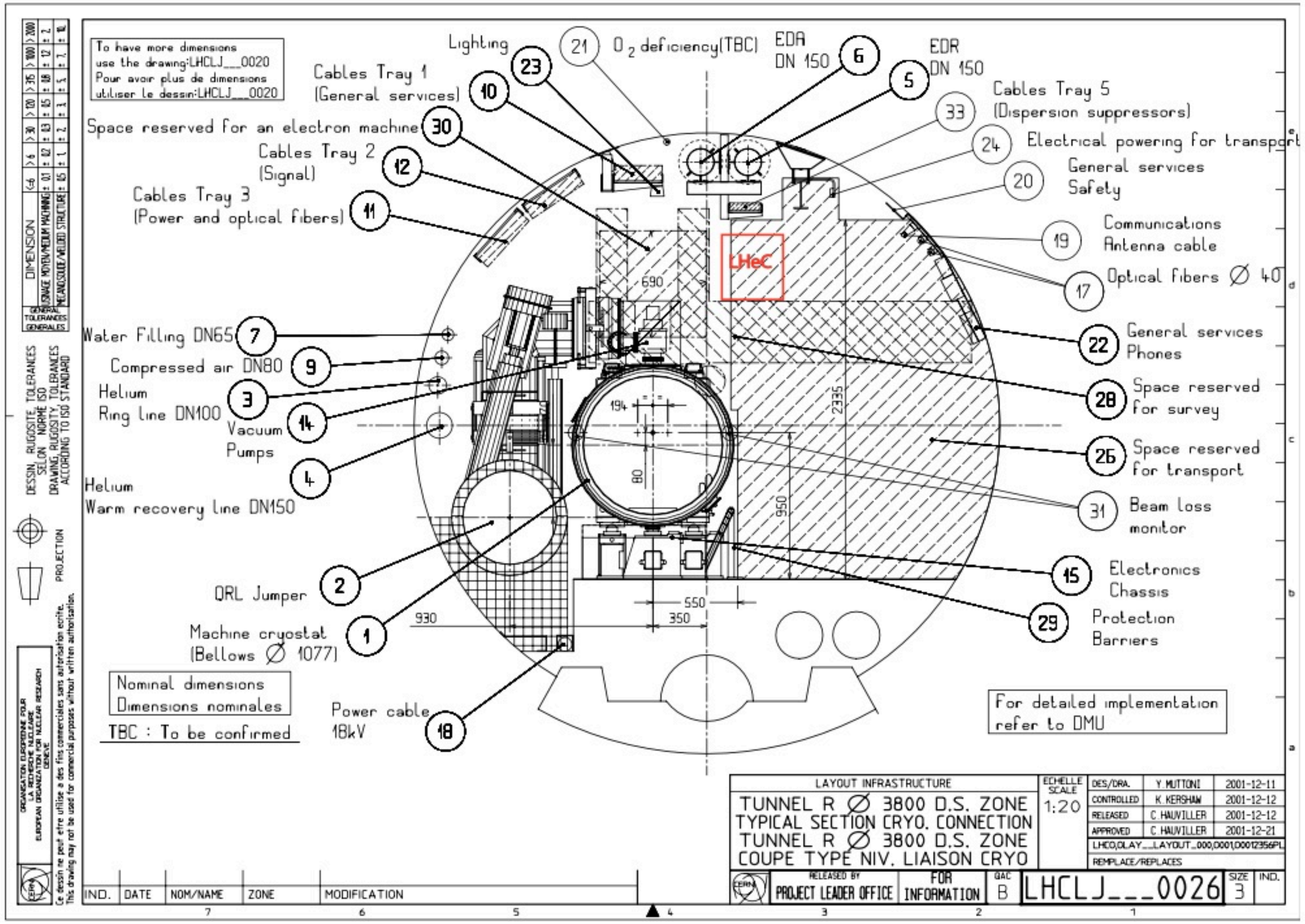


Figure 6.2: Representative cross section of the LHC tunnel. The location of the electron ring is indicated in red.

