QUANTUM DOTS

III

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w opracowaniu internetowej wersji wykładów.

1 Magnetic Field Effects

An external magnetic field essentially modifies the properties of electrons confined in QD's.

The increasing magnetic field leads to the changes of symmetry of N-electron ground state, which – in turn – cause the rapid jumps of the N-electron energy as a function of magnetic field.

These changes are associated with the rapid changes of the spatial configuration of N-electron ground state and can be called **phase transitions** in the N-electron system.

The phase transitions are observed by either transport or capacitance spectroscopy.

1.1 Theory

Hamiltonian of the single electron in the magnetic field $\mathbf{B} = (0, 0, B)$ (in symmetric gauge)

$$h = -\frac{\hbar^2}{2m_e}\nabla^2 + U_{conf}(\mathbf{r}) + \frac{m_e\omega_c^2}{8}(x^2 + y^2) + \frac{\hbar\omega_c}{2}l_z + \frac{1}{2}g^\star\mu_B B\sigma_z\ , \qquad (1)$$

$\omega_c = eB/m_e$ = cyclotron frequency
l_z = operator of z component of the orbital momentum
$g^\star$ = effective Lande factor
μ_B = Bohr magneton
σ_z = z component of Pauli spin matrix

The last term in (1) $\Longrightarrow$ **spin Zeeman splitting**. Even at high magnetic field it provides a small contribution to the energy.

1.2 Fock-Darwin states

We add to Hamiltonian (1) the two-dimensional parabolic confinement energy

$$U_{conf} = \frac{m_e \omega_0^2}{2}(x^2 + y^2) \tag{2}$$

and obtain the Fock-Darwin Hamiltonian

$$h_{FD} = h + U_{conf} \ . \tag{3}$$

Hamiltonian (3) provides a simple but fairly realistic model for the electron confined in the QD in the presence of external magnetic field.

The Schrődinger equation with Hamiltonian (3) admits analytical solutions.

These are **Fock-Darwin states** associated with energy eigenvalues

$$E_{nm} = (2n + |m| + 1)\hbar\Omega + m\hbar\omega_c \ , \tag{4}$$

where

$$\Omega = \sqrt{\omega_0^2 + \omega_c^2/4}$$

$n = 0, 1, 2, \ldots$
$m = 0, \pm 1, \pm 2, \ldots$

1.3 Many-electron system in QD in magnetic field: experiment and theory

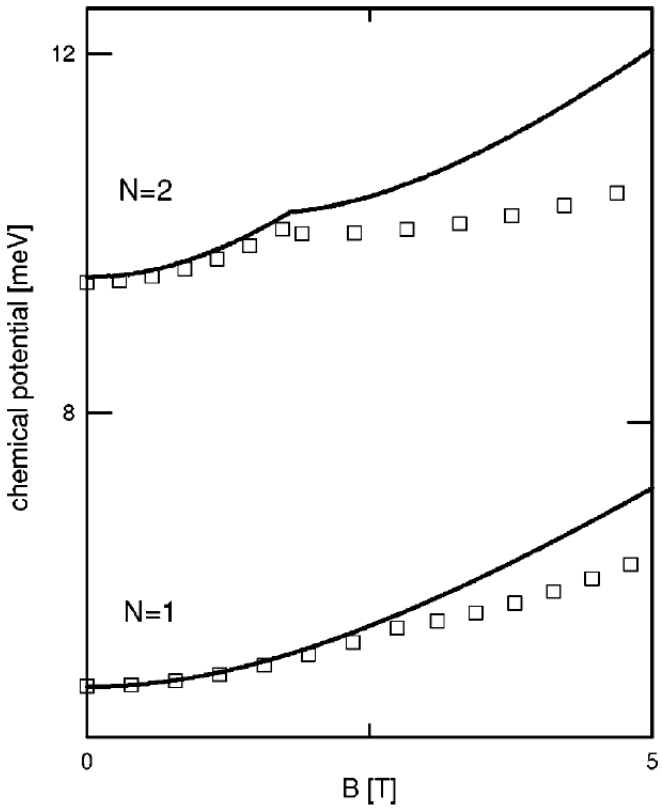

Figure 1: Chemical potential calculated for $N = 1$ and $N = 2$ electrons confined in two-terminal QD as a function of magnetic field B. Experimental results of R.C. Ashoori et al. are depicted by squares, solid curve shows the calculation results.

