

# **Struktura Mg<sub>2</sub>Al<sub>3</sub>, układu o gigantycznej komórce elementarnej**

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WFiIS AGH w Krakowie

Współpraca z siecią CMA

M. Duda (doktorat), B. Kozakowski - struktura

W. Sikora, A. Bartyzel – analiza symetryczna

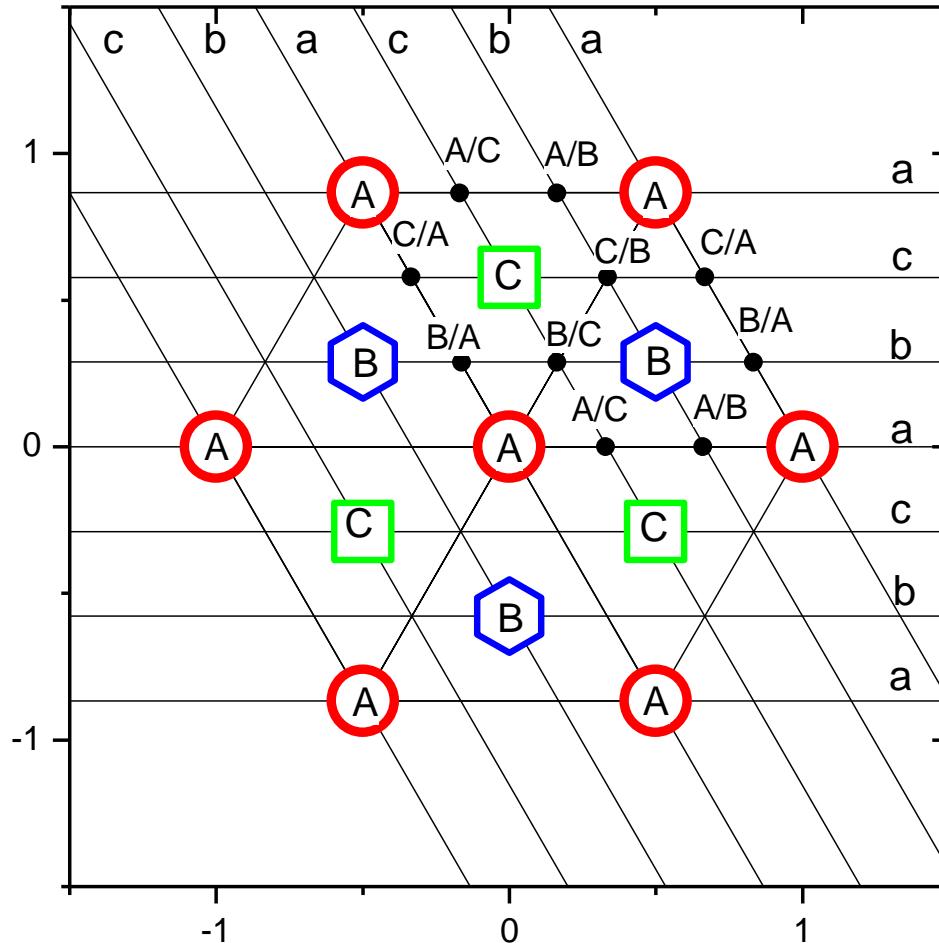
L. Pytlik, J. Adamowski, B. Łabno – stabilność klastrów

K. Wierzbanowski + zespół – naprężenia

Wstęp:

Jakie struktury mogą tworzyć stabilne konfiguracje dla izotropowych oddziaływań między atomami?

# Struktury gęsto upakowane w D2 - warstwa heksagonalna



# Struktury gęsto upakowane w D3

Możemy wyróżnić następujące sekwencje warstw:

- **AB** – struktura HCP,

na heksagonalną komórkę elementarną przypadają dwa węzły w pozycjach  $(0,0,0)$  oraz  $(2/3,1/3,1/2)$ ;  $c/a = 1.633$ , np. struktura Mg .....

**Inwersja nie jest** elementem symetrii węzłów sieci.

- **ABC** – struktura FCC,

na kubiczną komórkę elementarną przypadają cztery węzły sieci:  $(0,0,0)$  oraz  $(0,0,1/2)$  ....., np. struktura Cu .....

**Inwersja jest** elementem symetrii każdego węzła sieci.

- **ABAC** – struktura DHCP,  
na heksagonalną komórkę elementarną przypadają cztery węzły sieci. Dla dwóch węzłów lokalna symetria zawiera inwersję (są to tzw. węzły kubiczne) a dla pozostałych dwóch węzłów inwersja nie jest elementem symetrii lokalnej (są to tzw. węzły heksagonalne). Rozróżnienie symetrii lokalnej węzłów ma zasadnicze znaczenie w przypadku pojawienia się uporządkowania magnetycznego, co zostało zaobserwowane dla neodymu i prazeodymu.
- Bardziej skomplikowane sekwencje warstw (mogą być periodyczne lub aperiodyczne) – politypy.

Do opisu struktur z układu regularnego, heksagonalnego i romboedrycznego stosujemy współrzędne heksagonalne.

Przeliczniki:

$$a_h = \frac{a_{\text{FCC}}}{\sqrt{2}}$$

$$c_h = a_{\text{FCC}} \sqrt{3}$$

$$d_h = \frac{a_{\text{FCC}}}{\sqrt{3}}$$

$$\frac{d_h}{a_h} = \sqrt{\frac{2}{3}}$$

$$a_h = \sqrt{2}a_{\text{sc}}$$

$$c_h = a_{\text{sc}} \sqrt{3}$$

$$d_h = \frac{a_{\text{sc}}}{\sqrt{3}}$$

$$\frac{d_h}{a_h} = \sqrt{\frac{1}{6}}$$

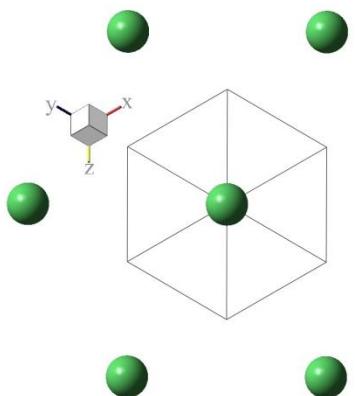
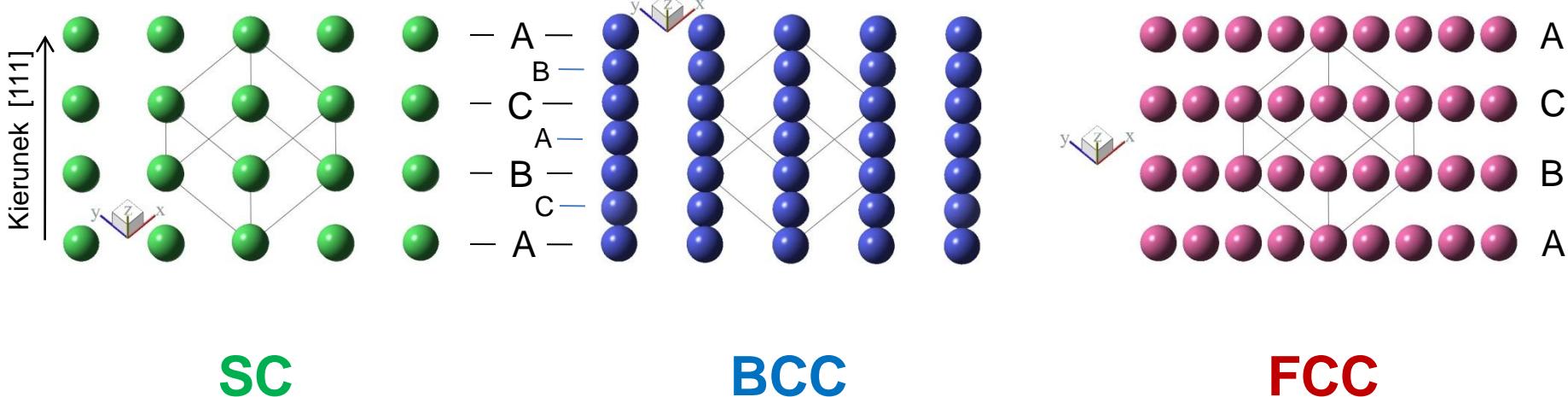
$$a_h = \sqrt{2}a_{\text{BCC}}$$

$$c_h = a_{\text{BCC}} \sqrt{3}$$

$$d_h = \frac{a_{\text{BCC}}}{2\sqrt{3}}$$

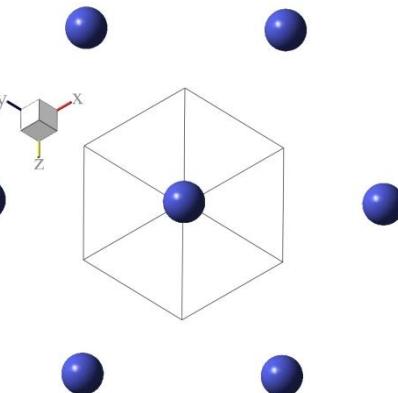
$$\frac{d_h}{a_h} = \sqrt{\frac{1}{24}}$$

# Każda sieć regularna składa się z warstw heksagonalnych: ABC

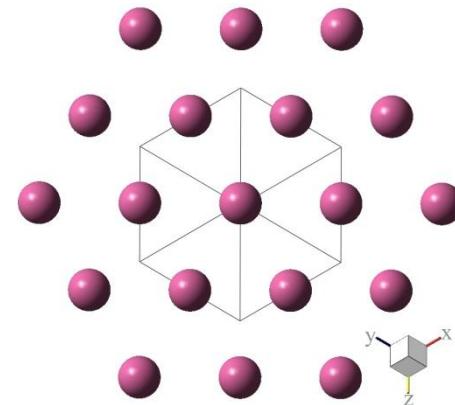


(52%, może być stabilna,  
np. Polon)

Płaszczyzna (111)

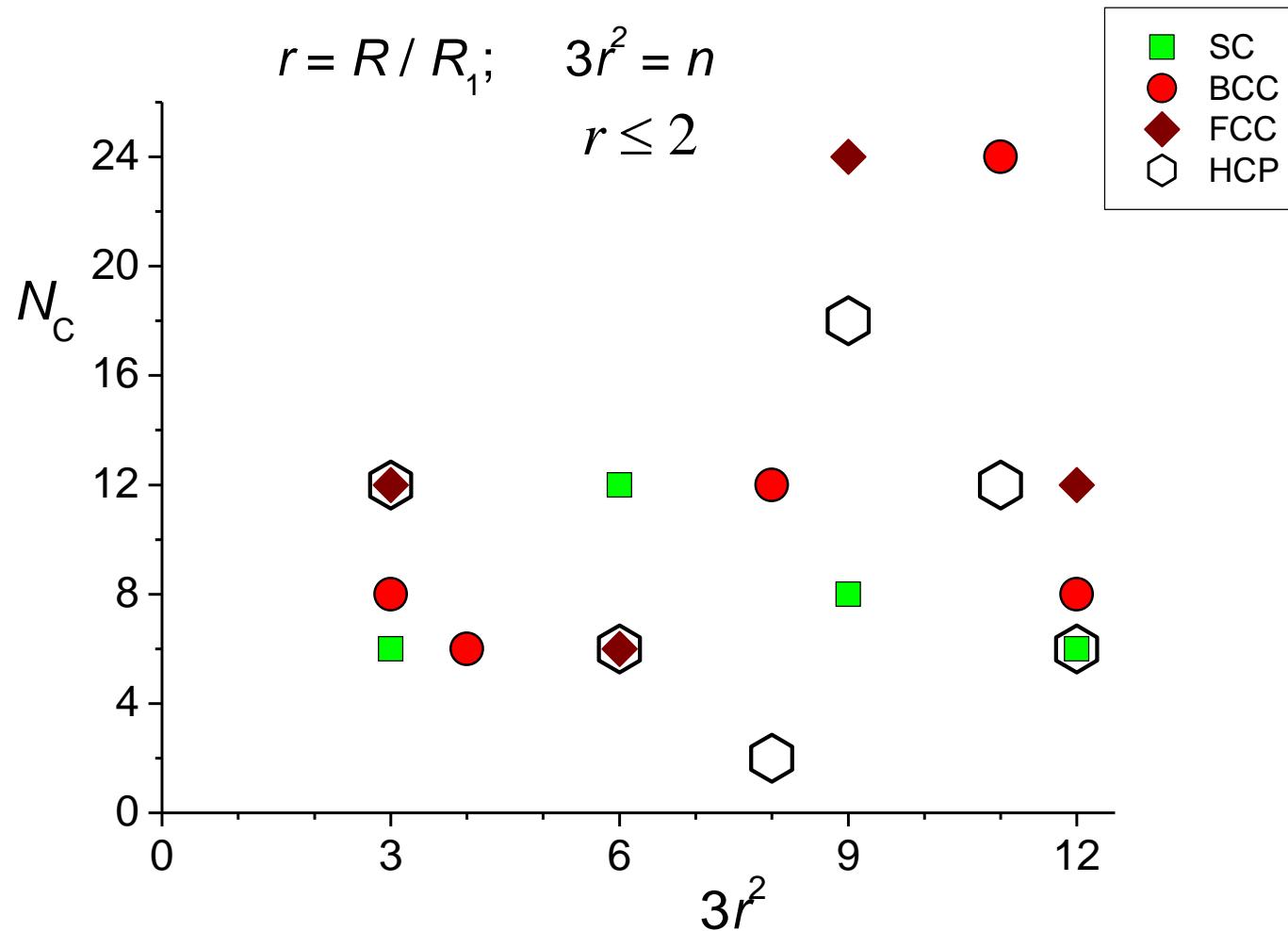


(68%, jednak energetycznie  
porównywalna z BCC)

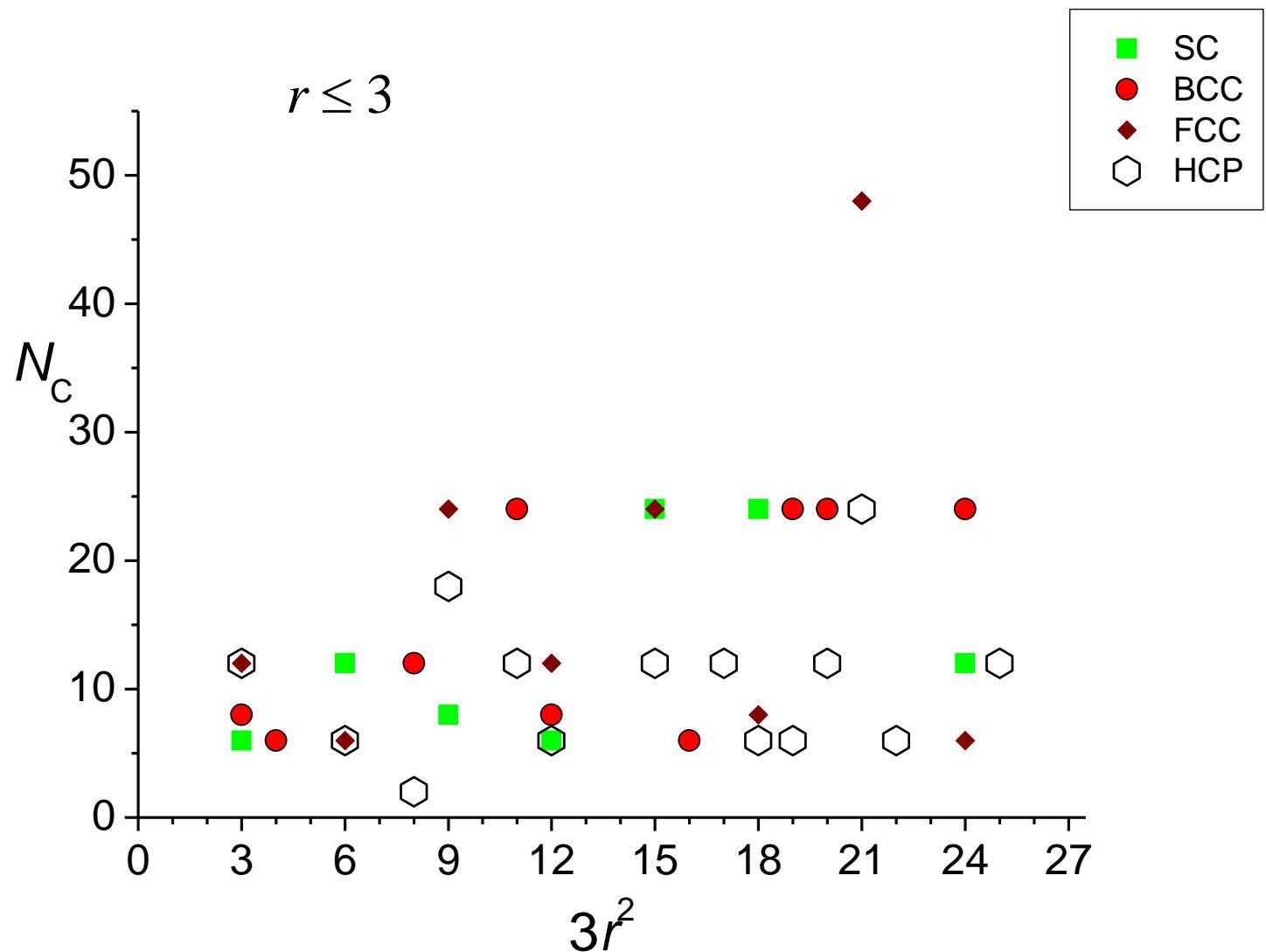


(74% - struktura gęstego  
upakowania)