LHeC Ring_Ring Version

	HA	HL
electron beam 60 GeV		
IP β function $\beta^*_{x,y}$ [m]	0.4, 0.2	0.18, 0.1
syn rad power (interaction region) [kW]	51	33
critical energy [keV]	163	126
proton beam 7 TeV		
IP β function $\beta^*_{x,y}$ [m]	4.0, 1.0	1.8, 0.5
collider		
Lum e^-p (e^+p) [10^{32}cm^{-2}s^{-1}]	9 (9)	18 (18)
rms beam spot size $\sigma_{x,y}$ [μm]	45, 22	30, 16
crossing angle θ [mrad]	1	
$L_{ep}(\theta)$ [10^{32}cm^{-2}s^{-1}]	7.3 (7.3)	13 (13)
$L_{eN} = A\ L_{eA}$ [10^{32}cm^{-2}s^{-1}]	0.45	

Table 6.3: Parameters of the RR interaction region.

Beam Disruption

… more adequate in linear colliders

$$D_{x,y} = \frac{2\ N\ r_0\ \sigma_z}{\gamma\ \sigma_{x,y}\ (\sigma_x + \sigma_y)}$$

	HERA	LEP	LHeC	EIC	CLIC
Energy (GeV)	27.5	100	50	10	250
Beam size @ IP (μm)	112 / 30	180 / 7	5.8 / 5.8	91 / 7	0.2 / 0.0023
N_e /bunch (10^9)	40	400	3.1	150	6.8
βx, βy (cm)	63 / 26	125 / 5	10 / 10	42 / 5	0.8 / 0.01
bb-parameter	0.03 / 0.04	0.07	1.64 / 1.64	0.06 / 0.10	
Disruption parameter, x / y	0.13 / 0.49		14.28 / 14.28	0.10 / 1.24	0.1 / 12

Keep disruption limited / beam quality sufficiently high to ensure energy recovery performance.

The Agora Questionnaire

CoM Energy and upgrades	$E_e = 50 GeV,\ \ E_p = 7 TeV \ \rightarrow\ \ E_{cm} \approx 1.3 TeV$
Peak Luminosity (10^{34} cm^{-2} s^{-1})	$\approx 10^{34} cm^{-2} s^{-1}$
IP Challenges	*MDI, Synchr. rad. background*
Length of facility, km	*9km (ERL) +27 km (LHC)*
Length of new accelerators, km	*9km ... for the 1/3 LHC version*
Beam parameters challenges	*20 mA in 3 pass ERL —> PERLE as prototype*
Special technologies	*ERL technology —> PERLE as prototype*
R&D/validation (yrs. needed); constr. start year	*special sc. magnet with field-free aperture for the e- beam ≈ 4 years R&D, NbTi / Nb_3 Sn*
Construction time, yrs.	*10 + 2 years (estimate)*
Cost (wrt ILC) (+/-, %), level of maturity	$1.3 \cdot 10^9$ *CHF* $\longrightarrow$ 1/10 *ILC*
Environment issues: AC power consumption of facility, resources (Nb, LHe…) needed	*AC power < 100 MW, delib. limited*

