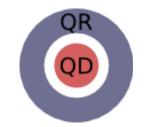
Wave function engineering in quantum dot-ring structures

## Nanostructures with highly controllable electronic properties

E. Zipper, M. Kurpas, M. M. Maśka Instytut Fizyki, Uniwersytet Sląski w Katowicach, Poland

## Talk outline



Szafran B, Peeters F M and Bednarek S 2004 *Phys. Rev. B 70 125310* 

1. **DRN** structure - quantum dot (QD) surrounded by a quantum ring (QR).

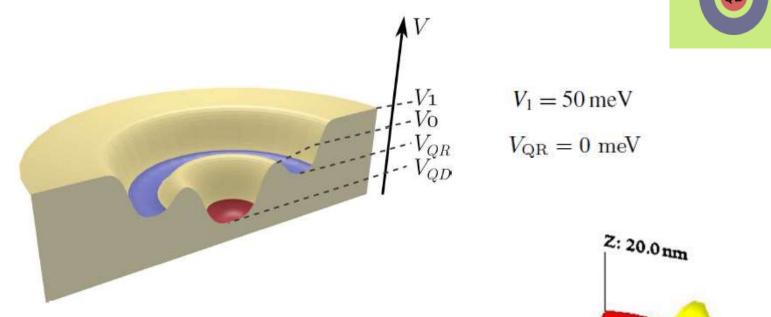
2. Relaxation times of spin qubits and memory devices built on DRNs.

- 3. Engineering of optical absorption of DRNs.
- 4. Conducting properties of arrays of DRNs.
- 5. Summary.

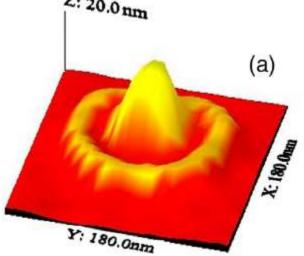
E.Z., M.Kurpas, M.Maśka

arXiv:1203.1025v1, New Journal of Physics 14,(2012)093029

• 2D semiconductor nanostructure (DRN) in the form of quantum dot (QD) surrounded by a quantum ring (QR).

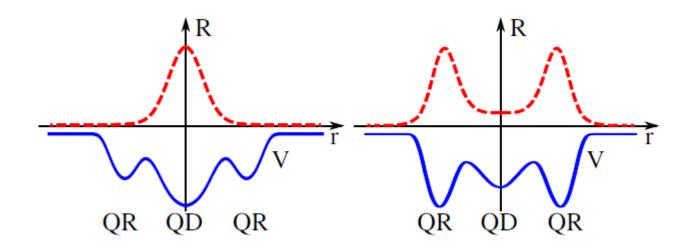


Electron can tunnel from QD to QR via the barrier  $V_0$ 



Somaschini et al., Nanotechnology 22, 185602 (2011

•By changing the barrier parameters and/or potential well offset one changes the shape and distribution of wave functions – wave function engineering



- Such manipulations can change:
- spin relaxation times of DRNs by orders of magnitude
- cross section for optical intraband absorption from strong to negligible
- conducting properties of a DRN and of an array of DRNs from highly conducting to insulating

### **Basic theoretical formulas**

• Single electron in a DRN in a static in-plane magnetic field B

$$H_{\mathbf{R}} = \frac{\mathbf{p}^2}{2m^*} + \frac{e\hbar}{2m_e} \hat{\boldsymbol{\sigma}} \cdot \boldsymbol{B} + V(r),$$

•The single electron energy spectra – set of discrete states –  $E_{nl}$ , n=0,1,..., l=0, ±1 ...

$$\Delta_{nl} = E_{nl} - E_{00}$$

$$\Psi_{nl} = R_{nl}(r) \exp(il\phi) \chi_{\sigma},$$

Overlap factor:  $\Xi_{nl} = \int_0^\infty R_{00}^* R_{nl} r^2 dr$   $l = \pm 1.$ 

#### Spin relaxation time for dot-ring nanostructure

- Nanostructure with a single electron in magnetic field B a natural two level system – spin qubit or spin memory device
- InGaAs with  $R_{DRN}$ =80nm,  $m^* = 0.067m_e$ ,  $|g_s| = 0.8$  B = 1T

GS with  $E_{00} - |g\rangle$ 

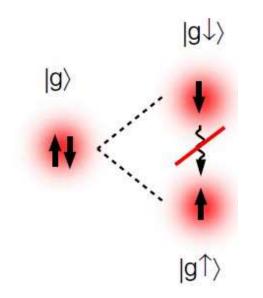
$$\Delta_Z = g_s \mu_B B,$$

$$k_BT << \Delta_Z << \Delta_{01}$$

• Modification of relaxation times in DRNs

#### Relaxation for spin qubit ( $B_{II}=1T$ )

• At 0.5T<B<10T relaxation mainly due to one-phonon admixture of orbital to spin state



Amasha et al., PRL100, 046803, (2008)

drastic reduction of spin relaxation rates due to strong quantum confinement;

- T<sub>1</sub> calculated by Khataetski and Nazarov for circularly symmetric nanostructures with very strong confinement in the z direction (A.V.Khaetskii, Y.Nazarov, Phys.Rev. B 64, 125316,(2001)
- The most important mechanism is due to spin-orbit mediated interaction with piezoelectric phonons

$$H = H_R + \frac{e\hbar}{2m^*}\hat{\sigma} \cdot B + H_{\rm SO} + V_{\rm ph} \tag{1}$$

where

$$H_{\rm SO} = \beta \left( -\hat{\sigma}_x p_x + \hat{\sigma}_y p_y \right) \tag{2}$$

is the Dresselhaus SO interaction due to the bulk inversion asymmetry,

$$H_R = \frac{1}{2m^*} \left( \mathbf{p} + e\mathbf{A} \right)^2 + V(r), \tag{3}$$

