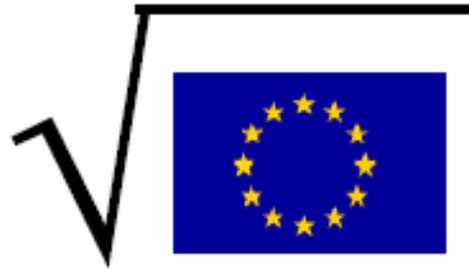


# **Why square root?**

## **On voting systems in the European Council**



**Wojciech Słomczyński**  
**Karol Życzkowski**  
Jagiellonian University (Kraków)

# Voting systems for the Council of the European Union

- a two-tier decision-making system:
  - the **Member States** at the lower level
  - the **European Union** at the upper level

The Council of the EU  
votes by a **qualified  
majority voting**:  
a decision of the Council  
is taken, if it is approved  
by a qualified majority

## Indirect voting in the Council

- A representative of a member state with a population **N** goes to Brussels and says **yes** according to the will of the majority of his co-patriots...
- How many of them are satisfied, **N** or **N/2** ? (since the representative followed their will).
- ***We do not know!*** These numbers will be different in each cases. Mathematics is needed to compute the average and to prove that the difference **satisfied** - **dissatisfied** scales as ... **Sqrt (N)**

# How to analyse voting systems?

- 27 Members States:  
more than **134 mln** possible coalitions



- **voting power** (capacity to affect EU Council decisions)
- **voting weight** (number of votes)
- **voting power** held by a given state depends not only on its **voting weight** but also on the distribution of the **weights** among all the remaining states
- the **voting power** needs not to be proportional to the **voting weight**

# Voting power vs. voting weight

*the **voting power** needs not to be proportional to the **voting weight** !*

- A simple example: shareholders' assembly takes decisions by a simple majority vote
  - shareholder **X** - 51% of stocks of a company (voting weight = 51%)
  - shareholder **Y** - 49% of stocks of a company (voting weight = 49%)
  - shareholder **X** - 100% of the **voting power**
  - shareholder **Y** - 0% of the **voting power**

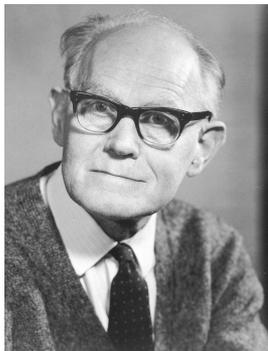


# How to measure voting power?

- **power index** - probability that the vote of a country will be decisive in a hypothetical ballot measures the potential voting power

natural assumption:

*all potential coalitions are equally likely*



**Penrose-Banzhaf index**



# Indirect voting in the Council

- A representative of each country has to vote **yes** or **no** and cannot split his vote
- *example:* if **30** millions of Italians support a decision, and **29 M** are against, an Italian minister says **yes** (on behalf of **59** millions).
- Thus **30 M** Italians can overrule **39 M** Poles (+**29** millions of opposing Italians...)
- **One person-one vote** system would be perfect ... if all citizens of each country had the same opinion.

# Is voting power important?

- potential (*a priori*) **voting power** vs. actual **voting power**
- *value of stocks of a company* -
  - *How many stocks give an investor full control over the company?*  
(the answer depends on the distribution of the shares...)
  - *How much should he pay for them?*

# How to compute the Banzhaf index ?

( <i>Banzhaf, 1965</i> ): number of players	$n$
# of coalitions	$2^n$
# of winning coalitions	$w$
# of coalitions with $i$ -th player	$2^{n-1}$
# of winning coalitions with $i$ -th player $X_i$	$w_i$
# of coalitions, for which the vote of $X_i$ is critical	$c_i := w_i - (w - w_i) = 2 \cdot w_i - w$

**Banzhaf index** =  $c_i / 2^{n-1}$

probability that vote of  $X_i$  will be decisive

## Penrose-Banzhaf index (normalised)

$\beta_i = c_i / \sum_i c_i$  (*Penrose, 1946*):  $p_i = (1 + \beta_i) / 2$

probability, that player  $X_i$  is going to win

---

# Council of Ministers of European Economic Community 1958-1972

# of countries:  $n = 6$   
 sum of all votes (weights):  $S = 17$   
 quota:  $q = 12$   
 # of coalitions  $T = 2^6 = 64$   
 # of coalitions with state X  $32$   
 # of winning coalitions:  $w = 14$

State	votes	Winning coal. with X	Winning coal. Without	Difference	Banzhaf index	Banzhaf Normalis. Index
		$w_i$	$W - w_i$	$c_i$	$c_i/2^{n-1}$	$\beta_i$
Germany	4	12	2	10	5/16	5/21 ~ 0.24
France	4	12	2	10	5/16	5/21 ~ 0.24
Italy	4	12	2	10	5/16	5/21 ~ 0.24
Holland	2	10	4	6	3/16	3/21 ~ 0.14
Belgium	2	10	4	6	3/16	3/21 ~ 0.14
Luxemb.	1	7	7	0	0	0

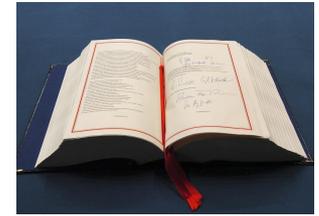
# Treaty of Nice



**345** votes are distributed among **27** member states on a *degressively proportional* basis, e.g.:

- **DE, FR, IT, UK** – **29** votes (weight)
- **ES, PL** – **27** votes (weight), *etc.*
- a) the sum of the weights of the Member States voting in favour is at least **255 (~73.9% of 345)**
- b) a majority of Member States (i.e. at least **14** out of **27**) vote in favour
- c) the Member States forming the qualified majority represent at least **62%** of the overall population of the European Union
- **‘triple majority’**