# O stabilności struktury decyduje liczba kolejnych sąsiadów

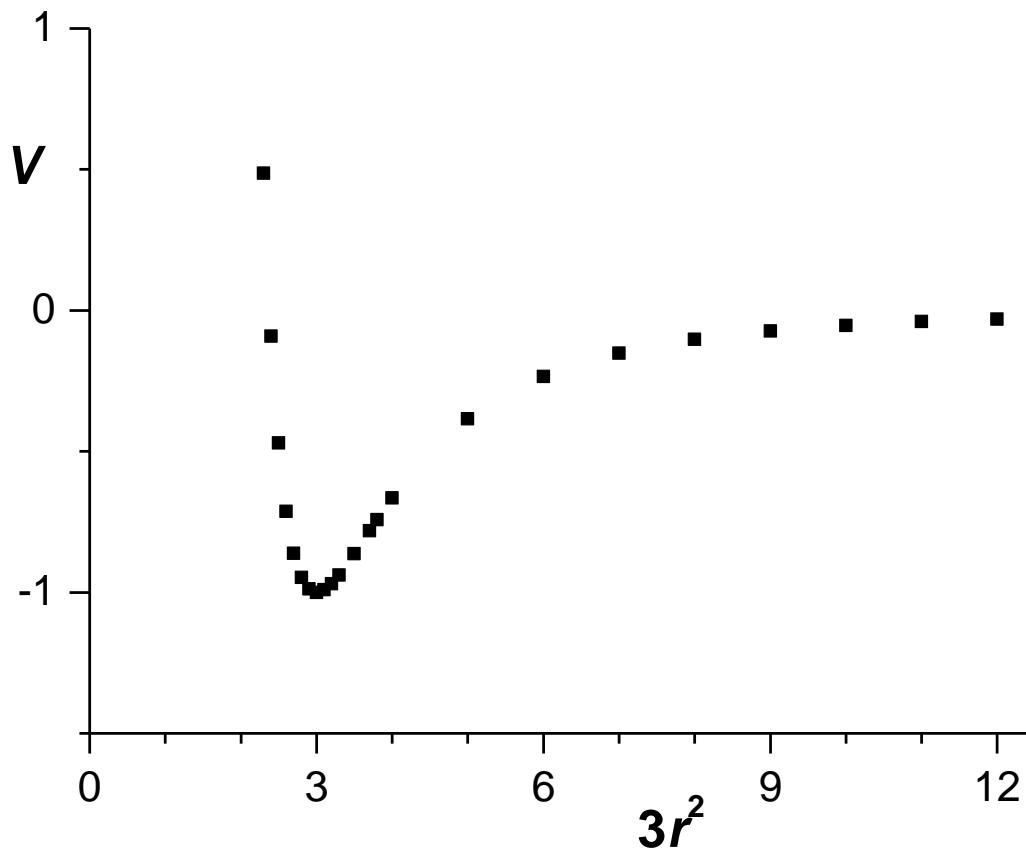


# Liczba kolejnych sąsiadów

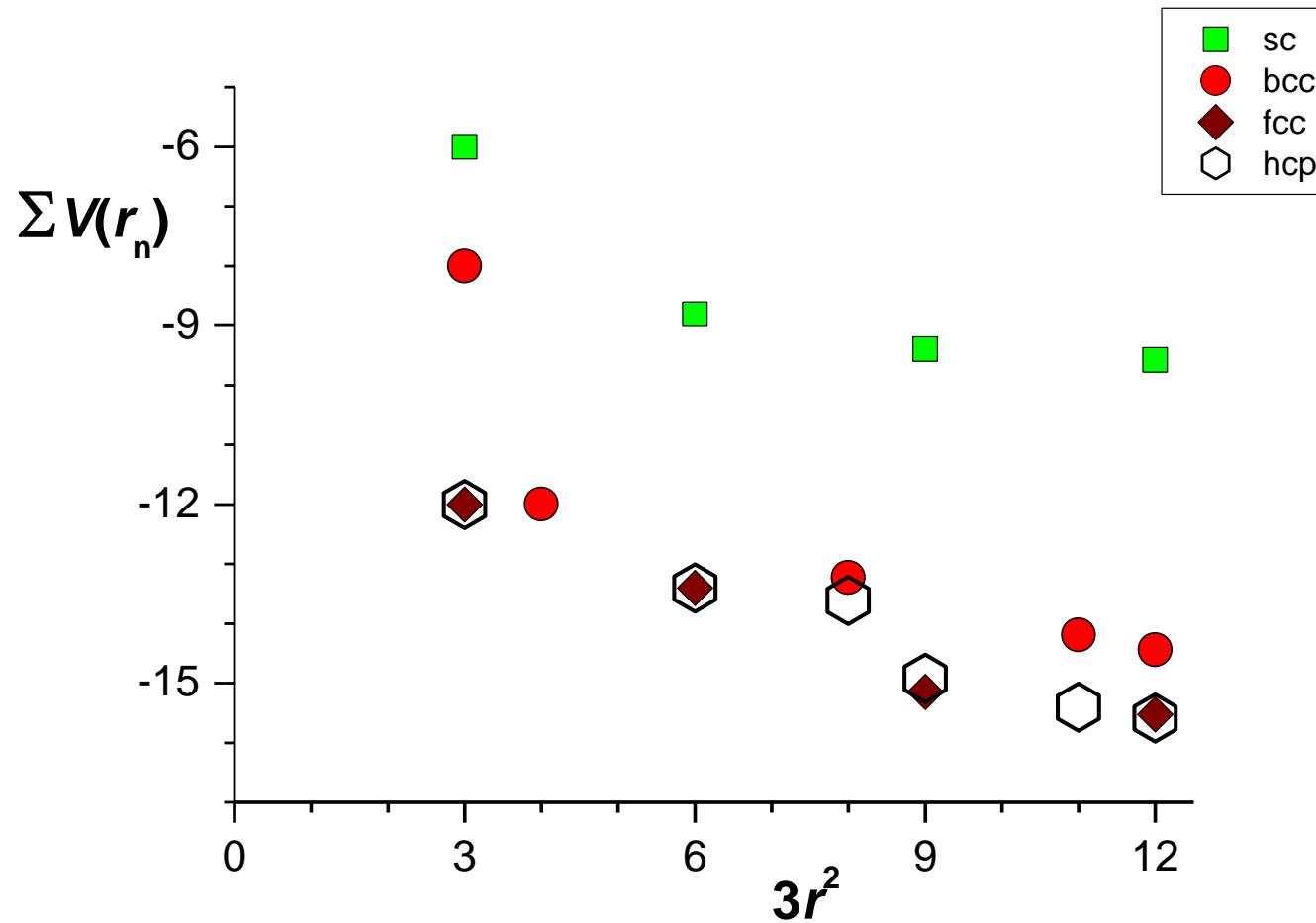


Znormalizowany potencjał Lennarda-Jonesa:

$$V = 1/r^{12} - 2/r^6$$



# Sumaryczna energia układu modelowego



Dla upakowania sztywnych kul najbardziej stabilne są fazy FCC, HCP, DHCP itp.

**Dla potencjału typu Lennarda-Jonesa stabilność faz FCC, HCP i BCC jest porównywalna.**

W ekstremalnych warunkach (wysokie ciśnienie) stabilna może być faza SC (np. Polon). Taką możliwość stwarza mała liczba najbliższych sąsiadów (6) i dwa razy większa liczba kolejnych sąsiadów (12) dla  $n=6$ .

W układach gęsto upakowanych, stosunek odległości między warstwami heksagonalnymi do stałej sieci heksagonalnej wynosi  $d_h/a_h = (2/3)^{1/2} \approx 0.816$ .

Sekwencja warstw **dla wszystkich sieci regularnych jest ABC**.  
Stosunek  $d_h/a_h$  wynosi odpowiednio:

0.816 dla FCC

0.408 dla SC

0.204 dla BCC;

Inna wartość stosunku prowadzi do sieci heksagonalnej (dowolna sekwencja warstw) lub romboedrycznej (sekwencja AB).

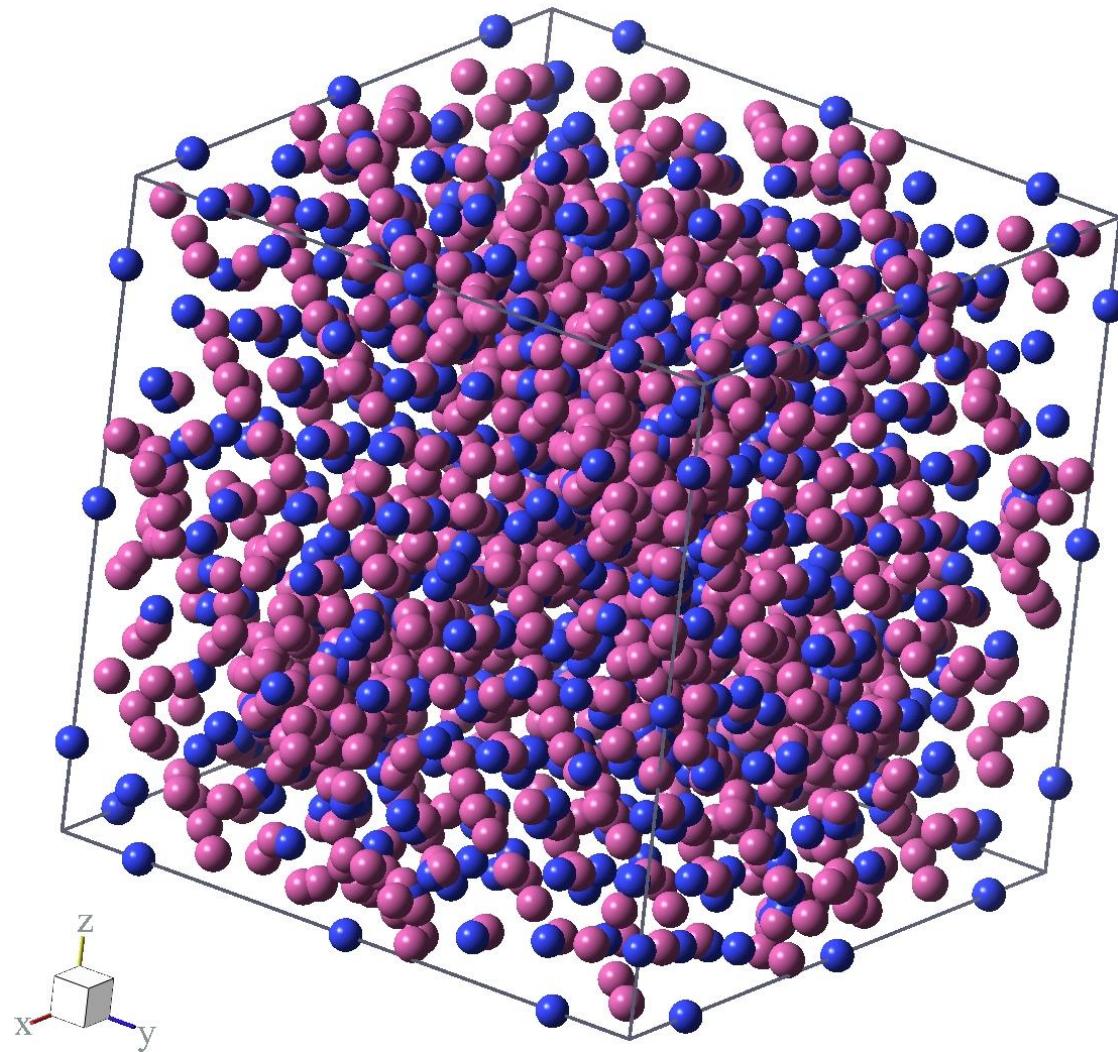
## $\beta$ - Mg<sub>2</sub>Al<sub>3</sub>

The Samson phase is one of the most complex intermetallic structures. The first description of the  $\beta$ -Mg<sub>2</sub>Al<sub>3</sub> structure was provided by S. Samson in 1965. Its cubic elementary cell (Fd-3m, no. 227 space group) contains **1168 atoms** which are distributed over 1832 atomic positions. About 75% of atoms (879 to be exact) form the firm framework of the structure - 528 of them are Al and 351 are Mg. By “the framework” or “skeleton atoms” we understand a set of Samson’s positions which are occupied by atoms with the probability of 100%. The remaining 289 (25%) atoms partially occupy 953 positions with the average occupation probability of 30%. They form clusters arranged in an elementary cell in a tetrahedral lattice.

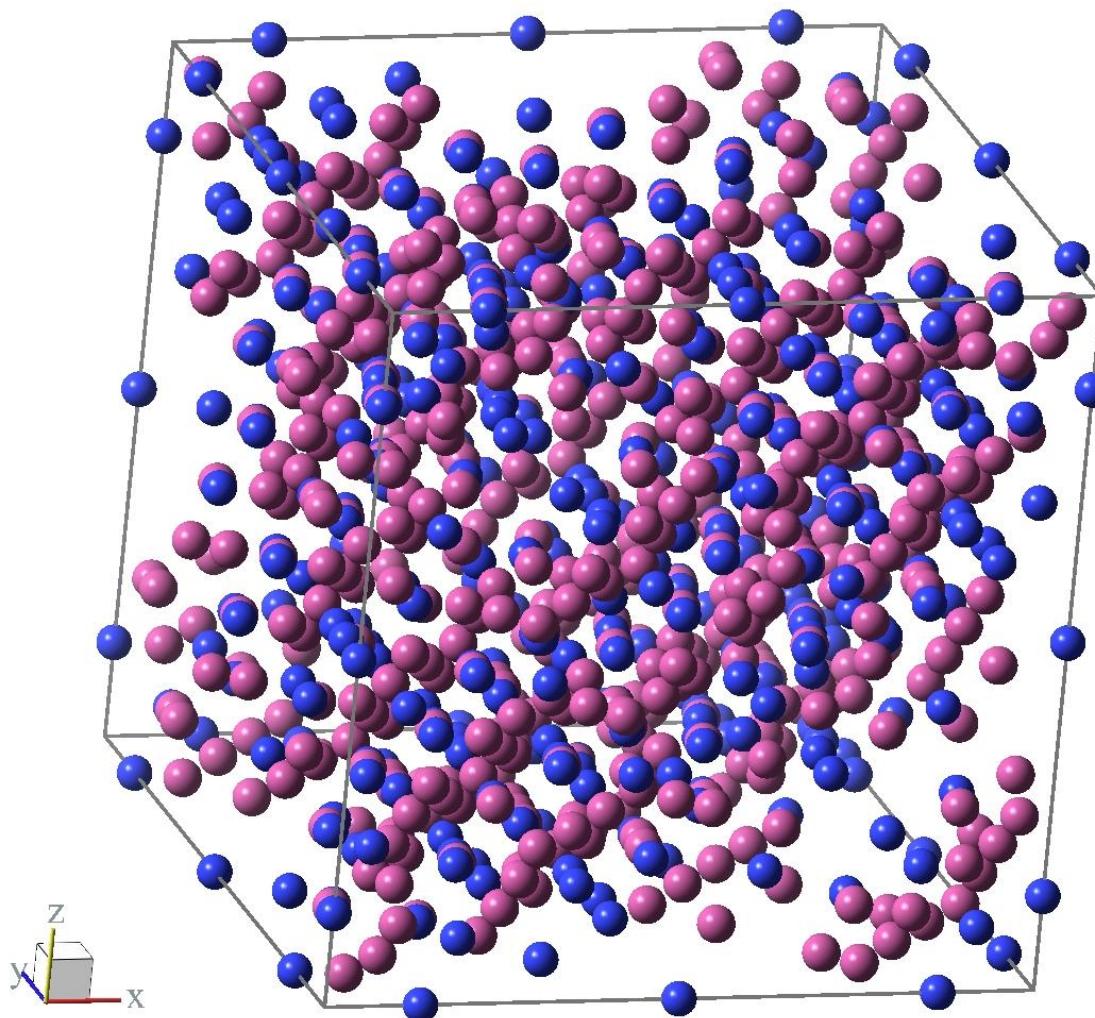
The lattice constant of the Samson structure is gigantic:  $a_c=2.8242(1)$  nm.