• For an electron occupying the nth orbital level

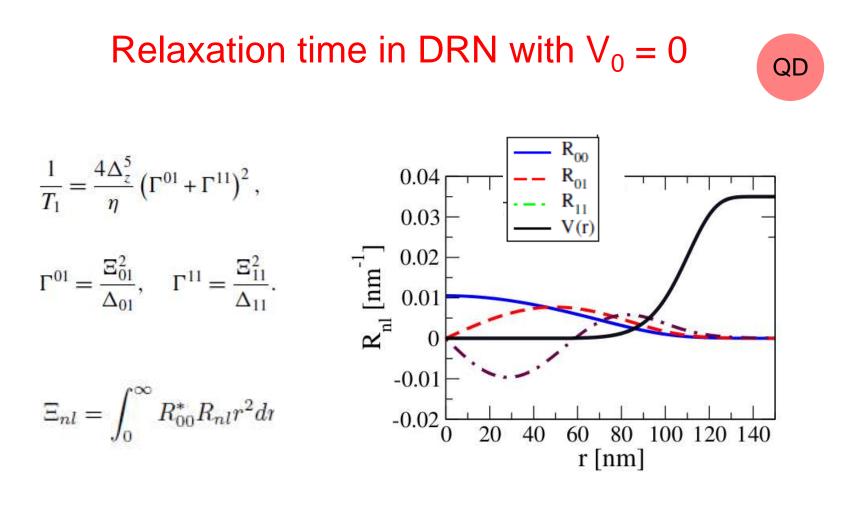
$$T_1^{-1} = C_{ph} \left( \alpha_{xx}^2 + \alpha_{yy}^2 \right) \Delta_z^5$$

$$\alpha_{xx} = 2e^2 \sum_{m \neq n} \frac{|\langle n | x | m \rangle|^2}{E_m - E_n} = \alpha_{yy}$$

 T<sub>1</sub> calculated by Khaetskii and Nazarov for circularly symmetric nanostructures due to spin-orbit mediated interaction with piezoelectric phonons is given by:

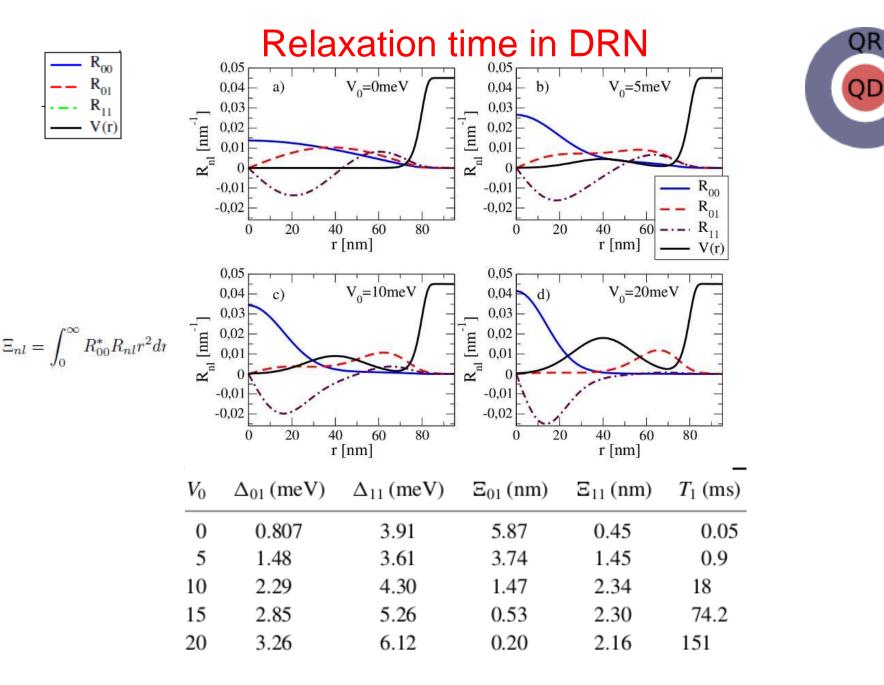
$$\frac{1}{T_1} = \frac{\Delta_z^5}{\eta} \left[ \Gamma^{01} + \Gamma^{11} \right]^2,$$
  
$$\Xi_{nl} = \int_0^\infty R_{00}^* R_{nl} r^2 dr$$
  
$$\Gamma^{01} = \frac{\Xi_{01}^2}{\Delta_{01}}, \quad \Gamma^{11} = \frac{\Xi_{11}^2}{\Delta_{11}}.$$
  
$$\Delta_{nl} = E_{nl} - E_{00}$$

$$\eta = rac{\hbar^5}{\Lambda_p (2\pi)^4 (m^*)^2}, \qquad \Lambda_p = 0.007 \quad ext{for GaAs type systems}$$



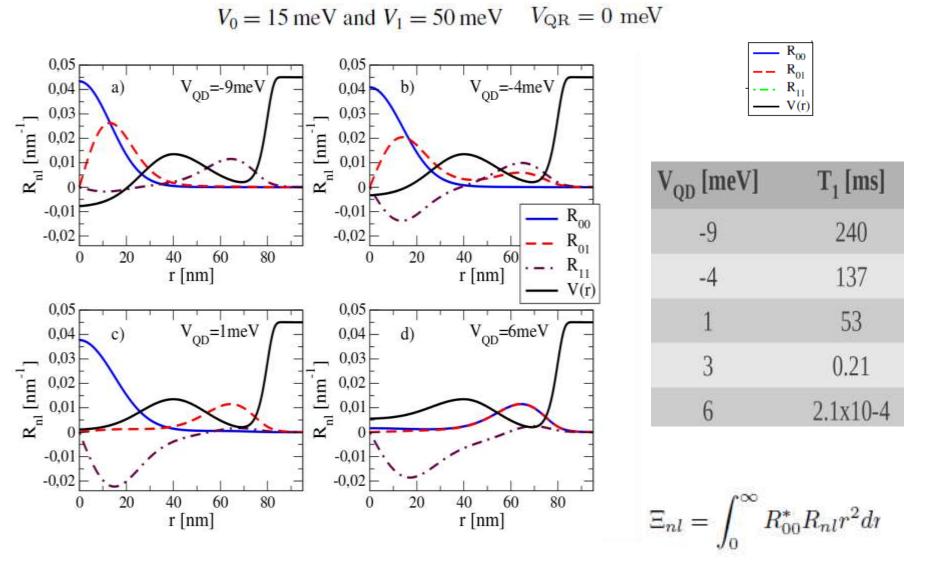
$V_0$	$\Delta_{01} [{\rm meV}]$	$\Delta_{11} [{\rm meV}]$	$\Xi_{01} \; [\rm{nm}]$	$\Xi_{11} \; [\rm{nm}]$	$T_1[ms]$
0	0.807	3.91	5.87	0.45	0.05