# Constitutional Treaty

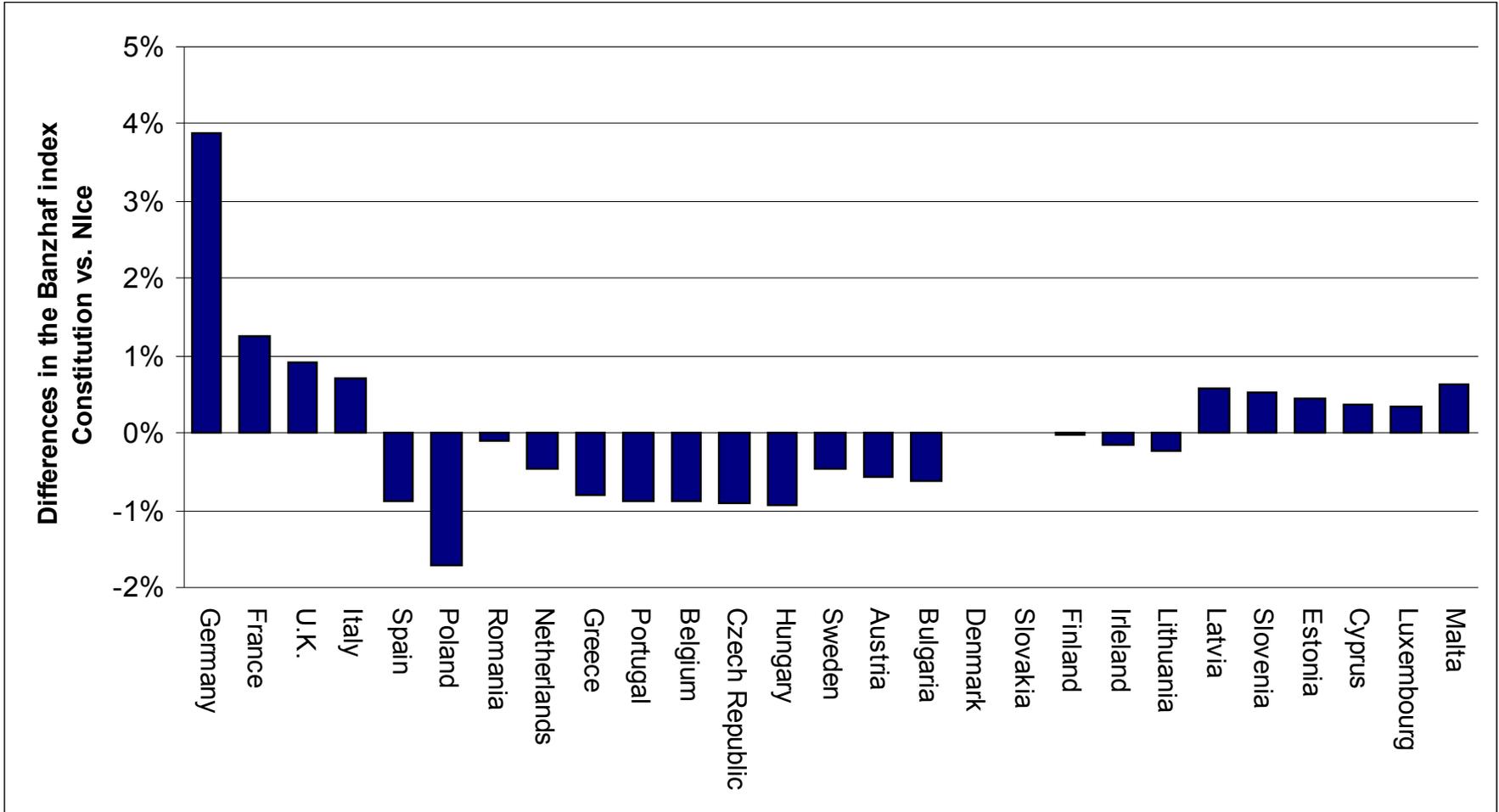


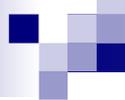
a) -----

- b) at least **55%** of Member States (i.e. at least **15** out of **27**) vote in favour
- c) the Member States forming the qualified majority represent at least **65%** of the overall population of the European Union
- c') a blocking minority must include at least **four** Council members
- 'double majority'



# 'Nice' vs. 'Constitution'





(...) if two votings were required for every decision, one on a per capita basis and the other upon the basis of a single vote for each country, the system would be inaccurate in that it would tend to favour large countries.

**[L. Penrose, 1952]**

# How the linear voting weights enhance the power of the largest states?

*A model example:* 160 M people living in one large state and 8 small. Assume that in both groups **51% of the population** votes **yes** in a certain case

- **group A):** One state with 80 M people
- **51%** say **yes**
- so does its minister in the Council
- result: **80M for**

- **group B):** 8 states with 10 millions each
- **51%** of people in this group say **yes**, **but majority in each state varies**
- 8 ministers in the Council may vote as 4:4 (or 5:3...)
- result: **40M for** (*less likely 50,60,70 or 80M*)

# Penrose square root law:

Voting power of a citizen in a country with population  $N$  is proportional to  $N^{-1/2}$

**Bernoulli scheme for  $k=N/2$  and  $p=q=1/2$   
+ Stirling expansion gives probability**

$$P_k = p^k q^{1-k} \binom{N}{k}$$

$$\begin{aligned} P_k &= \left(\frac{1}{2}\right)^{N/2} \left(\frac{1}{2}\right)^{N/2} \frac{N!}{(N/2)!(N/2)!} \approx \\ &\approx \frac{1}{2^N} \frac{(N/e)^N \sqrt{2\pi N}}{\left[(N/2e)^{N/2} \sqrt{2\pi N/2}\right]^2} = \sqrt{\frac{2}{\pi N}} \sim \frac{1}{\sqrt{N}} \end{aligned}$$

# ***Square root weights - Penrose law***

- this degressive system is distinguished by the ***Penrose square root law*** (1952)

$$1/\sqrt{N}$$

Voting power of a single citizen  
of a state with population **N**

$$\sqrt{N}$$

Voting power of its representative  
in EU Council

---

implies that

***each citizen of each country has  
the same potential voting power !***

# Square root weights - example

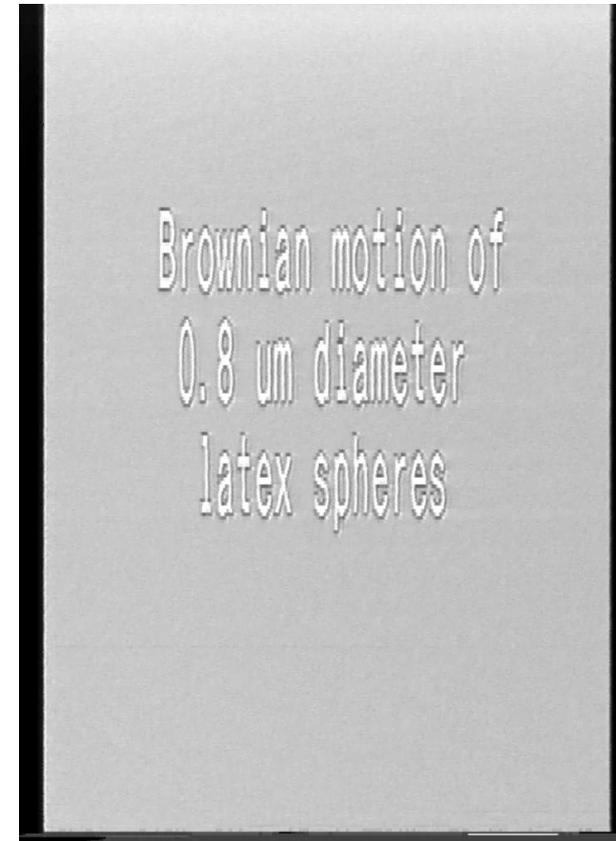
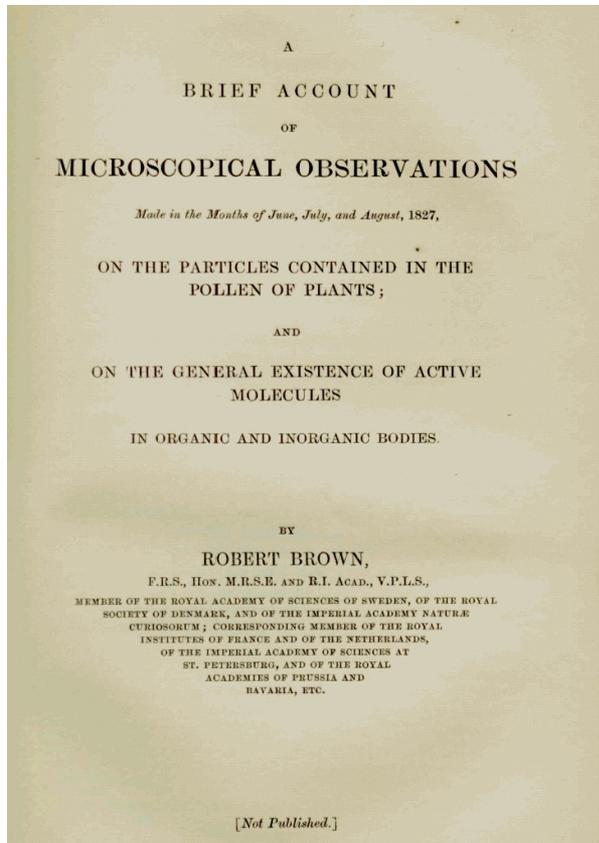