$\beta\text{-Mg}_2\text{Al}_3$ ; 1168 atoms;  $a_c=2.8242$  nm

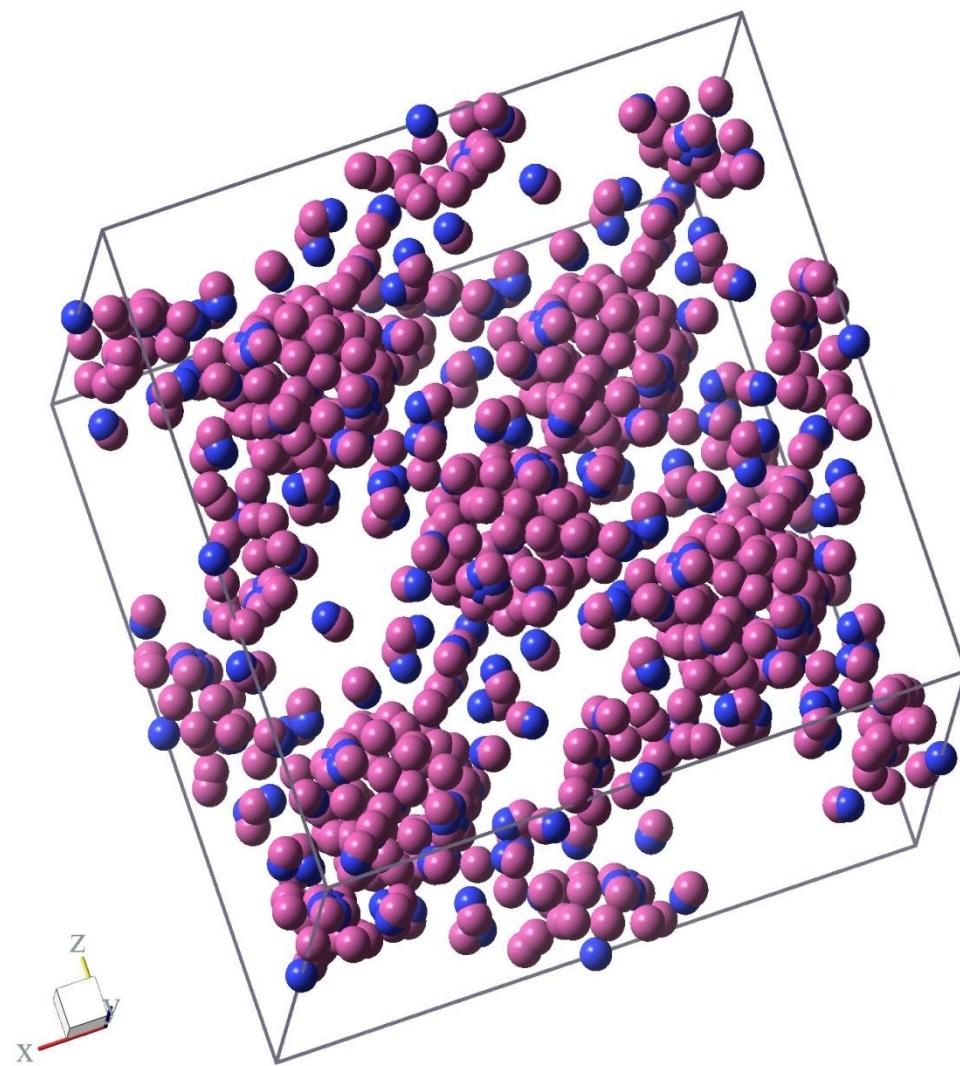
all the data used in this presentation come from Feuerbacher M. *et al.*, Z. Krist. 222 (2007) 259



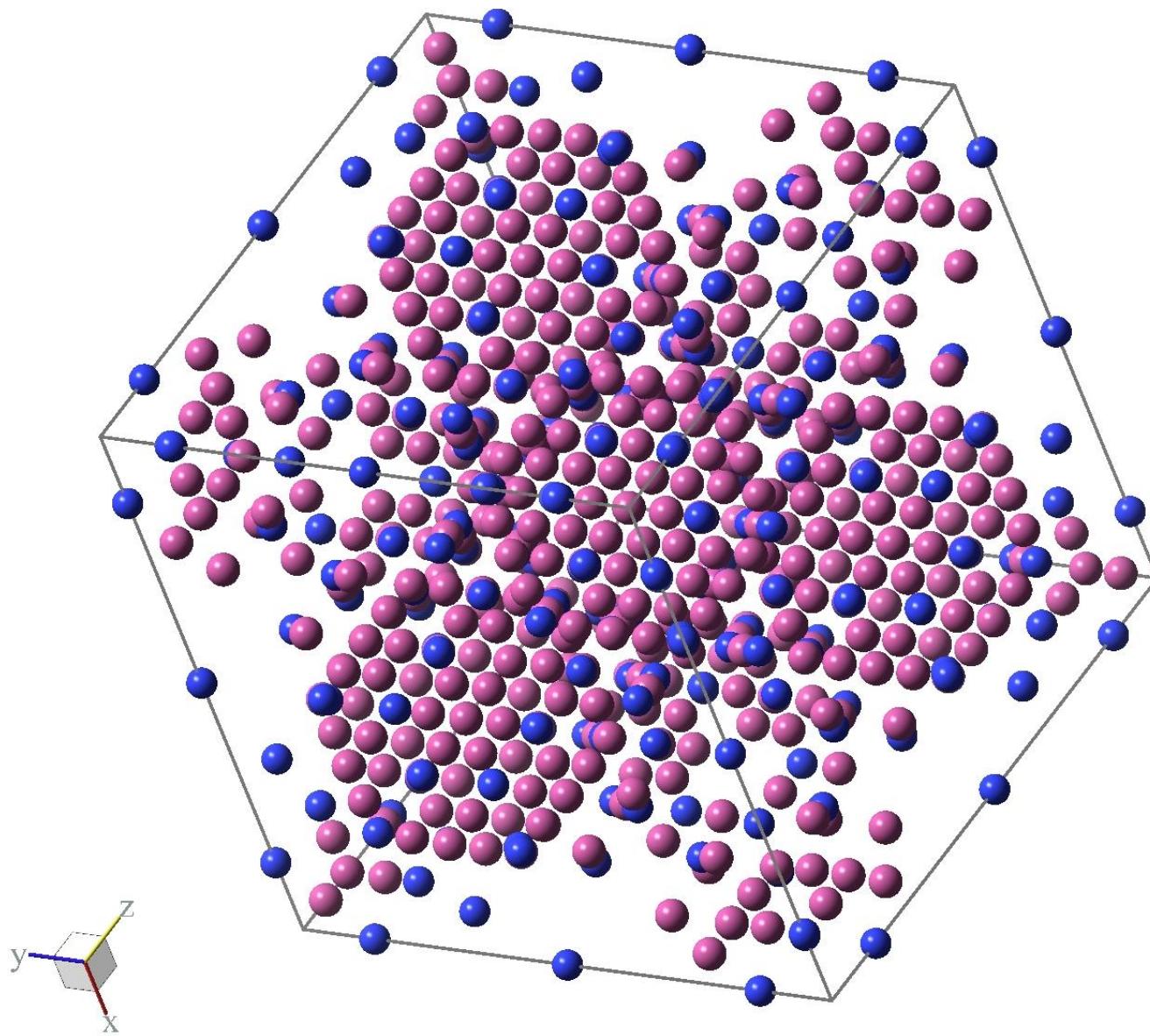
$\beta$  -  $\text{Mg}_2\text{Al}_3$ ; SOF=1; 879 skeleton atoms (they form 75% of atoms)



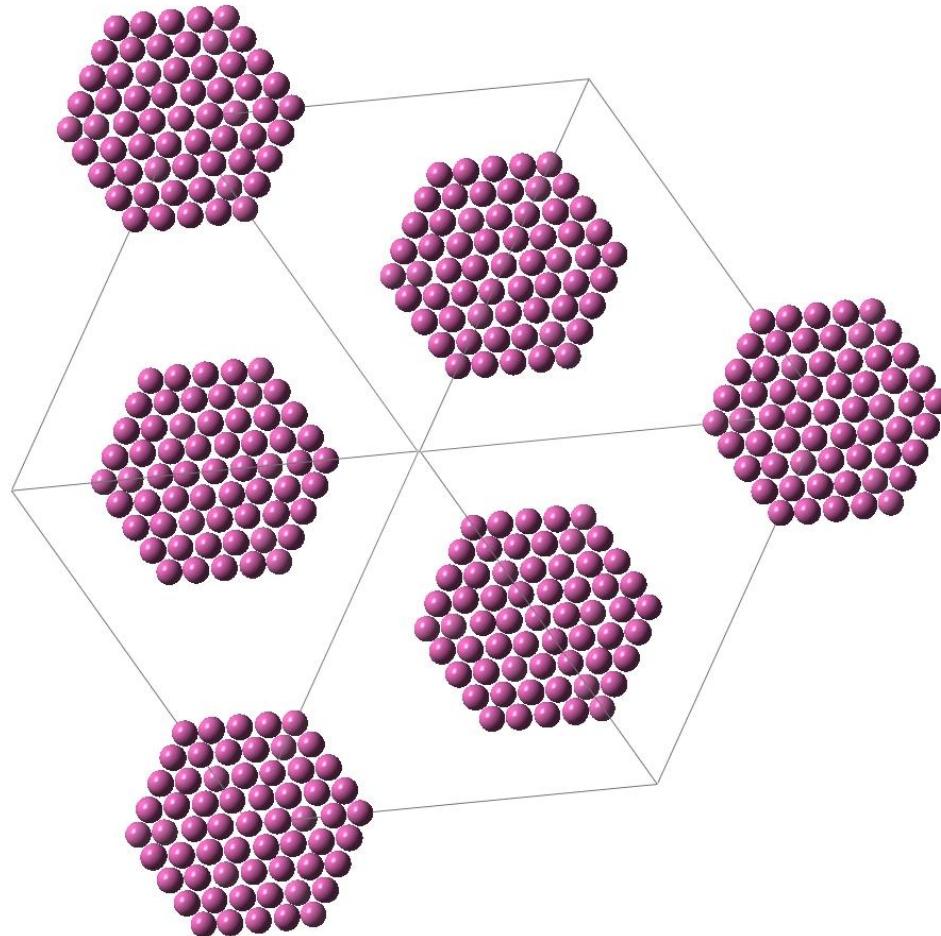
$\beta$  -  $\text{Mg}_2\text{Al}_3$  ; SOF < 1 ; 289 atoms (25%)



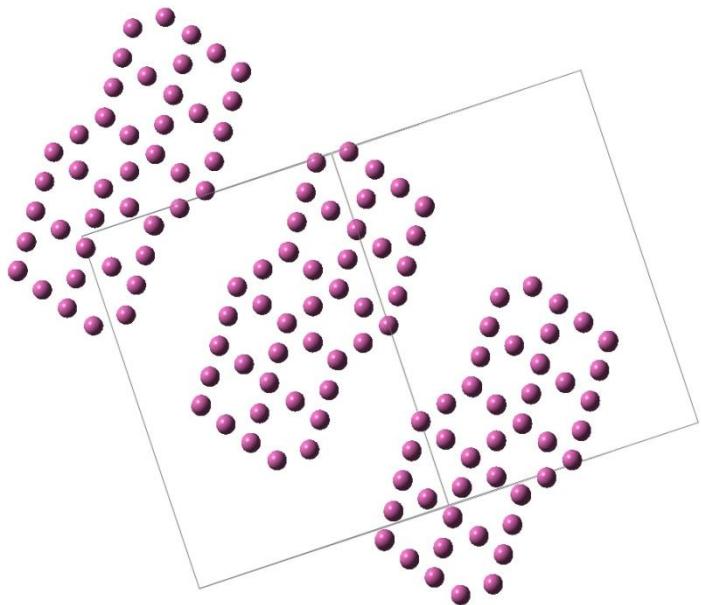
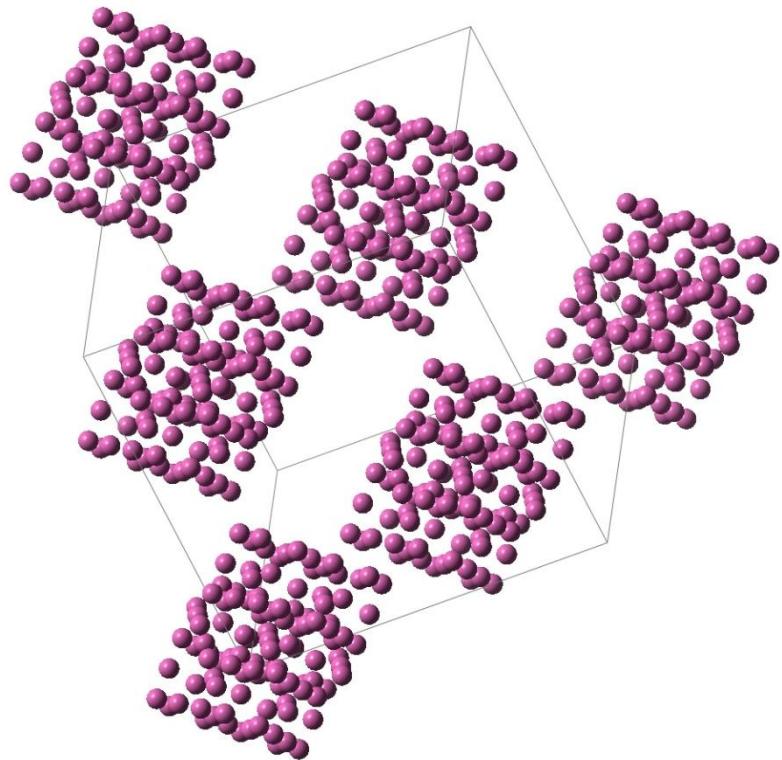
$\beta\text{-Mg}_2\text{Al}_3$ ; skeleton atoms along [111]



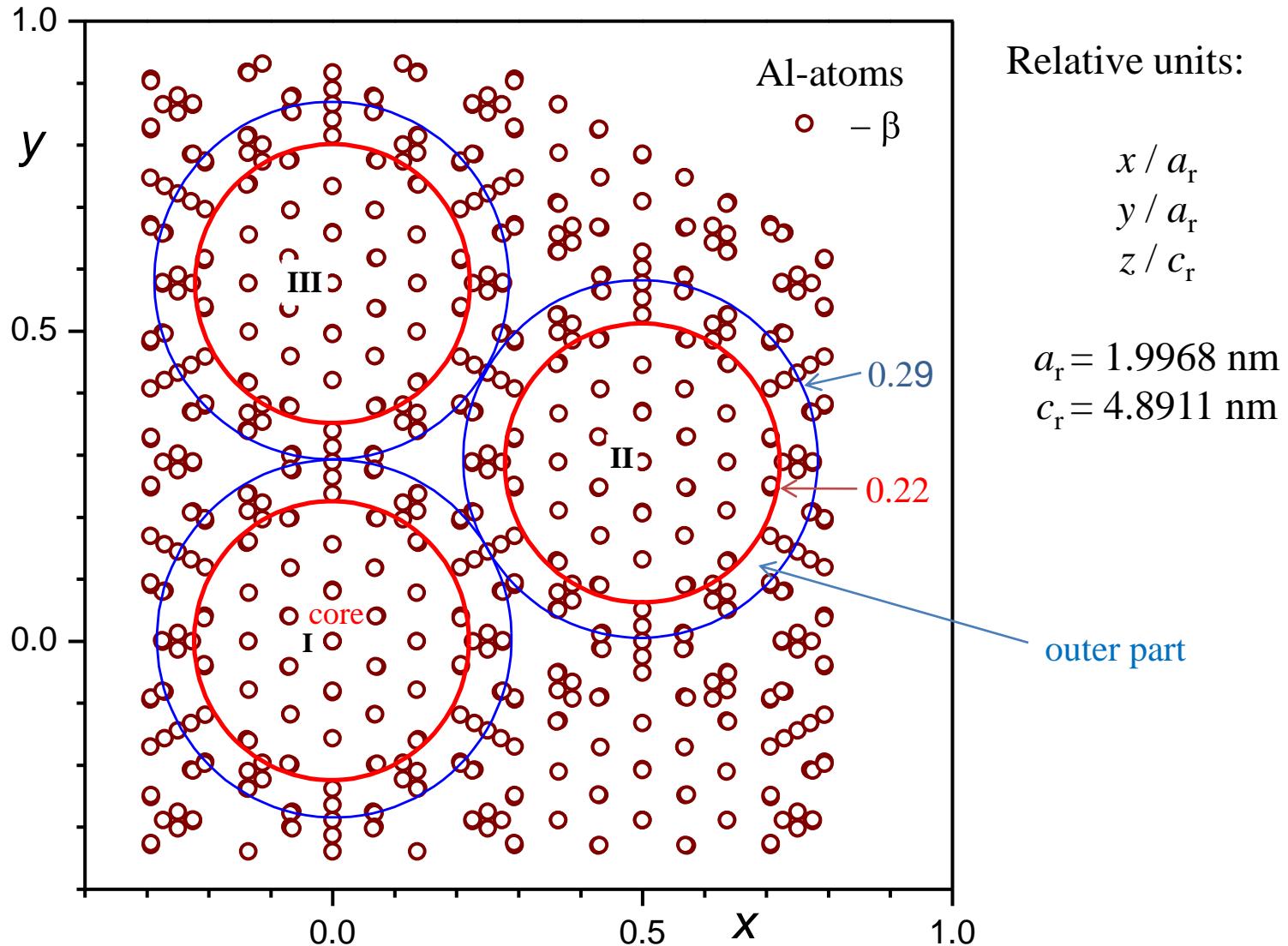
$\beta$  -  $\text{Mg}_2\text{Al}_3$ ; along [111] – hexagonal lattice of domains