• Pure QD:  $T_1$  is entirely determined by excitations to the first orbital state • Small  $T_1$  because of large  $\Xi_{01}$ 



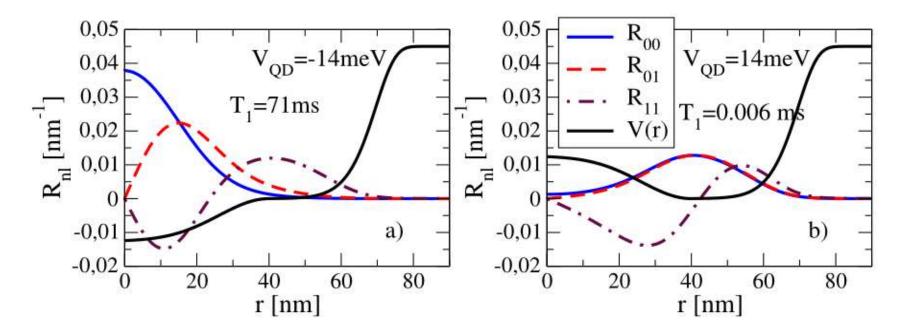
By increasing the barrier height excitations to the second orbital state determine  $T_1$ 

#### Wave function engineering by changing the potential well offset





Wave function engineering by changing the potential well offset at  $V_0 = 0$ 



The increase of  $T_1$  by introducing a small disk shaped gate below a central part of initial quantum dot

Presented results for InGaAs R<sub>DRN</sub>=80nm,  $m^* = 0.067m_e$ ,  $|g_s| = 0.8$ 

•T<sub>1</sub> strongly increases with decreasing the radius of DRN

• e.g. for InGaAs R<sub>DRN</sub>=40nm

 $T_1(V_0=0) = 7.5ms$  (0.05ms),  $T_1(V_0=20meV) = 2.5s$  (151ms),

•T<sub>1</sub> strongly increases with decreasing the  $g_s$ 

e.g. for GaAs/AlGaAs with  $g_s = 0.4 \text{ R}_{\text{DRN}} = 80 \text{ nm}$ ,

 $T_1$  (GaAs/AlGaAs) = 32  $T_1$  (GaAs/InGaAs) 0.16s - 5s

• Important feature – not only the value of  $T_1$  itself but the possibility to change  $T_1$  by electrostatic gating

#### **Optical intraband absorption of DRNs**

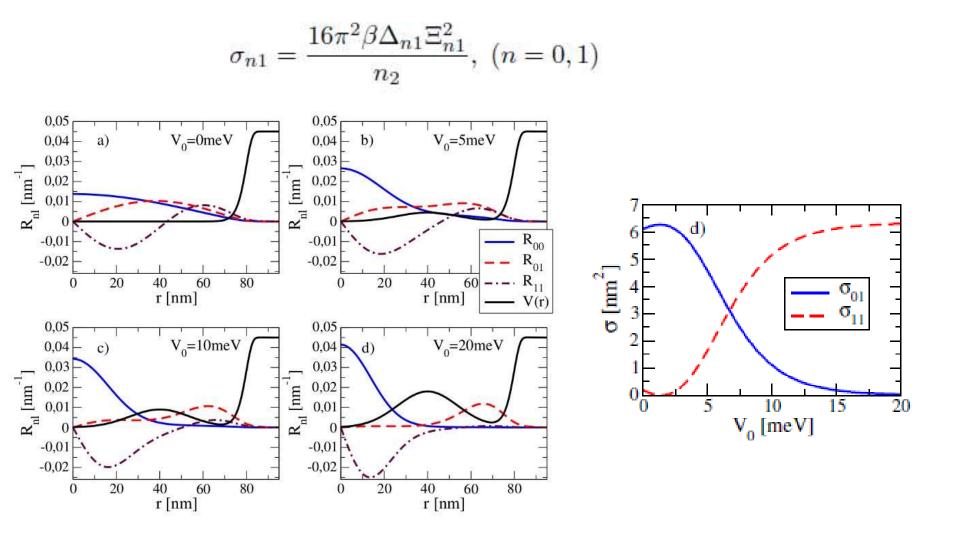
$$\sigma_{i,f} = \frac{16\pi^2 \beta \hbar \omega \Xi_{i,f}^2}{n_2} \delta(E_f - E_i - \hbar \omega) F_{\rm FD}(E_i, E_f)$$

where  $\beta = 1/137$  is a fine structure constant,  $l_f = l_i \pm 1$ ,  $n_2$  is the refractive index. B=0

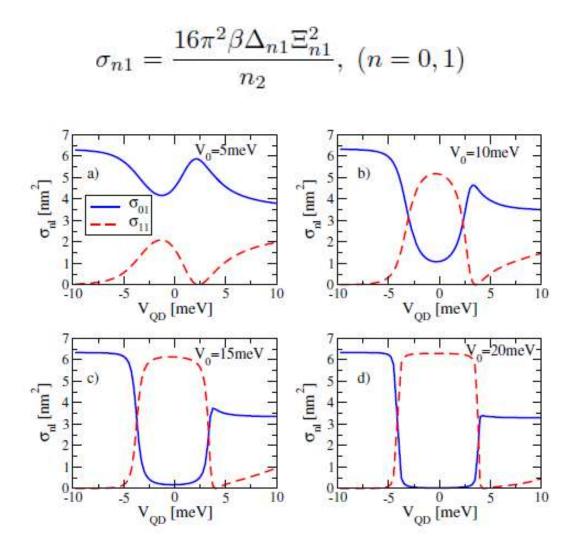
Resonant absorption at  $\Delta_{n1}$  with  $\Gamma$  = 1meV

$$\sigma_{n1} = \frac{16\pi^2 \beta \Delta_{n1} \Xi_{n1}^2}{n_2}, \ (n = 0, 1)$$

#### Optical absorption of DRNs for different V<sub>0</sub>



#### Optical absorption of DRNs for different V<sub>QD</sub>



# One can engineer DRNs according to their applications:

1. One can design DRNs to get strong effective absorption

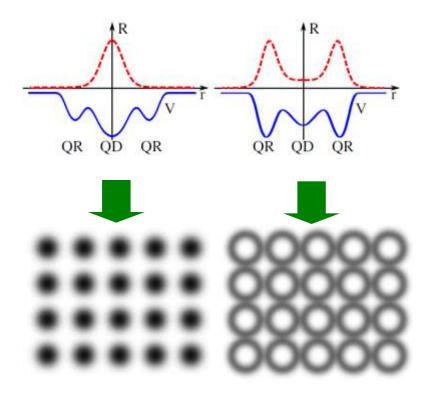
2, One can design DRNs with negligible absorption.