- the **'square root'** weights attributed to Member States are proportional to the sides of the squares representing their populations

# Brownian motion (1827)

## ■ *Clarkia pulchella*

This plant was *Clarkia pulchella*, of which the grains of pollen, taken from antheræ full grown, but before bursting, were filled with particles or granules of unusually large size, varying from nearly  $\frac{1}{4000}$ th to about  $\frac{1}{5000}$ th of an inch in length, and of a figure between cylindrical and oblong, perhaps slightly flattened, and having rounded and equal extremities. While examining the form of these particles immersed in water, I observed many of them very evidently in motion; their motion consisting not only of a change of



# Albert Einstein

(1879-1955)

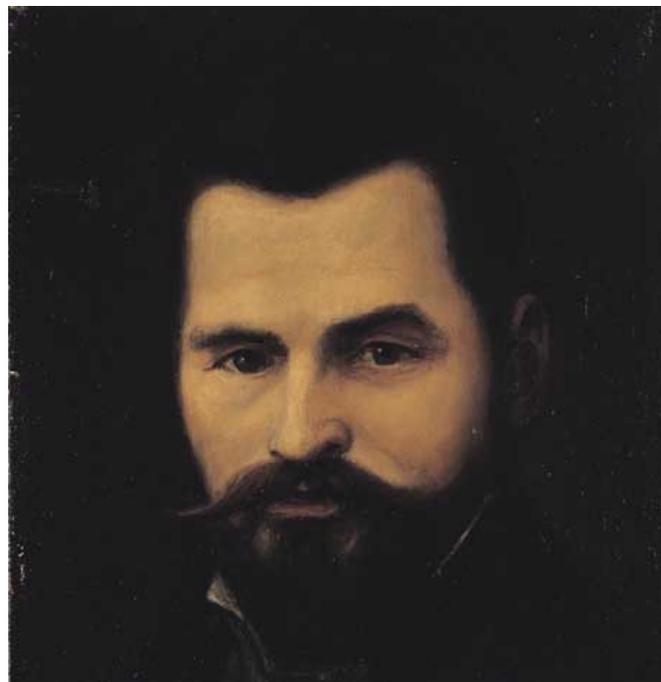


5. *Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen;*  
*von A. Einstein.*

In dieser Arbeit soll gezeigt werden, daß nach der molekularkinetischen Theorie der Wärme in Flüssigkeiten suspendierte Körper von mikroskopisch sichtbarer Größe infolge der Molekularbewegung der Wärme Bewegungen von solcher Größe ausführen müssen, daß diese Bewegungen leicht mit dem Mikroskop nachgewiesen werden können. Es ist möglich, daß die hier zu behandelnden Bewegungen mit der sogenannten „Brownschen Molekularbewegung“ identisch sind; die mir erreichbaren Angaben über letztere sind jedoch so ungenau, daß ich mir hierüber kein Urteil bilden konnte.

# Marian Smoluchowski

(1872-1917)

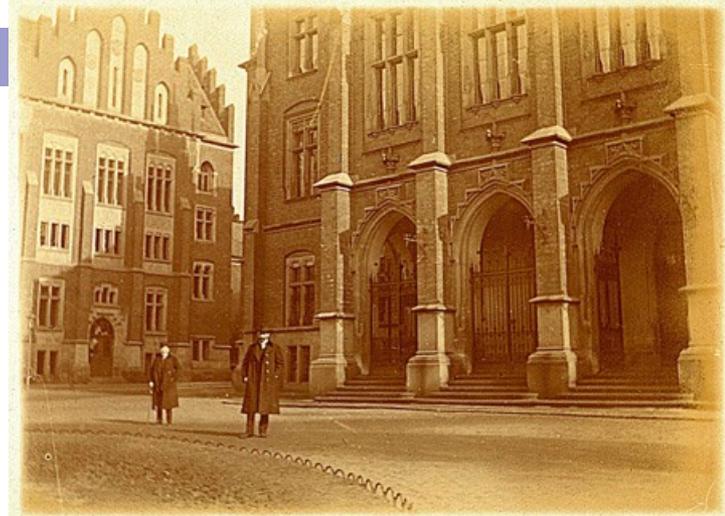


XXIX. ZARYS KINETYCZNEJ TEORJI RUCHÓW BROWNA  
I ROZTWORÓW MĘTNYCH.

(Rozprawy Wydziału matematyczno-przyrodniczego Akademii Umiejętności w Krakowie. T. XLVI. Serja A. 1906; str. 257—281).

§ 1. Ruch, polegający na dygotaniu i trzęsieniu się, który odbywają drobne, w silnem powiększeniu jeszcze widzialne cząstki, znajdujące się w stanie zawieszenia w cieczach, były często badane od r. 1827, w którym zwrócił na nie uwagę botanik Robert Brown, aż do dzisiaj; a jednak zjawisko to nie zostało jeszcze dostatecznie objaśnione. Żadna z pomiędzy różnych proponowanych teoryj nie przyjęła się powszechnie. Niepewność ta pochodzi częściowo z nie-

# Marian Smoluchowski (1906)



## XXIX. ZARYS KINETYCZNEJ TEORJI RUCHÓW BROWNA I ROZTWORÓW MĘTNYCH.

(Rozprawy Wydziału matematyczno-przyrodniczego Akademii Umiejętności w Krakowie. T. XLVI. Serja A. 1906; str. 257—281).