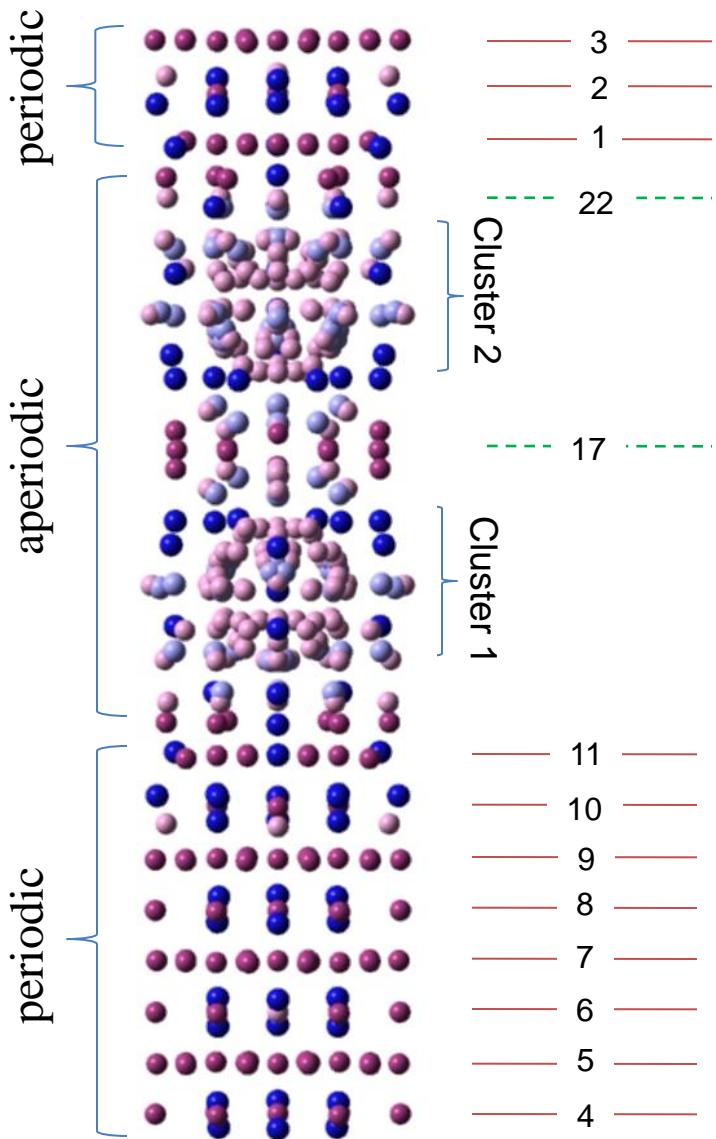
$\beta$  -  $\text{Mg}_2\text{Al}_3$ ; 6 domains-III (Al - skeleton atoms)



# Projection of Al-skeleton atoms into base hexagonal plane

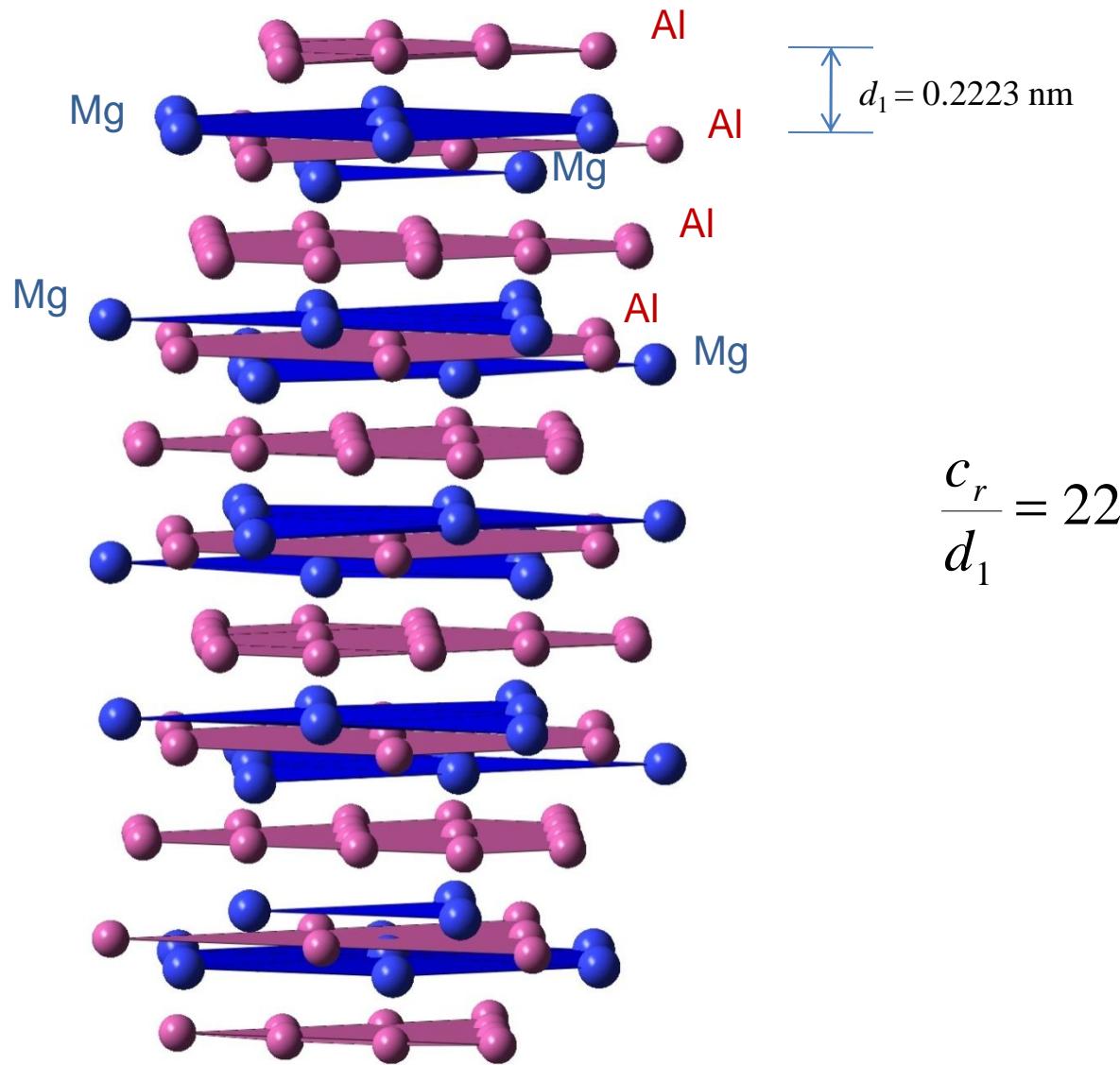


# $\beta\text{-Mg}_2\text{Al}_3$ ; single domain (Al + Mg along [111])

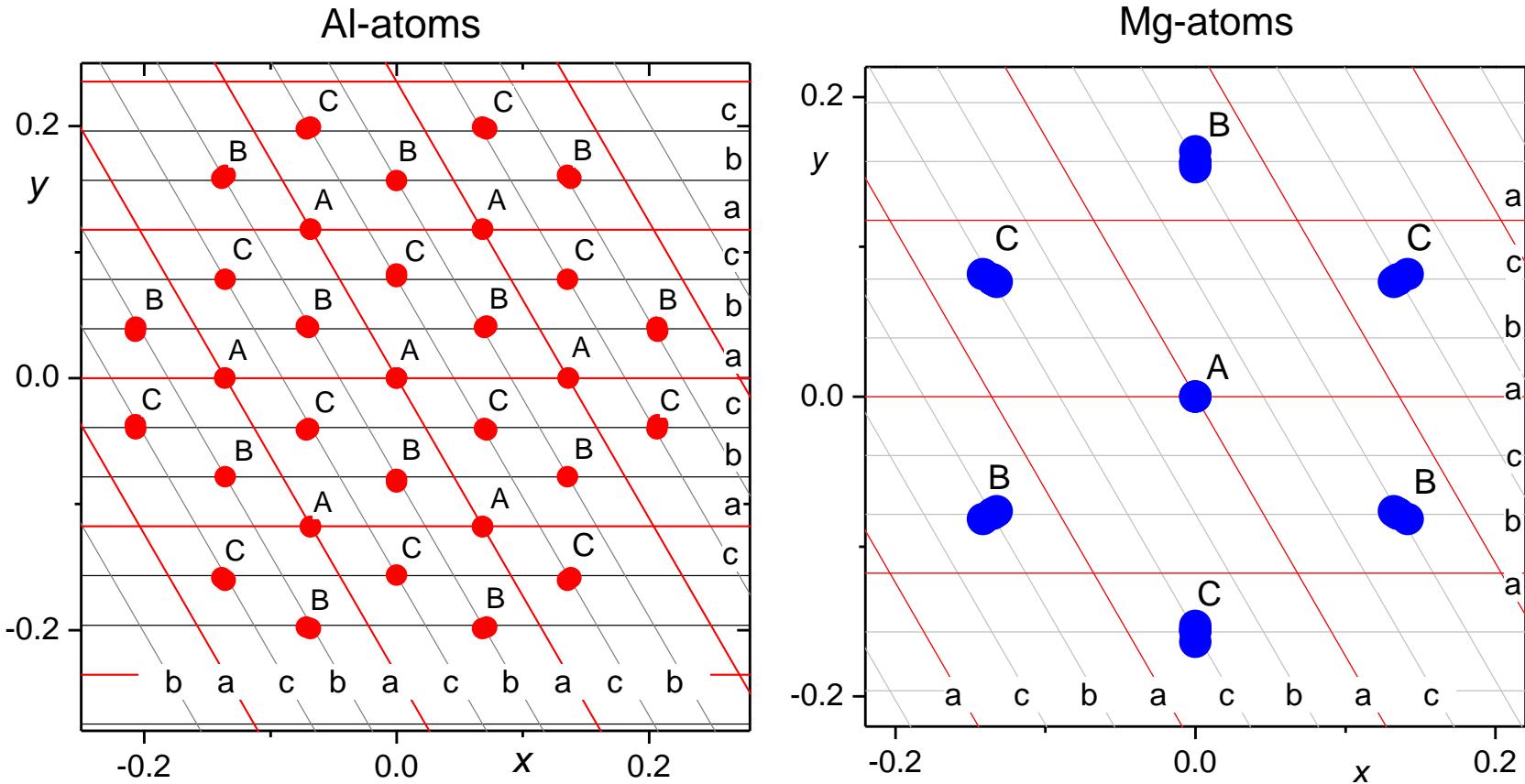


- eleven parallel and evenly distant hexagonal Al layers occupied by skeleton atoms
- two aperiodic clusters occupied by atoms with  $\text{SOF} < 1$

$\beta$  -  $\text{Mg}_2\text{Al}_3$ ; skeleton atoms of single domain (11 Al +10 Mg layers)