3.By changing  $V_{QD}$  one can move over from highly absorbing to almost transparent DRNs.

4. The absorbed energy can be changed form infra-red to microwaves by changing the material, the radius of the nanostructure and the barrier parameters.

#### Conducting properties of arrays of DRNs

from insulating to conducting system

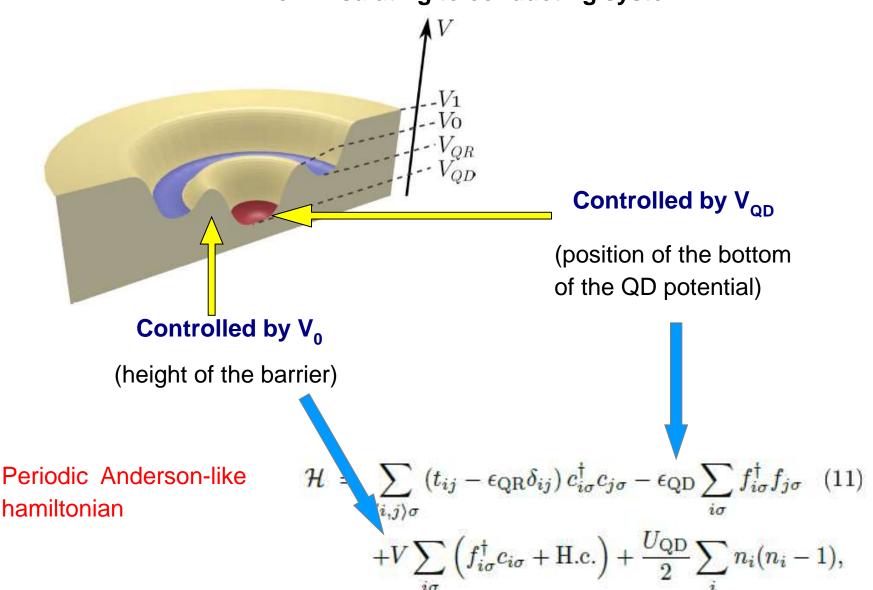


Periodic Anderson-like hamiltonian

$$\mathcal{H} = \sum_{\langle i,j \rangle \sigma} (t_{ij} - \epsilon_{\text{QR}} \delta_{ij}) c_{i\sigma}^{\dagger} c_{j\sigma} - \epsilon_{\text{QD}} \sum_{i\sigma} f_{i\sigma}^{\dagger} f_{j\sigma} \quad (11)$$
$$+ V \sum_{i\sigma} \left( f_{i\sigma}^{\dagger} c_{i\sigma} + \text{H.c.} \right) + \frac{U_{\text{QD}}}{2} \sum_{i} n_i (n_i - 1),$$

#### Conducting properties of arrays of DRNs

from insulating to conducting system



#### Array of DRNs in external electric field

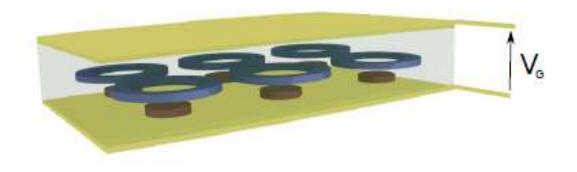


FIG. 10: (color online) Example of a spatial configuration of QR and QD that allows for controlling  $V_{\text{QR}}$  and  $V_{\text{QD}}$  by an external electrostatic field.

#### **Controlling individual DRNs**

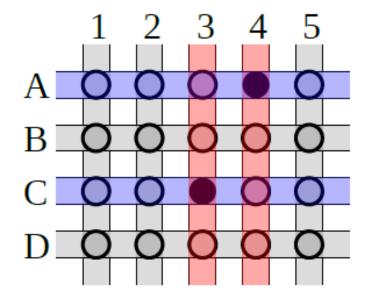
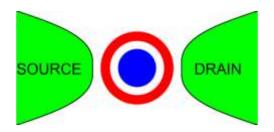
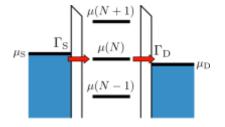


FIG. 11: (color online) Schematic illustration of the setup allowing for controlling individual DRNs. Stripes along the xaxis located above the DRNs are labelled with letters, whereas the stripes along the y axis located below the DRNs are labelled with numbers. The full black circles indicate DRNs where the difference  $\epsilon_{\rm QD} - \epsilon_{\rm QR}$  is largests when voltage is applied to pairs of stripes A4 and C3

#### Single electron tunneling through DRN





Current I flows through the nanostructure at low bias if:

 $\mu_{\rm D} \le \mu_{\rm N} \le \mu_{\rm S}$ 

$$\mu_{\rm N} \equiv U(N) - U(N-1)$$

U(N) – total energy of nanostructure with N electrons including discreate quantum states separated by  $\Delta_{nl}$  and charging energy  $E_c = \frac{e^2}{C}$ 

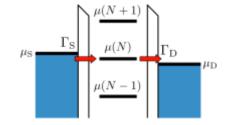
- otherwise Coulomb blockade

$$I = -e^2 \frac{\Gamma_{\rm S} \Gamma_{\rm D}}{\Gamma} f'_{\rm FD} \left(\mu_{\rm N} - \mu\right) \left(V_{\rm S} - V_{\rm D}\right)$$

(Kouwenhoven, Beenaker)

$$\mu_{\mathrm{S}(\mathrm{D})} = \mu \mp |e| V_{\mathrm{S}(\mathrm{D})}, \quad \Gamma = \Gamma_{\mathrm{S}} + \Gamma_{\mathrm{D}}$$

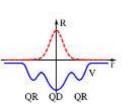
I depends on tunnel rates  $\Gamma_{\rm S}, \Gamma_{\rm D}$ 