§ 1. Ruch, polegający na dygotaniu i trzęsieniu się, który odbywają drobne, w silnem powiększeniu jeszcze widzialne cząstki, znajdujące się w stanie zawieszenia w cieczach, były często badane od r. 1827, w którym zwrócił na nie uwagę botanik Robert Brown, aż do dziś dnia; a jednak zjawisko to nie zostało jeszcze dostatecznie objaśnione. Żadna z pomiędzy różnych proponowanych teoryj nie przyjęła się powszechnie. Niepewność ta pochodzi częściowo z nie-

# Smoluchowski's explanation of the Brownian motion...

Stąd znajdujemy wartość przeciętnego zboczenia w jedną lub drugą stronę:

$$v = 2 \sum_{m=\frac{n}{2}}^n \frac{2^m - n}{2^n} \binom{n}{m},$$

jeżeli dla uproszczenia liczbę  $n$  przyjmiemy za parzystą. Wyrażenie to można przekształcić przez zastosowanie twierdzenia dwumianowego w formę dogodniejszą:

(1)

$$v = \frac{n}{2^n} \binom{n}{\frac{n}{2}},$$

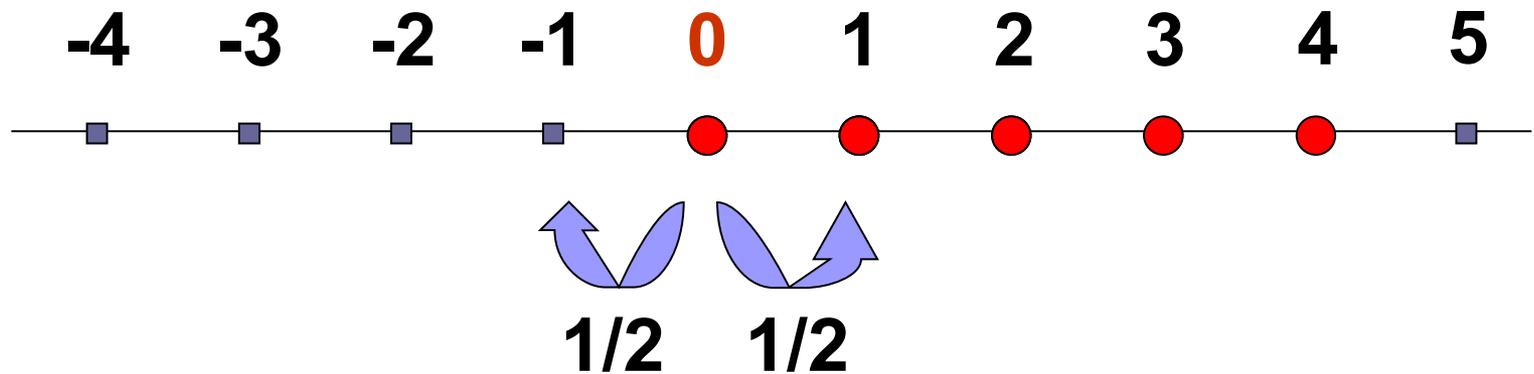
która dla dużych liczb  $n$  przechodzi w

(2)

$$v = \sqrt{\frac{2n}{\pi}}.$$



# Random walk on the line

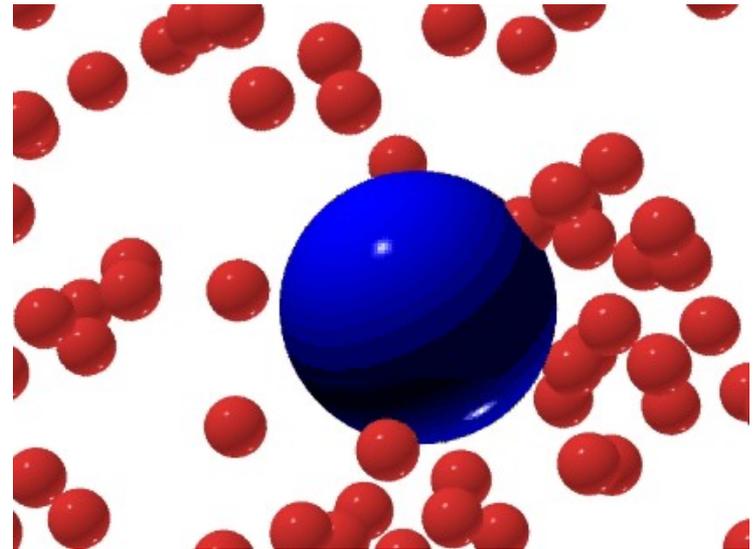


- If particle moves from point different than 0 its **mean** distance to 0 does not change
- if particle moves from the point 0 its **mean** distance to 0 grows by 1

# Random walk : a diffusion law

- Probability that a particle returns to its initial point after  $k$  steps scales as  $k^{-1/2}$
- Thus the mean distance  $\langle Dx \rangle$  from 0 grows with the time  $n$  as

$$\langle Dx(n) \rangle \sim \sum_{k=1}^n k^{-1/2} \sim n^{1/2}$$



The **Penrose square root law** is closely related to **diffusion law** !



# ***Qualified majority threshold***

- The choice of an appropriate decision-taking **quota (threshold)** affects both the distribution of voting power in the Council (and thus also the **representativeness** of the system) and the voting system's **effectiveness** and **transparency**.
- Different authors have proposed different quotas for a square root voting systems, usually varying from 60% to 74%.
- The **optimal quota** enables the computed voting power of each country to be practically equal to the attributed voting weight.

# Optimal threshold

physics.org

## Physics for

Distributing votes fairly between Europe to a European Constitution. Karol Zyczkowski and Jacek Jurek describe how physicists are helping politicians.

In October 2004 representatives of the European Union's 25 member states signed a treaty to establish a European Constitution. Since then the treaty has endured a bumpy ride, with citizens in France and the Netherlands voting against ratification last year. One of the sticking points has been the way votes are distributed between member states in the Council of Ministers, the main decision-making body of the European Union (EU). Physicists and mathematicians are applying their statistical know-how to propose a solution to this problem.

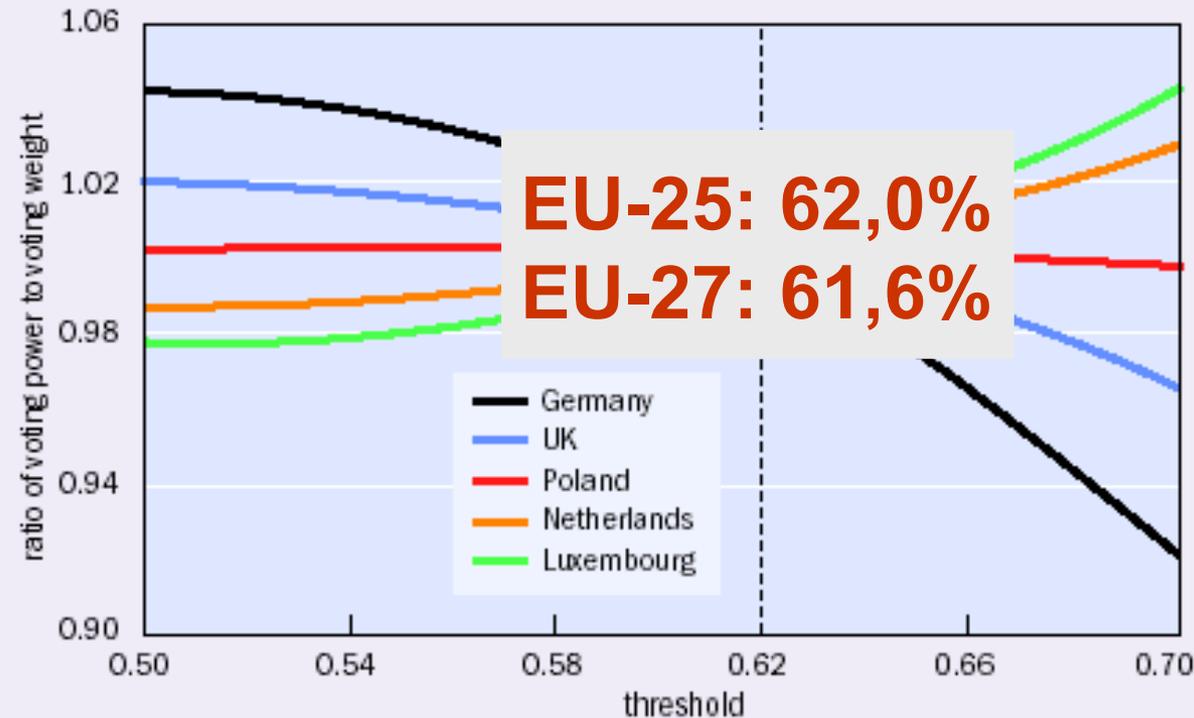
The Council of Ministers consists of politicians from each member state who vote on behalf of their respective countries. To ensure that the influence of each country's vote reflects the size of its population without overwhelming the voice of smaller countries, the current voting rules – set out in 2000 in the Treaty of Nice – are based on a complicated system of “qualified majority vote”. Each country is assigned a voting weight loosely based on its population, and approximately 72% of the total weight must be behind a proposal to be passed. In addition, at least 13 of the 25 states must support the proposal and the population of those states must exceed 62% of the total population of the EU.