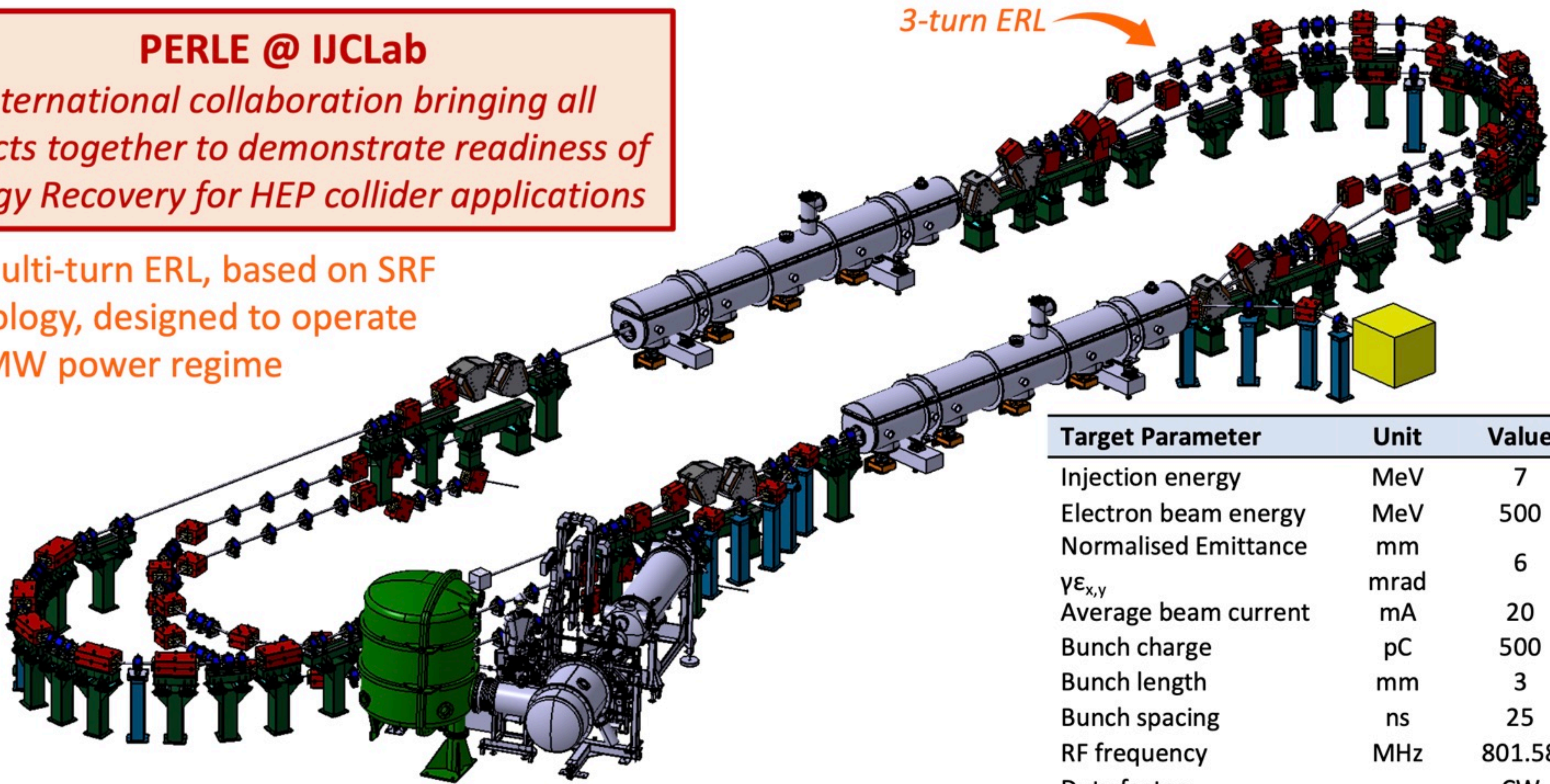
Upcoming facilities for Energy Recovery R&D

complementary in addressing the R&D objectives for Energy Recovery

PERLE @ IJCLab

international collaboration bringing all aspects together to demonstrate readiness of Energy Recovery for HEP collider applications

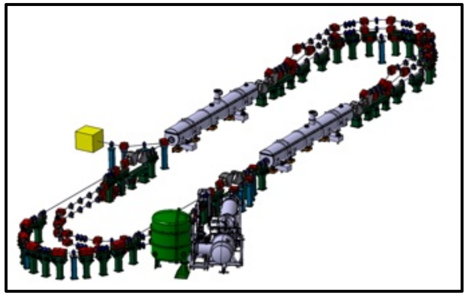
first multi-turn ERL, based on SRF technology, designed to operate at 10MW power regime