In DRN additional possibility to control transport of single electrons by changing  $\Gamma_{\rm S}, \, \Gamma_{\rm D}$ 

a) GS in QR

b) GS in QD

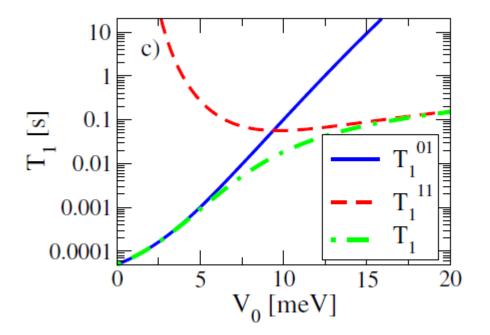


 $\frac{I^{\rm ring}}{I^{\rm dot}} = 500$ 

One can change the magnitude of the current and the coupling strength between DRN and S,D electrodes from large (Kondo regime) to weak by electrostatic gating

## Summary

- Systematic studies of the effect of the shape and the height of the barrier and / or the potential well offset on measurable properties of DRNs.
- By manipulating the confinement parameters one can alter the overlap of the electron wave functions so that the transition probabilities will be enhanced or suppresed on demand.
- Such modifications strongly influence the relaxation times, the optical absorption spectra and the conducting properties of DRNs.
- The wavelength range may be expanded from microwaves to infra-red by utilizing DRNs of different size and different material.
- The basic issues of quantum mechanics can be explored to optimize the specific properties of nanostructures by means of sophisticated design.



 $T_1$  as a function of the barrier height :

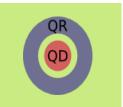
from the first excited state (QR)

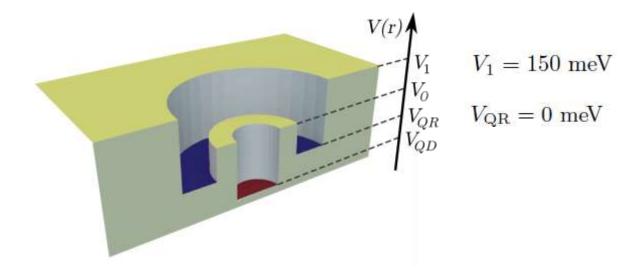
- from the second excited state (QD)
- from both states

For small  $V_0$  excitations to the first orbital state determine  $T_1$ 

By increasing the barrier height excitations to the second orbital state determine  $T_1$ 

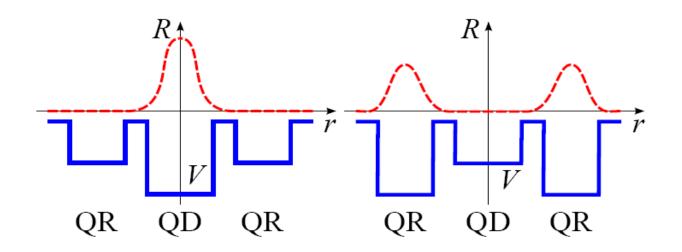
 2D semiconductor nanostructure (DRN) in the form of quantum dot (QD) surrounded by a quantum ring (QR).





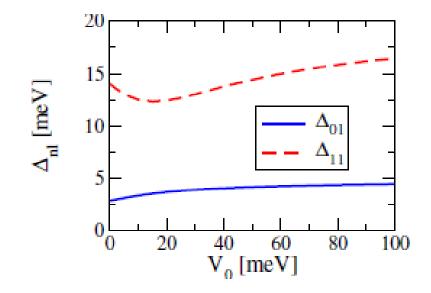
Electron can tunnel from QD to QR via the barrier  $V_0$ 

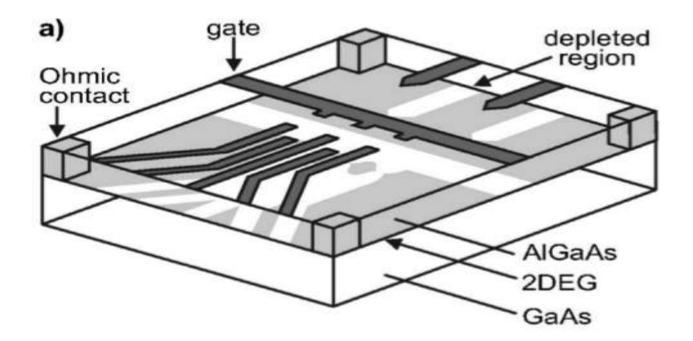
•By changing the barrier parameters and/or potential well offset one changes the shape and distribution of wave functions – wave function engineering



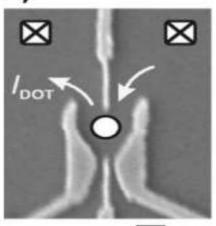
- Such manipulations can change:
- spin relaxation times of DRNs by orders of magnitude
- cross section for optical intraband absorption from strong to negligible
- conducting properties of an array of DRNs from highly conducting to insulating

#### Energy gaps to the first two excited states



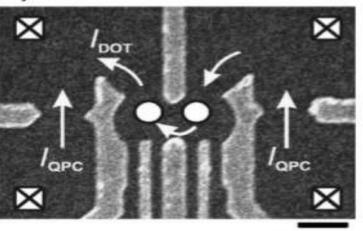


b)



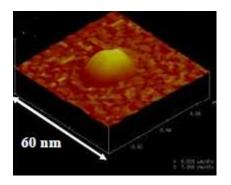
200 nm

c)



200 nm

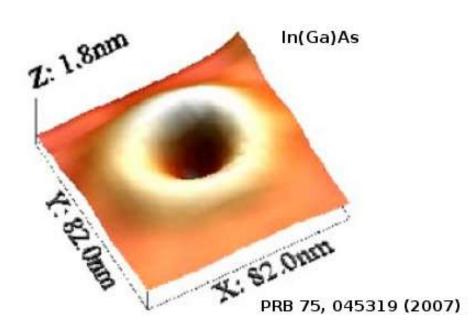
## Kropki kwantowe



- Obiekty 0-wymiarowe (sztuczne atomy)zbudowane z półprzewodników o rozmiarach d ~ 5-100 nm
- · Można precyzyjnie umieszczac zadaną liczbę elektronów
- · Ciekawe zastosowania gdy w kropce 1 elektron
- Obliczenia kwantowe (quantum computing)

$$|Q\rangle = a|0\rangle + b|1\rangle, |a|^2 + |b|^2 = 1$$

- Question:
- Are quantum rings good for qubit implementations?