During the drafting of the new constitution, ministers decided to simplify this system by dropping the voting weights. Instead, decisions would rely purely on the number of states voting for a proposal and the proportion of the EU population comprised by those states. Under the constitution, a qualified majority would require at least 55% of member states and 50% of the total population to agree.

Although the proposal for the new constitution is away with the voting weights, which have no objective basis and tend to assign too much power to certain countries, it has flaws of its own. Large states, Germany in particular, would gain from the direct link to population, while small countries would derive disproportionate power from the increase in the number of states needed to support a proposal. The combined weight of small states would sap influence away from medium-sized countries like Spain and Poland.

Is it possible to objectively design a voting system that overcomes these deficiencies, in which each citizen of each member state would have the same power to influence the decisions made on their behalf? Can it be done in a way that is transparent, easy to implement, efficient to use, and will readily accommodate any future changes to the EU (such as the inclusion of Romania and Bulgaria in 2007)? The answer, according to research carried out by the present authors, is “yes”.

## 1 The critical point



By plotting the ratio of the voting power of each country to its voting weight against the threshold chosen for majority, a “critical point” emerges. At a threshold of 62% each country achieves a ratio of one, meaning its voting power is equal to its voting weight. Since the weights are chosen to be proportional to the square root of population, the voting power of each citizen is equal. With a lower threshold, larger countries have disproportionate power, while for a higher threshold, smaller countries have more influence.

physics.org

It is possible to design a voting system in which each citizen of each member state would have the same power to influence decisions made on their behalf?

The sum of weights of the countries voting in favour of a motion exceeds 62% of the total weight (arXiv.org/cond-mat/0405396).

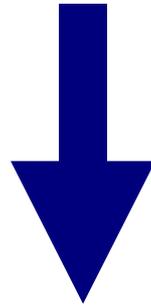
This system has a pleasing simplicity, but how was the magic number of 62% obtained? We first calculated the voting power for each state depends on the threshold chosen for a majority, and we observed the “critical point”. As the threshold approaches 62%, the voting power of each country, irrespective of its size, converges on the ideal square-root value (see figure 1). The Penrose law is thus fulfilled, and every citizen’s voting power is equalized, with a simpler system than that of the official versions under discussion. Furthermore, any further enlargement of the EU would involve only one recalculation of the threshold for the qualified majority rule – which would become 61.4% for a 27-state EU. Indeed, the critical-point behaviour emerges for almost any number of member states or coalition distribution.

Our proposed voting system has stimulated considerable interest among experts in voting theory, and has been dubbed the “Jagiellonian compromise” by the media. Prior to the EU summit in Brussels in June 2004, an open letter in support of square-root voting was presented to the Council of Ministers that was endorsed by more than 40 scientists in 10 European countries sent to EU institutions and the governments of member states.

The reaction of politicians has been varied, but inevitably depends more on how the Jagiellonian compromise affects an individual country’s share of the vote (see figure 2) than on universal criteria such as simplicity and objectivity. When a similar system was put forward by Swedish diplomats in 2000, Sweden’s prime minister Göran Persson said, “Our formula has the intangibility of being easy to understand by public opinion and practical to use in an enlarged Europe... it is transparent, logical and loyal. Maybe that is why it does please everybody.” The former Irish prime minister Bertie Ahern also made numerous positive references to voting systems based on Penrose’s law, and the Jagiellonian compromise has been endorsed by a number of leading politicians in Poland and was scrutinized by UK government researchers in 2004.

As now seems increasingly likely, the European situation fails to come into force, the question of voting in the EU Council of Ministers will be revisited. The Jagiellonian compromise offers future negotiators a simple but objective system based on rational principles that grants equal voting power to all citizens of the EU.