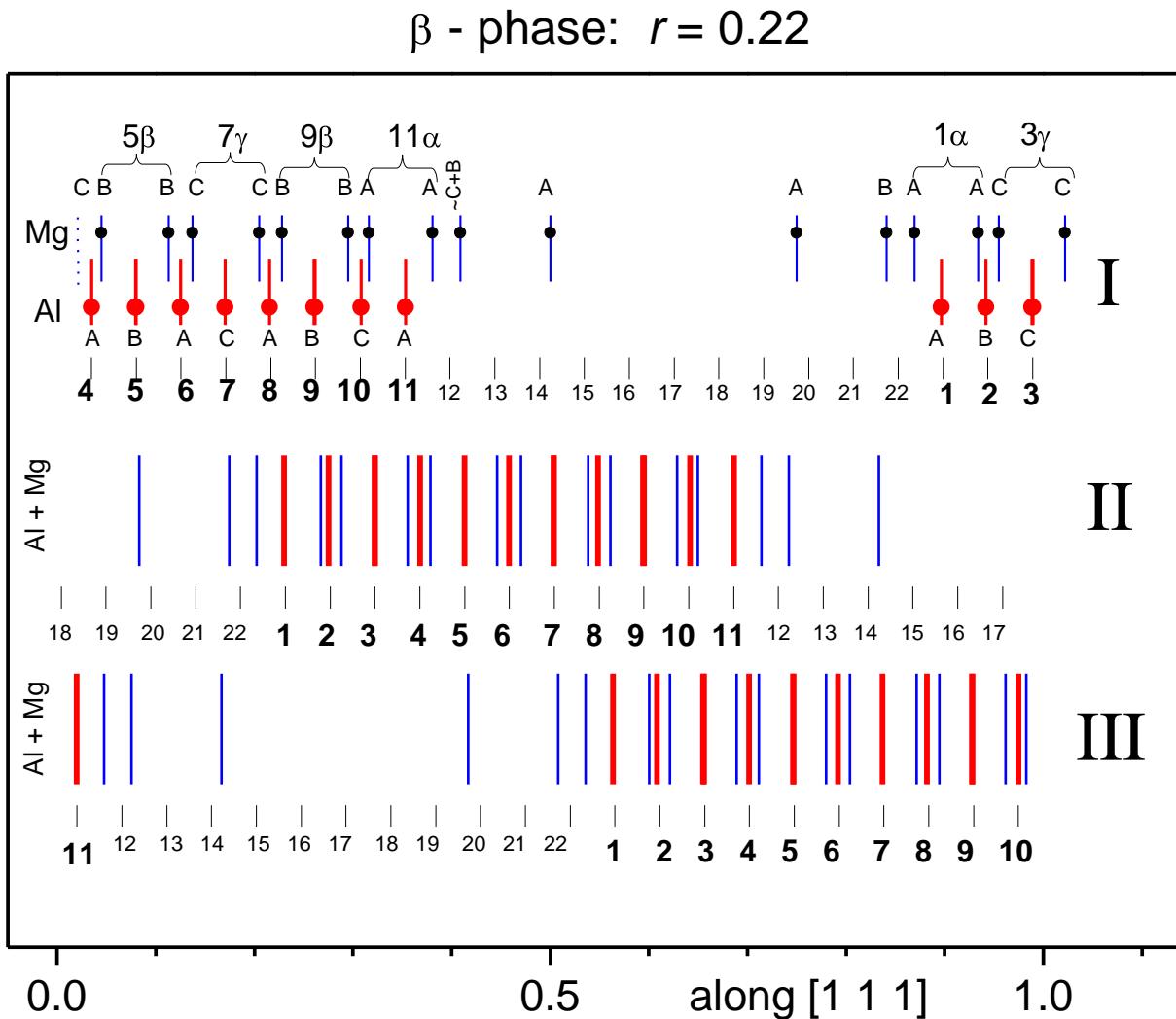


# Projection of skeleton core atoms of domain-I for $\beta\text{-Mg}_2\text{Al}_3$ into xy plane

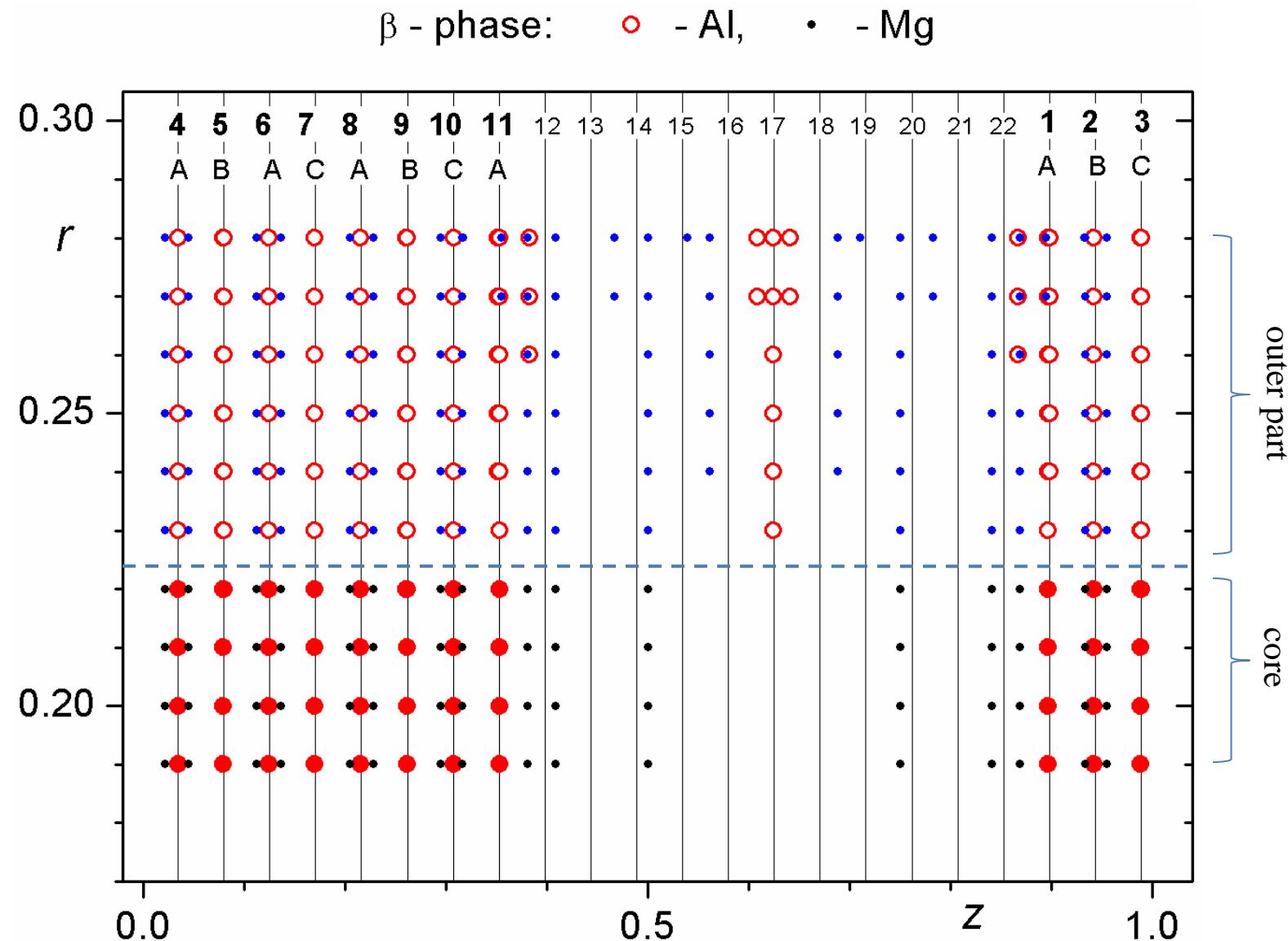


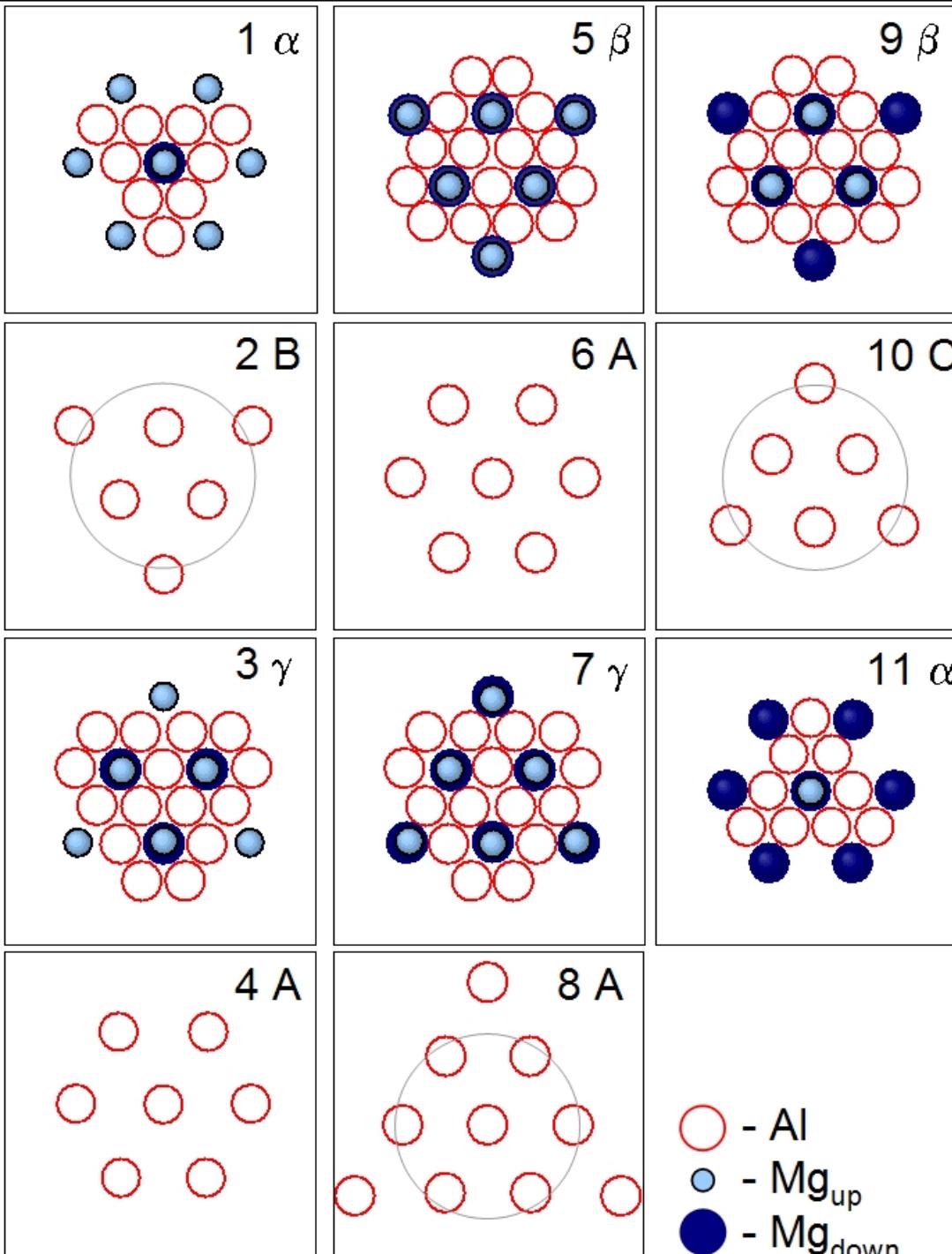
Only A, B or C positions of hexagonal lattice are occupied

# Three domains of skeleton atoms of $\beta\text{-Mg}_2\text{Al}_3$ along the main diagonal



# Z-component of $\beta$ -Mg<sub>2</sub>Al<sub>3</sub> skeleton-atoms decorating domain-I





**Core: 11 Al + 10 Mg layers**

The nominal composition is

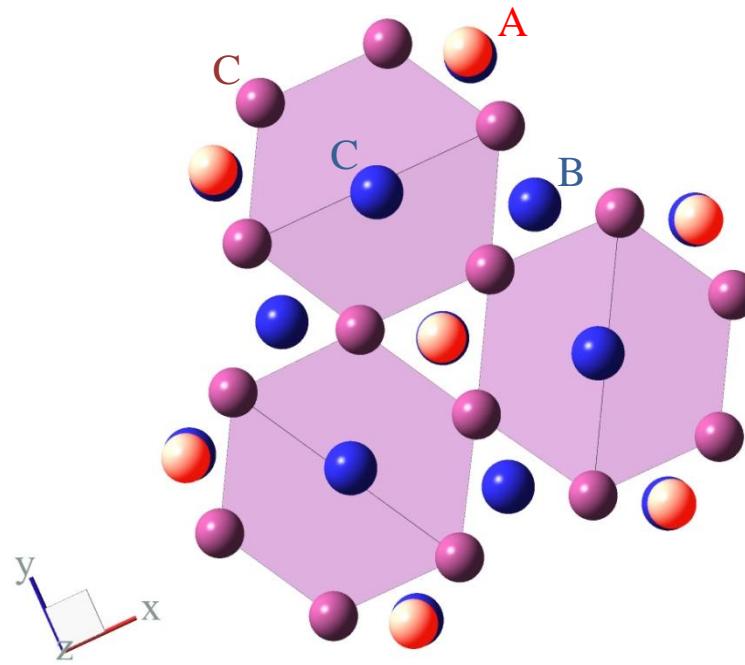
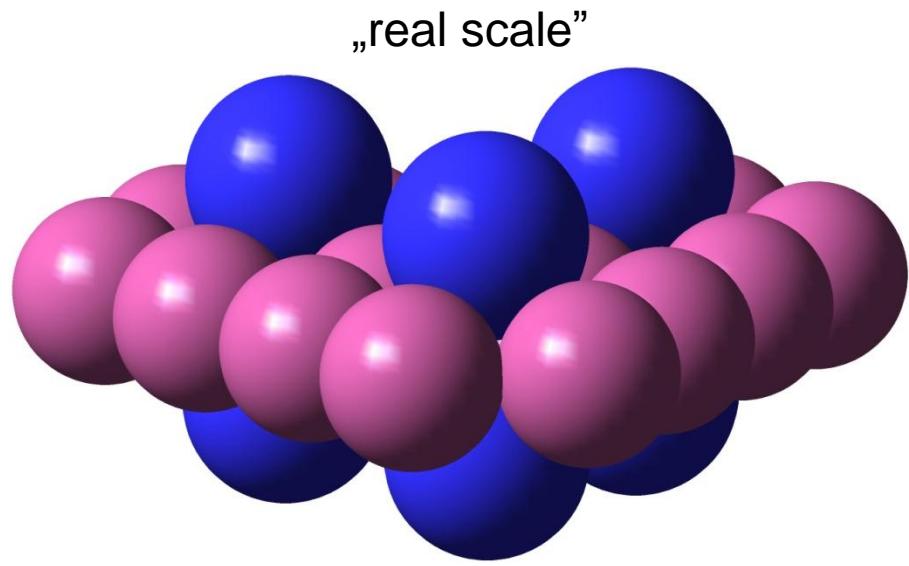
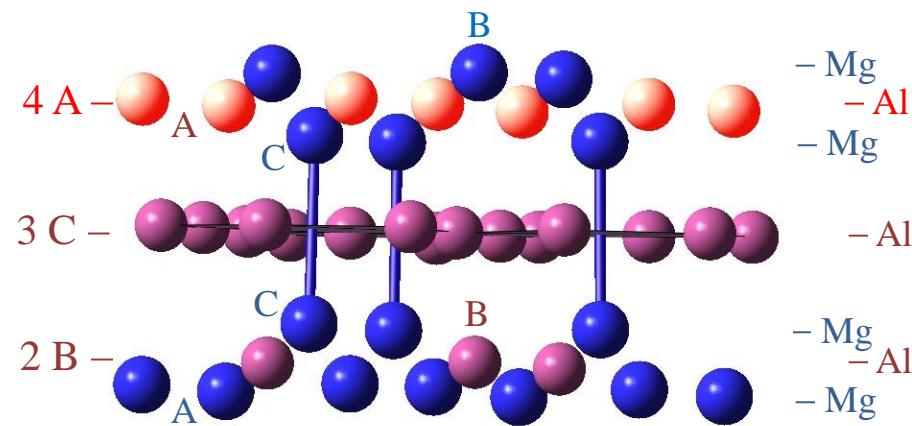
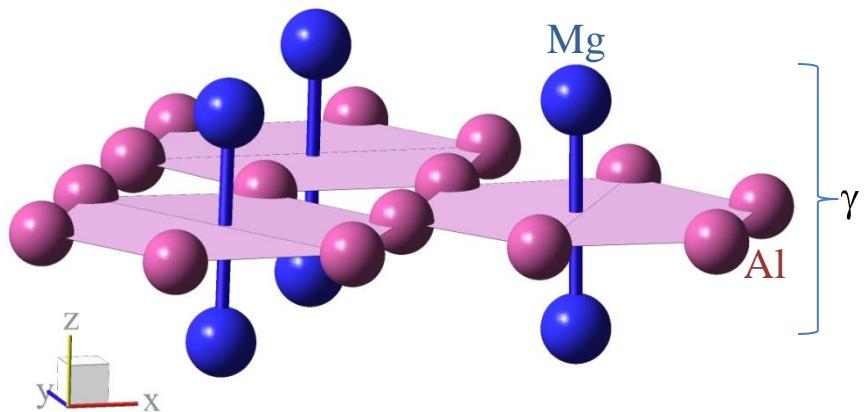


The real composition for layers is

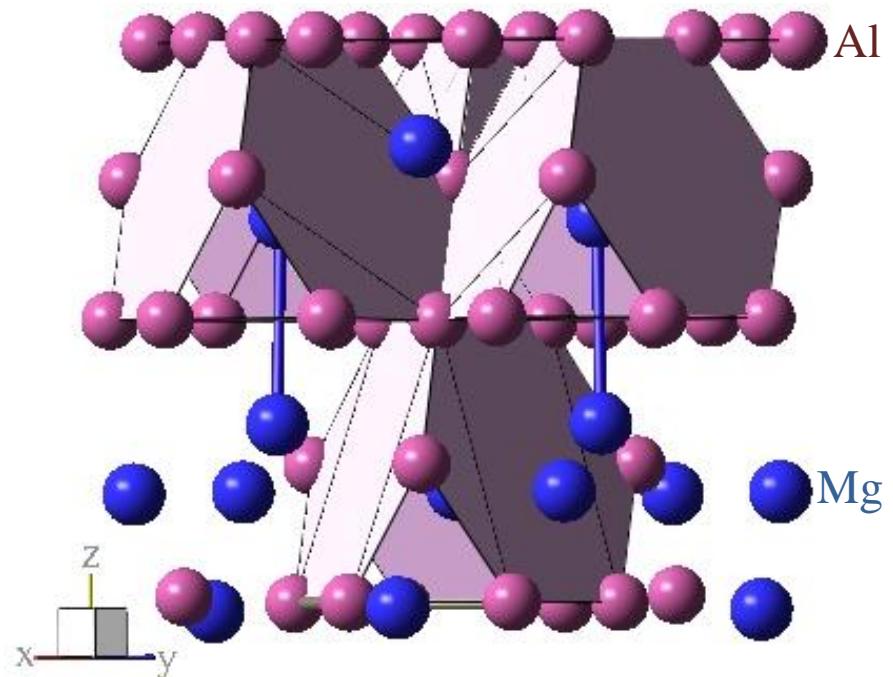


- the magnesium atoms are in deficiency,
- the chemical balance is restored by Mg atoms scattered within outer parts of the structural domains

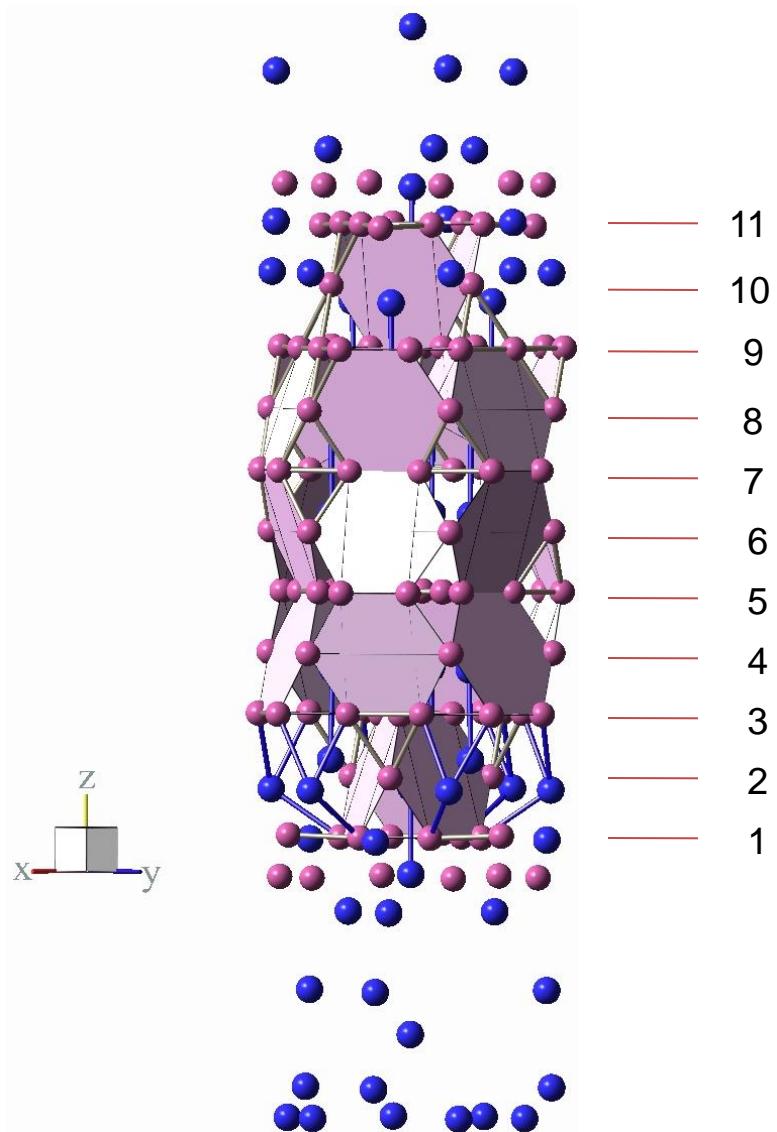
# Position of Mg atoms in respect to 11 hexagonal Al-layers



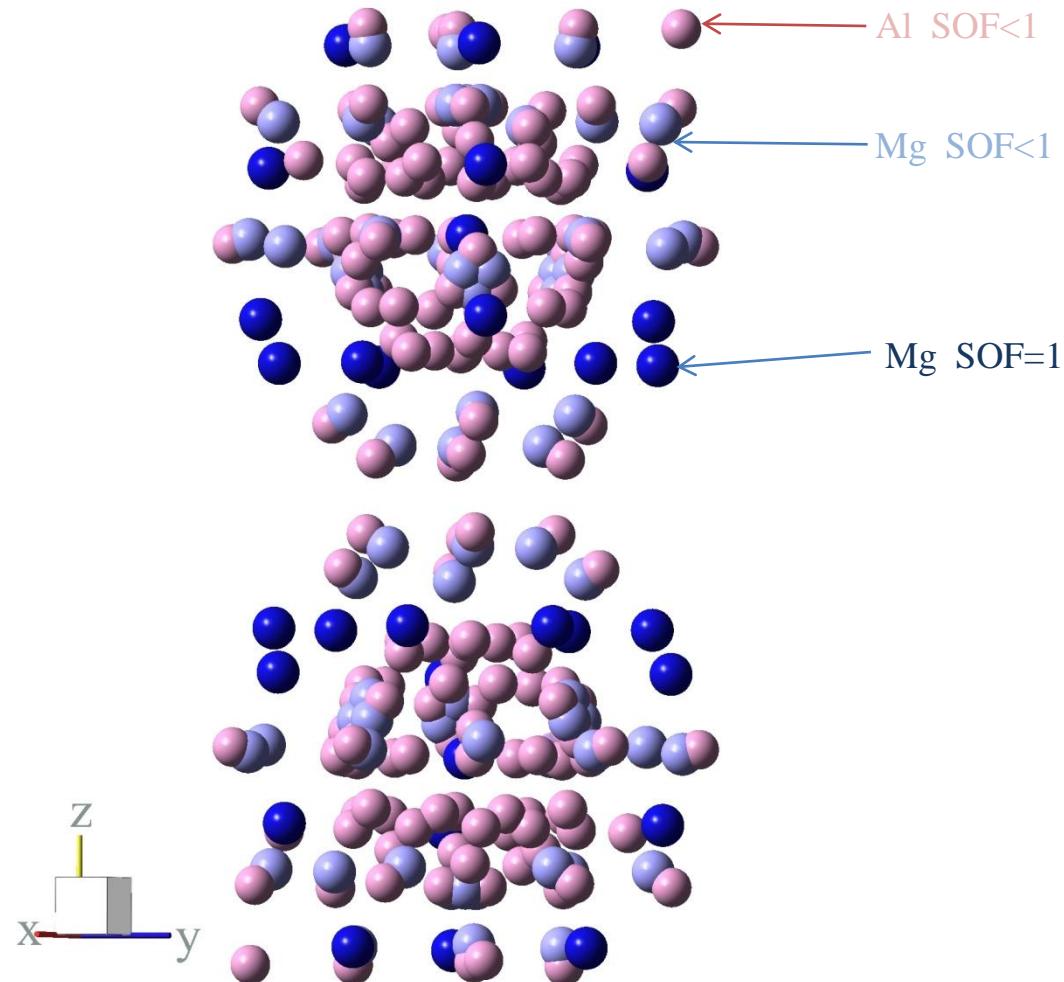
# Skeleton atoms of $\beta\text{-Mg}_2\text{Al}_3$ : hexagonal layers and Friauf polyhedra