#### Self-assambled In(Ga)As quantum rings Benito Allen et al

Further increase of  $T_1$  by decreasing  $\Xi_{11}$  by changing the barrier shape from rectangular to gaussian

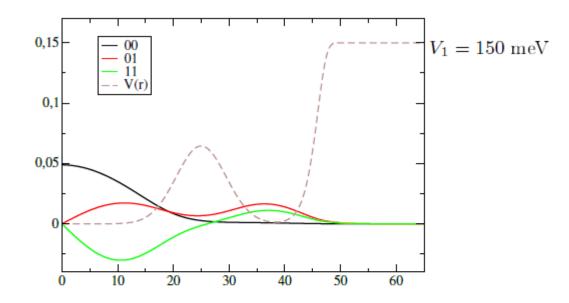


FIG. 7: The distribution of the wave functions for the first three lowest energy states for the gaussian-like confining potential. The relaxation time  $T_1$  for DRN in this case is of the same order as for a small (inner) quantum dot.

$$V_0 = 30 \text{ meV}, \quad T_1 = 1.75 \text{ s}, \quad (T_1 = 0.61 \text{ s})$$

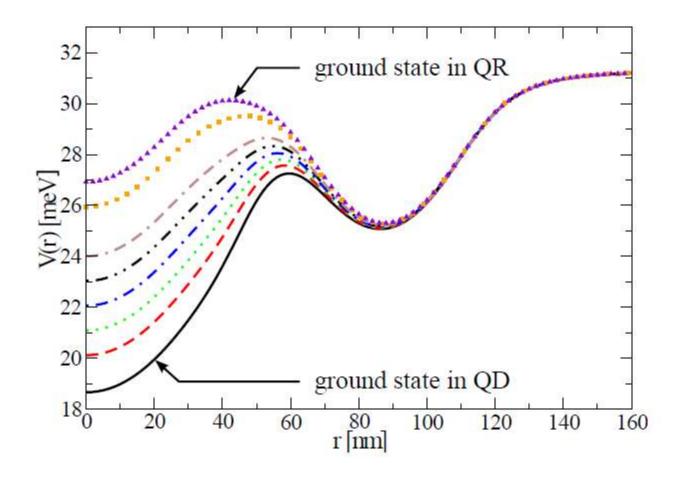
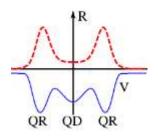
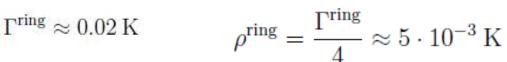


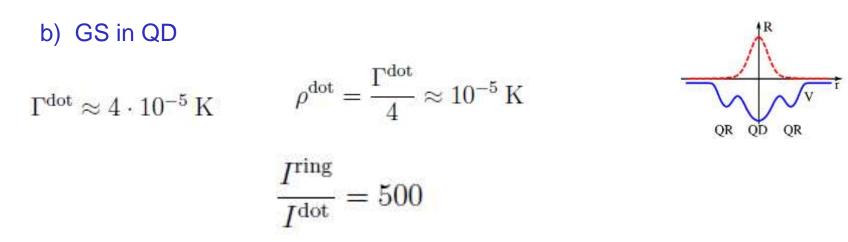
Figure 3. Cross section of the confining potential for different voltage applied to the electrodes.

$$I = -e^{2}\rho f_{\rm FD}' \left(\mu_{\rm N} - \mu\right) \left(V_{\rm S} - V_{\rm D}\right)$$
$$\rho = \frac{\Gamma_{\rm S}\Gamma_{\rm D}}{\Gamma}$$

a) GS in QR







One can change the magnitude of the current and the coupling strength between DRN and S,D electrodes from large (Kondo regime) to weak by electrostatic gating

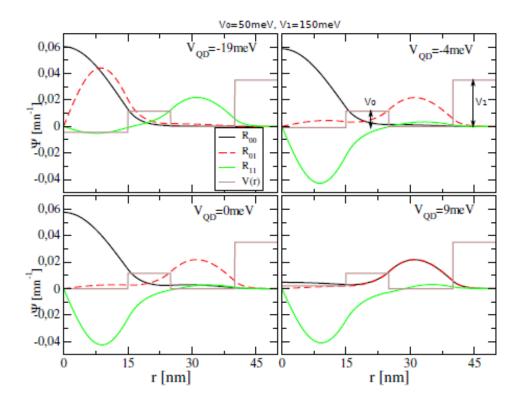


FIG. 5: The distribution of the wave functions for the first three energy states. Each panel corresponds to a different value of  $V_{QD}$ .

#### Coherent manipulations of spin qubits

- Initialization
- Measurement by spin to charge conversion and detection by QPC
- Universal set of quantum logic gates :
- Single qubit rotations by ESR or electrical gates
- Quantum controlled CNOTgate:
- The value of T is negated if and only if C has the value 1:
- $0_C 0_T \rightarrow 0_C 0_T$ ,  $0_C 1_T \rightarrow 0_C 1_T$
- $1_C 0_T \rightarrow 1_C 1_T$ ,  $1_C 1_T \rightarrow 1_C 0_T$
- E.g. two coplanar rings with the tunable barrier $H = J\sigma_z^{spin,1}\sigma_z^{spin,2} + \Delta_{s1}\sigma_z^{spin,1} + \Delta_{s2}\sigma_z^{spin,2})$

## Coherent manipulations of qubits

• The qubit hamiltonian

$$H_Q = -\frac{1}{2}\Delta_z \widehat{\sigma}_z - \frac{1}{2}\Delta_x \widehat{\sigma}_x.$$

where  $\hat{\sigma}_z, \hat{\sigma}_x$  denote Pauli spin matrices.

For the orbital qubit

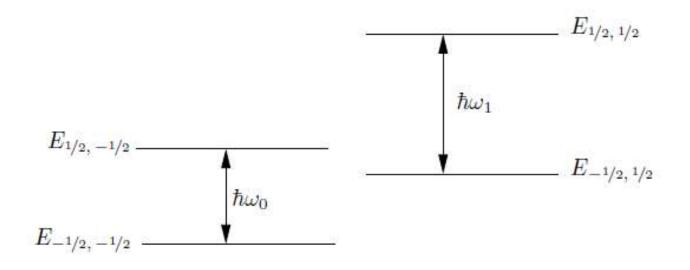
 $\Delta_{z,orb}(\phi') = E_{n_{\uparrow},s} - E_{m_{\downarrow},s}$  can be tuned by varying the magnetic flux

 $\Delta_x = \hbar \omega_{Q,orb}$  describes the amplitude of tunneling through a barrier and can be tuned by changing the barrier parameters.