Target Parameter	Unit	Value
Injection energy	MeV	7
Electron beam energy	MeV	500
Normalised Emittance $\gamma\varepsilon_{x,y}$	mm mrad	6
Average beam current	mA	20
Bunch charge	pC	500
Bunch length	mm	3
Bunch spacing	ns	25
RF frequency	MHz	801.58
Duty factor		CW

PERLE – Powerful Energy Recovery Linac for Experiments [CDR: *J.Phys.G* 45 (2018) 6, 065003]

PERLE Timeline for TDR phase and beyond



Technical Design phase (TDR)

Prepare to Build phase (P2B)

Installation phases

Phase 1: Installation of the injection line

Phase 2: PERLE 250 MeV version

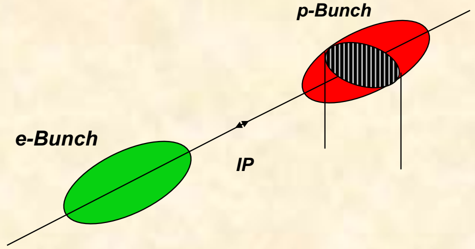
Phase 3: PERLE 500 MeV version

2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031

court. Achille Stocchi

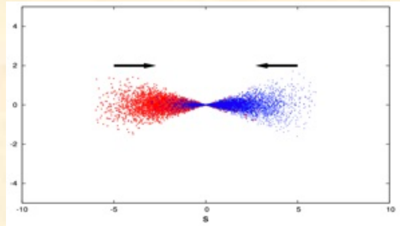
and … the Luminosity Limits:

$$L = \frac{N_e \cdot N_p \cdot n_b \cdot f_{rev}}{2\pi\sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \cdot \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}} \cdot \Sigma_i H_i$$



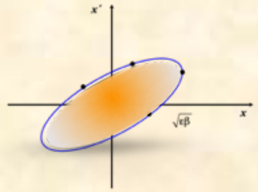
Liouville & Aperture

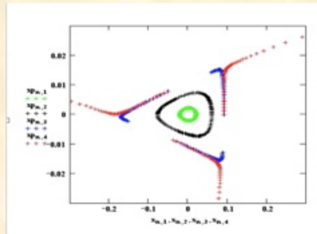
$$\beta(L) = \beta_0 + \frac{L^2}{\beta_0} \quad , \quad \sigma = \sqrt{\varepsilon\beta}$$



Beam Beam Tuneshift

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$





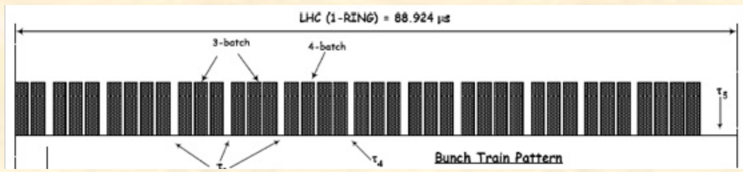
Chromaticity & sextupole strength

$$Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)ds$$

$$S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}} \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}} .$$

Crossing Angle

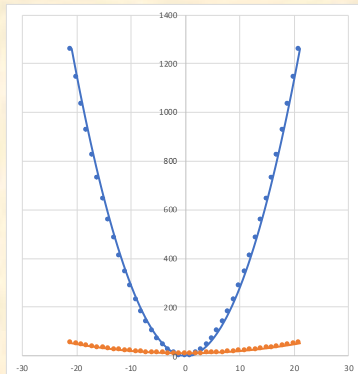
Bunch Fill Factor H3 ≃ 0.8



Hourglass Factor, H1 ≃ 0.9

$$A = \frac{\sin^2\frac{\phi}{2}}{(\sigma_x^*)^2[1 + (\frac{s}{\beta^*})^2]} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} .$$

$\beta^* = 35\ cm / 10\ m$



ERL projects world wide