# Jagiellonian Compromise

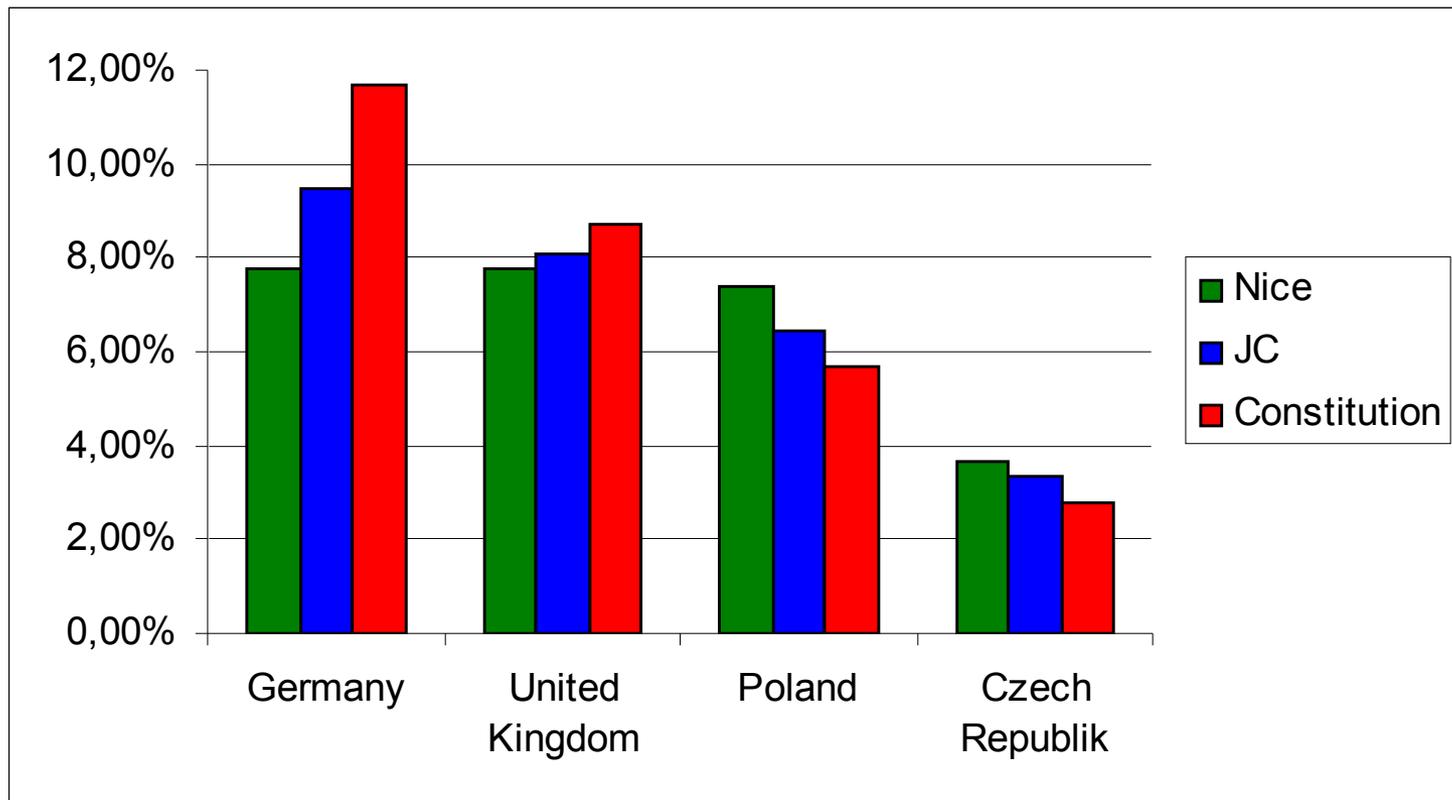


**square root weights**

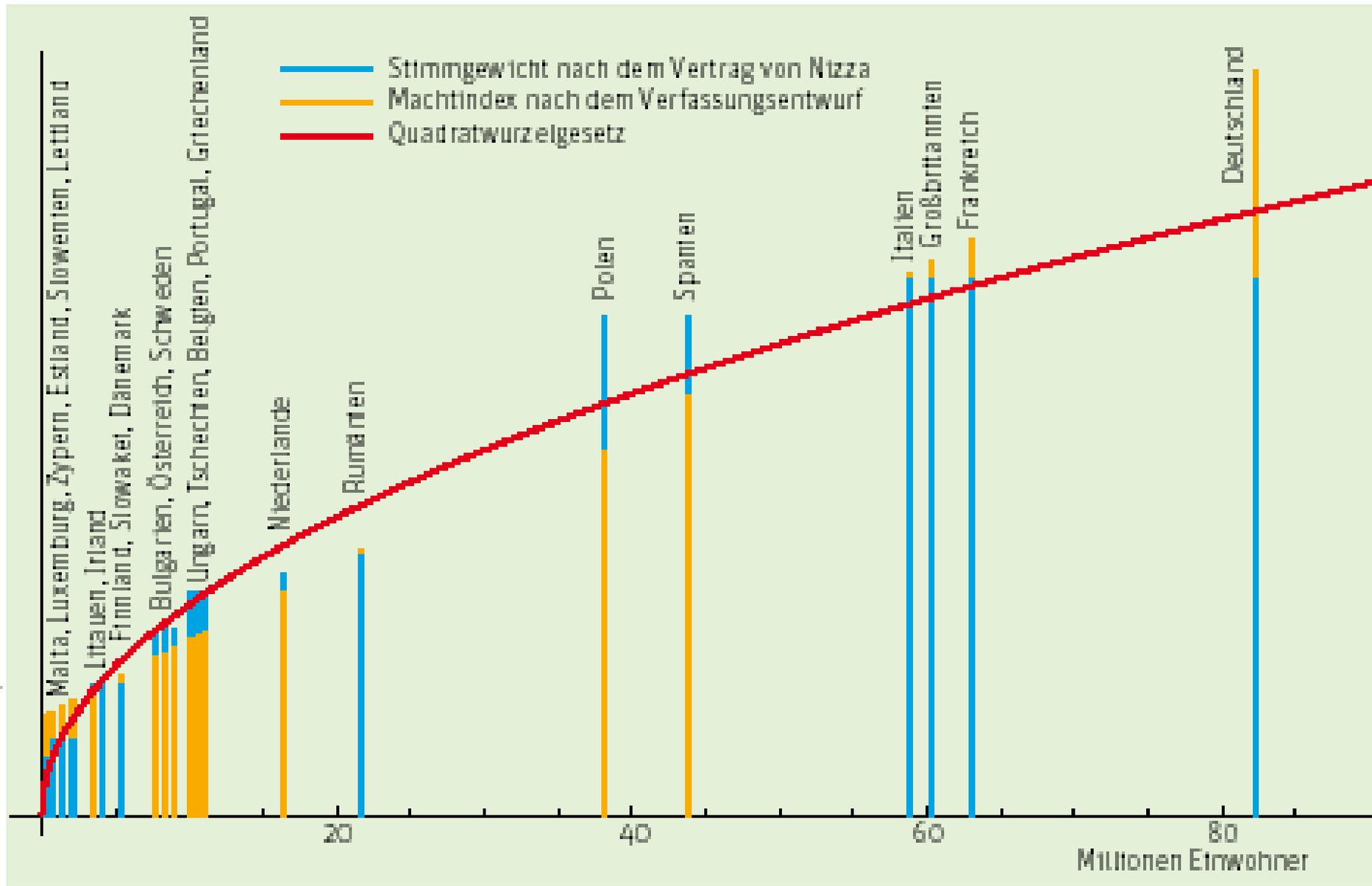
**+**

**optimal quota**

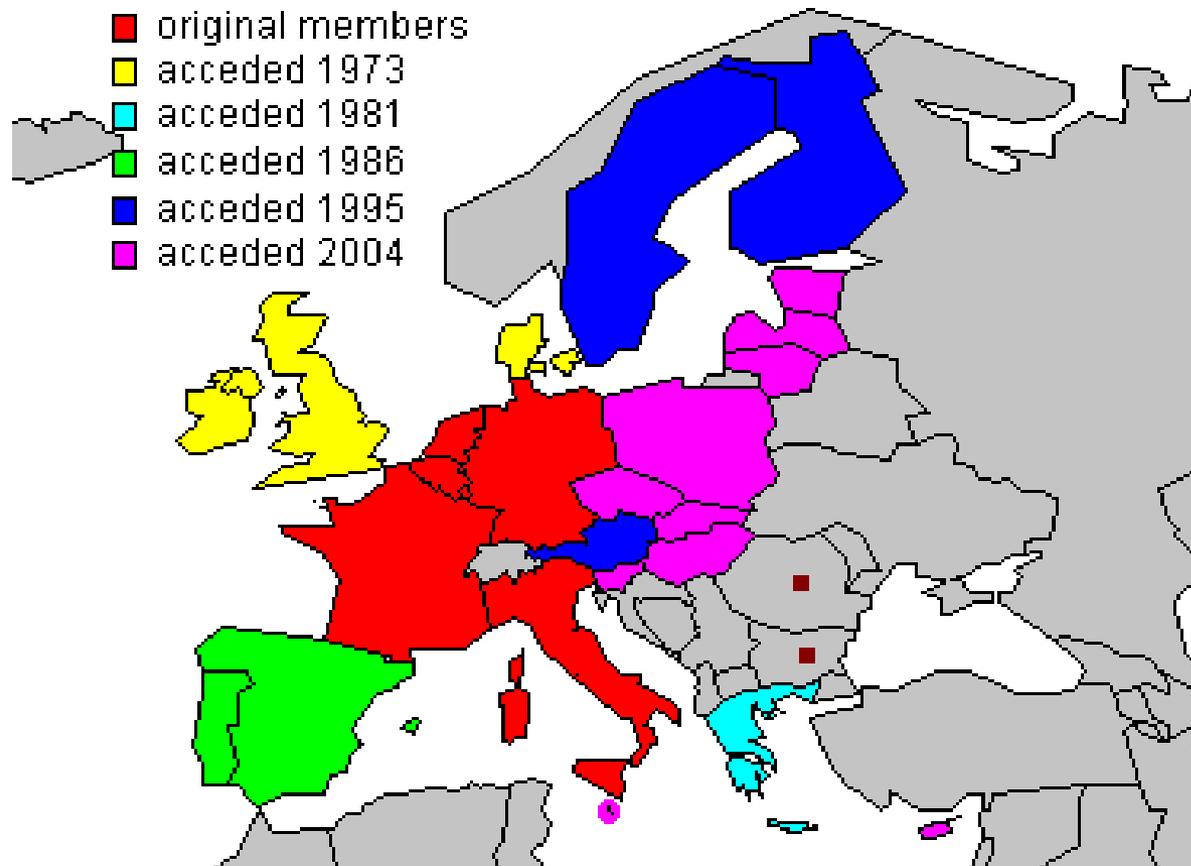
# 'Nice' / Jagiellonian Compromise / Constitution



