

(1)

(7)

(72)

(891)

(12,672)

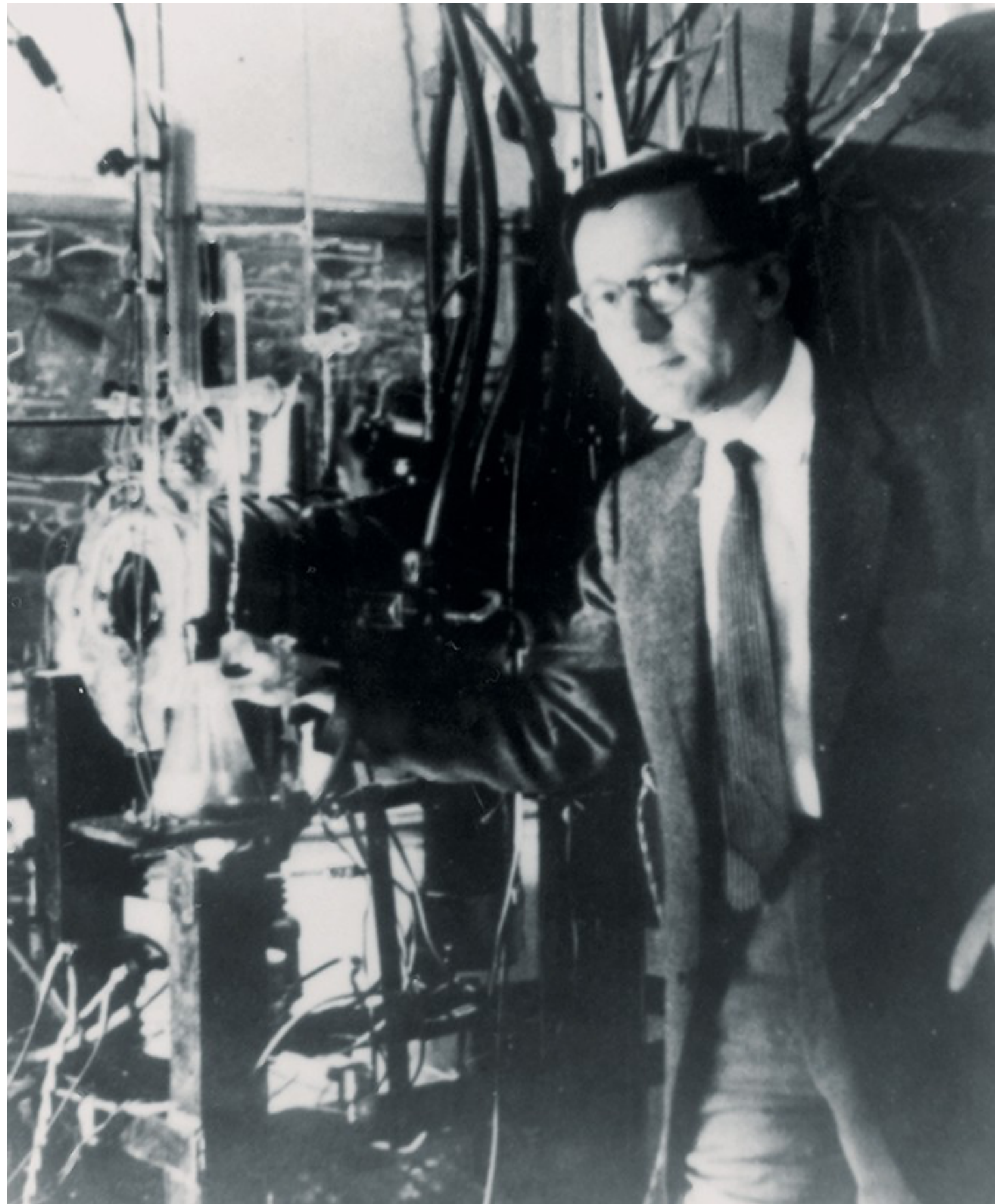
# Nowe pomiary momentów magnetycznych naładowanych leptonów

Mateusz Dyndał (KOiDC WFiIS AGH)

Seminarium WFiIS AGH, 26.01.2024



# Historical perspective



**P. Kusch** and H. M. Foley (1947)





# New papers published in 2023

PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search

Featured in Physics Editors' Suggestion

Measurement of the Electron Magnetic Moment

X. Fan, T. G. Myers, B. A. D. Sukra, and G. Gabrielse  
Phys. Rev. Lett. **130**, 071801 – Published 13 February 2023

Physics See Viewpoint: [Searching for New Physics with the Electron's Magnetic Moment](#)

PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Edit

Editors' Suggestion Open Access

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm

D. P. Aguillard *et al.* (The Muon  $g - 2$  Collaboration)  
Phys. Rev. Lett. **131**, 161802 – Published 17 October 2023

PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Edit

Editors' Suggestion Open Access

Observation of the  $\gamma\gamma \rightarrow \tau\tau$  Process in Pb + Pb Collisions and Constraints on the  $\tau$ -Lepton Anomalous Magnetic Moment with the ATLAS Detector

G. Aad *et al.* (ATLAS Collaboration)  
Phys. Rev. Lett. **131**, 151802 – Published 12 October 2023



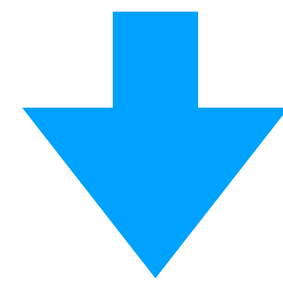
# Magnetic (dipole) moment in classical physics

$$\vec{M} = \vec{\mu} \times \vec{B}.$$

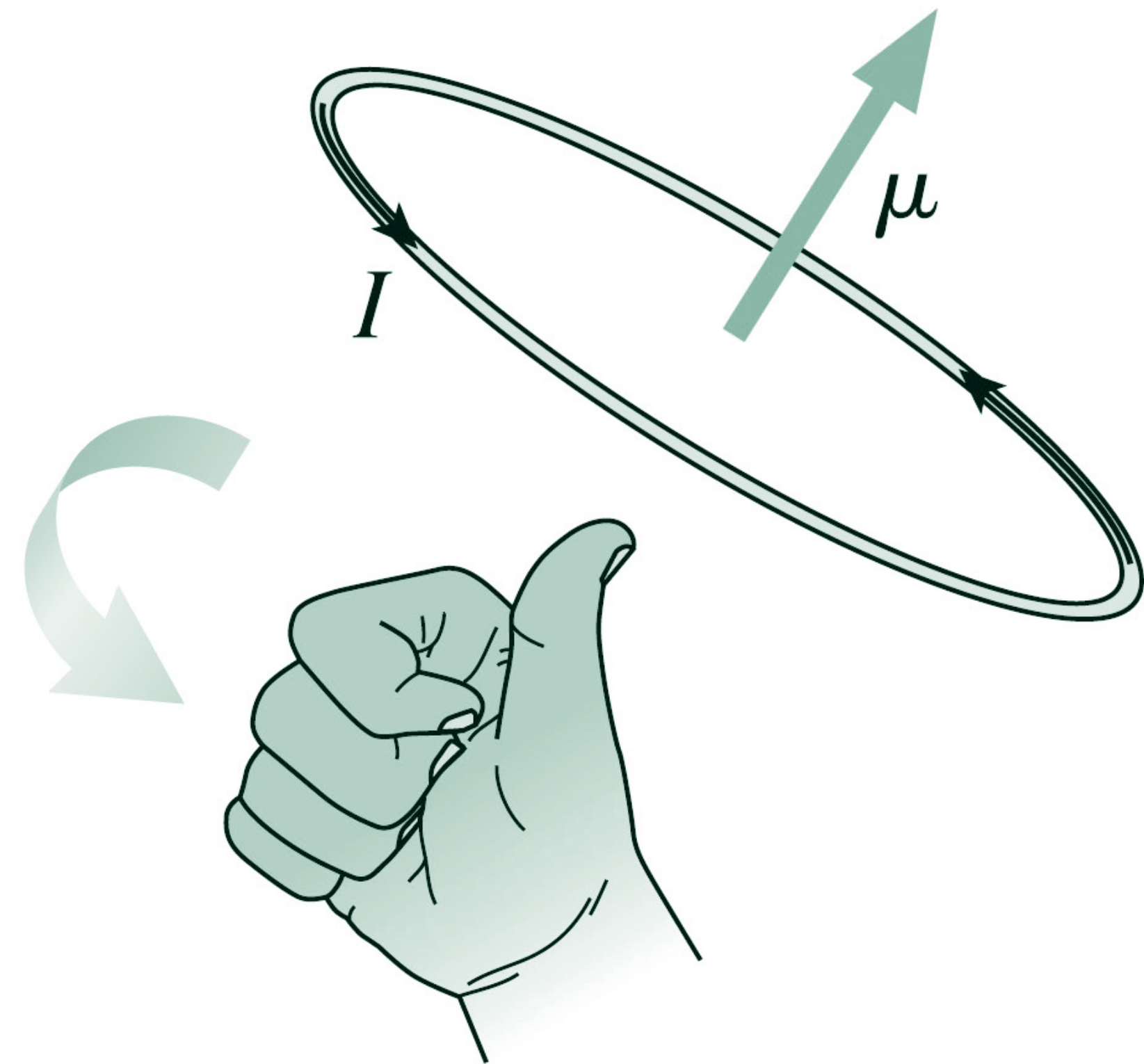
For single electron:

$$\mu = IA = \frac{e}{2\pi r / v} \pi r^2 = \frac{evr}{2}.$$

$$|\vec{L}| = |\vec{r} \times \vec{p}| = rp \sin \theta = rp = rm_e v.$$



$$\mu = \frac{e}{2m_e} L.$$





# QM: Magnetic (dipole) moment and the g-factor

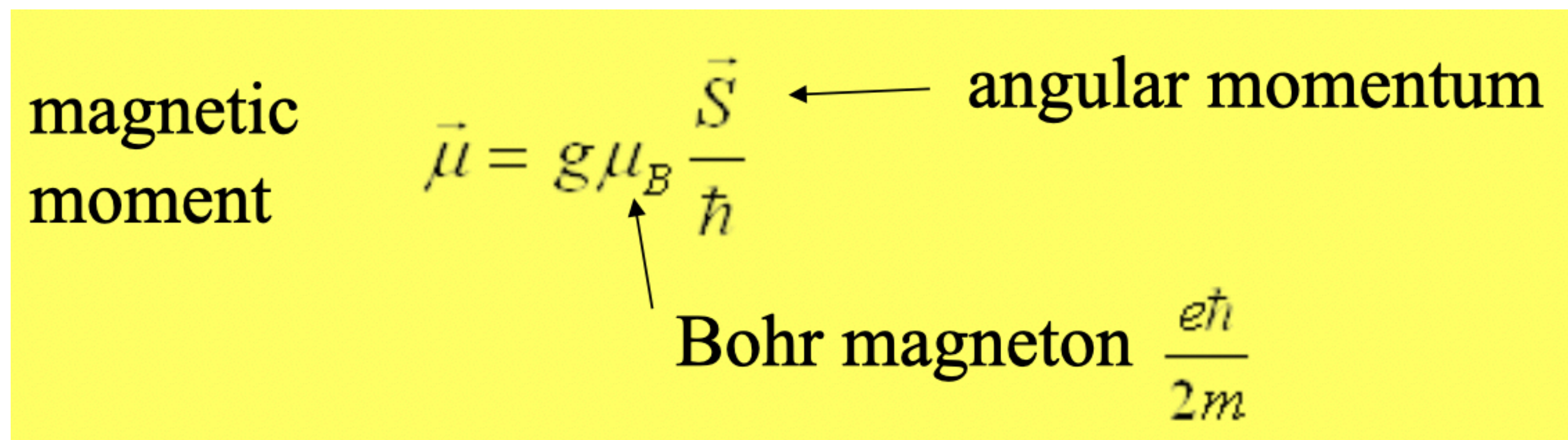
- $g$  = proportionality constant between **spin** and **magnetic moment**

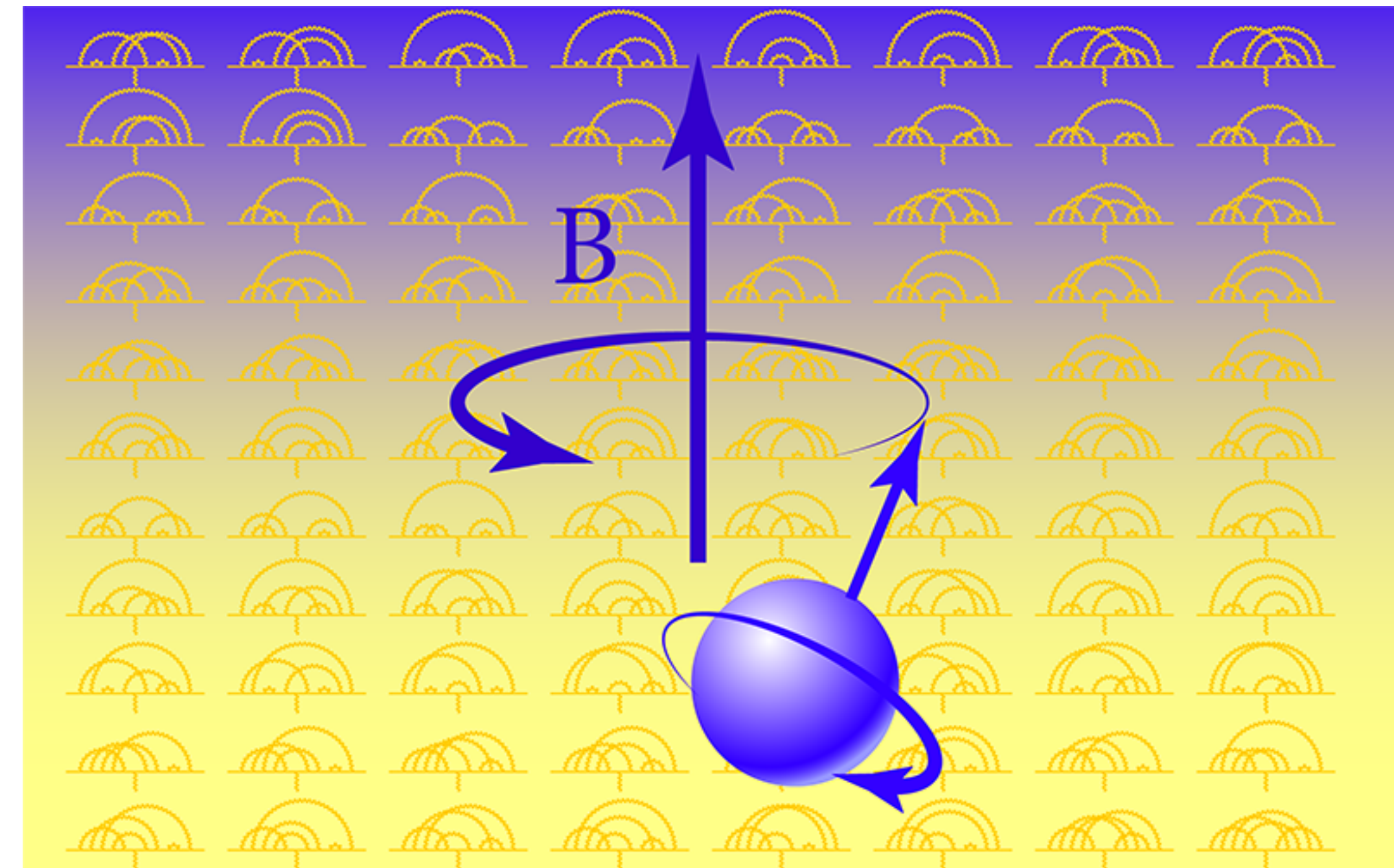
magnetic moment

$$\vec{\mu} = g \mu_B \frac{\vec{S}}{\hbar}$$

Bohr magneton  $\frac{e\hbar}{2m}$

angular momentum

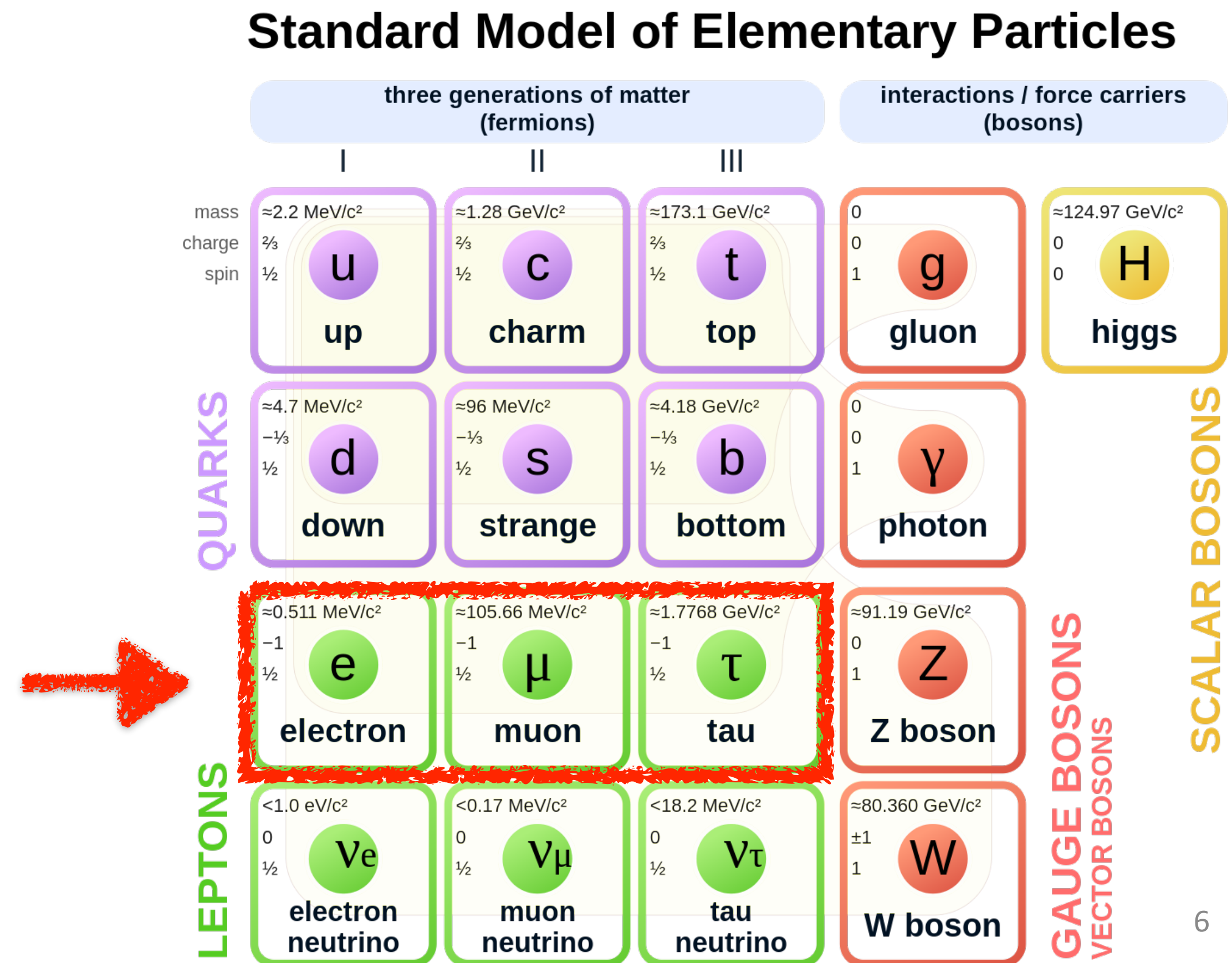
A yellow rectangular box containing the equation  $\vec{\mu} = g \mu_B \frac{\vec{S}}{\hbar}$ . To the left of the equation is the text 'magnetic moment'. To the right is 'angular momentum' with an arrow pointing to  $\vec{S}$ . Below the equation is 'Bohr magneton' with an arrow pointing to  $\mu_B$  and the formula  $\frac{e\hbar}{2m}$  below it.