# Single domain of skeleton atoms of $\beta\text{-Mg}_2\text{Al}_3$ : 11 hexagonal layers and Friauf polyhedra

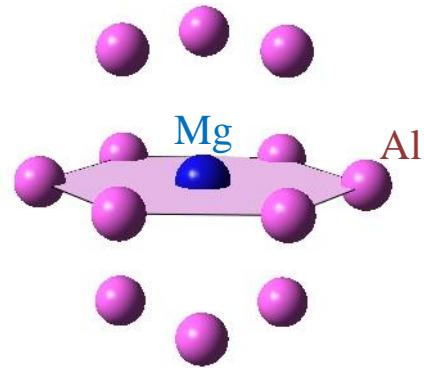
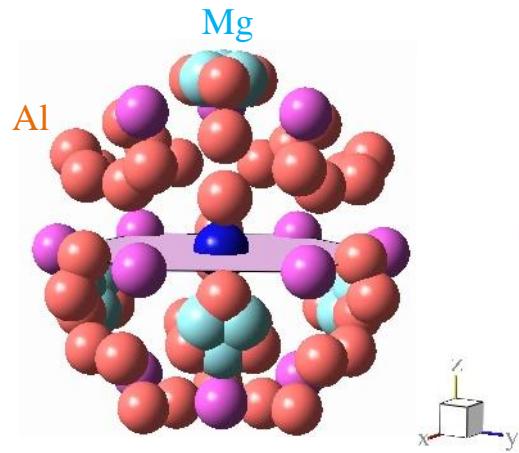


# Clusters in $\beta\text{-Mg}_2\text{Al}_3$

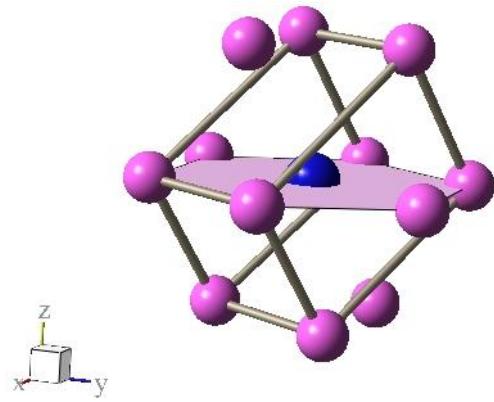
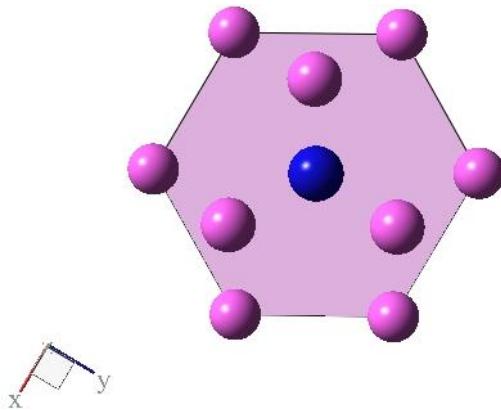


# Cluster 2 of $\beta$ -Mg<sub>2</sub>Al<sub>3</sub>

SOF<1

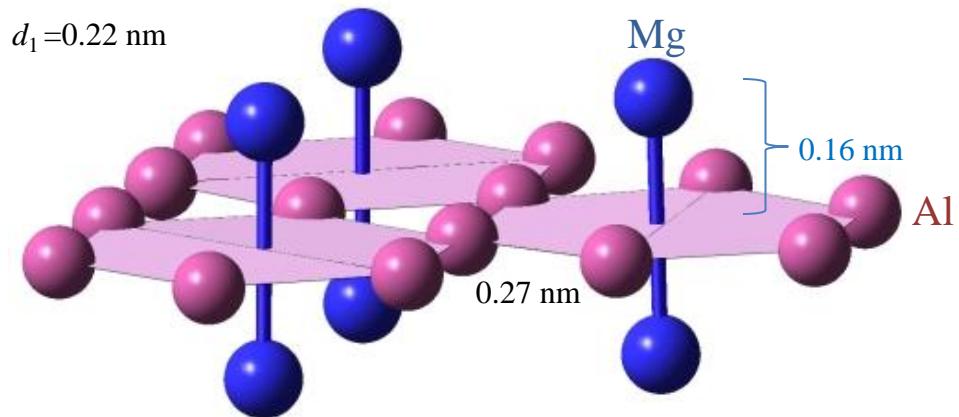


Deformed hexagonal ring



# Position of Mg atoms in respect to hexagonal Al-layers

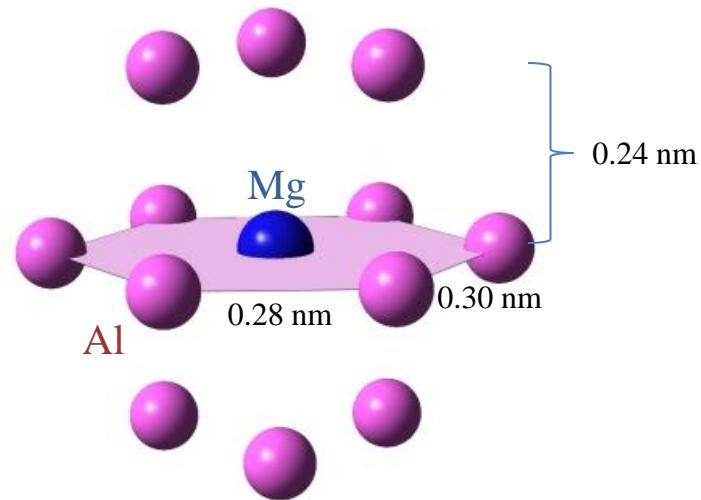
- for periodic set of 11 hexagonal layers



Very stable and unique configuration  
(Friauf polyhedron)

Corresponding composition:  $\text{Mg}_1\text{Al}_2$

- for aperiodic clusters



Plenty of room in a ring (size effect). Energetically less stable; at higher temperature stabilized by entropy

$\text{Mg}_1\text{Al}_1$

## $\beta'$ - Mg<sub>2</sub>Al<sub>3</sub>

At a temperature of 214°C (for cooling rate of 5 °C/min), the structure undergoes a phase transformation to the rhombohedral  $\beta'$ -Mg<sub>2</sub>Al<sub>3</sub> (space group *R*3*m*, no. 160, which is a subgroup of the *Fd*-3*m* group - index 4) with  $a_r=1.9968(1)$  nm,  $c_r=4.89114(8)$  nm.

# Important relations between cubic and rhombohedral lattice constants

$$\frac{c_r}{a_c} = \sqrt{3}$$

and

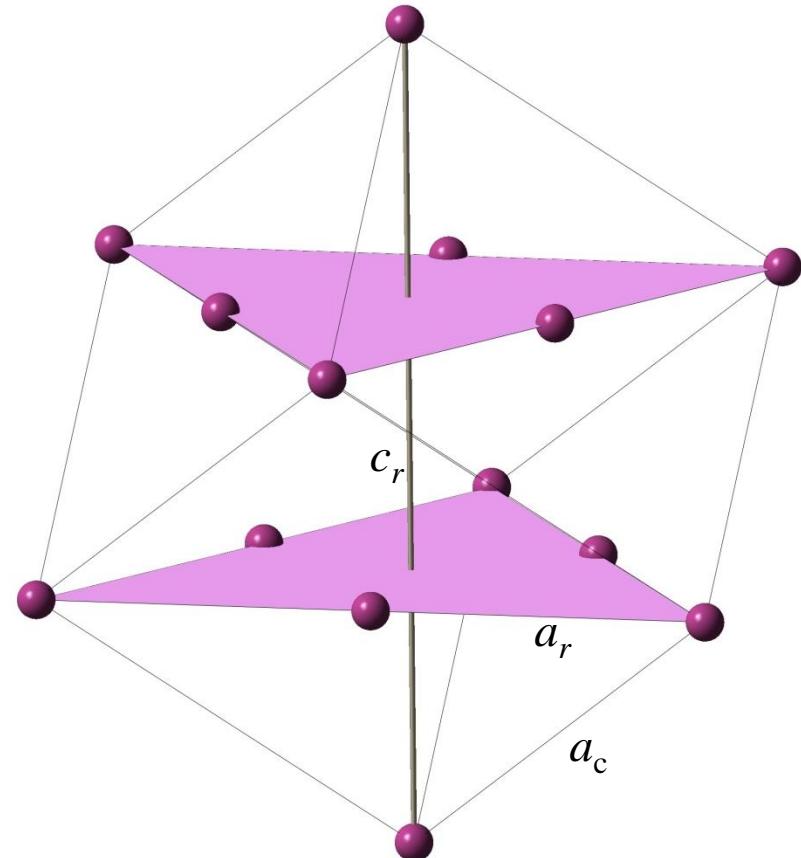
$$\frac{a_c}{a_r} = \sqrt{2}$$

The size of unit cell of  $\beta$  and  $\beta'$  structures is exactly the same, the symmetry – not.

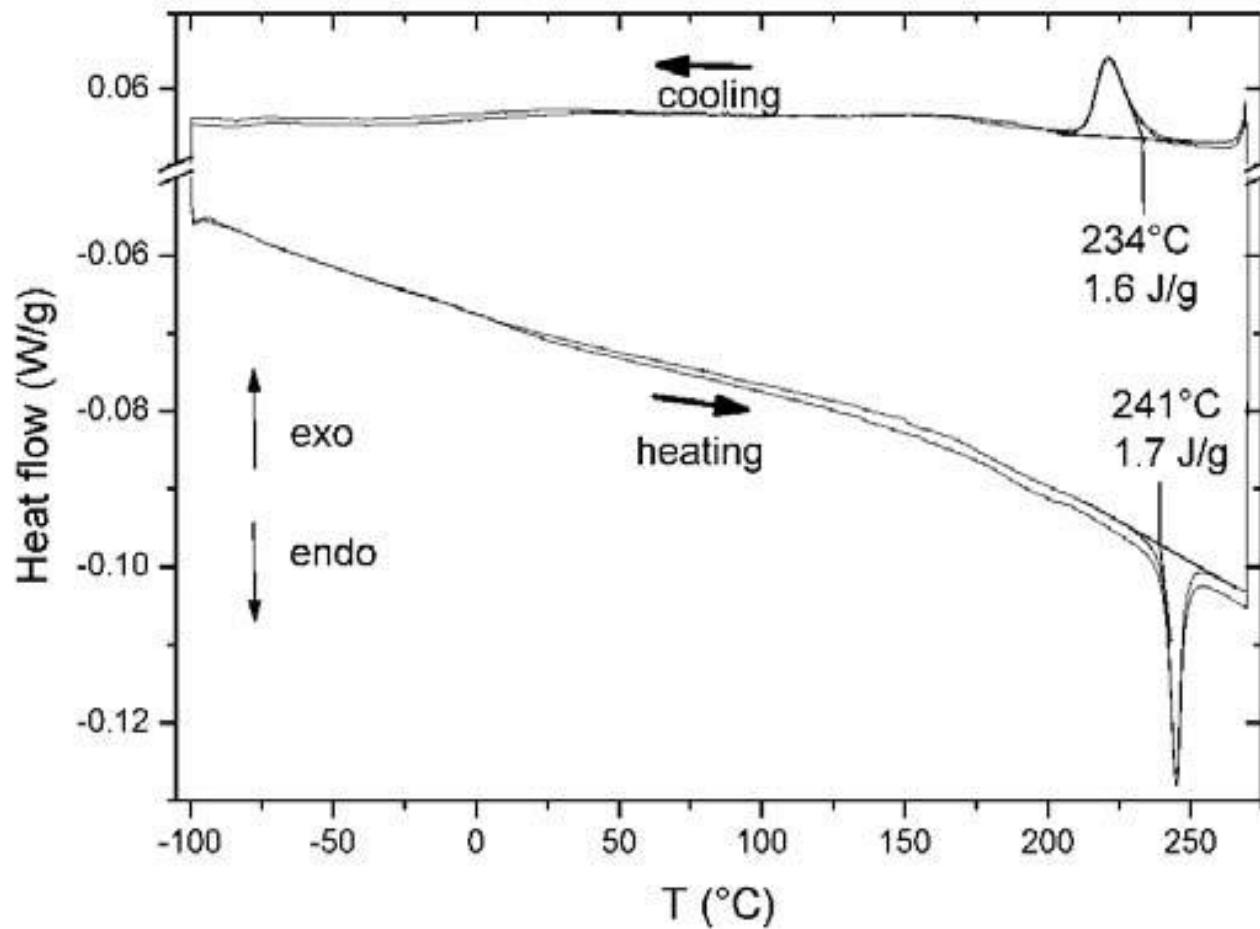
$$V_r = 75\% \cdot V_c$$

from the experiment:

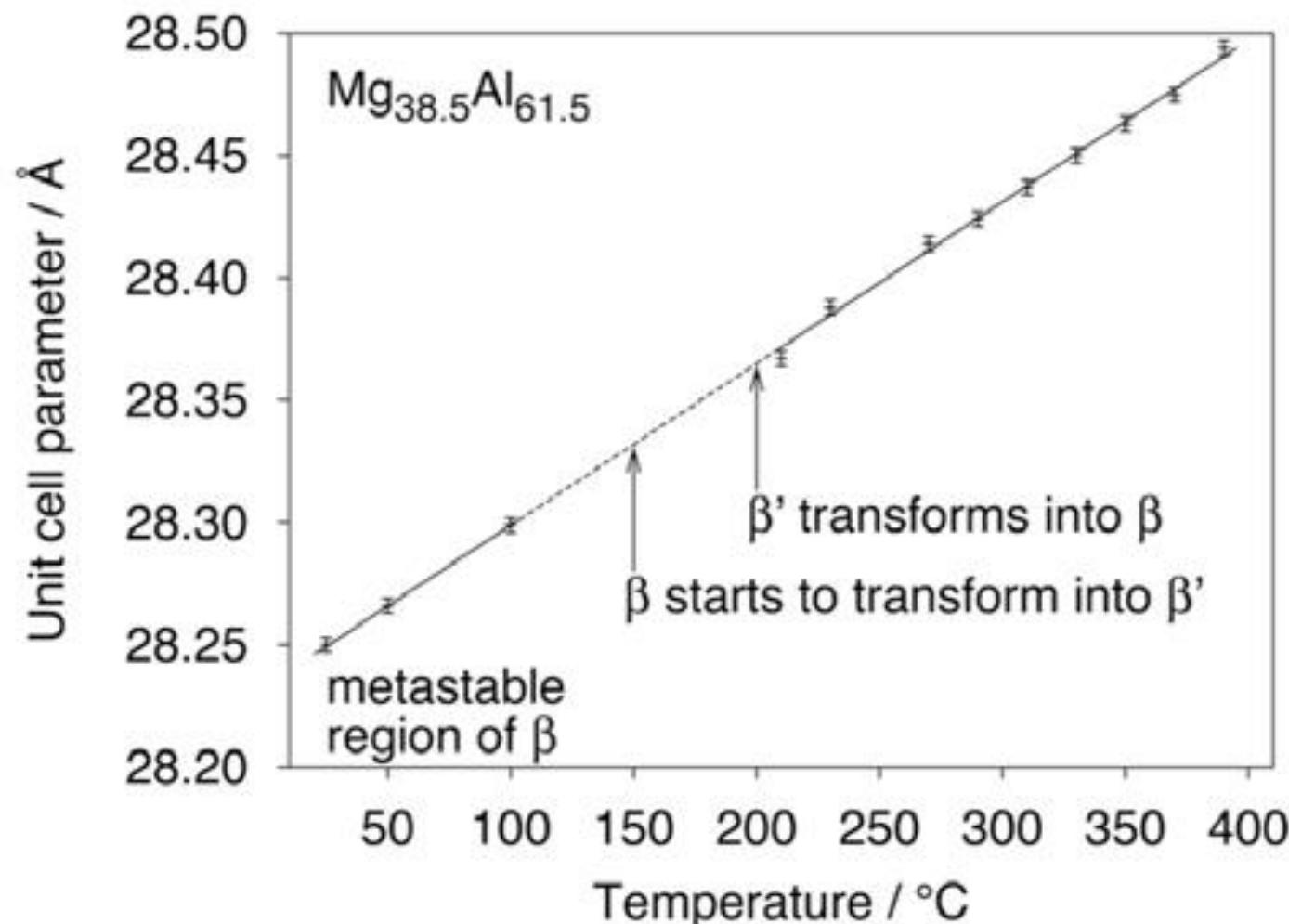
$$16889/22519 = 0.74999$$



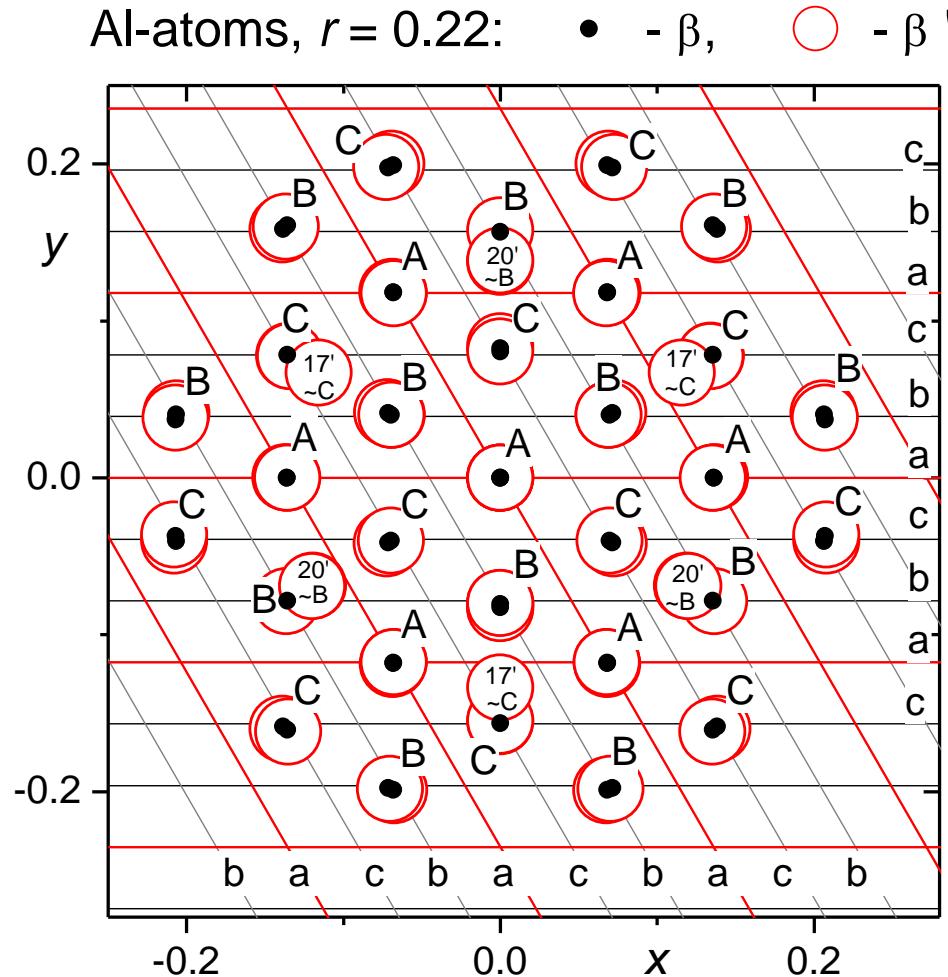
$\beta$  - Mg<sub>35</sub>Al<sub>65</sub> DSC scans [Z. Krist. 222 (2007) 259]