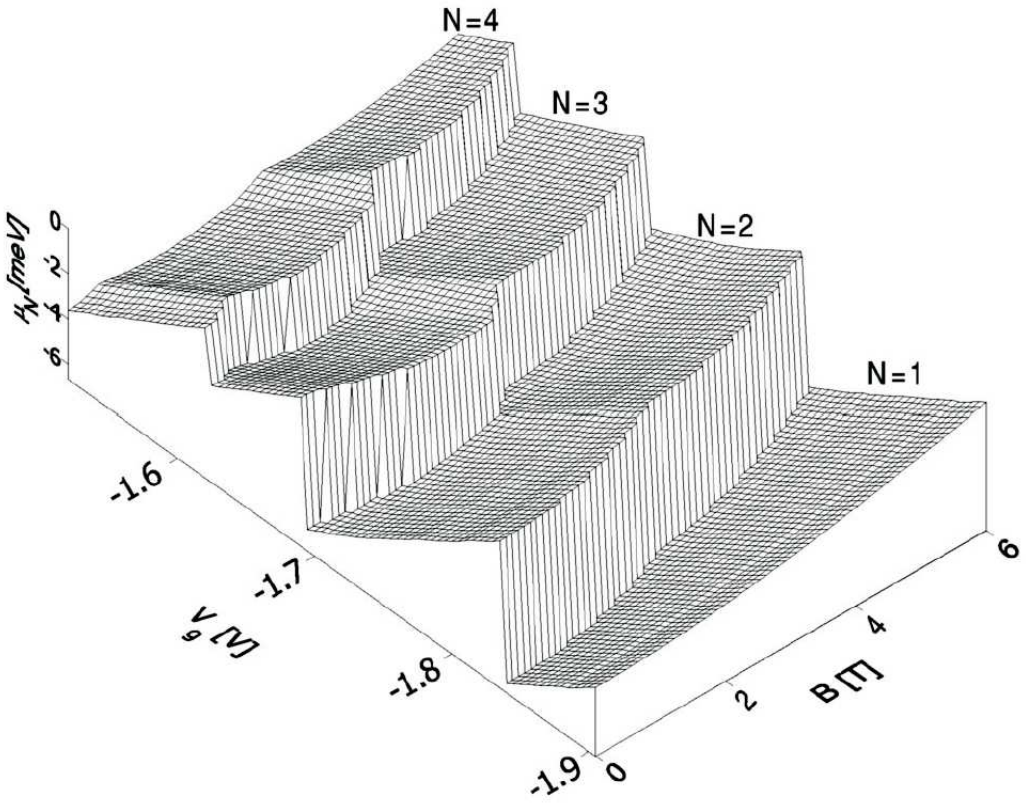

Figure 2: Chemical potential μ_N of N-electron system confined in three-terminal QD (S. Tarucha, NTT) as a function of magnetic field B and gate voltage V_g for $V_{sd} = 0$. Zero on energy scale corresponds to the Fermi energy.

Notation of orbitals

We denote the one-electron orbitals in the QD in magnetic field by the letters

s, p, d, f, g

that correspond to the magnetic quantum numbers

$m = 0, 1, 2, 3, 4.$

Indices $\pm$, for example, $p_{\pm}$, correspond to the signs of l_z eigenvalues.

1.4 Phase transitions in magnetic field

For each orbital the z component of the total spin (S_z) takes on the maximum value that is allowed for the considered orbital.

The increasing magnetic field induces the subsequent phase transitions after which S_z increases (electron system becomes spin polarized).

The last phase transition leads to the full polarization of electron spins, which means that $S_z = N\hbar/2$.

The N-electron phase created after this transition is called the **Maximum Density Droplet (MDD)**.