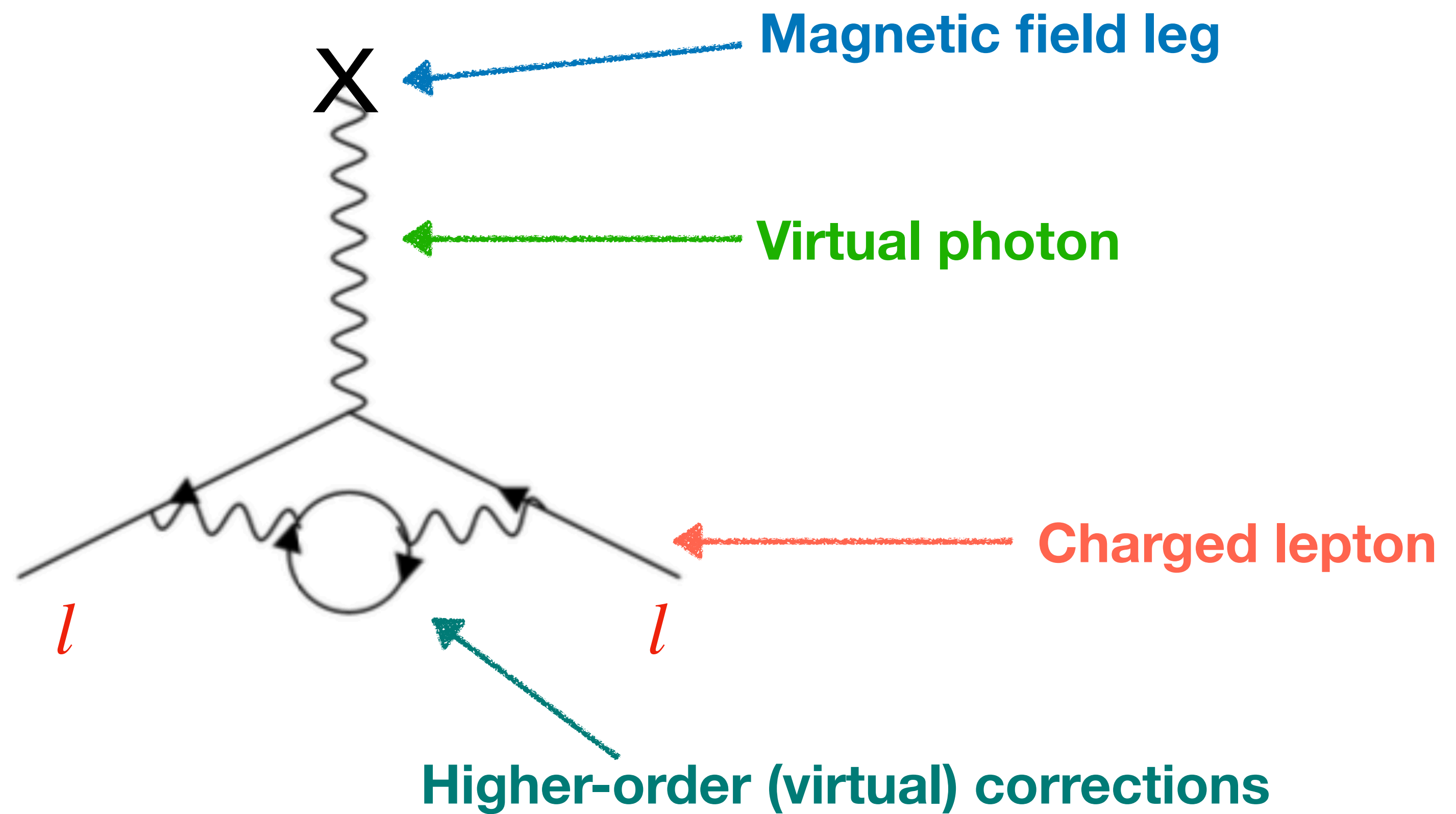
# Charged leptons in the Standard Model

- Standard Model (SM) = our current understanding of fundamental interactions
- Predicts **three generations** of matter particles (fermions)
  - Including three types of charged leptons:
    - > **electron**
    - > **muon**
    - > **tau**
  - Charged leptons interact ~identically in SM; they only differ in mass





# Interaction of charged leptons with magnetic field in QED

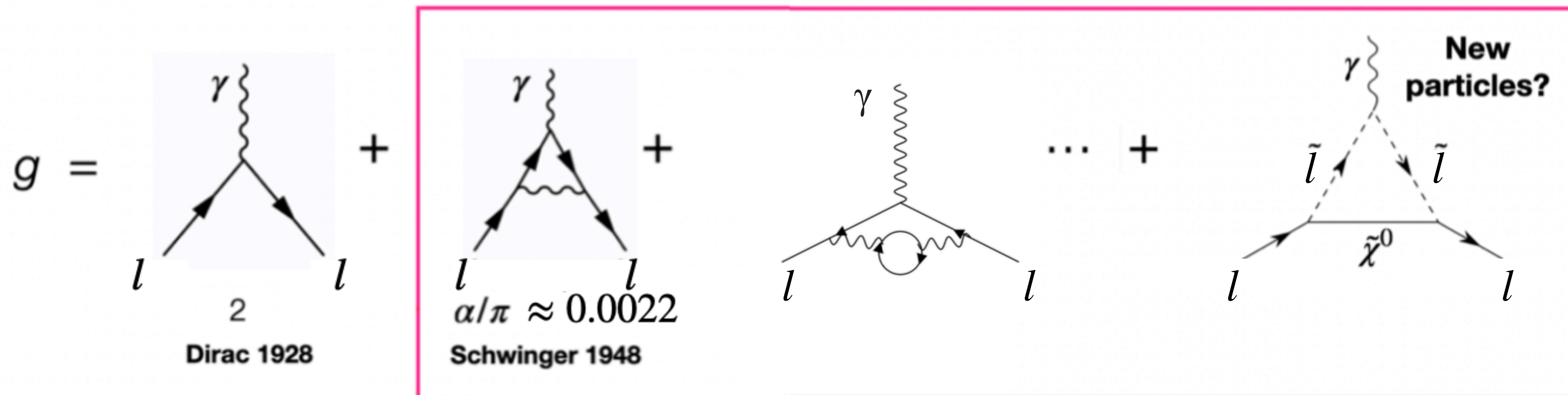




# Magnetic (dipole) moment of charged leptons

- For electrons/muons/taus Standard Model predicts  $g \approx 2$
- Note: quantum fluctuations make it slightly larger than 2...
- Anomalous magnetic moment
  - $a_\ell = (g_\ell - 2)/2$
  - Shows how much  $g$  differs fractionally from 2!

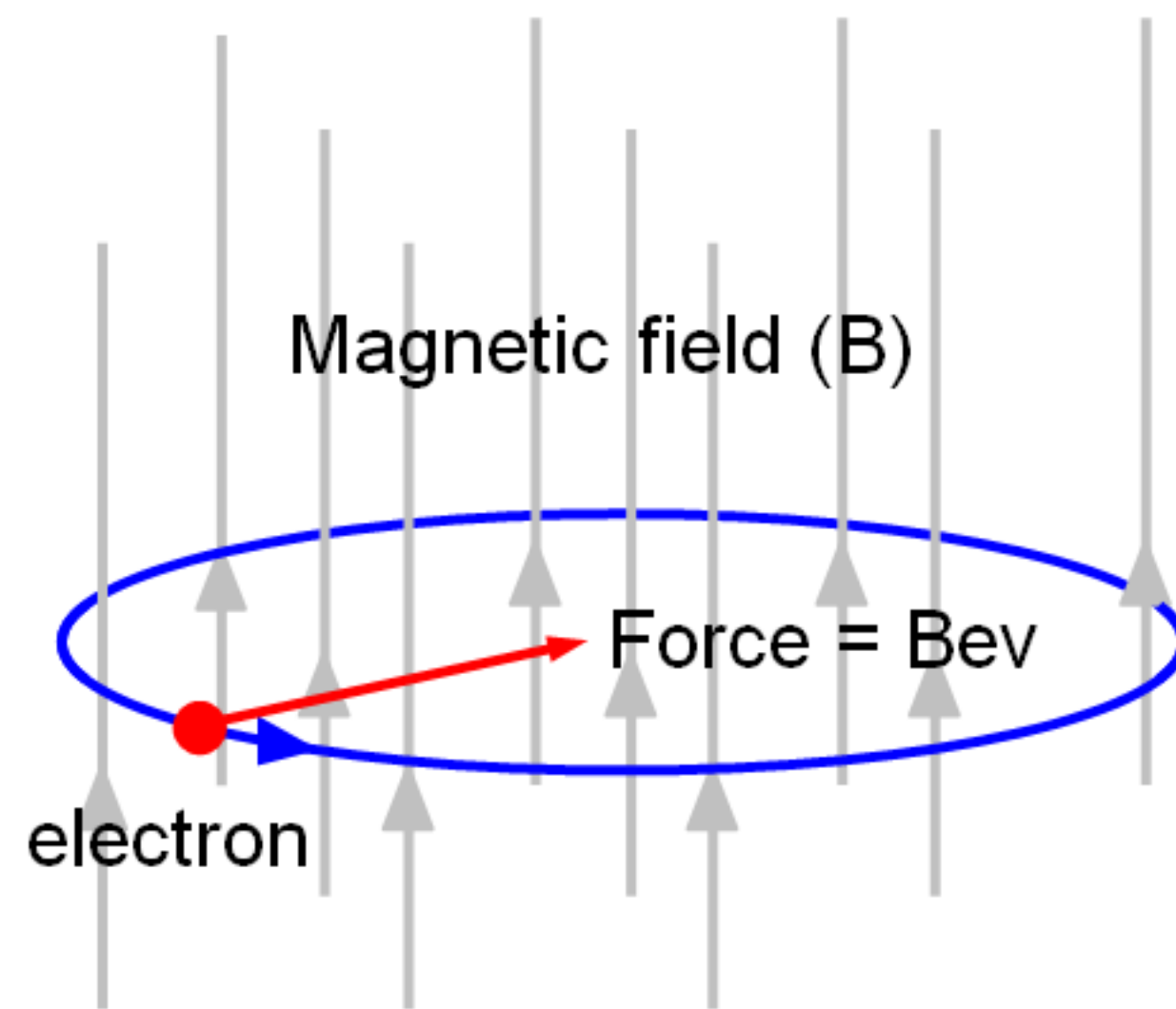
$a_\ell$





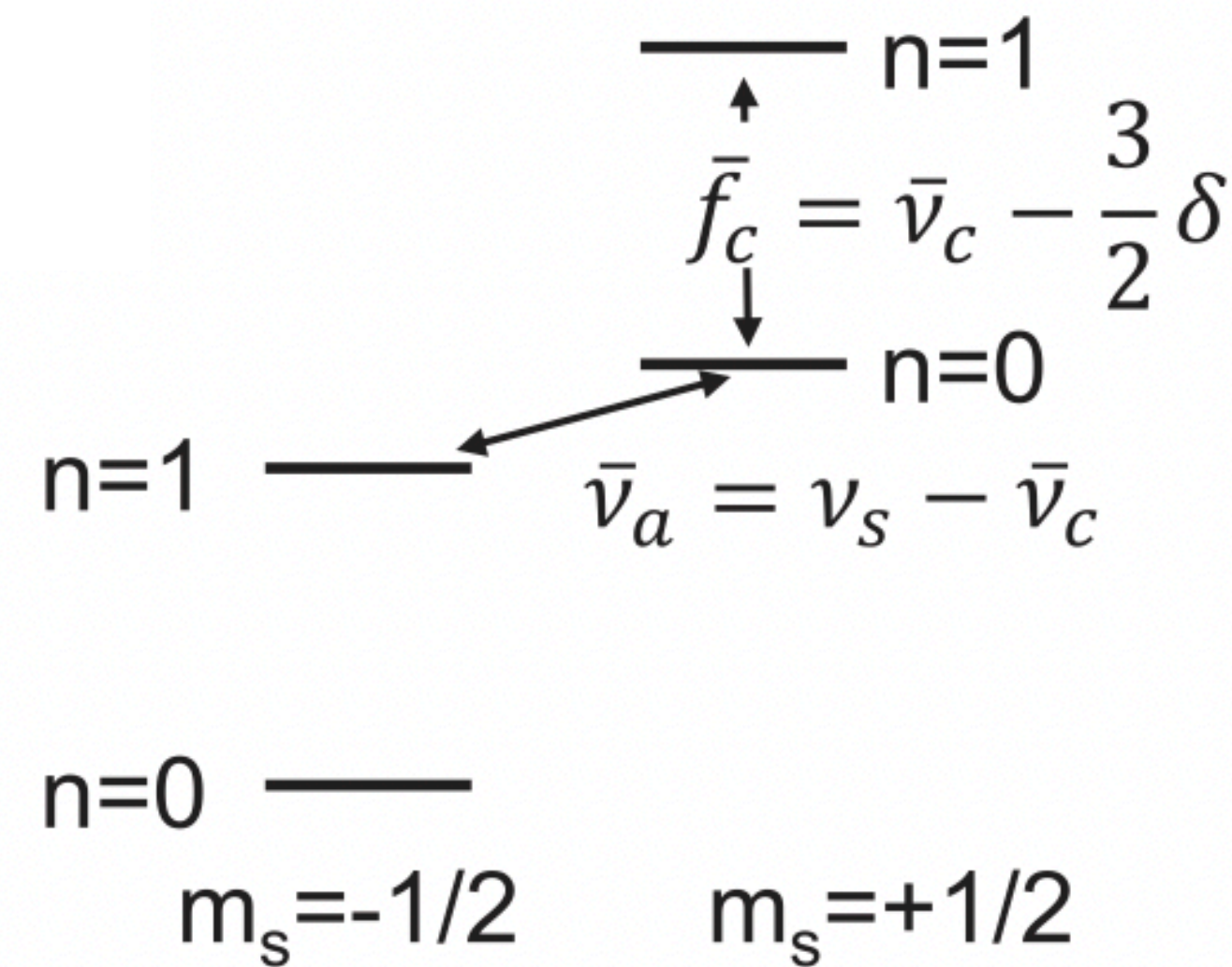
# Electron in a magnetic field

## Classically



$$\frac{mv^2}{r} = qBv \rightarrow \nu_c = eB/(2\pi m)$$

## In QM



$$E = h\nu_s m_s + h\nu_c \left( n + \frac{1}{2} \right)$$

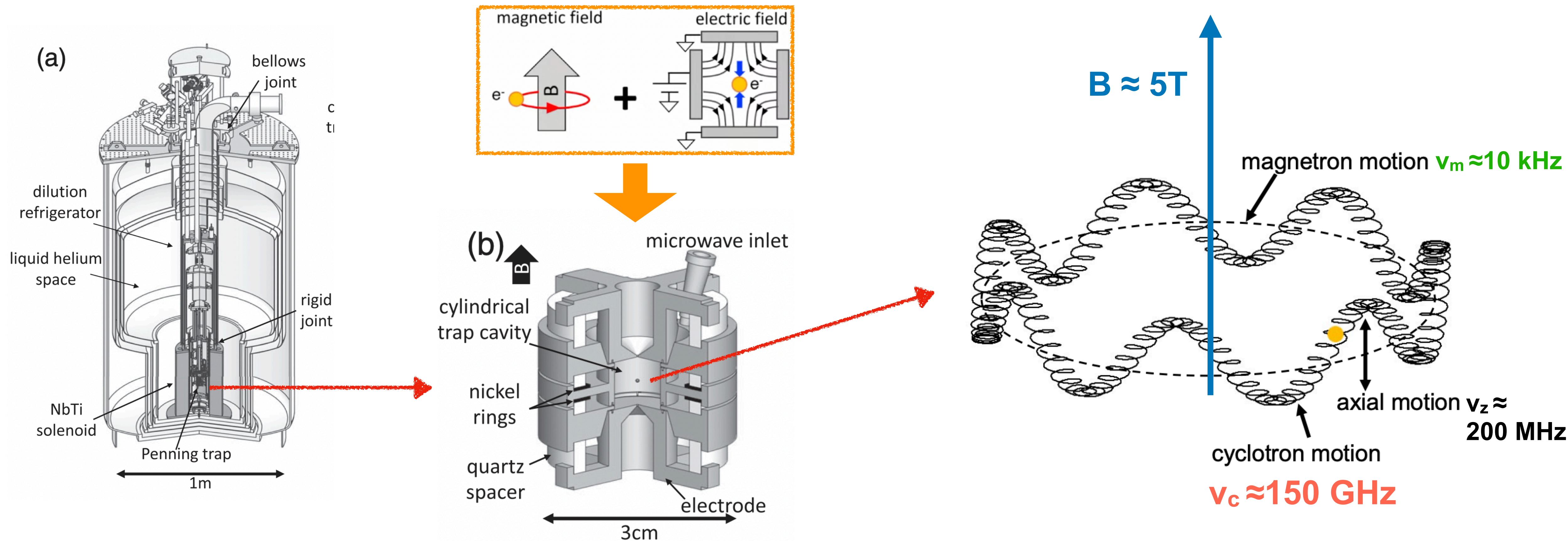
$$\frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_a}{\nu_c}$$

Note there is no B dependence!