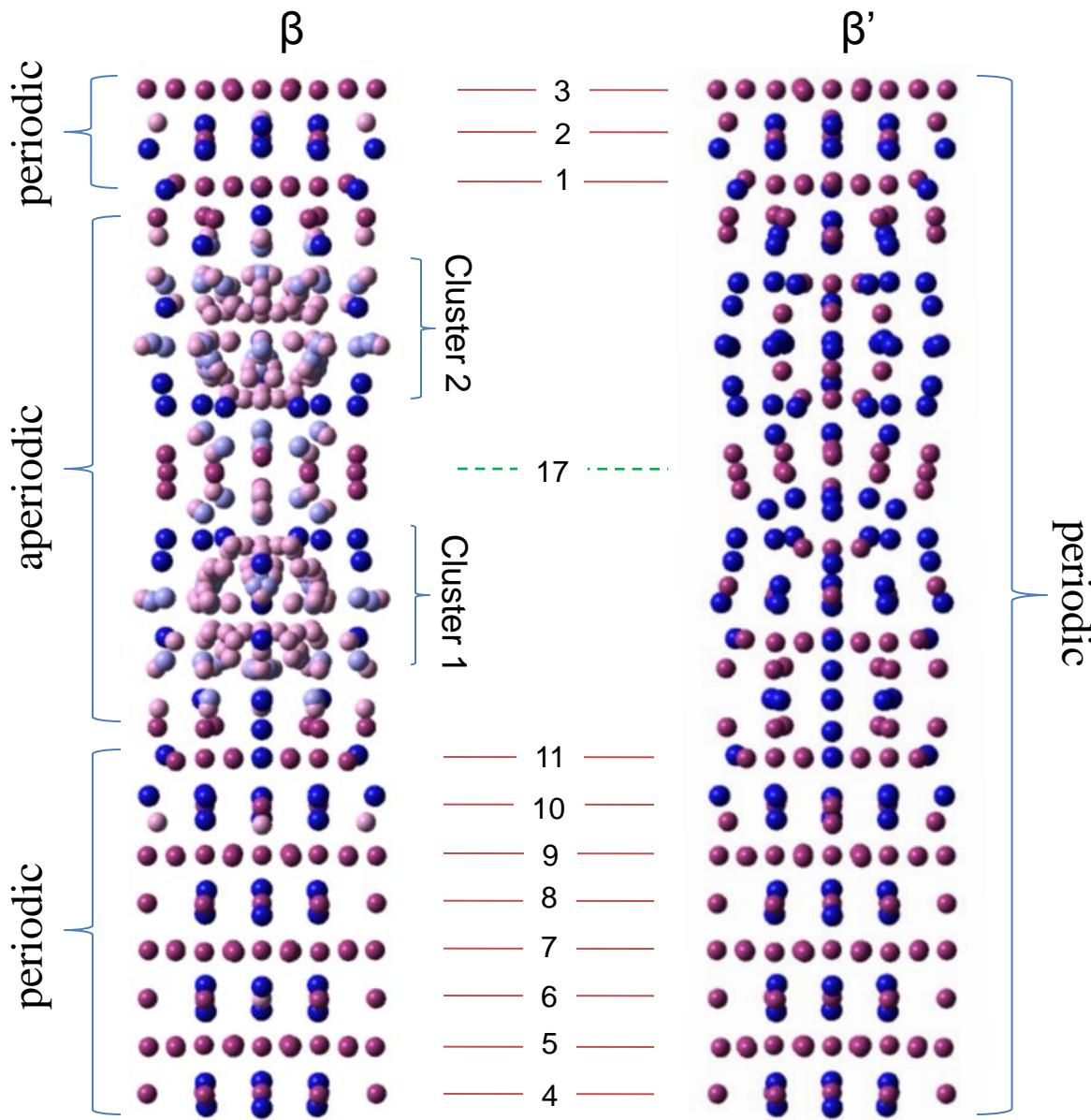
$\beta$  -  $\text{Mg}_{38.5}\text{Al}_{61.5}$  annealed at 400 C [Z. Krist. 222 (2007) 259]



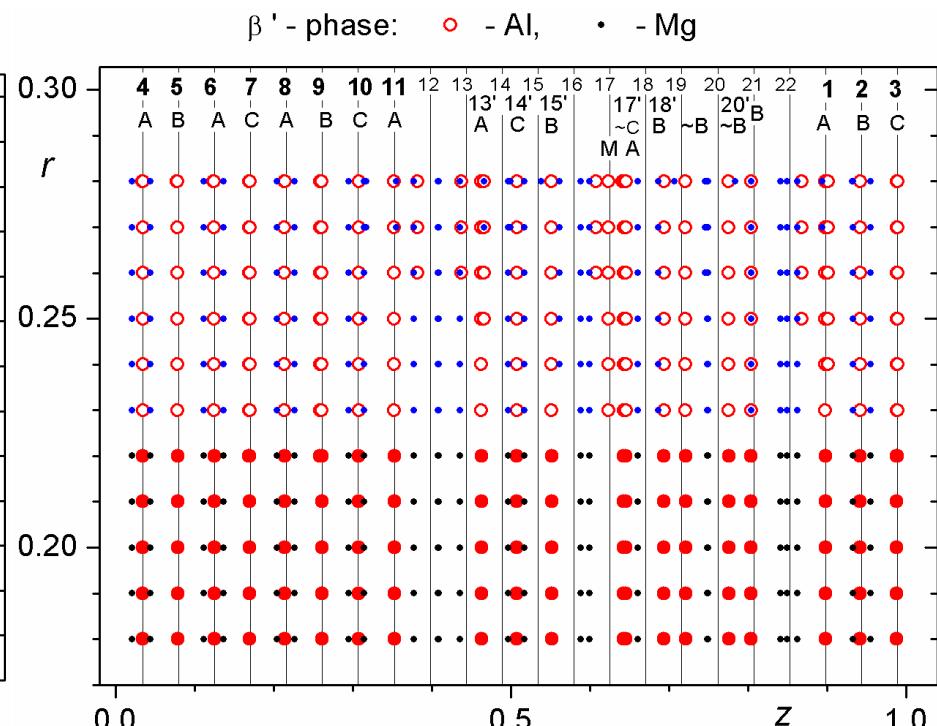
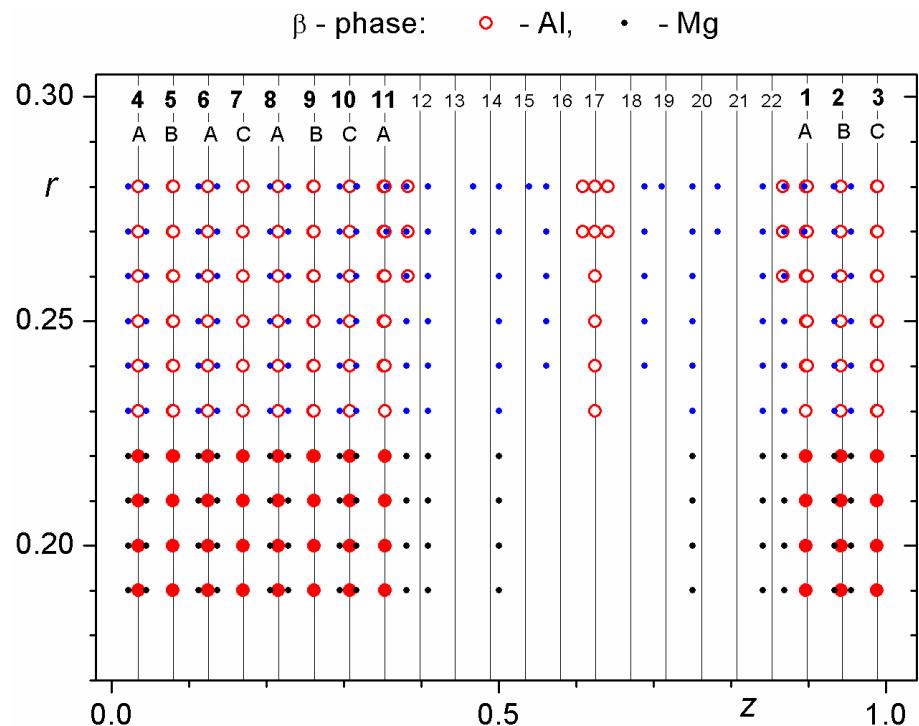
# Projection of skeleton atoms of domain-I into xy plane for $\text{Mg}_2\text{Al}_3$



$\text{Mg}_2\text{Al}_3$ ; domain-I (Al + Mg)