For a spin qubit  $\Delta_z$  and  $\Delta_x$  are energy splittings in real magnetic fields  $B_z$  and  $B_x$ 

Both types of qubits can be driven by microwave pulses resonant with the level separation

#### **CNOT** gate on spin qubits



Microwave pulse of energy  $\hbar\omega_1$  realizes the CNOT gate

## To work with qubits one must be able to

- Prepare qubit in an initial state
- Coherently manipulate the superposition states
- Couple qubits together
- Measure their state
- Decouple from the environment that leads to decoherence

Any two state quantum system that can be addressed, controlled, coupled to its neighbors and maintained in a coherent state is potentially useful for quantum communication

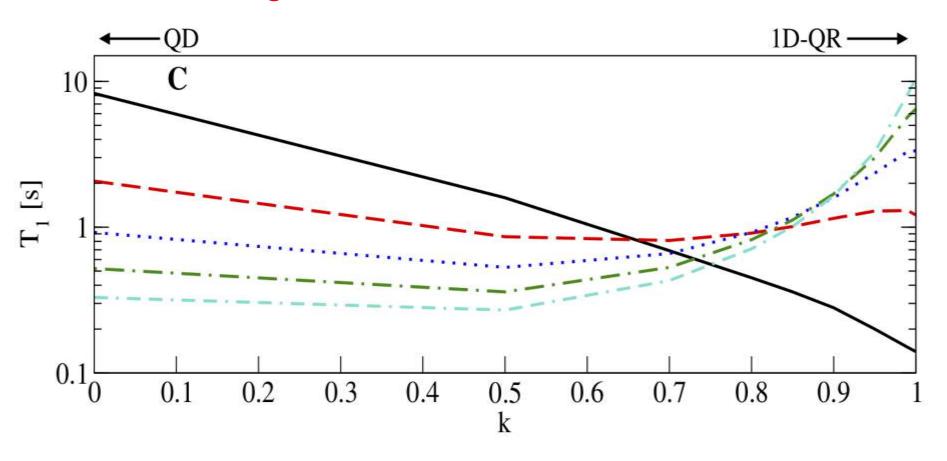
# Dispersion relation of an electron on the ring of radius R $E_{n,s} = \frac{\hbar^2}{2m^*R^2} \left[ \left( n + \phi' \right)^2 + 2g_s \frac{m^*}{m_e} s\phi' \right],$ $n = 0, \pm 1, \pm 2 \dots \quad s = \frac{1}{2}, -\frac{1}{2}, \phi' = \frac{\phi}{\phi_0}, \phi_0 = \frac{h}{e},$ R=20nm

Orbital and spin degrees of freedom to be used for quantum manipulations

1. N<sub>e</sub> ~`200 electrons, n<sub>F</sub>>>1 orbital states occupied successively by spin-up and spin –down electrons ; e-e interaction well described by the constant interaction model which shifts energy spectra by  $E_c=e^2/C$ 

E.Z. M.Kurpas, M.Szelag, J.Dajka, M.Szopa Phys. Rev. B 74, 125426 (2008), M.Kurpas, J.Dajka, E.Z. Physica Status Solidi b244, 2470, (2007).

## Single electron case (black solid line)



$$T_1^l = rac{\eta}{\Delta_z^5} \; rac{\Delta_l^2}{\Xi_l^4},$$

#### **QUANTUM COMPUTING**

- Prepare qubit in an initial state
- Coherently manipulate the superposition states
- Couple qubits together
- Measure their state
- Decouple from the environment that leads to decoherence

Any two state quantum system that can be addressed, controlled, coupled to its neighbors and maintained in a coherent state is potentially useful for quantum communication

## **Quantum dots and rings**

- Quantum dot (QD)- small semiconductor quasi 2D structure
- (R~5-50nm) which can be filled with one or few electrons or holes.
- QD with single electron in magnetic field B a natural two level system – spin qubit

$$---- \downarrow \qquad \Delta_Z = g_s \mu_B B,$$

- We change the geometry of QD to QR the dispersion relation changes - we get two additional (orbital – I) degrees of freedom to be used for quantum manipulations
- Applying the magnetic field B we can change the GS from I=0 to that with higher I

# Decoherence

Nano-systems are expected to behave according to the laws of quantum mechanics if they are separated well from the external degrees of freedom

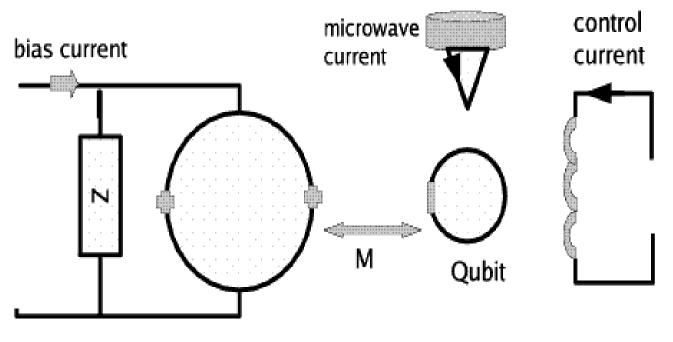
The important constraints on the device are relaxation and dephasing effects due to various decoherence sources. We have assumed that:

$$kT \ll \hbar\omega_{\alpha,\beta} \ll \Delta$$

system is put in a shield, that screens it from unwanted radiation.