# Electron g-2: measurement idea

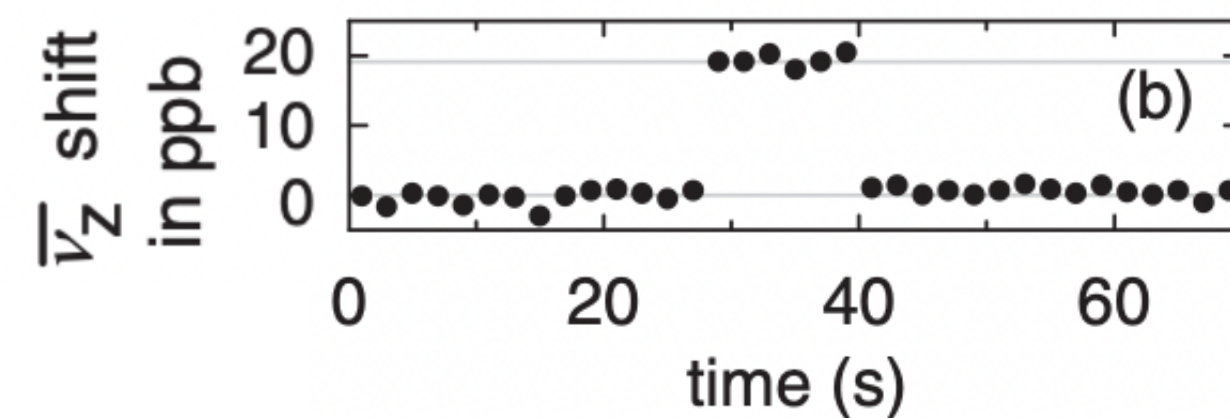
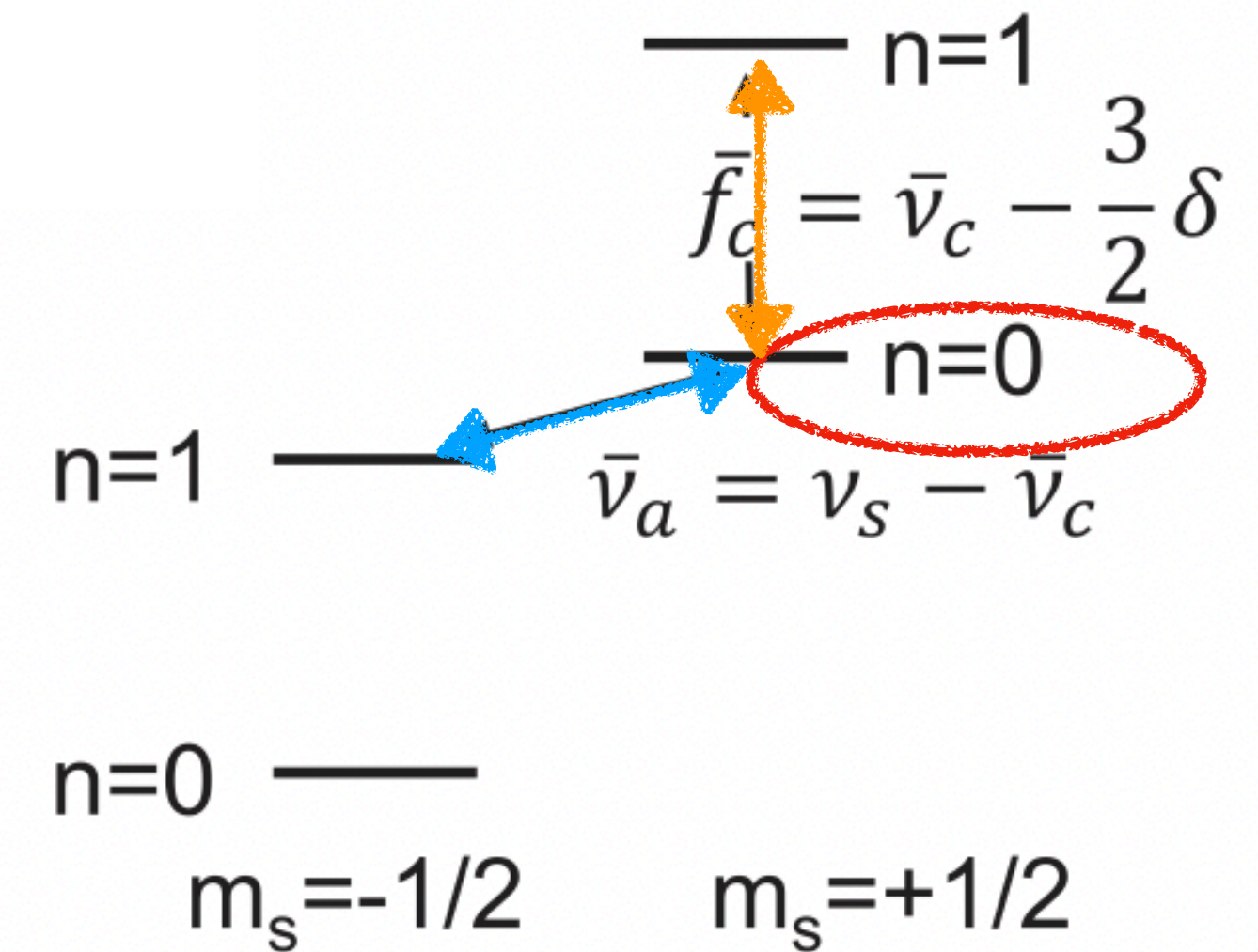
- Using one-electron quantum cyclotron (**Penning trap**), with  $T < 100$  mK



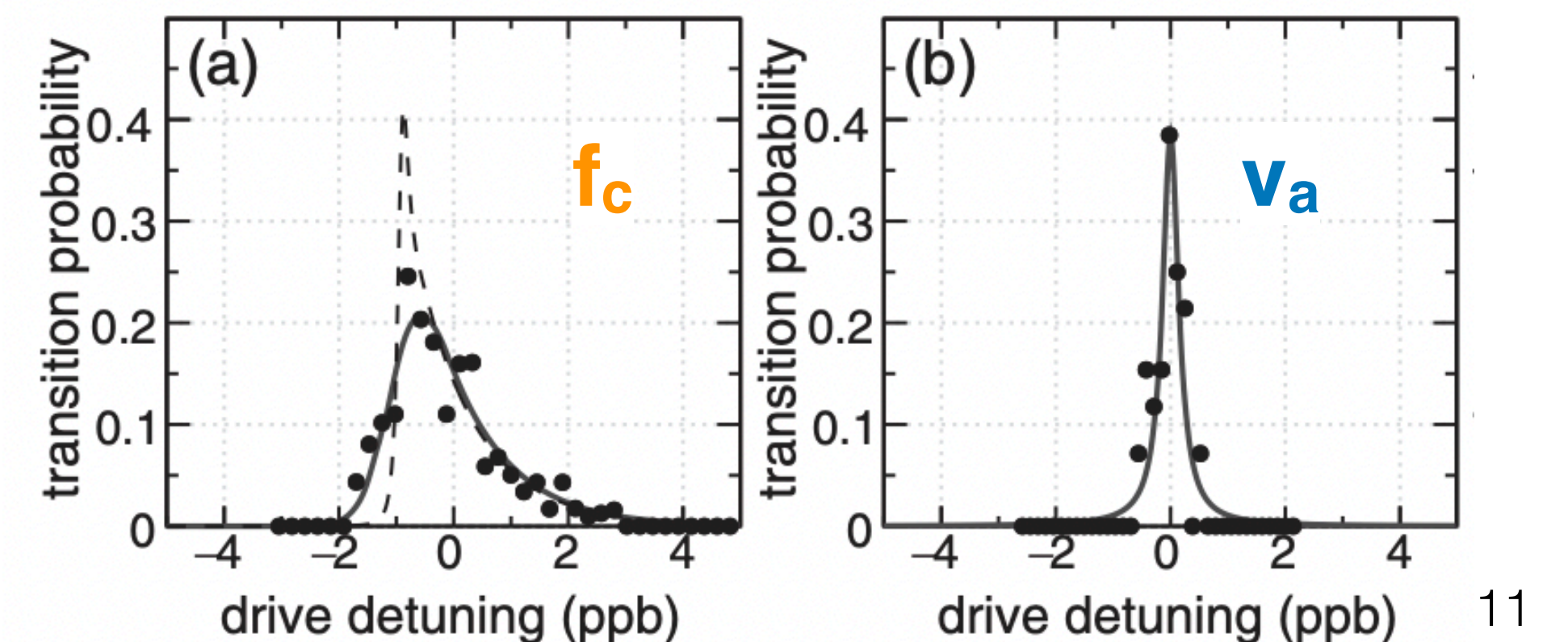


# Electron g-2: Measurement procedure

- Cyclotron quantum jump spectroscopy
- Each cyclotron and anomaly quantum jump trial starts with preparing the electron in the spin-up ground state,  $|n=0, m_s=+1/2\rangle$
- Quantum jumps detected via shifts in the axial frequency



- Multiple attempts at different frequencies are binned in a histograms to reveal the frequency lines:

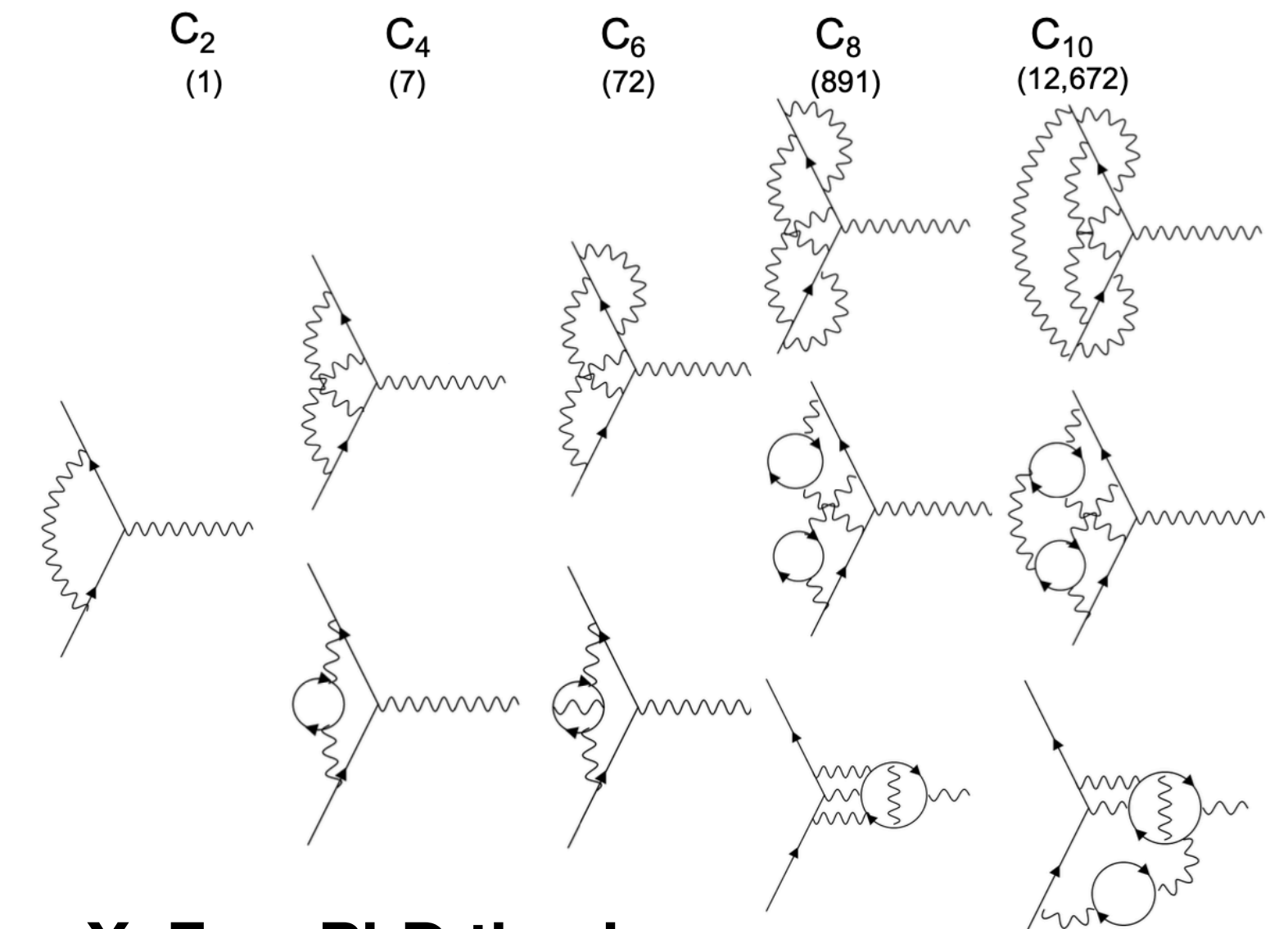




# Electron g-2: status of theory calculations

- QED provides the asymptotic series in powers of  $\alpha$ , along with the muon and tau contributions ( $a_{\mu\tau}$ )
- The constants  $C_2, C_4, C_6, C_8$  calculated exactly, but require measured lepton mass ratios as input
- The measurement is so precise that a numerically calculated tenth order  $C_{10}$  is required
- Alternative theory evaluation of  $C_{10}$  differs slightly for reasons not yet understood

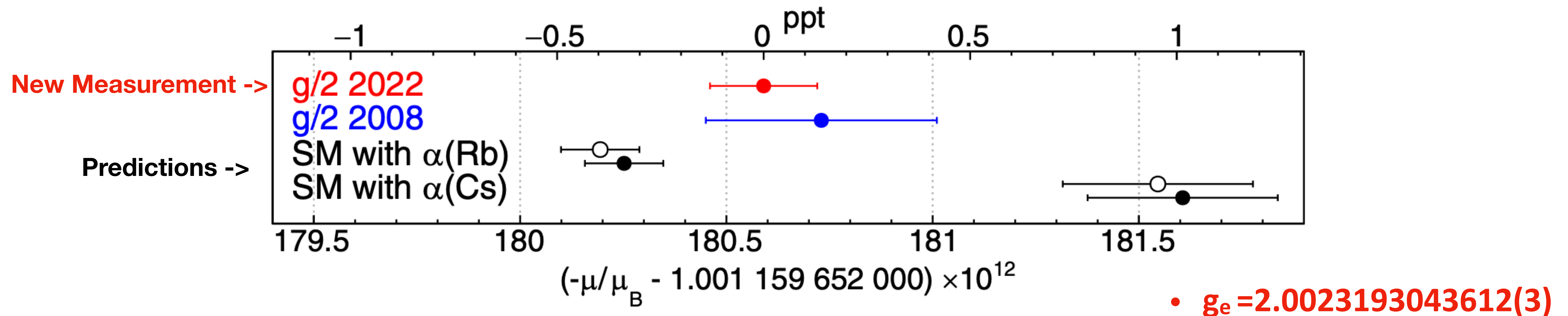
$$\frac{g}{2} = 1 + C_2 \left( \frac{\alpha}{\pi} \right) + C_4 \left( \frac{\alpha}{\pi} \right)^2 + C_6 \left( \frac{\alpha}{\pi} \right)^3 + C_8 \left( \frac{\alpha}{\pi} \right)^4 + C_{10} \left( \frac{\alpha}{\pi} \right)^5 + \dots + a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}}.$$





# Electron g-2: Results

Phys.Rev.Lett. 130 (2023) 7, 071801



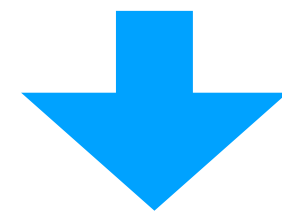
- Significant uncertainty reduction wrt previous measurement (blue), reaching **0.13 ppt** relative precision (ppt is  $10^{-12}$ )
- SM predictions (solid and open black points for slightly differing  $C_{10}$  calculations) are functions of discrepant  $\alpha$  measurements (independent measurements use Rb or Cs atom recoil)
- New measurement with positron (CPT symmetry test) underway



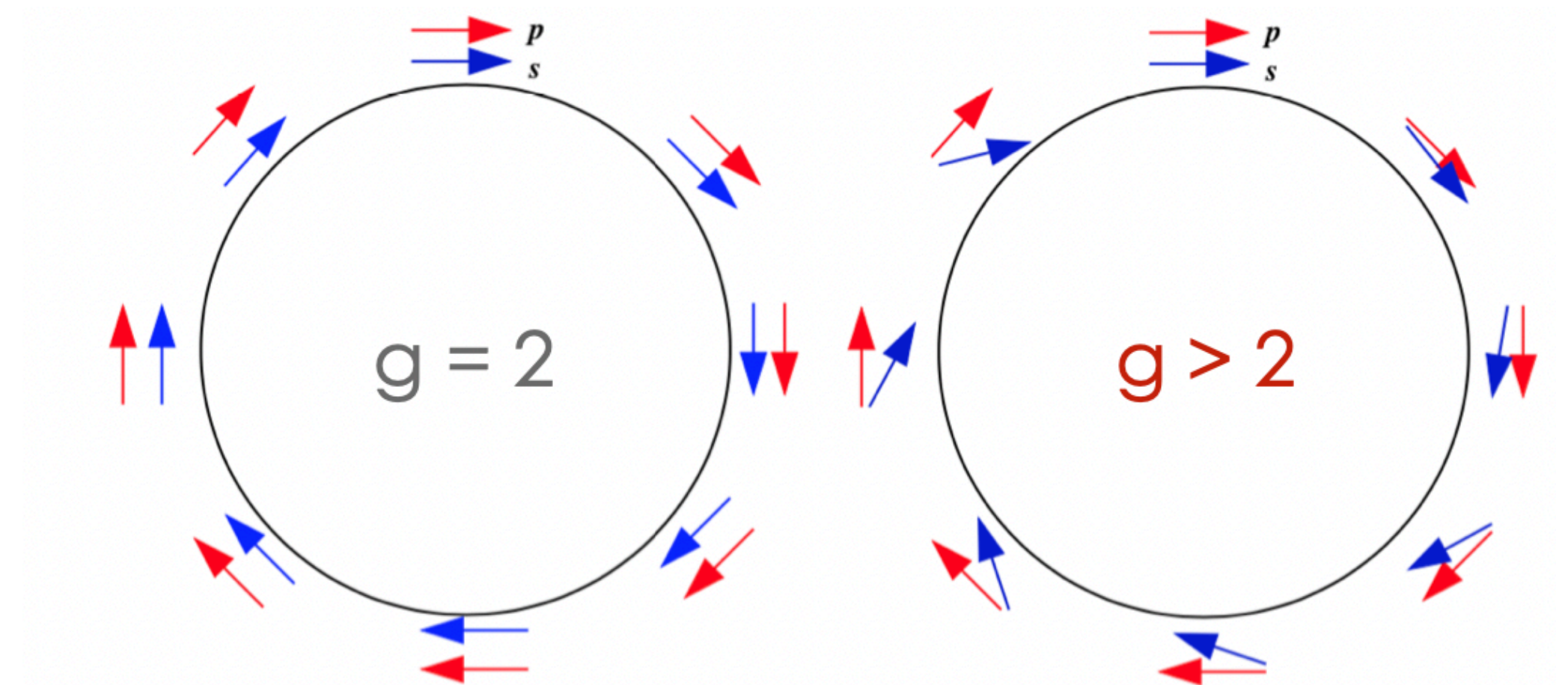
# How to measure muon g-2?