<b>Member State</b>	<b>Population (in millions)</b>	<b>Voting power Constitution</b>	<b>Voting weight (JC)</b>	<b>Voting power (JC)</b>
<b>Germany</b>	<b>82.44</b>	<b>11.66</b>	<b>9.47</b>	<b>9.45</b>
<b>France</b>	<b>62.89</b>	<b>9.02</b>	<b>8.27</b>	<b>8.27</b>
<b>United Kingdom</b>	<b>60.39</b>	<b>8.69</b>	<b>8.10</b>	<b>8.10</b>
<b>Italy</b>	<b>58.75</b>	<b>8.49</b>	<b>7.99</b>	<b>7.99</b>
<b>Spain</b>	<b>43.76</b>	<b>6.55</b>	<b>6.90</b>	<b>6.91</b>
<b>Poland</b>	<b>38.16</b>	<b>5.71</b>	<b>6.44</b>	<b>6.45</b>
<b>Romania</b>	<b>21.61</b>	<b>4.15</b>	<b>4.85</b>	<b>4.85</b>
<b>Netherlands</b>	<b>16.33</b>	<b>3.50</b>	<b>4.21</b>	<b>4.21</b>
<b>Greece</b>	<b>11.13</b>	<b>2.88</b>	<b>3.48</b>	<b>3.48</b>
<b>Portugal</b>	<b>10.57</b>	<b>2.80</b>	<b>3.39</b>	<b>3.39</b>
<b>Belgium</b>	<b>10.51</b>	<b>2.80</b>	<b>3.38</b>	<b>3.38</b>
<b>Czech Rep.</b>	<b>10.25</b>	<b>2.77</b>	<b>3.34</b>	<b>3.34</b>



From Christoph Pöppe, Spektrum der Wissenschaft 2007



<b>EU-<math>M</math></b>	<b>6</b>	<b>9</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>25</b>	<b>27</b>
<b><math>R_{opt}</math></b>	<b>73.0%</b>	<b>67.4%</b>	<b>65.5%</b>	<b>64.5%</b>	<b>64.4%</b>	<b>62.0%</b>	<b>61.4%</b>

Tab. 2 shows the value of the critical quota  $R_{opt}$  as a function of the number  $M$  of members of the EU.

# Optimal quota – the normal approximation

$w_k$  ( $k = 1, \dots, M$ ) - voting weights,  $\sum_{i=1}^k w_i = 1$ ,  $[q; w_1, \dots, w_M]$

$$n(z) := \frac{\text{card}\{I \subset \{1, \dots, M\} : \sum_{i \in I} w_i = z\}}{2^M}, \quad N(q) := \sum_{z \leq q} n(z) \approx \Phi\left(\frac{q - m}{\sigma}\right)$$

$m := \frac{1}{2} \sum_{i=1}^k w_i = \frac{1}{2}$ ,  $\sigma^2 := \frac{1}{4} \sum_{i=1}^k w_i^2$ ,  $\Phi$  - standard normal cumulative distribution function

$$\psi_k(q) \approx \Phi\left(\frac{q - m + w_k / 2}{\sqrt{\sigma^2 - w_k^2 / 4}}\right) - \Phi\left(\frac{q - m - w_k / 2}{\sqrt{\sigma^2 - w_k^2 / 4}}\right) \quad (k = 1, \dots, M)$$

$$q_n := m + \sigma = \frac{1}{2} \left(1 + \sqrt{\sum_{i=1}^k w_i^2}\right)$$

$$\psi_k(q_n) \approx \sqrt{\frac{2}{\pi e}} v_k + o(v_k^4), \quad v_k := \frac{w_k}{\sqrt{\sum_{i=1}^k w_i^2}} \ll 0 \quad \leftarrow \text{ASSUMPTION}$$

$$\beta_k(q_n) \approx w_k$$

$$q = m = 1/2$$

$$\psi_k(1/2) \approx \sqrt{\frac{2}{\pi}} v_k + o(v_k^2)$$

# Optimal quota – solution of the problem

$w_k$  ( $k = 1, \dots, M$ ) - voting weights,  $\sum_{i=1}^k w_i = 1$ ,  $[q; w_1, \dots, w_M]$

$$q^* \cong q_n(w_1, \dots, w_M) = \frac{1}{2} \left( 1 + \sqrt{\sum_{i=1}^k w_i^2} \right)$$

$$q_s(M) = \frac{1}{2} \left( 1 + \frac{1}{\sqrt{M}} \right) \leq \frac{1}{2} \left( 1 + \frac{1}{\sqrt{M_{\text{eff}}}} \right) = q_n(w_1, \dots, w_M),$$

where  $M_{\text{eff}} := \frac{1}{\sqrt{\sum_{i=1}^k w_i^2}}$  - effective number of players (Laakso, Taagepera (1979))

In particular for Penrose voting system ( $w_k \sim \sqrt{N_k}$ ) we get

$$q_n(N_1, \dots, N_M) = \frac{1}{2} \left( 1 + \frac{\sqrt{\sum_{i=1}^k N_i}}{\sum_{i=1}^k \sqrt{N_i}} \right)$$

The efficiency of the system does not decrease, when the number of players  $M$  increases

$$A(q_s) \geq A(q_n) \approx 1 - \Phi(1) \approx 15.9\%$$

- For the **Council of Ministers of EU-27** the optimal quota equals **61.6%**.
- For **EU-M** the optimal quota  $q$  can be approximated by a simple mathematical formula:

$$q = \frac{1}{2} \left( 1 + \frac{\sqrt{N_1 + \dots + N_M}}{\sqrt{N_1} + \dots + \sqrt{N_M}} \right)$$

where  $N_i$  stands for the population of the  $i$ -th country.