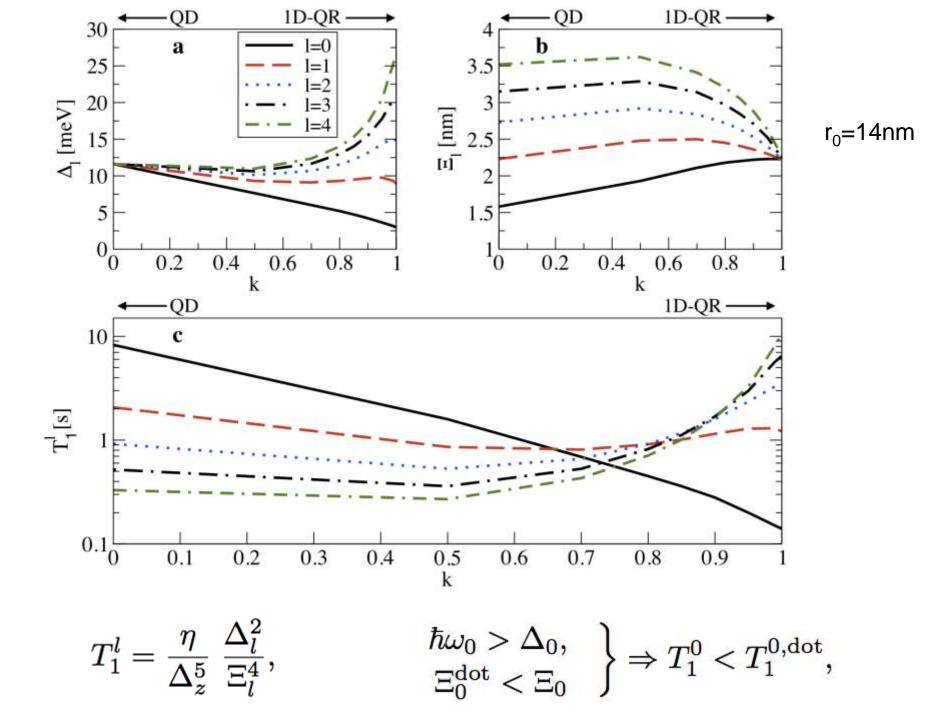
#### Decoherence

- Main decoherence mechanism for electron spins is the interaction with nuclear spins, T<sub>2</sub>~10 -100µs;
- Several strategies to reduce randomness of the nuclear-spin system (Hanson et al., 2007);
- For qubit on a single hole spin  $T_2 \sim 1000 \mu s$  due to the p-symmetry;
- For semiconductors of group IV (e.g.Si, SiGe) and CN with zero nuclear spin decoherence time should be longer ( of the order of the relaxation time);
- Time of coherent spin manipulations ~ ns
- During T<sub>2</sub> more than 10<sup>4</sup> coherent operations can be performed allowing for QEC beeing efficient



DC-SQUID

Experimental setup for measurements on flux qubit. (Caspar H.van der Wal et al, Eur.Phys.J.31, 111,(2003))



$$\alpha_{xx} = \alpha_{yy} = 2e^2 \sum_{nl} \frac{\Xi_{nl}^2}{\Delta_{nl}}$$

The formula for the relexation time:

 $\Delta_{nl} = E_{nl} - E_{00}$ 

$$\frac{1}{T_1} = \frac{4\Delta_z^5}{\eta} \left(\Gamma^{01} + \Gamma^{11}\right)^2,$$
$$\Xi_{nl} = \int_0^\infty R_{00}^* R_{nl} r^2 dr \quad l = \pm 1$$

$$\Gamma^{01} = \frac{\Xi_{01}^2}{\Delta_{01}}, \quad \Gamma^{11} = \frac{\Xi_{11}^2}{\Delta_{11}}.$$

 $\eta = \frac{\hbar^5}{\Lambda_p (2\pi)^4 (m^*)^2}, \qquad \Lambda_p = 0.007 \qquad \mbox{for GaAs based systems}$ 

### Conclusions

•Quantum rings with one or few electrons can be effectively reduced to a two- state system with two external control parameters.

• Orbital qubit:

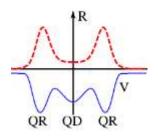
• The two states carry opposite persistent currents and are coupled by tunneling which leads to a quantum superposition of states.

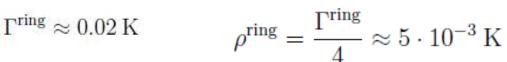
- The qubit can be driven by microwave pulses .
- •Flux state can be measured by a separate SQUID magnetometer.

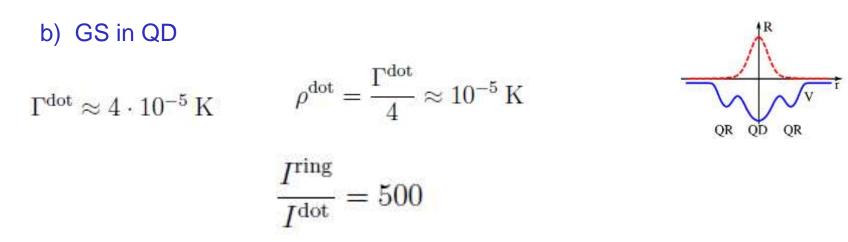
• Two or more qubits can be coupled by the flux the circulating currents generate.

$$I = -e^{2}\rho f_{\rm FD}' \left(\mu_{\rm N} - \mu\right) \left(V_{\rm S} - V_{\rm D}\right)$$
$$\rho = \frac{\Gamma_{\rm S}\Gamma_{\rm D}}{\Gamma}$$

a) GS in QR







One can change the magnitude of the current and the coupling strength between DRN and S,D electrodes from large (Kondo regime) to weak by electrostatic gating

$$I = -e^2 \frac{\Gamma_{\rm S} \Gamma_{\rm D}}{\Gamma} f'_{\rm FD} \left(\mu_{\rm N} - \mu\right) \left(V_{\rm S} - V_{\rm D}\right)$$

(Kouwenhoven, Beenaker)

$$\mu_{\mathrm{S}(\mathrm{D})} = \mu \mp |e| V_{\mathrm{S}(\mathrm{D})}, \quad \Gamma = \Gamma_{\mathrm{S}} + \Gamma_{\mathrm{D}}$$

I depends on tunnel rates  $\Gamma_{\rm S}, \Gamma_{\rm D}$ 

via 
$$\rho = \frac{\Gamma_{\rm S}\Gamma_{\rm D}}{\Gamma}$$

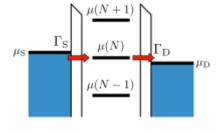
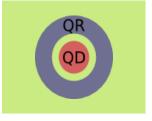


Fig.2

In DRN additional possibility to control transport of single electrons by changing  $~\Gamma_{\rm S},~\Gamma_{\rm D}$ 

$$\Gamma_{\rm S} = \Gamma_{\rm D} = \frac{\Gamma}{2}, \quad \Gamma \ll T, \ \Delta, \ E_C$$



We assume:

• *Fig.2* with e.g. N =1, T~ 0.1K

Electrical control of optical, coherent and conducting properties of quantum nanostructures

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