- Why muon g-2?
  - The muon is 200 times more massive than electron  $\rightarrow (m_e/m_\mu)^2 \sim 40\,000$  times more sensitive to new massive particles
  - In a **magnetic storage ring**, the muon spin precesses slightly faster than the cyclotron frequency:

$$\underline{\vec{\omega}_s} = -\frac{ge\vec{B}}{2m} - (1 - \gamma)\frac{e\vec{B}}{m\gamma} \quad \underline{\vec{\omega}_c} = -\frac{e\vec{B}}{m\gamma}$$



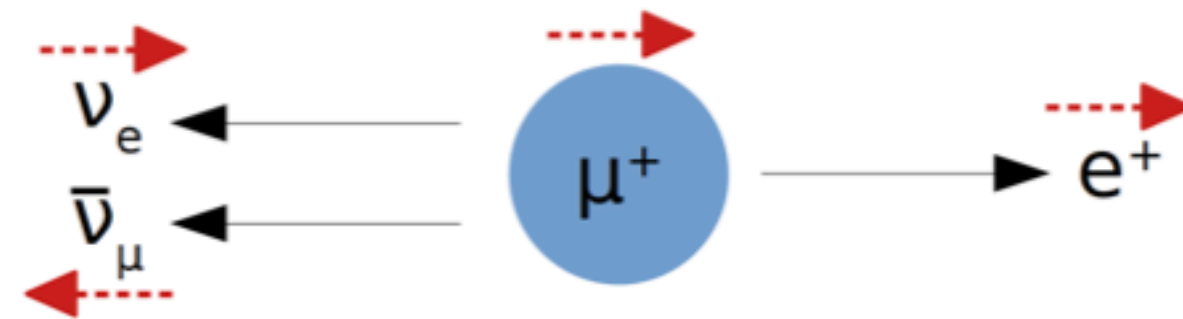
$$\vec{\omega}_a = \underline{\vec{\omega}_s} - \underline{\vec{\omega}_c} = -\left(\frac{g-2}{2}\right)\frac{e\vec{B}}{m} \equiv -\boxed{a_\mu}\frac{e\vec{B}}{m}$$



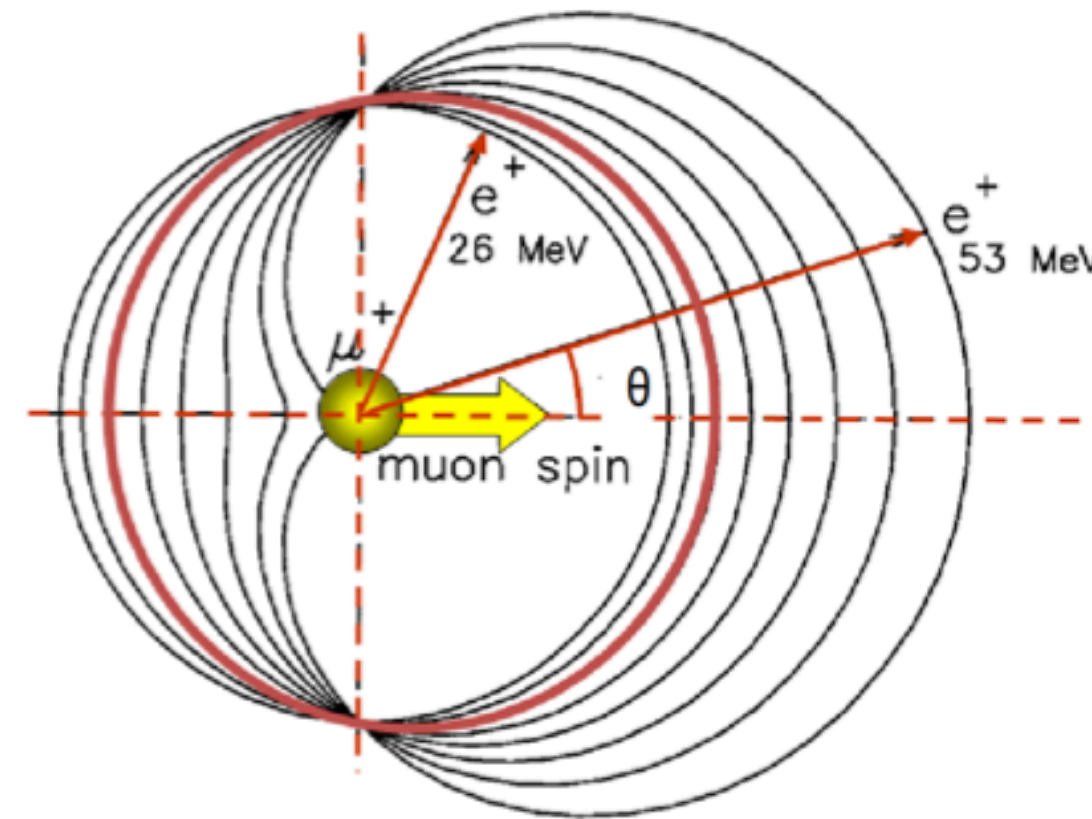
- Measure  $\omega_a$  and  $\mathbf{B} \rightarrow$  obtain  $a_\mu$



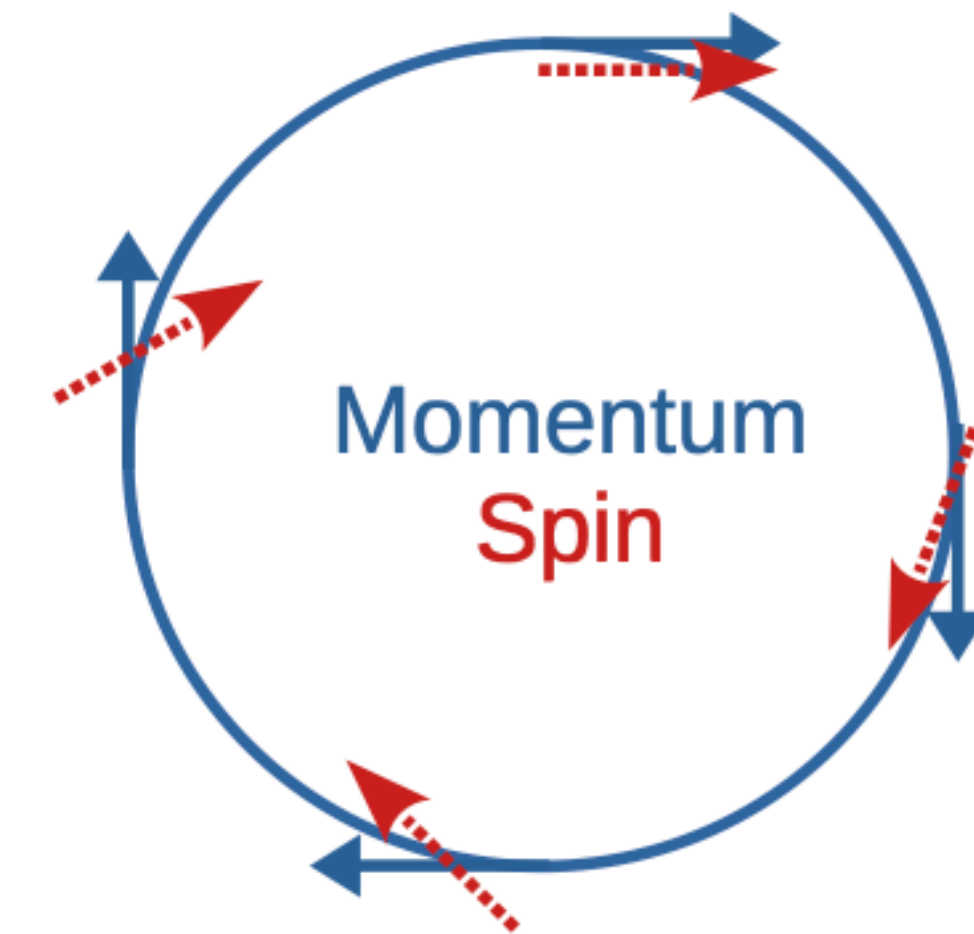
# How to measure $\omega_a$ for muons?



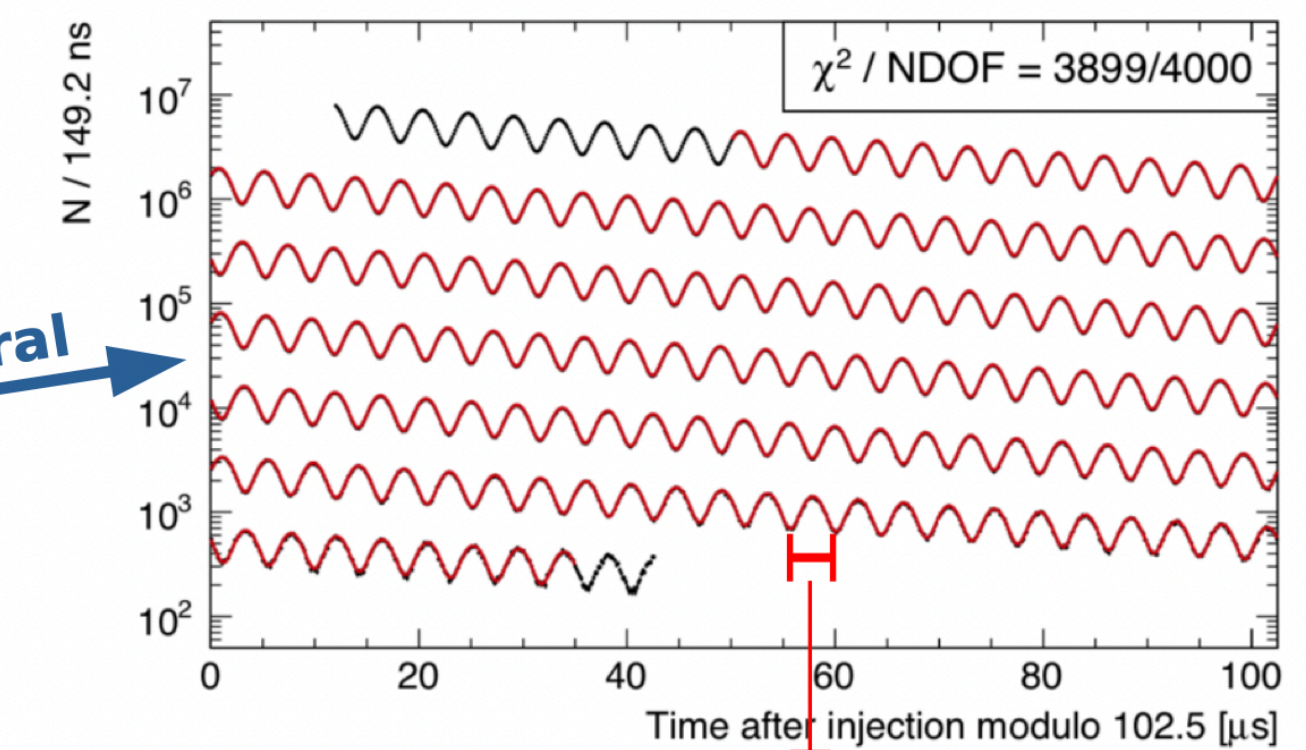
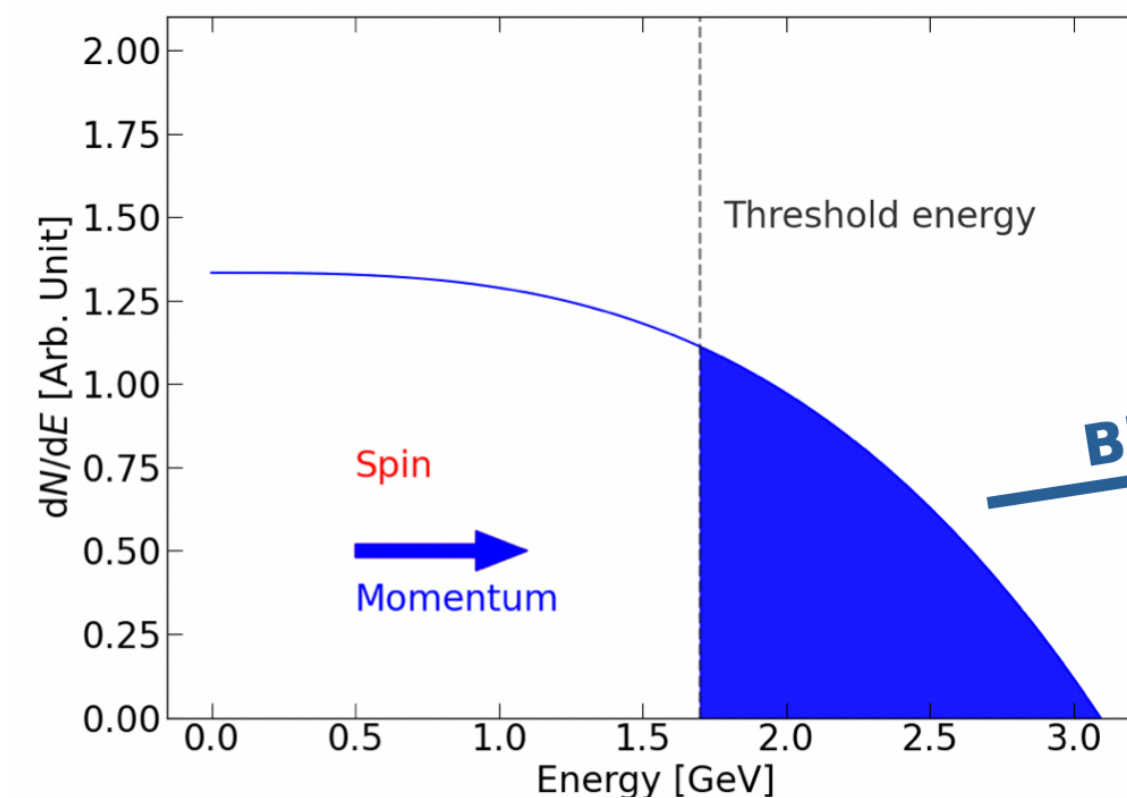
Muon decays in a positron and 2 neutrinos



Parity violation  $\rightarrow$  positrons in CM preferably in the direction of the muon spin



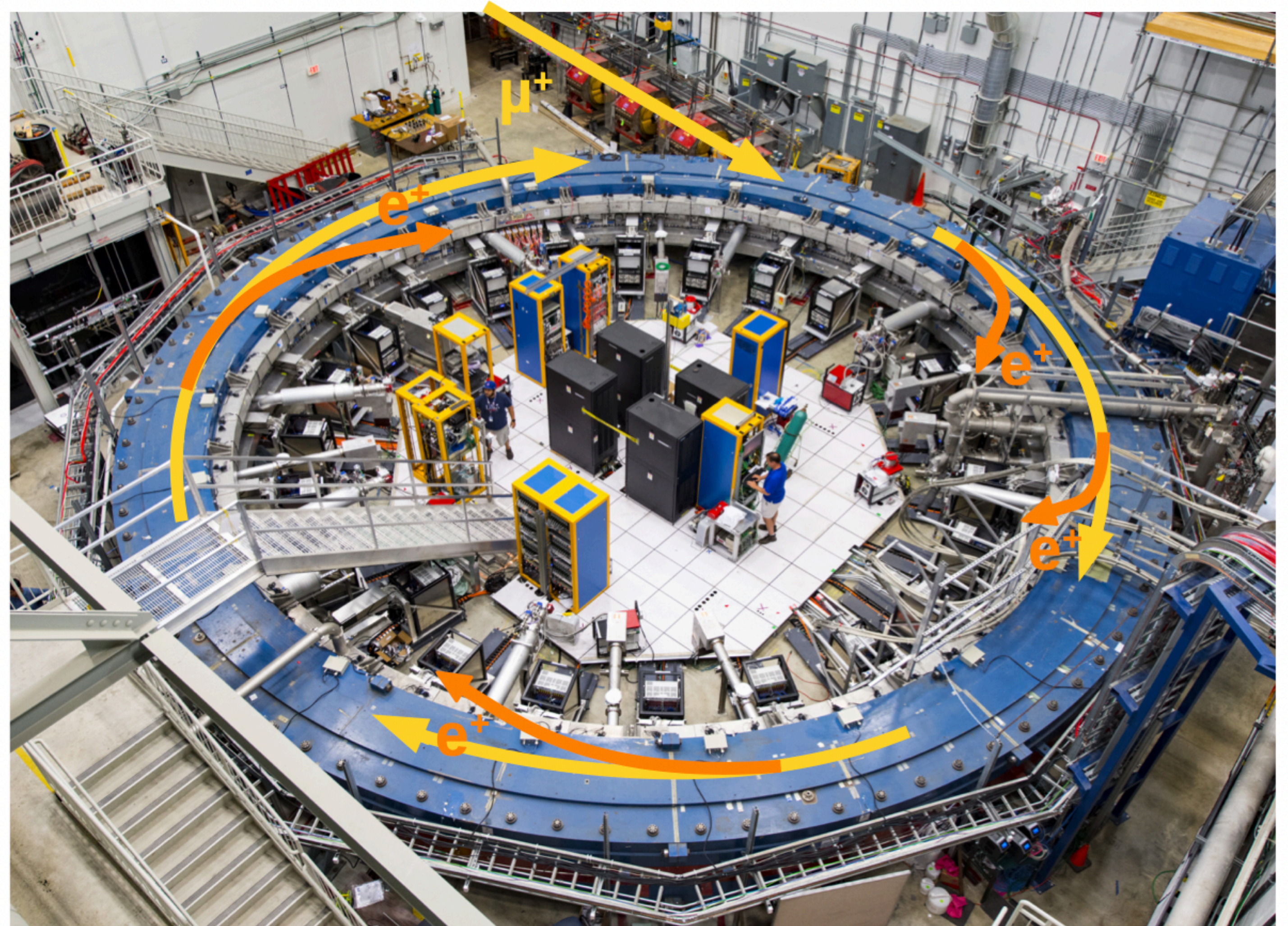
Spin precession  $\rightarrow$  the energy spectrum in the lab frame **oscillates** through time