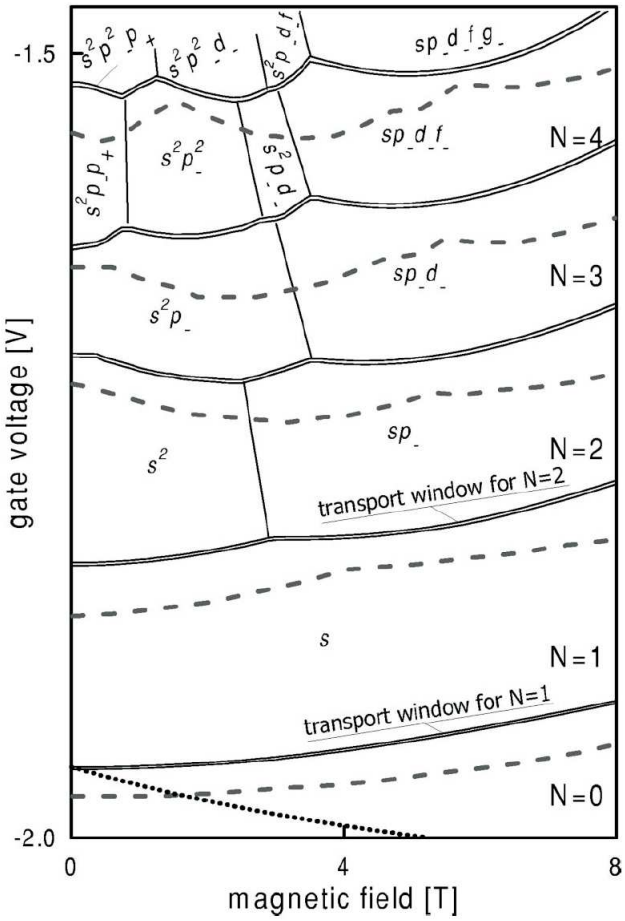

Figure 3: Transport windows as functions of gate voltage and magnetic field for $V_{sd} = 0.1$ mV in the vertical QD (S. Tarucha, NTT). Solid (dashed) curves display the calculated (measured) gate voltage for which the single electron tunnels through the QD. Number N of electrons is fixed in Coulomb blockade regions. In transport windows, the number of electrons oscillates between N and $N + 1$. Symbols denote the ground-state orbitals, thin lines separate regions of different phases.

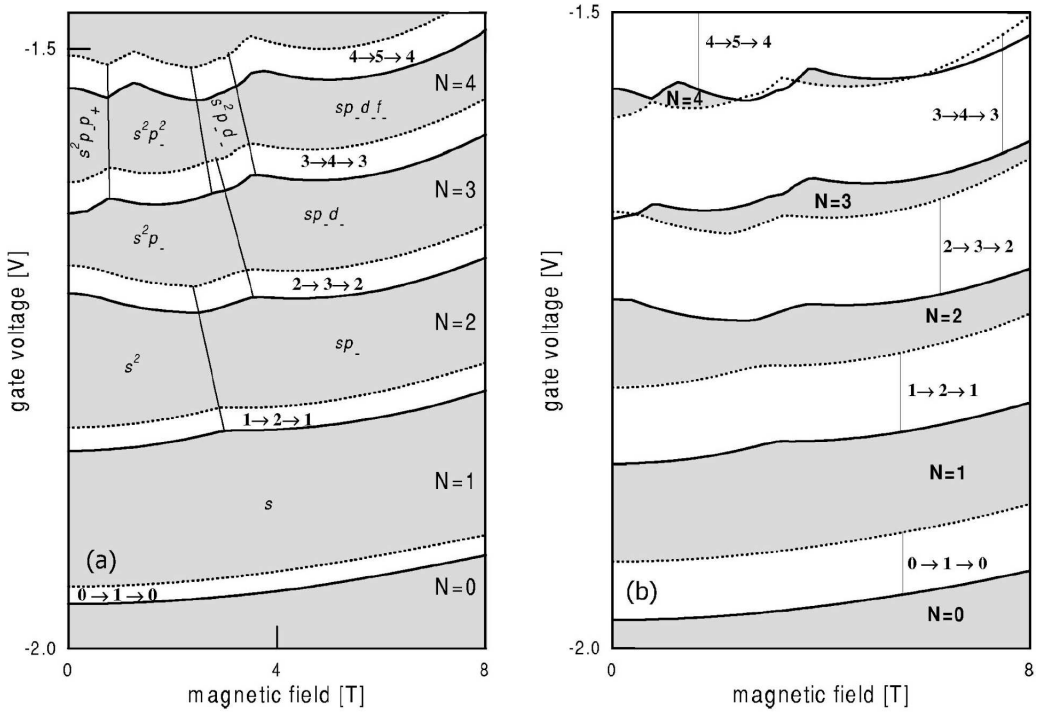

Figure 4: Transport windows (white areas) and Coulomb blockade regions (gray areas) as functions gate voltage and magnetic field for $V_{sd} = 1$ mV (a) and 3 mV (b) in the vertical QD (S. Tarucha, NTT). Number N of electrons is fixed in Coulomb blockade regions. In transport windows (double solid lines), the number of electrons oscillates as follows: $N \longrightarrow N+1 \longrightarrow N$. Symbols denote the ground-state orbitals, thin lines separate regions of different phases.