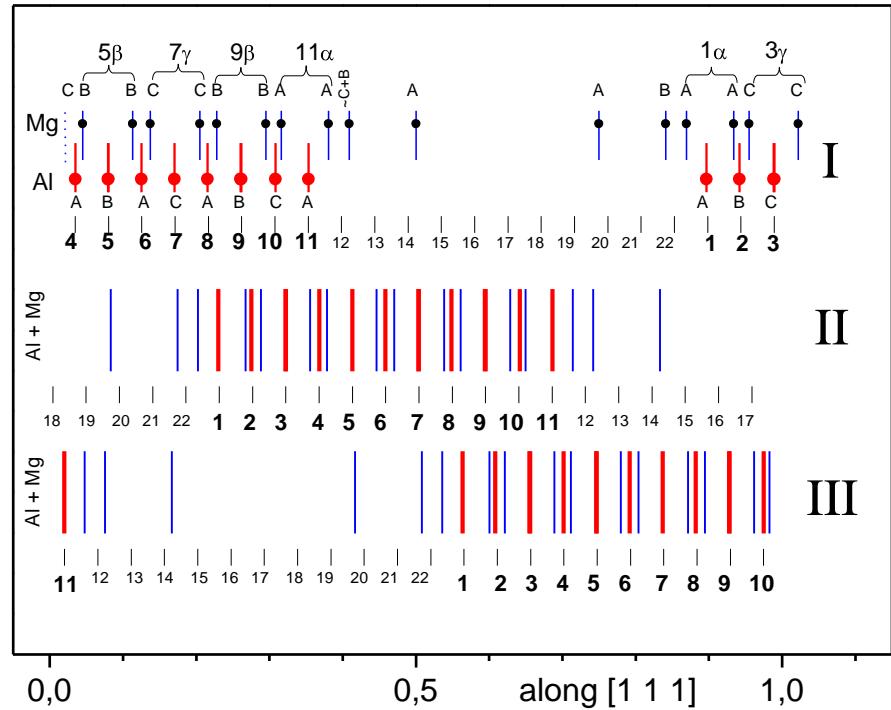


# Position of atoms decorating domain-I in perpendicular direction to the hexagonal plane

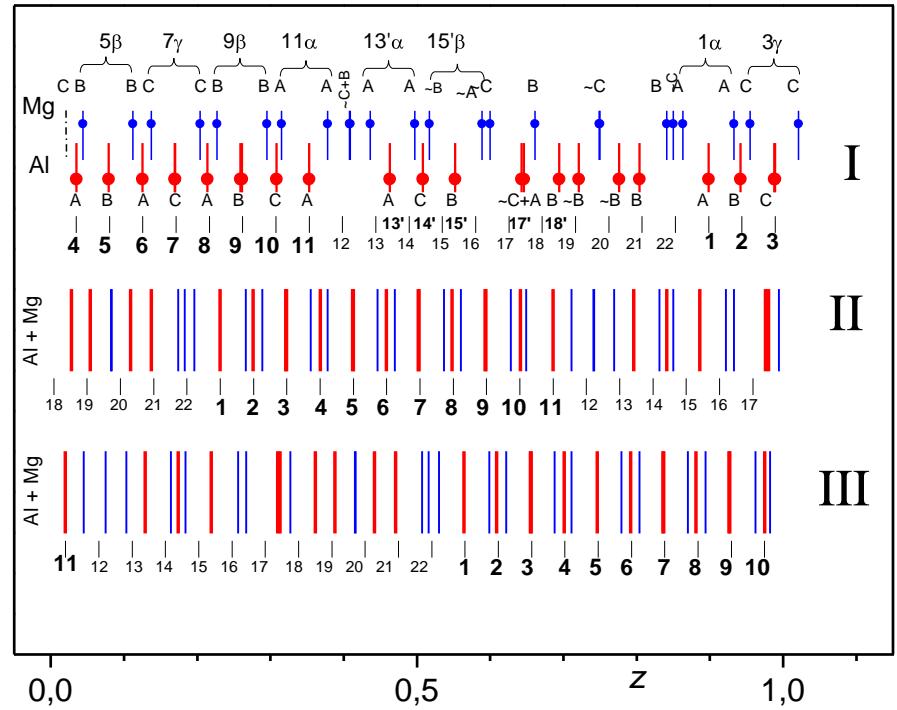


# Three domains of skeleton atoms of $\text{Mg}_2\text{Al}_3$ along the main diagonal

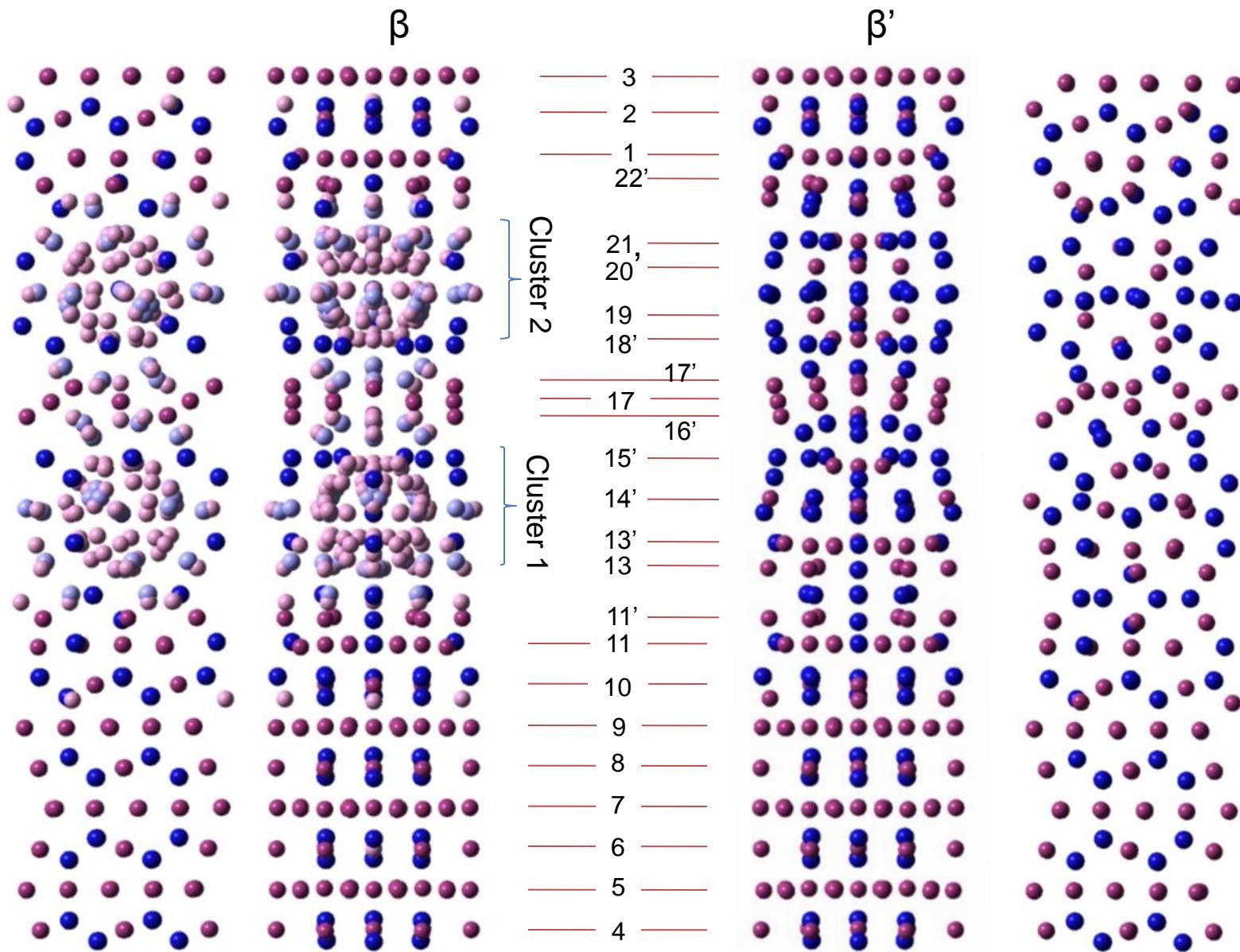
$\beta$  - phase:  $r = 0.22$



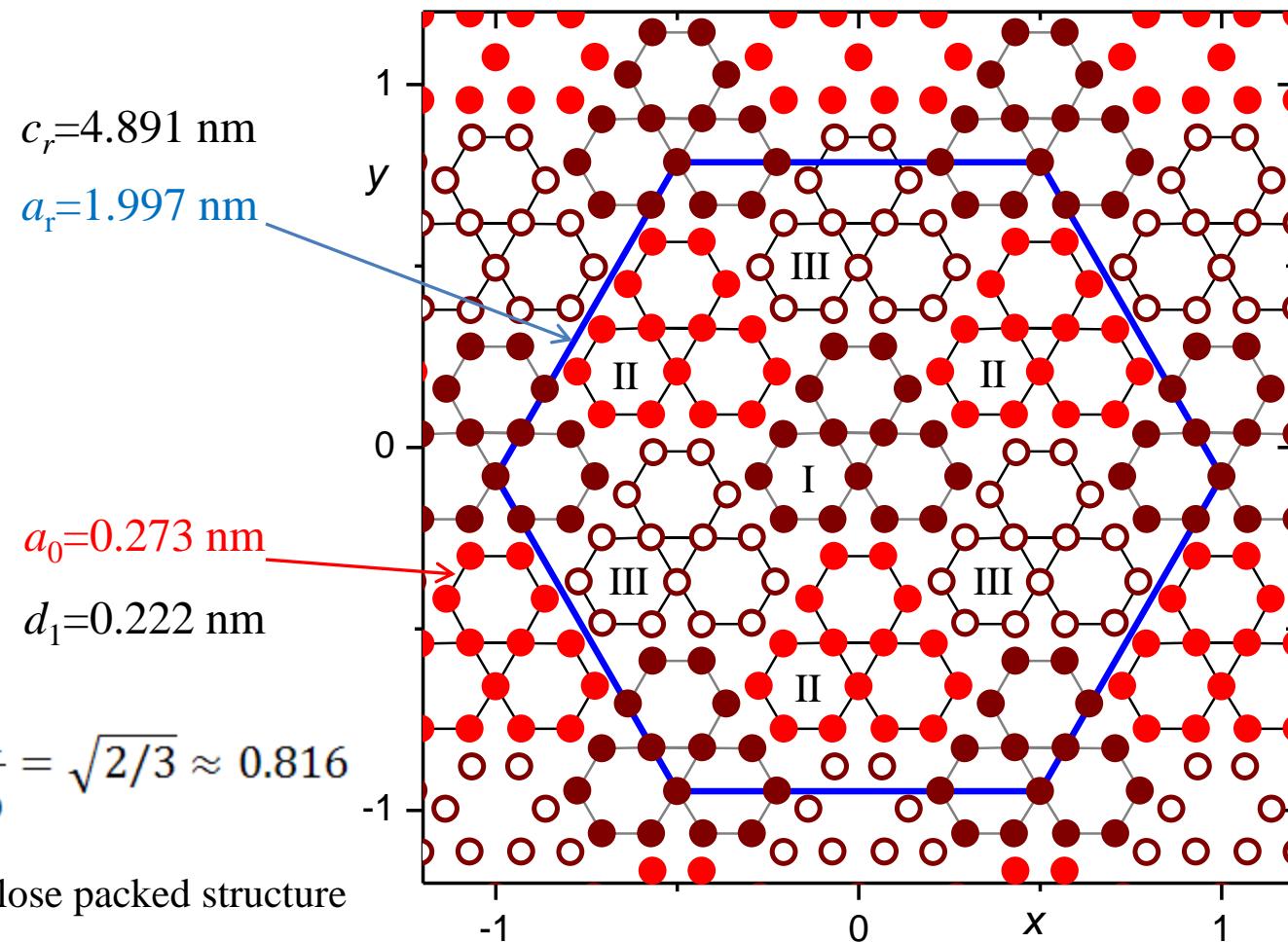
$\beta'$  - phase:  $r = 0.22$



# $Mg_2Al_3$ ; domain-I (Al + Mg)



# Superstructure of domains in $xy$ plane



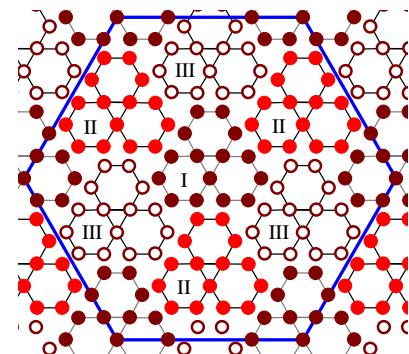
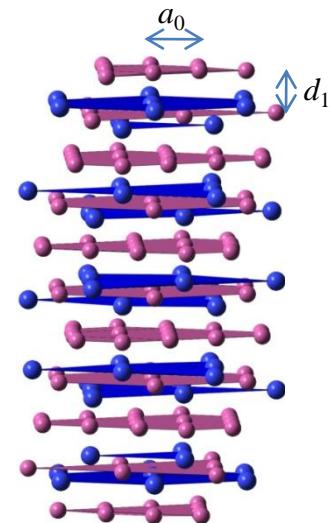
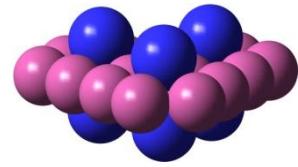
$$\frac{a_r}{a_0} = \frac{22}{3} \approx 7.3$$

$$\frac{c_r}{d_1} = 22$$

Layer 9B

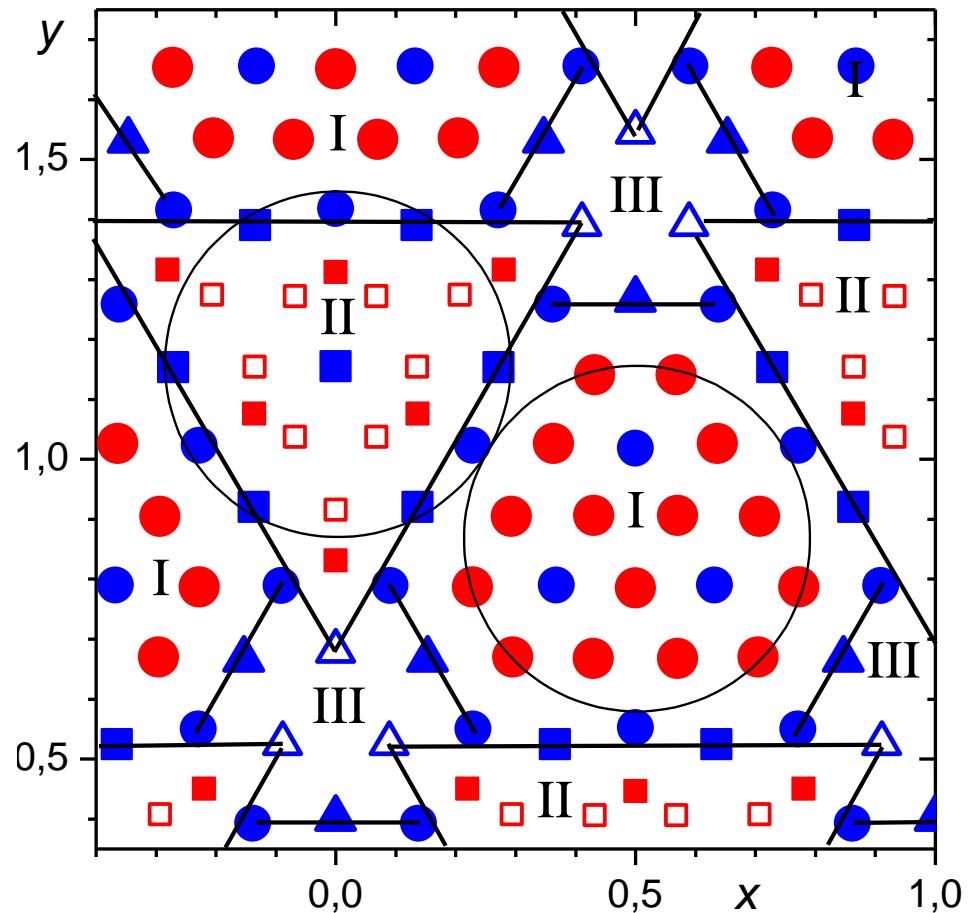
# Conclusions

1. The most stable building element of Mg-Al compounds is a hexagonal ring of Al atoms ( $a_0 = 0.273 \text{ nm}$ ) filled with a couple of Mg atoms which are pushed out from the base plane (equivalent to the Friauf polyhedron) of a real local composition  $\text{Mg}_1\text{Al}_2$ .
2. Close packing of Al atoms brings to distance between hexagonal layers equal to  $d_1 = 0.222 \text{ nm} = (2/3)^{1/2}a_0$ . Additional:  $22 \cdot d_1 = c_r$ .
3. The core of each domain consists of a main chain of 11 hexagonal layers. It doesn't change during the  $\beta$ - $\beta'$  transition.
4. The unit cells of  $\beta$  and  $\beta'$  phases of  $\text{Mg}_2\text{Al}_3$  can be also regarded as the superstructures with the modulation vector equal to  $(3/22)$  (as the multi- $q$  modulated structures).

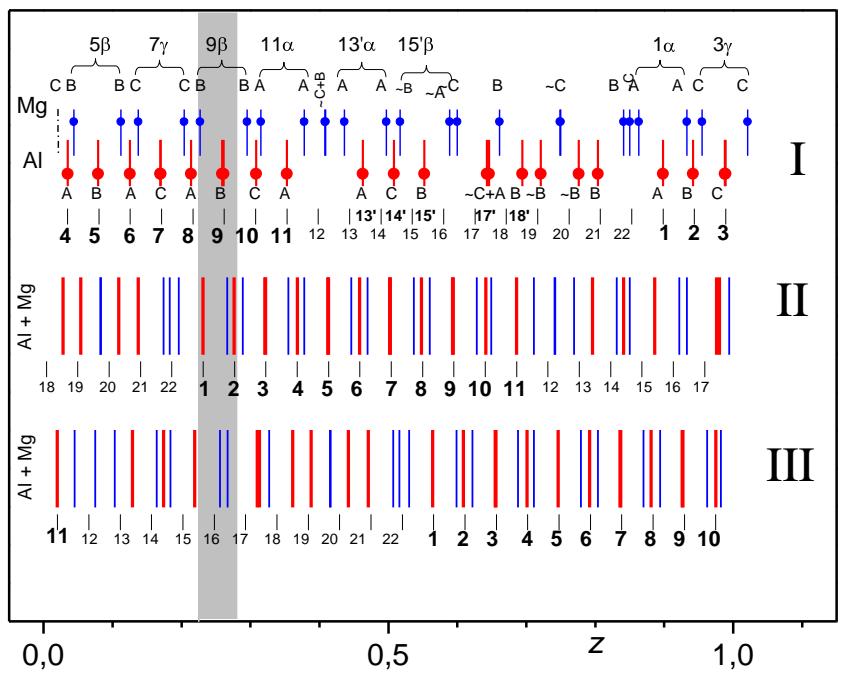




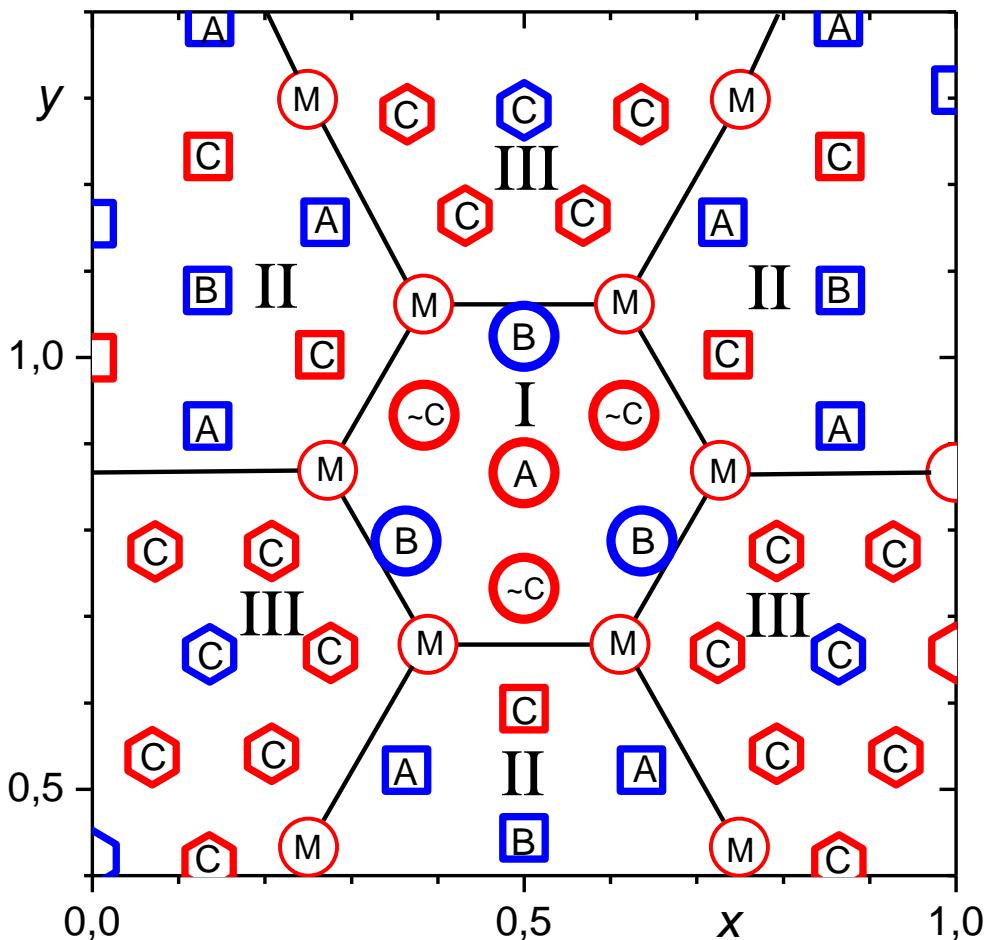
# Crosssection through the $\beta'$ -Mg<sub>2</sub>Al<sub>3</sub> in xy plane: domain-III almost disappeared



$\beta'$  - phase:  $r = 0.22$

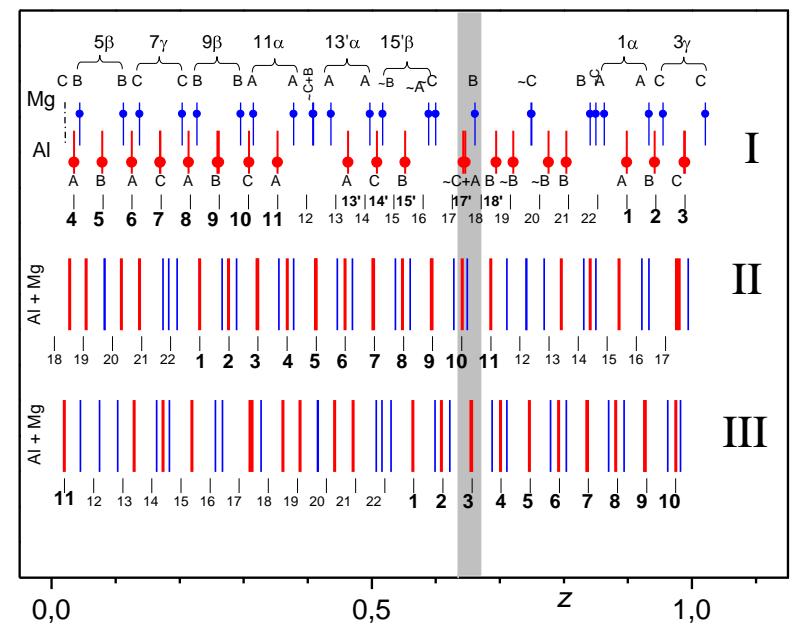


# Crosssection through the $\beta'$ -Mg<sub>2</sub>Al<sub>3</sub> in xy plane: near I-17'



○ Al (I)  
 ○ Mg (I)  
 □ Al (II)  
 □ Mg (II)  
 ○ Al (III)  
 ○ Mg (III)

$\beta'$  - phase:  $r = 0.22$



# Crossection through the $\beta'$ -Mg<sub>2</sub>Al<sub>3</sub> in xy plane: intergrowth of domains I and III