# The Muon g-2 Experiment

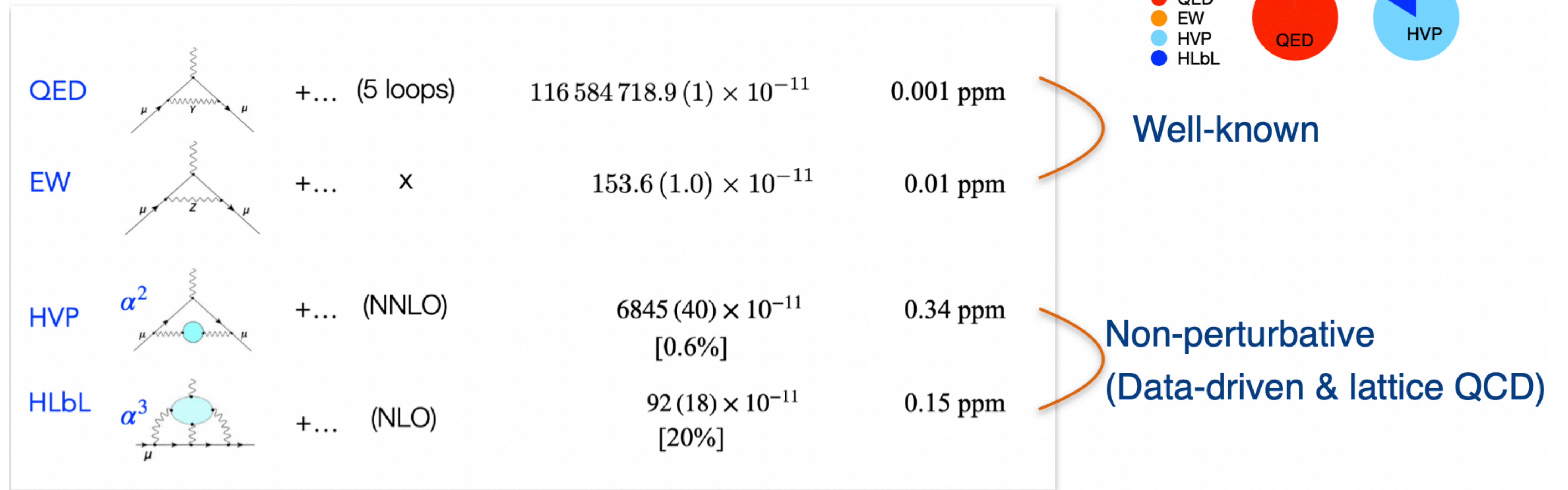
- Located in Fermilab, Chicago (continuing the experiment conducted at Brookhaven)
- **15 m-diameter superconducting magnet** with an exceptionally uniform magnetic field, used as a storage ring
- **24 EM calorimeters** to measure decay positrons (on the inside of the storage ring)
- B-field value actively mapped using an **NMR probe**





# Muon g-2: status of theory calculations

$$a_\mu = a_\mu(QED) + a_\mu(EW) + a_\mu(hadronic)$$



- QED and EW contributions are very well-known with small uncertainties
- **Hadronic vacuum polarisation (HVP)** contribution error dominates the uncertainty budget

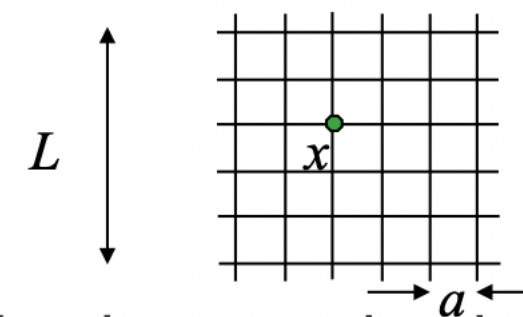


# Muon g-2: most recent result

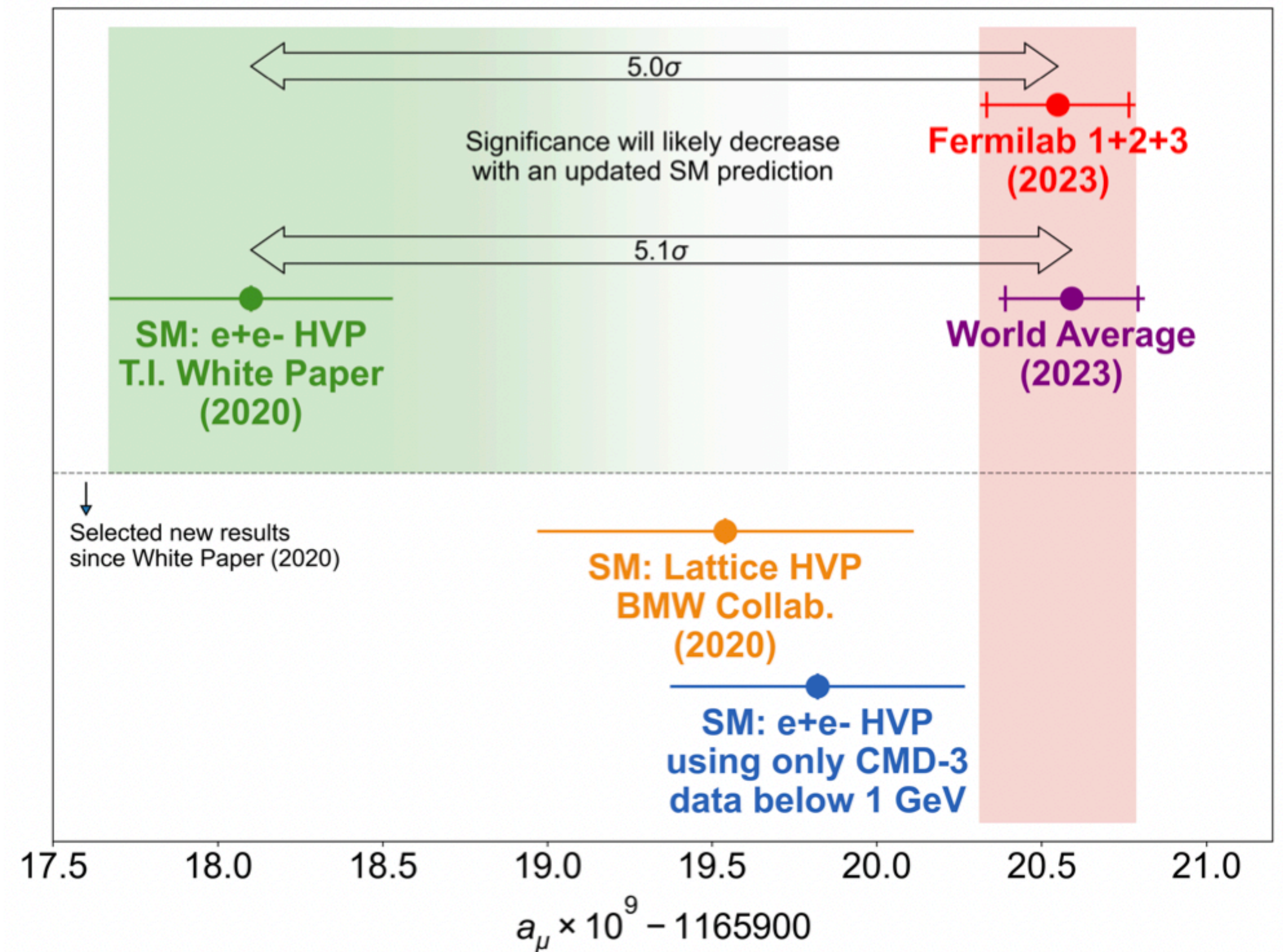
Phys.Rev.Lett. 131 (2023) 16, 161802

- Theory calculation differ in HVP calculations:
  - **lattice QCD** (Ab-initio)
  - **dispersive (e+e-) method** (data-driven)
  - this results in “theory-theory” tensions...

lattice



dispersive

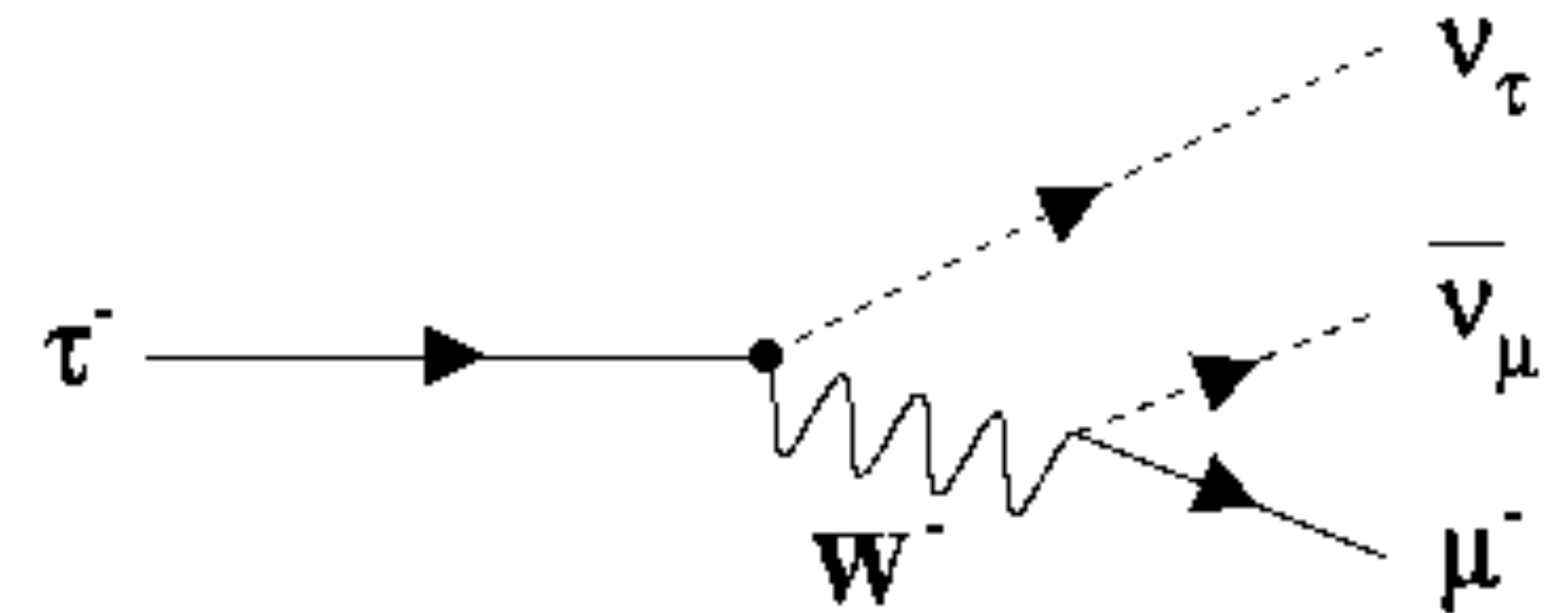


- $g_\mu = 2.0023318411(5)$



# The tau lepton

- Discovered in 1970's, it's the heaviest charged lepton
  - $\approx 2000$  heavier than the electron
  - Due to large mass, it decays almost immediately (lifetime of  $3 \times 10^{-13}$  s)
- Because of extremely short lifetime impossible to make experiments with magnetic field interaction

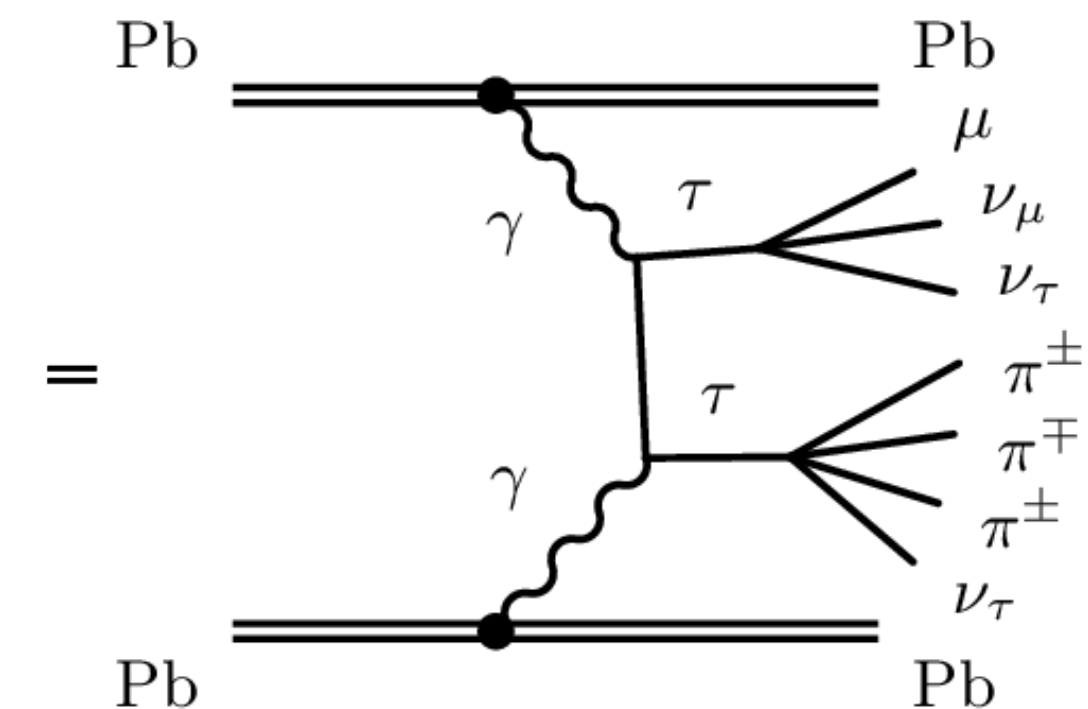
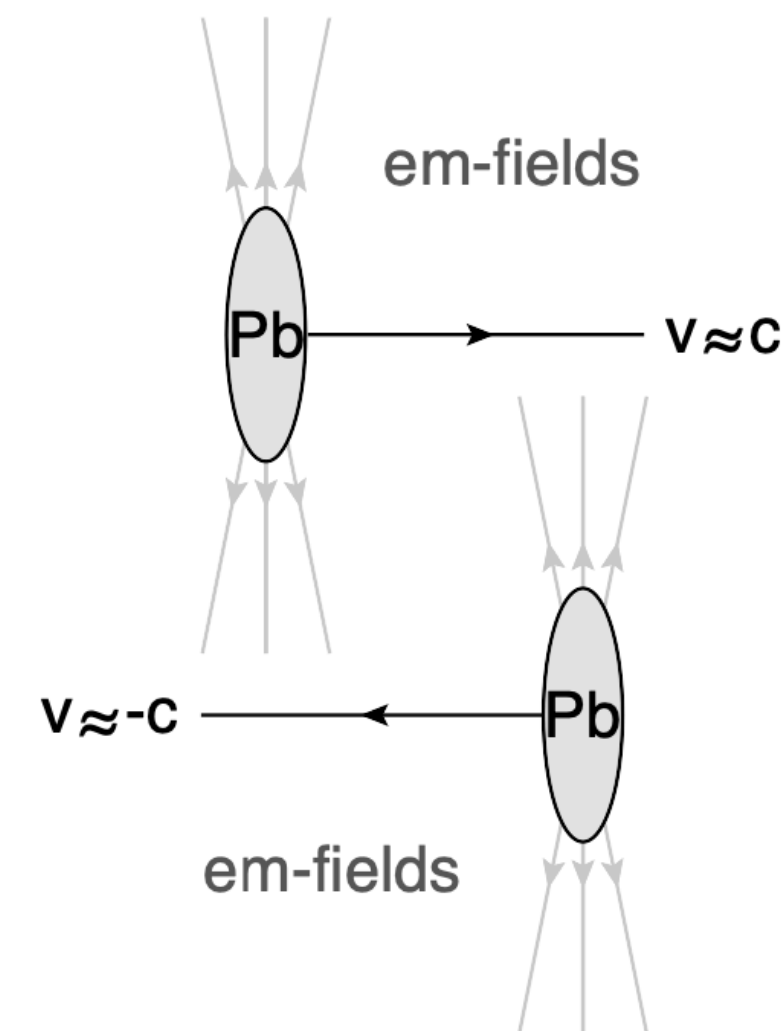
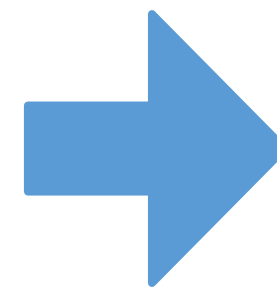
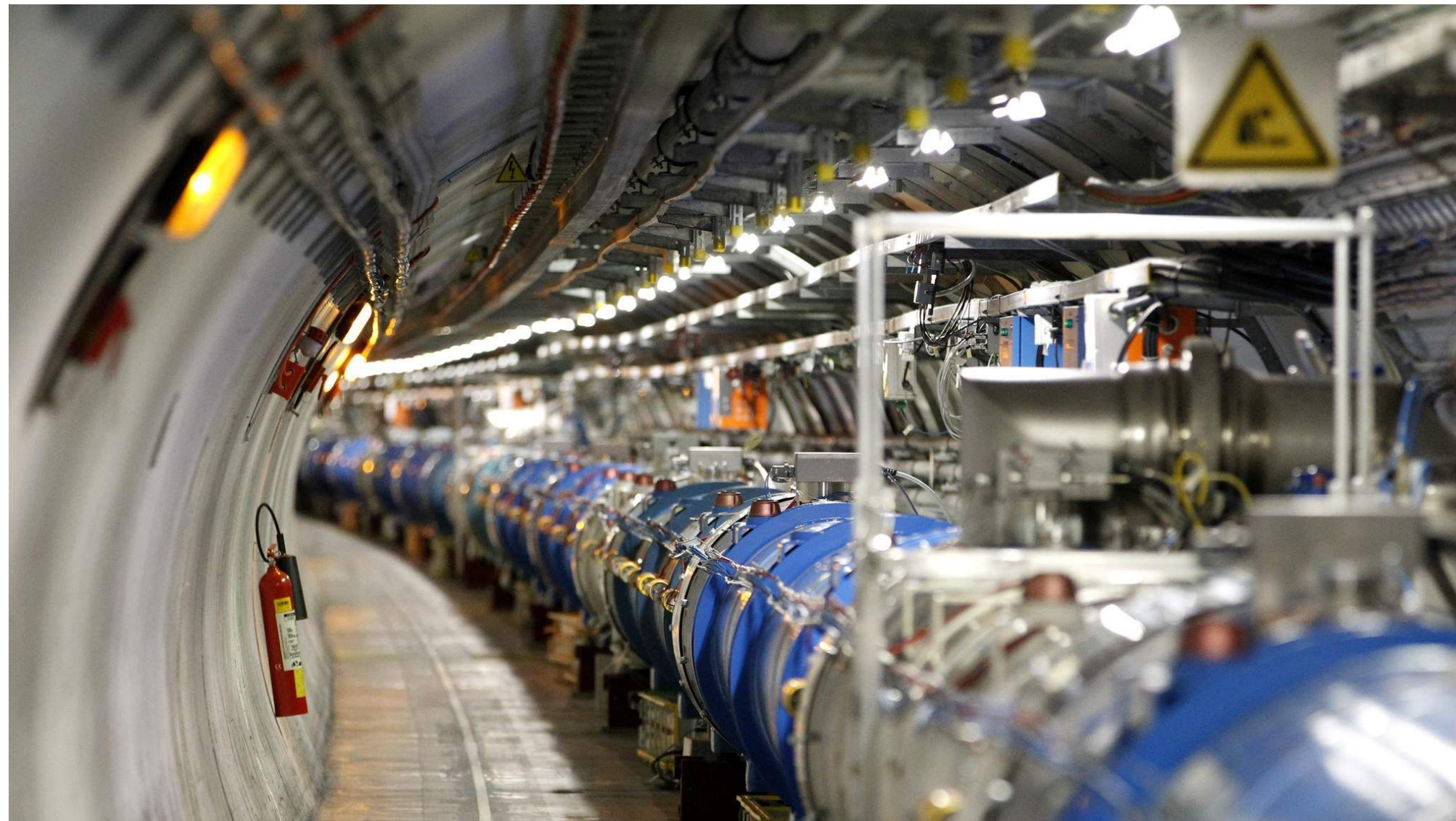
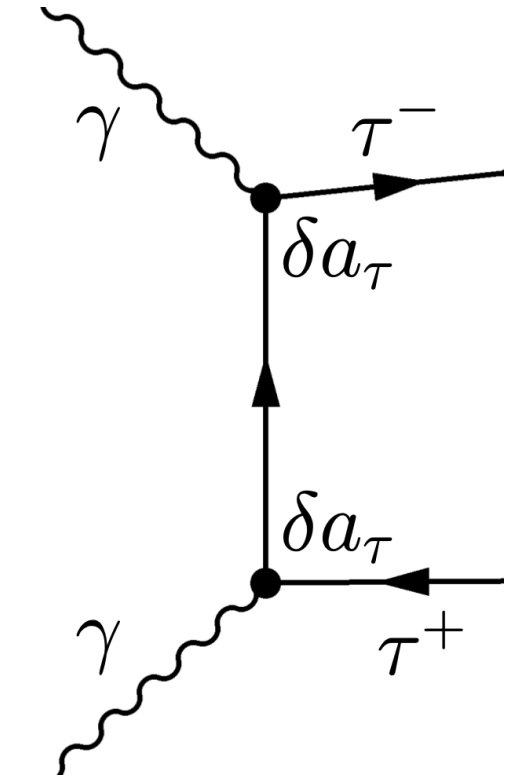