# Jagiellonian Compromise

- it is extremely **simple** since it is based on a single criterion, and thus it could be called a '**single majority**' system;
- it is **objective** (no arbitrary weights or thresholds), hence cannot *a priori* handicap any member of the European Union;
- it is **representative**: every citizen of each Member State has the same potential voting power;
- it is **transparent**: the voting power of each Member State is (approximately) proportional to its voting weight;
- it is **easily extendible**: if the number of Member States changes, all that needs to be done is to set the voting weights according to the square root law and adjust the quota accordingly;
- it is **moderately efficient**: as the number of Member States grows, the efficiency of the system does not decrease;
- it is also **moderately conservative**, that is, it does not lead to a dramatic transfer of voting power relative to the existing arrangements.

# ***Square root weights***

## ***- support from academics***

- advocated or analysed by Laruelle, Widgrén (1998), Baldwin, Berglöf, Giavazzi, Widgrén (2000), Felsenthal, Machover (2000-2004), Hosli (2000), Sutter (2000), Tiilikainen, Widgrén (2000), Kandogan (2001), Leech (2002), Moberg (2002), Hosli, Machover (2002), Leech, Machover (2003), Widgrén (2003), Baldwin, Widgrén (2004), Bilbao (2004), Bobay (2004), Kirsch (2004), Lindner (2004), Lindner, Machover (2004), Plechanovová (2004, 2006), Sozański (2004), Ade (2005), Koornwinder (2005), Pajala (2005), Maaser, Napel (2006), Taagepera, Hosli (2006)
- prior to the European Union summit in Brussels in June 2004, an **open letter** in support of square-root voting weights in the Council of Ministers endorsed by more than 40 scientists from 10 European countries

## Der Jagiellonische Kompromiss

Polen und ein neues Abstimmungs-system für den EU-Ministerrat

Von Friedrich Pakelshorn\*

Polen wirkt in der Europäischen Union darauf hin, das Abstimmungs-system im Ministerrat gegenüber dem in Volks-abstimmungen abgelehnten Verfassungs-vertrag und dem Vertrag von Nizza zu verbessern. Der Autor liefert einen sach-lichen Beitrag auch für ein eminent politisches Problem, das endemische Demokratiedefizit der EU.

Die Namensgebung ist kein Kotau der polnischen Presse vor ihrer Regierung, sondern honoriert den Beitrag von Wissenschaftlern der Jagielloni-schen Universität in Krakau. Das von der polni-schen Regierung vorgeschlagene System ist schon ein halbes Jahrhundert alt, wurde aber von der polnischen Presse erst jüngst mit dem geschichts-trächtigen Namen Jagiellonischer Kompromiss versehen. Der Jagiellonische Kompromiss garan-tiert allen Bürgern und Bürgerinnen der Union, dass ihnen über ihre Mandatsträger der gleiche Einfluss auf Ministerratsbeschlüsse zukommt. Er hilft, das Demokratiedefizit in der Europäischen Union zu vermindern, und stärkt die Legitima-tionskraft der Entscheidungen des Ministerrats.

### Transparenz und Einfachheit

Zudem ist der Jagiellonische Kompromiss das transparenteste unter allen gewichteten Abstim-mungssystemen, die bisher verwendet oder vorge-schlagen wurden. Die Stimmzahlen der Mit-gliedstaaten werden nicht in nichtelangen Ver-handlungsmarathons ausgepokert, sondern be-

Stimmengewichte der Mitgliedstaaten (EU-27)		
	Bevölkerung	Stimmen (Machtenteil in %)
Deutschland	82 311 600	9 073 944
Frankreich	65 336 700	7 058 828
Großbritannien	60 307 100	7 351 811
Italien	59 538 800	7 677 739
Spanien	44 494 300	5 670 694
Polen	38 104 800	5 373 642
Rumänien	21 590 600	4 644 483
Niederlande	16 346 700	4 043 421
Griechenland	11 769 700	3 042 348
Portugal	10 609 000	2 257 339
Belgien	10 590 500	3 251 338
Tschechische Republik	10 299 500	3 238 334
Ungarn	10 057 900	3 271 330
Schweden	9 119 800	3 020 314
Österreich	8 295 800	2 880 300
Bulgarien	7 665 500	2 769 288
Dänemark	5 445 700	2 334 243
Slowakei	5 391 600	2 322 242
Finnland	5 277 100	2 297 239
Irland	4 326 700	2 080 216
Litauen	3 385 700	1 640 191
Lettland	2 280 600	1 510 157
Slowenien	2 030 300	1 418 148
Estland	1 339 800	1 158 121
Japan	736 000	881 092
Luxemburg	464 400	681 071
Malta	407 300	639 067
<b>Total</b>	<b>494 674 000</b>	<b>96 017 10 000</b>
Quorum	50 333	61,57

Der Jagiellonische Kompromiss. Die Stimmengewichte sind die gerundeten Wurzeln der Bevölkerungszahlen (nach 1.1.2007, aus Eurostat 4/1/2007). Für die Gültigkeit eines Beschlusses braucht es diesen niedrigenen Quorum  $q = \lfloor \sqrt{494 674 000 + 96 017} / 2 \rfloor$  (Beldeans Quorum

sollte. Man mag die obige Argumentation kritisieren, aber ihre Grundlagen und Schlussfolgerungen haben sich bisher theoretisch wie praktisch hervorragend bewährt. Als erster Wissenschaftler studierte der englische Psychiater Lionel Penrose (1898–1972) Abstimmungs-systeme für die Uno-Generalsversammlung. Das Schrifttum spricht deshalb vom Quadratwurzel-Gesetz von Penrose und – statt Entscheidungsmacht – vom normalisierten Penrose-Index. Dieser Index wird auch nach dem Juristen John Banzhaf benannt, der zwanzig Jahre später das Thema neu aufgriff. Seither sind Abstimmungs-systeme mit all ihren Verästelungen eingehend untersucht worden.

Die Altmeister des Faches, der Philosoph Moshe Machover (London) und der Politikwissen-schafter Dan Pelsenthal (Haifa), haben zudem ergänzende Facetten beleuchtet, zum Beispiel die Sensitivität, mit welcher Erfolgsaussichten ein Mitgliedstaat eine Beschlussinitiative starten kann, oder die Resistenz, mit welchen Aussichten er die Initiative anderer abblocken kann. Alle ihre Studien führen zu dem System, das jetzt unter dem Namen Jagiellonischer Kompromiss firmiert.

Das Tipfchen auf dem i haben vor drei Jahren der Mathematiker Wojciech Słomczyński und der Physiker Karol Zyczkowski aus Krakau beige-steuert. Sie zeigten, dass die Wurzel der Gesamtbevölkerung und die Gesamtsumme der Bevölkerungswurzeln sich zum optimalen Quo-rum ausmitteln. Das spröde akademische Opti-mum wurde von polnischen Journalisten konziliant umschrieben als Jagiellonischer Kompromiss.

Nicht nur gut für Polen



**Reform Treaty**



*Treaties are like roses and young girls.  
They last while they last.*



*Charles de Gaulle*  
Time, 12th July, 1963

*Spektrum der Wissenschaft*  
August 2007

# Die Quadratwurzel, das Irrrationale und der Tod



Christoph Pöppe  
ist Redakteur bei  
Spektrum der  
Wissenschaft.

---



*Optimal quota for the union of  $M$  states:*

$$q_{opt} = (1/2 + M^{-1/2}/2).$$