Example tau decay into neutrinos and a muon



# Tau lepton EM interactions at the LHC

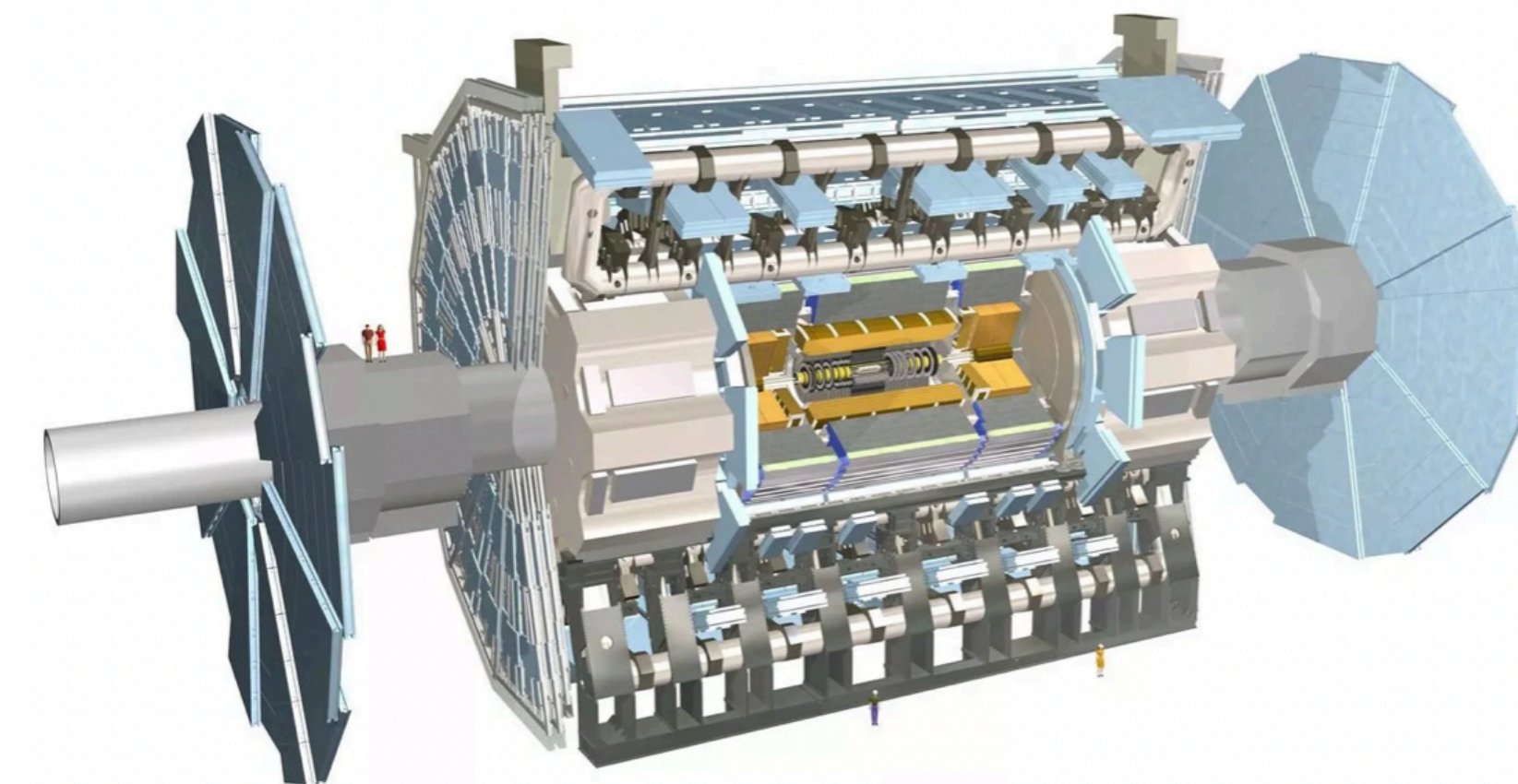
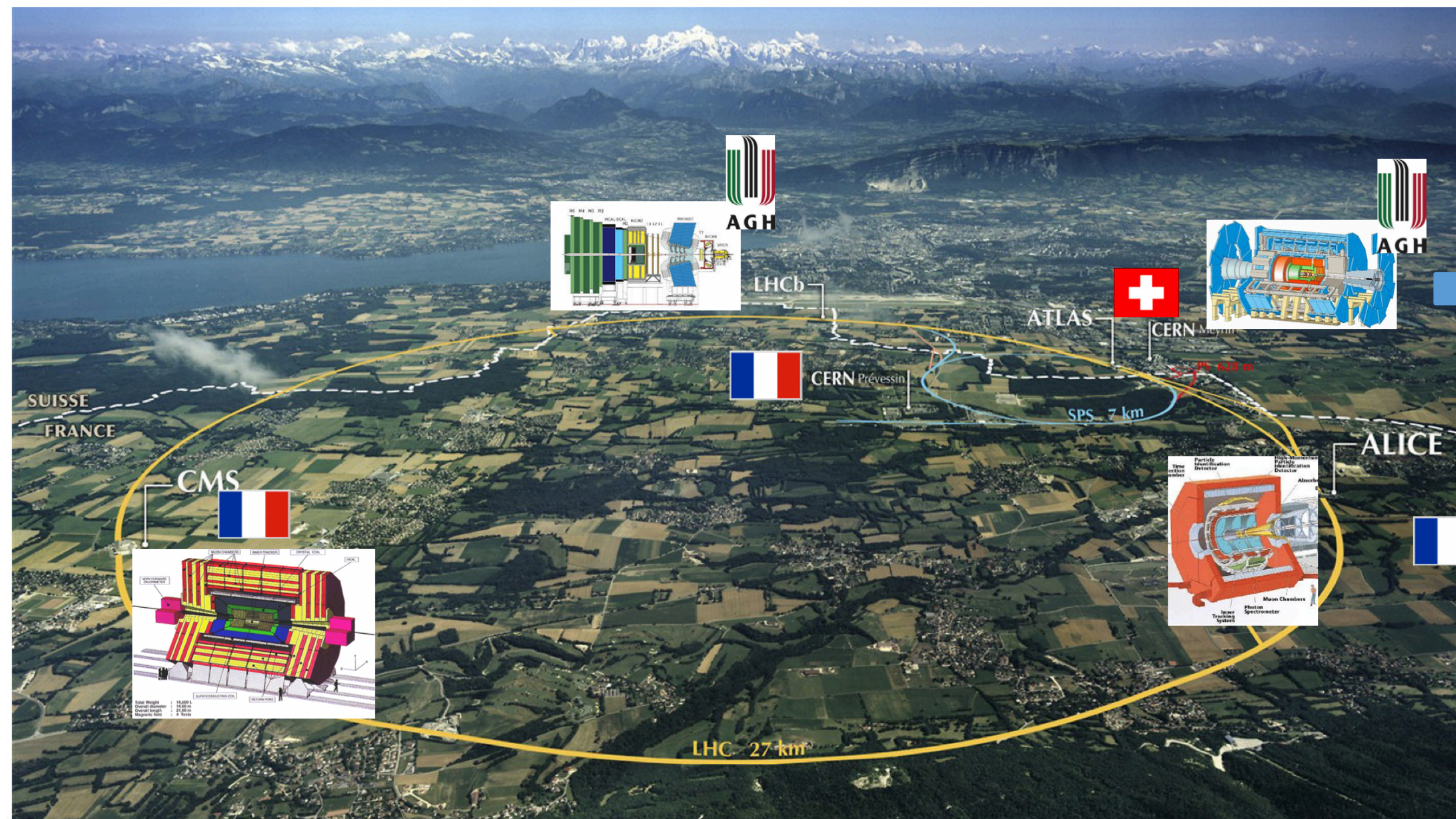
- We can measure the strength of EM interaction with tau lepton by studying the following process:
  - But: a powerful source of high-energy photons is needed...
- Heavy (charged) ions are intense source of photons
  - Fortunately we collide lead ions at the Large Hadron Collider!





# Large Hadron Collider and the ATLAS experiment

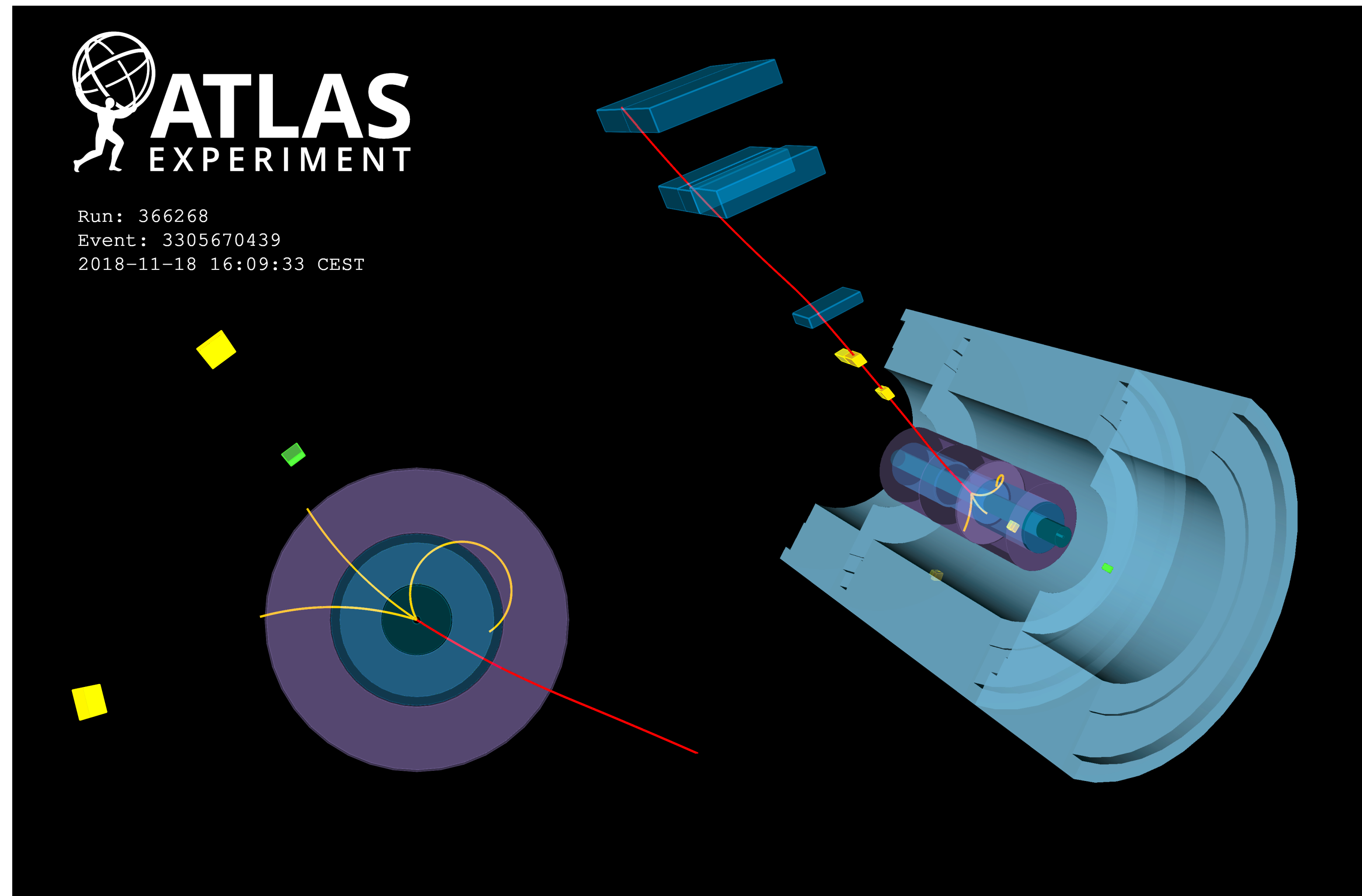
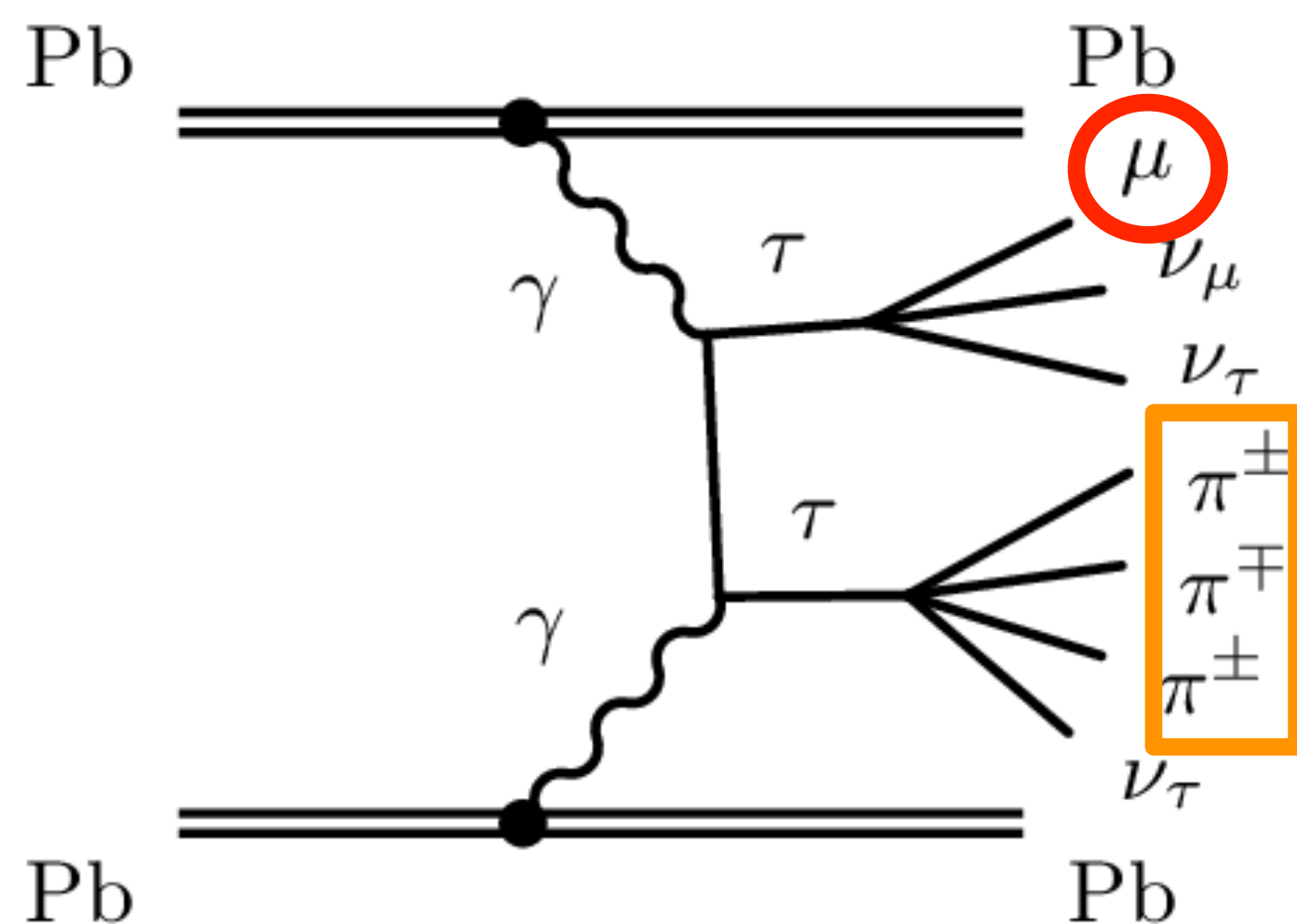
- LHC is the most powerful **particle accelerator** in the world
  - Four main experiments placed at the LHC ring: ALICE, **ATLAS**, CMS, LHCb





# Recording tau pairs in the ATLAS experiment

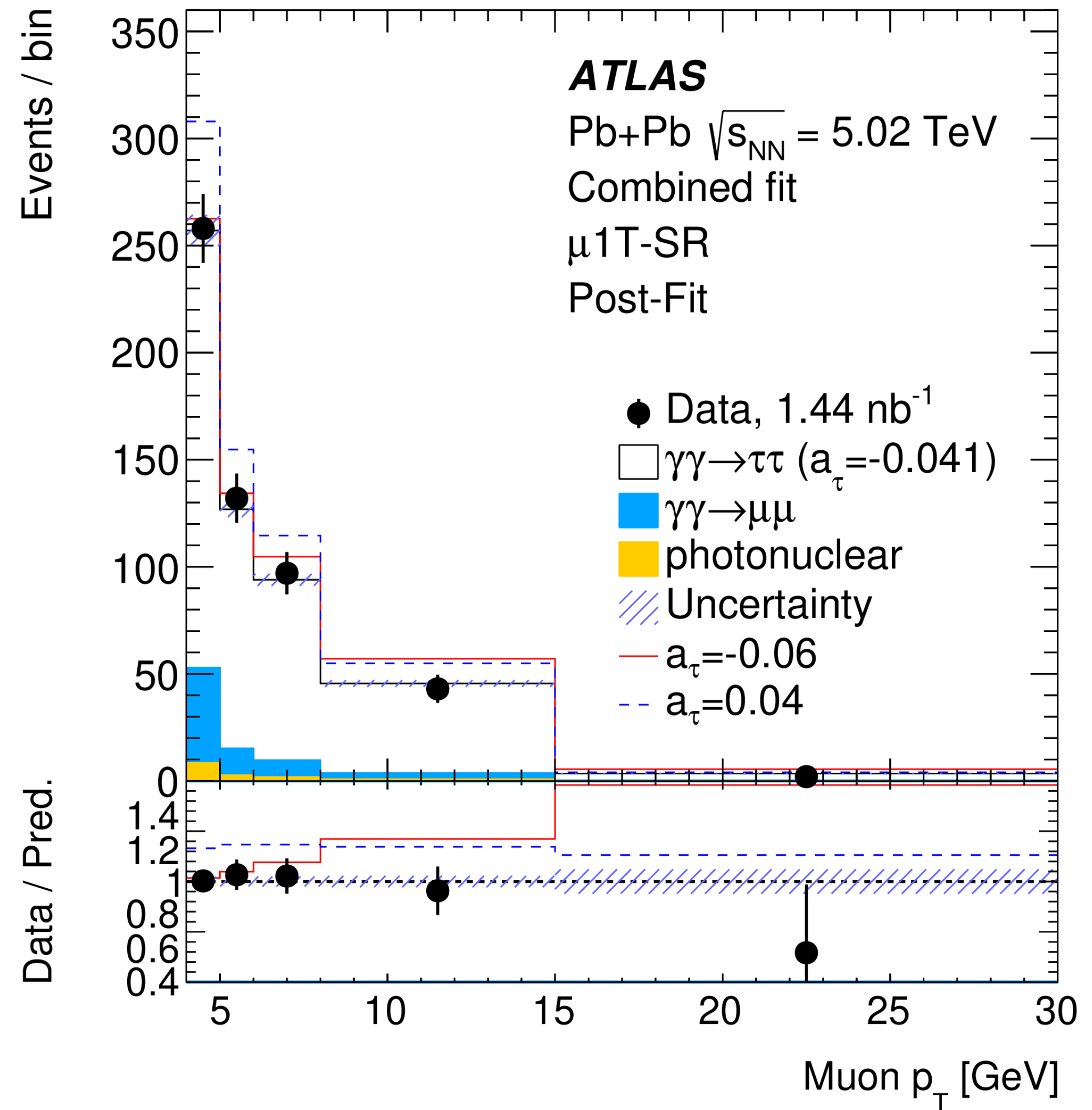
- In ATLAS, we reconstruct decay products of taus
- We observe taus **for the first time** in ion collisions!





# Tau g-2 measured in ATLAS

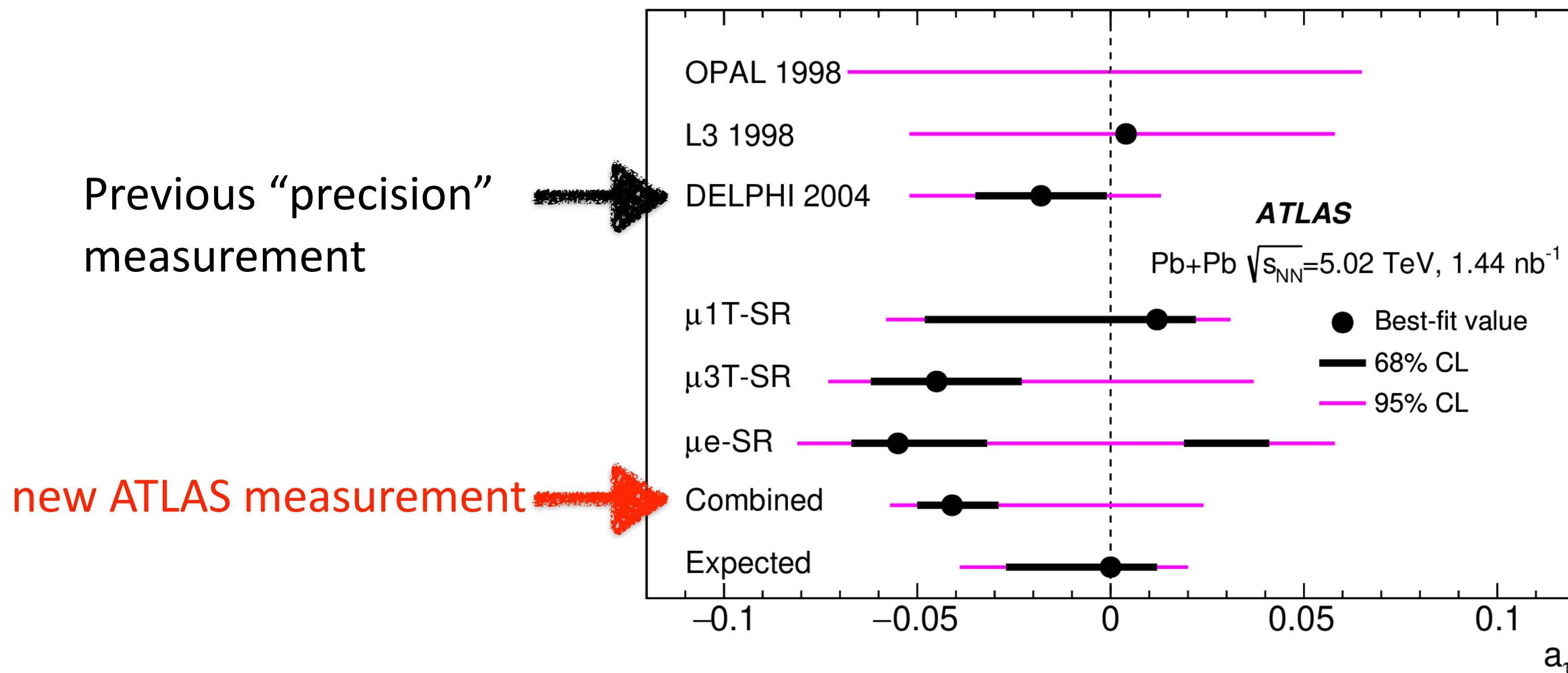
- The value of  $a_\tau = (g_\tau - 2)/2$  is sensitive to both cross-section variations and shapes of kinematic distributions





# Tau g-2 measured in ATLAS

- We measure  $a_\tau = (g_\tau - 2)/2$  Phys.Rev.Lett. 131 (2023) 15, 151802
- $g_\tau$  is found to be consistent with “2” ( **$1.94 < g_\tau < 2.02$  @95% CL**)
- Note there is also an alternative measurement from CMS experiment (but far less precise)





# Summary

- Magnetic dipole moments of charged leptons can be measured with high precision
  - Sensitive to ‘new’ particles via quantum fluctuations
  - Unprecedented accuracy achieved for electrons and muons:  
 $g_e = 2.0023193043612(3)$ ,  $g_\mu = 2.0023318411(5)$
  - Waiting for a clarification (of the theory): discrepant  $\alpha$  measurements (electrons), hadronic contributions (muons)
- Challenging to measure tau lepton magnetic dipole moment
  - ATLAS experiment has measured this recently (for the first time by using heavy-ion collisions)  
 $(1.94 < g_\tau < 2.02 \text{ @95\% CL})$  -> precision will be improved by studying more data
  - Major involvement of WFiS staff and students in the preparation of the ATLAS result!



# Backup



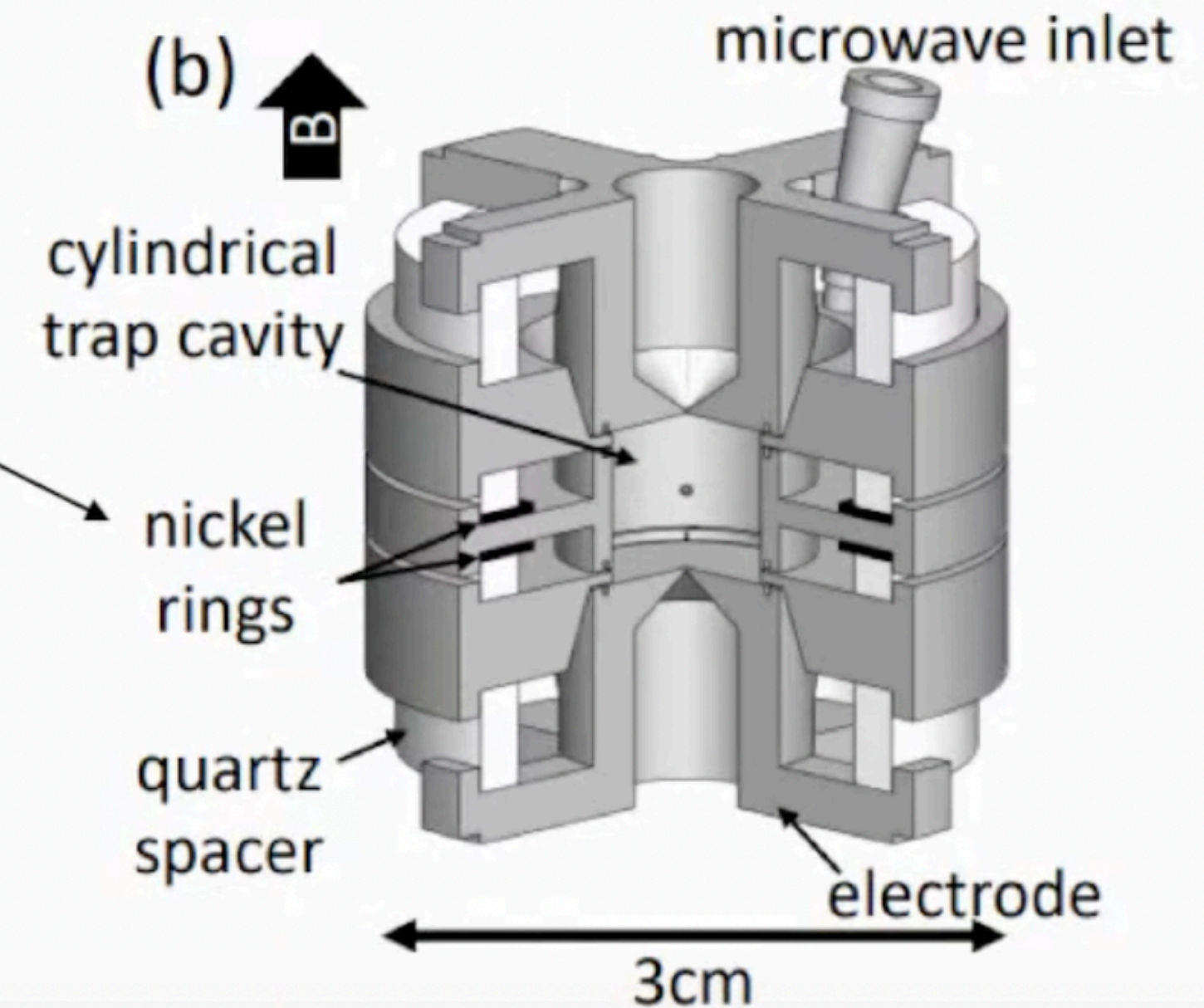
# Electron g-2: quantum non-demolition (QND)

Add a small magnetic gradient to couple the spin and cyclotron magnetic moments to the axial frequency  $\Delta\vec{B} = B_2 z^2$

Hamiltonian for the axial motion

$$H = \frac{1}{2} m \omega_z^2 z^2 - \mu B_2 z^2$$

↓  
shifts observed  $\omega_z$



QND → makes quantum transitions without causing them

$$\Delta \bar{\nu}_z \approx 1.3 (n + m_s) \text{ Hz}$$



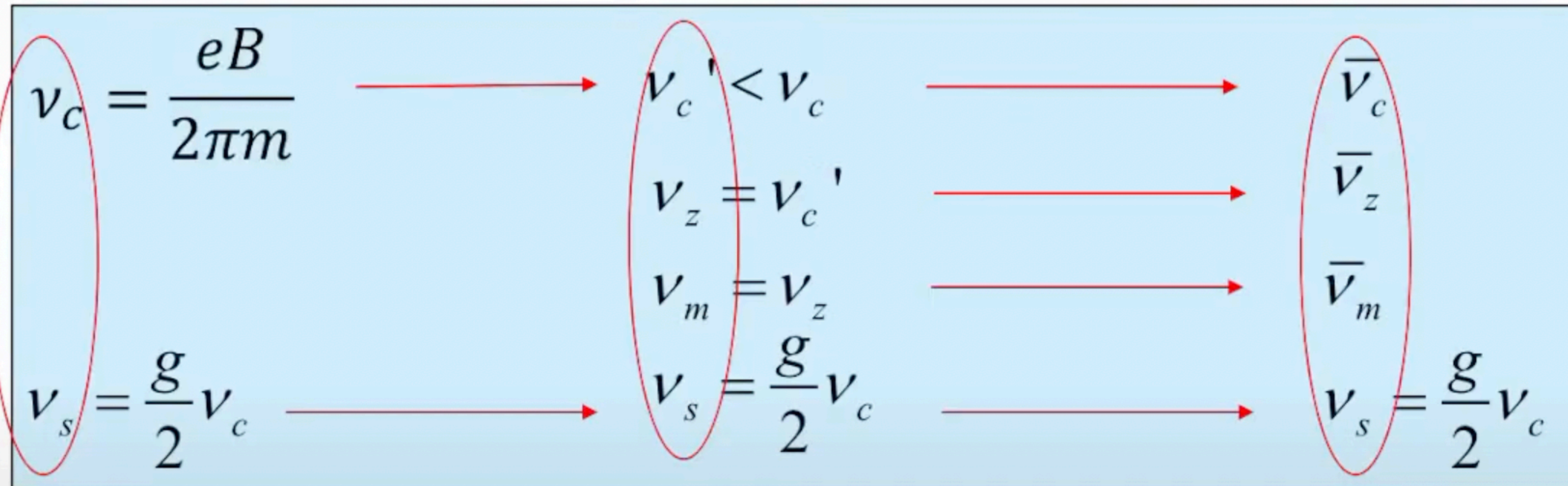
# Electron g-2: trap imperfections

**B in Free Space**

**Perfect Electrostatic  
Quadrupole Trap**

**Imperfect Trap**

- tilted B
- harmonic distortions to the trapping potential



Problem:  $\frac{g}{2} = \frac{\nu_s}{\nu_c}$  ← not a measurable eigenfrequency in an imperfect Penning trap

**Solution: Brown-Gabrielse Invariance Theorem**

$$\nu_c = \sqrt{(\bar{\nu}_c)^2 + (\bar{\nu}_z)^2 + (\bar{\nu}_m)^2}$$



$$-\frac{\mu}{\mu_B} = \frac{g}{2} \simeq 1 + \frac{\bar{\nu}_a - \bar{\nu}_z^2/(2\bar{f}_c)}{\bar{f}_c + 3\delta/2 + \bar{\nu}_z^2/(2\bar{f}_c)} + \frac{\Delta g_{cav}}{2}$$



# Electron predictions

term	contribution
tree level	1.000 000 000 000 000
$C_2 \left(\frac{\alpha}{\pi}\right)$	0.001 161 409 731 851 (000)(093)
$C_4 \left(\frac{\alpha}{\pi}\right)^2$	−0.000 001 772 305 060 (000)(000)
$C_6 \left(\frac{\alpha}{\pi}\right)^3$	0.000 000 014 804 204 (000)(000)
$C_8 \left(\frac{\alpha}{\pi}\right)^4$	−0.000 000 000 055 668 (000)(000)
$C_{10} \left(\frac{\alpha}{\pi}\right)^5$	0.000 000 000 000 456 (011)(000)
$a_{\mu,\tau}$	0.000 000 000 002 748 (000)
$a_{\text{hadron}}$	0.000 000 000 001 693 (012)
$a_{\text{weak}}$	0.000 000 000 000 031 (000)
total SM prediction	1.001 159 652 180 252 (011)(012)(093)

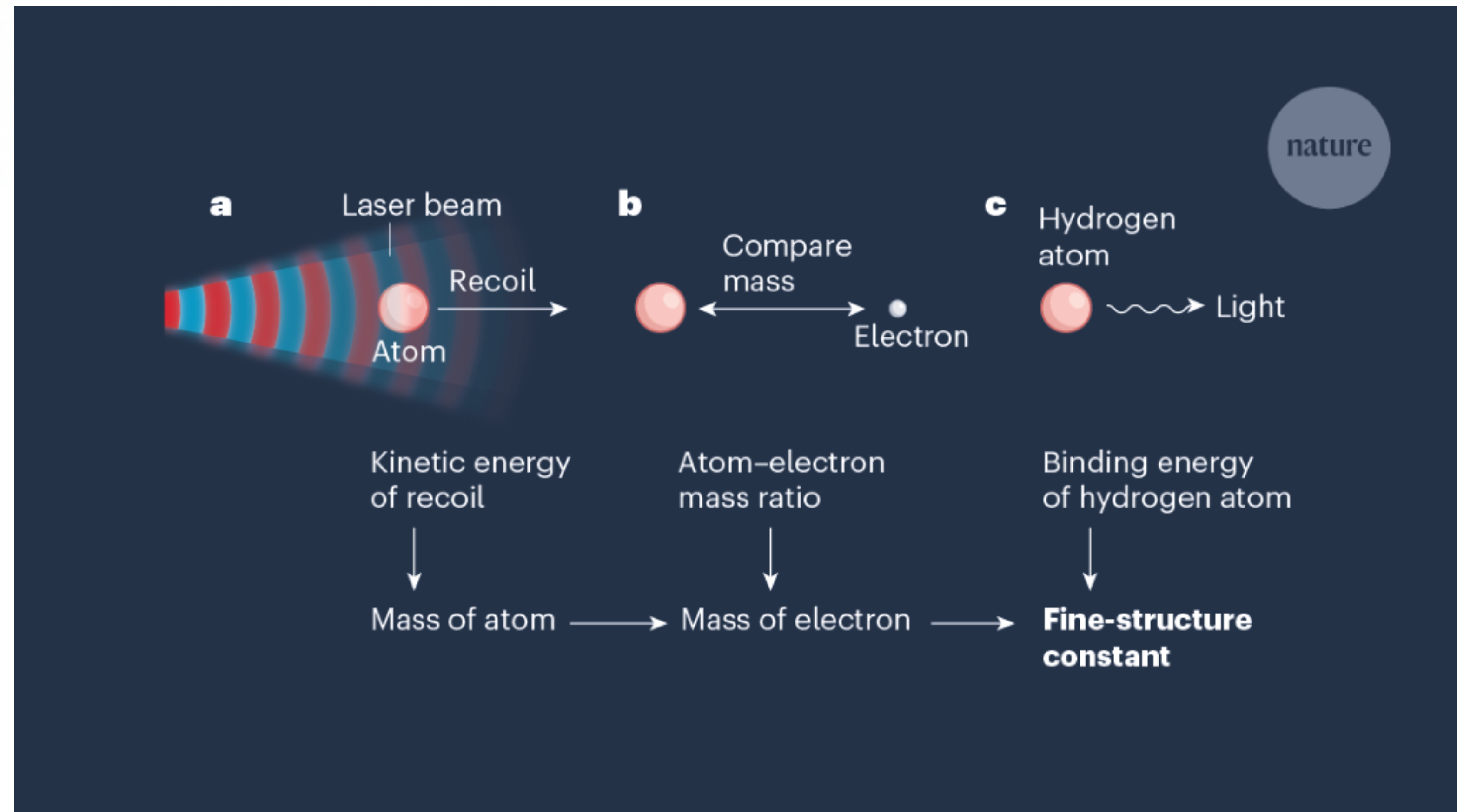
**X. Fan, PhD thesis**



# Fine structure constant measurement

successful independent approach is based on the measurement of the recoil velocity ( $v_r = \hbar k/m$ ) of an atom of mass  $m$  that absorbs a photon of momentum  $\hbar k$  (refs. [10,11](#)). Here  $\hbar$  is the reduced Planck constant ( $\hbar = h/(2\pi)$ ) and  $k = 2\pi/\lambda$  is the photon wave vector, where  $\lambda$  is the laser wavelength. Such a measurement yields the ratio  $h/m$  and then  $\alpha$  via the relation

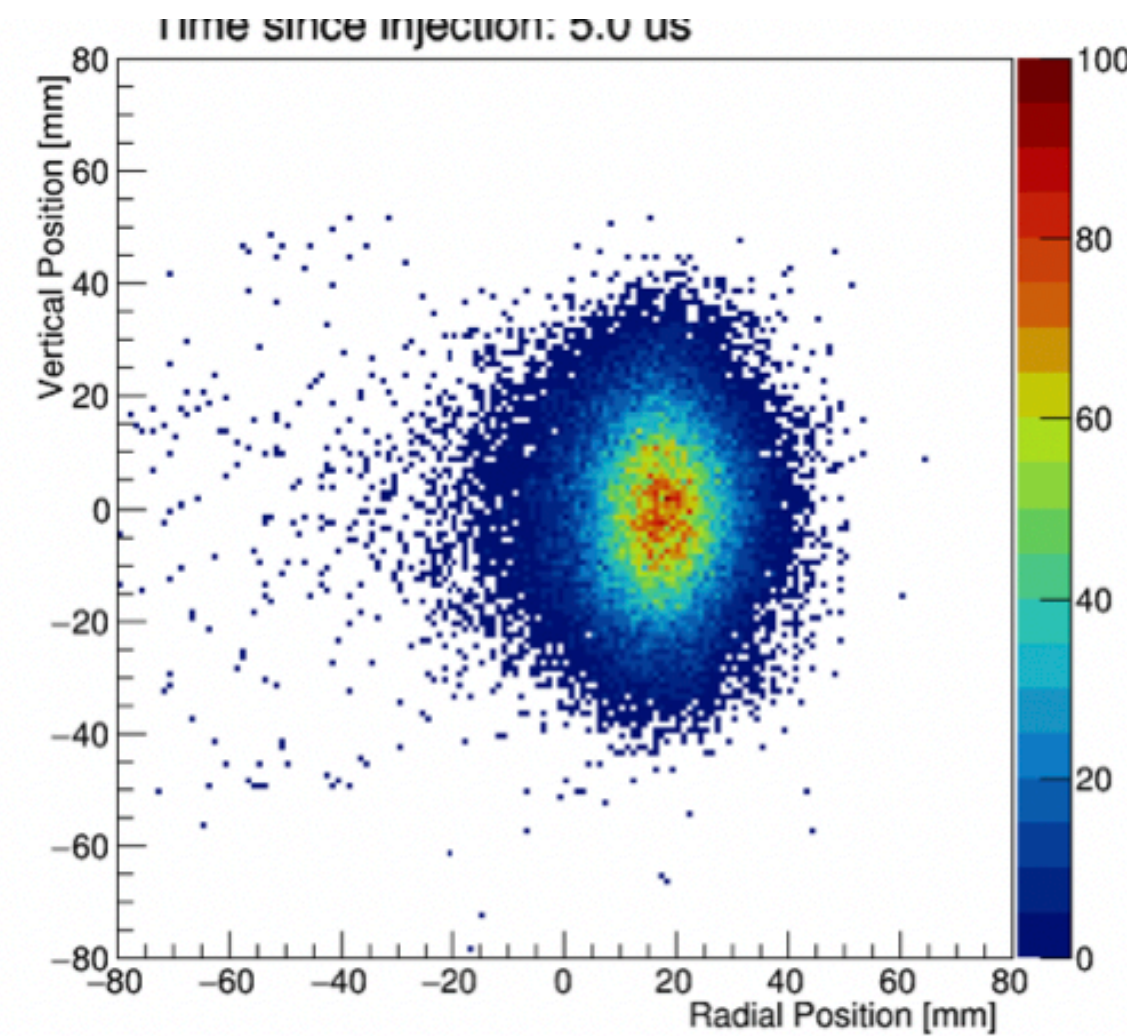
$$\alpha^2 = \frac{2R_\infty}{c} \times \frac{m}{m_e} \times \frac{h}{m}.$$





# Muon g-2: beam-dynamics effects

- The muon beam oscillates and breathes as a whole
- The full equation is more complex and corrections due to radial (x) and vertical (y) beam motion are needed



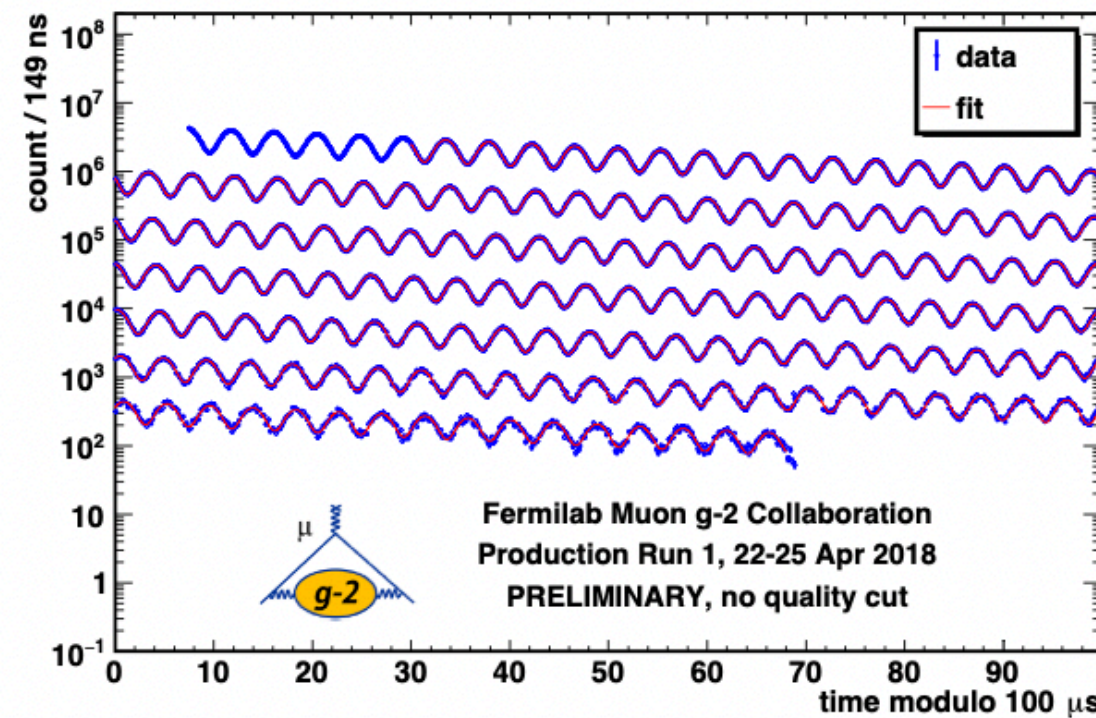
$$\begin{aligned}\vec{\omega}_a &= \vec{\omega}_s - \vec{\omega}_c = \\ &= -\frac{e}{mc} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} \right]\end{aligned}$$

- Running at  $\gamma_{\text{magic}}=29.3$  ( $p=3.094$  GeV/c) this coefficient is null
- Because of momentum spread ( $<0.2\%$ )  $\rightarrow$  **E-field Correction**

- Vertical beam oscillation  $\rightarrow$  **Pitch correction**



# Muon g-2: extracting $a_\mu$



$\omega_a$

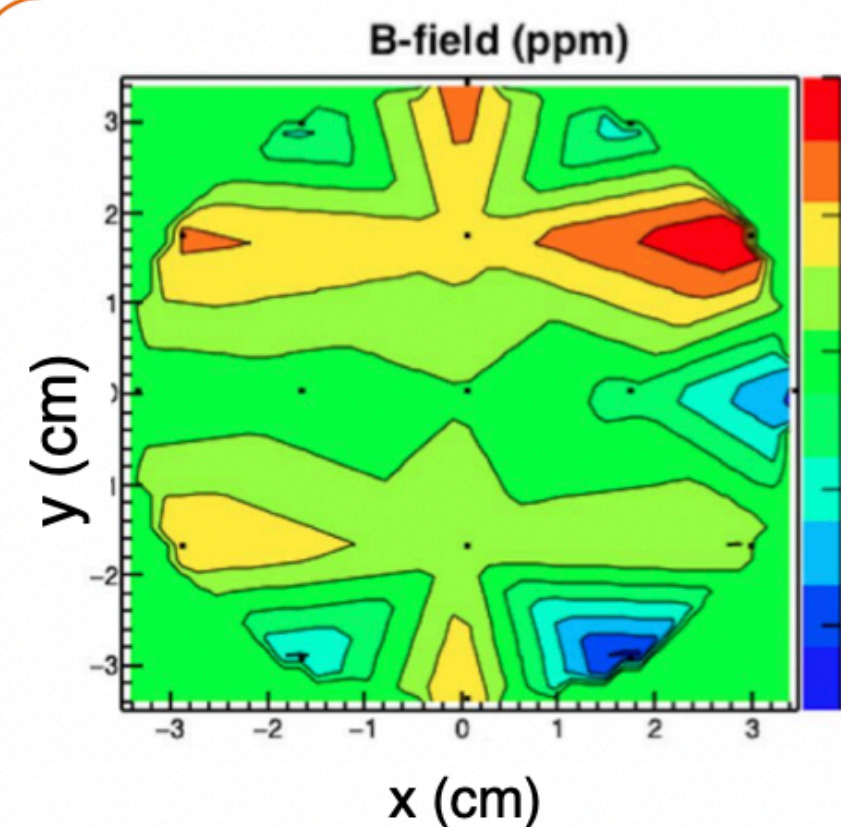
Extract from decay positron time spectra

$$N(t) = N_0 e^{-t/\tau_\mu} [1 + A \cos(\omega_a t + \phi)]$$

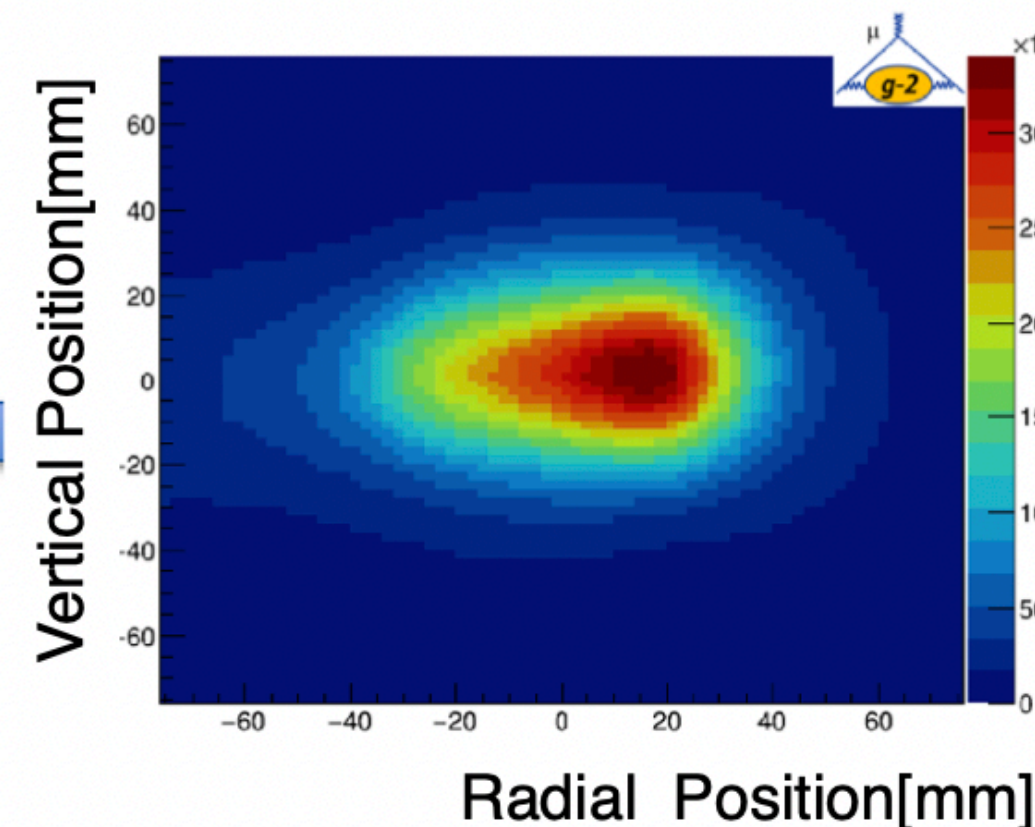
$$a_\mu = \left( \frac{g_e}{2} \right) \left( \frac{\omega_a}{\langle \omega_p \rangle} \right) \left( \frac{\mu_p}{\mu_e} \right) \left( \frac{m_\mu}{m_e} \right)$$

0.26 ppt      3 ppb      22 ppb      ⇒ 2017 CODATA

	Relative error (ppb)	Experiment
$g_e$	0.000 26	Quantum electron cyclotron. Hanneke et al. 2008.
$\mu_e/\mu_p$	3.0	Hydrogen spectroscopy. Winkler et al. 1972.
$m_\mu/m_e$	22	Muonium hyperfine splitting. Liu et al. 1999.



Map the magnetic field



Obtain muon distribution in the storage ring

$$\langle \omega_p \rangle \approx \omega_p \otimes \rho(r)$$

Average magnetic field weighted by muon distribution

$\omega_p$ : free proton precession frequency  
Using proton NMR  $\hbar\omega_p = 2\mu_p B$



# Tau g-2: a\_tau parametrisation

- Elementary  $\gamma\gamma \rightarrow \tau\tau$  cross section has explicit dependence on photon- $\tau$  vertex function:

$$i\Gamma_{\mu}^{(\gamma\ell\ell)}(p', p) = -ie \left[ \gamma_{\mu} F_1(q^2) + \frac{i}{2m_{\ell}} \sigma_{\mu\nu} \underline{q^{\nu}} \boxed{F_2(q^2)} + \frac{1}{2m_{\ell}} \gamma^5 \sigma_{\mu\nu} \underline{q^{\nu}} \boxed{F_3(q^2)} \right]$$

$\xrightarrow{\quad} = a_{\tau} (q^2=0) \qquad \qquad \qquad = d_{\tau} * 2m_{\tau} / e (q^2=0)$