2 Wigner Localization

In low-density electron systems, spatially ordered structures can be created, called the **Wigner phases**.

Wigner phases:

(1) Wigner crystal:
formed in many electron system, e.g., on the surface of liquid helium

(2) Wigner molecule = electronic molecule:
formed in few electron system confined in QD in high magnetic field

2.1 Theoretical results for circular QD's

Notation used to Wigner molecules

Different space configurations (**phases, isomers**) of Wigner molecules are designated using the classical notation.

The classical notation determines the space configuration of the system of classical charges, which corresponds to either global or local potential energy minimum.

The notation used for different Wigner molecule isomers:

$$N_1 - N_2 - N_3$$

where N_1, N_2, and N_3 = numbers of electrons occupying the inner, middle, and outer shell, respectively,

whereby $N_1 + N_2 + N_3 = N$ = number of electrons forming the Wigner molecule.

In the case of two shells occupied, we are using the notation:

$$N_1 - N_2$$

omitting 0 for the unoccupied shell.

Maximum Density Droplet phase = MDD phase

In the MDD phase, the spins of all electrons are identical (the total spin polarization), the spatial distribution of electrons is homogeneous (in the occupied

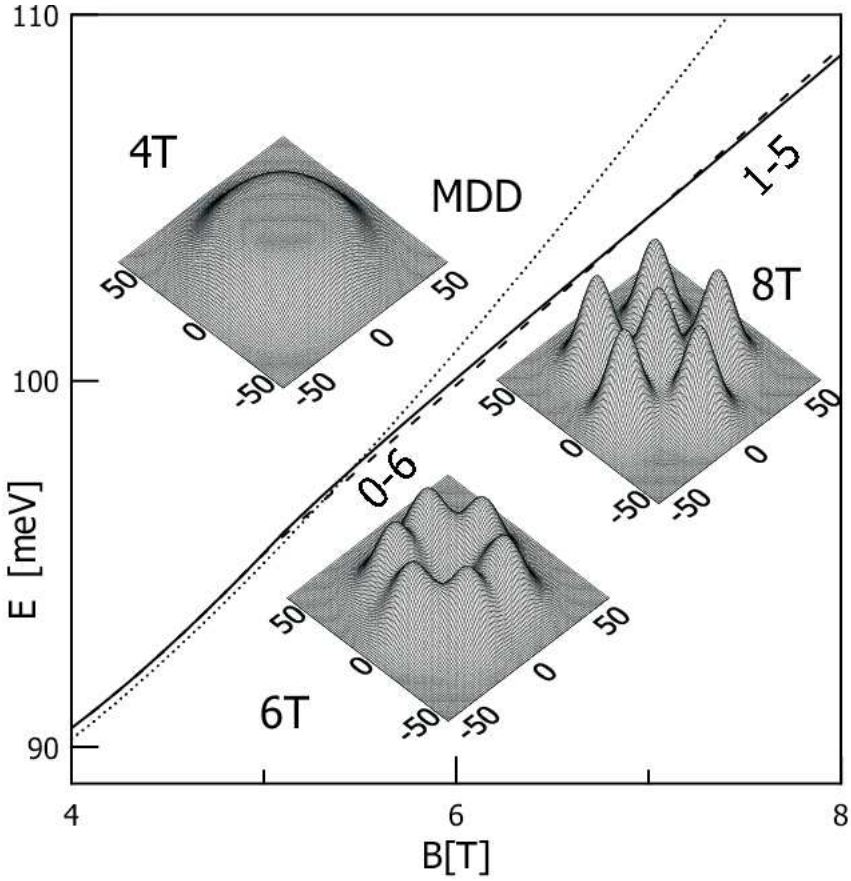

Figure 5: Energy E of six-electron ground state in circular QD with parabolic confinement as a function of magnetic field B. Dotted curve display the results for the MDD phase and solid (dashed) curve – for isomer $0-6$ $(1-5)$. Inset: electron density on the $x-y$ plane.

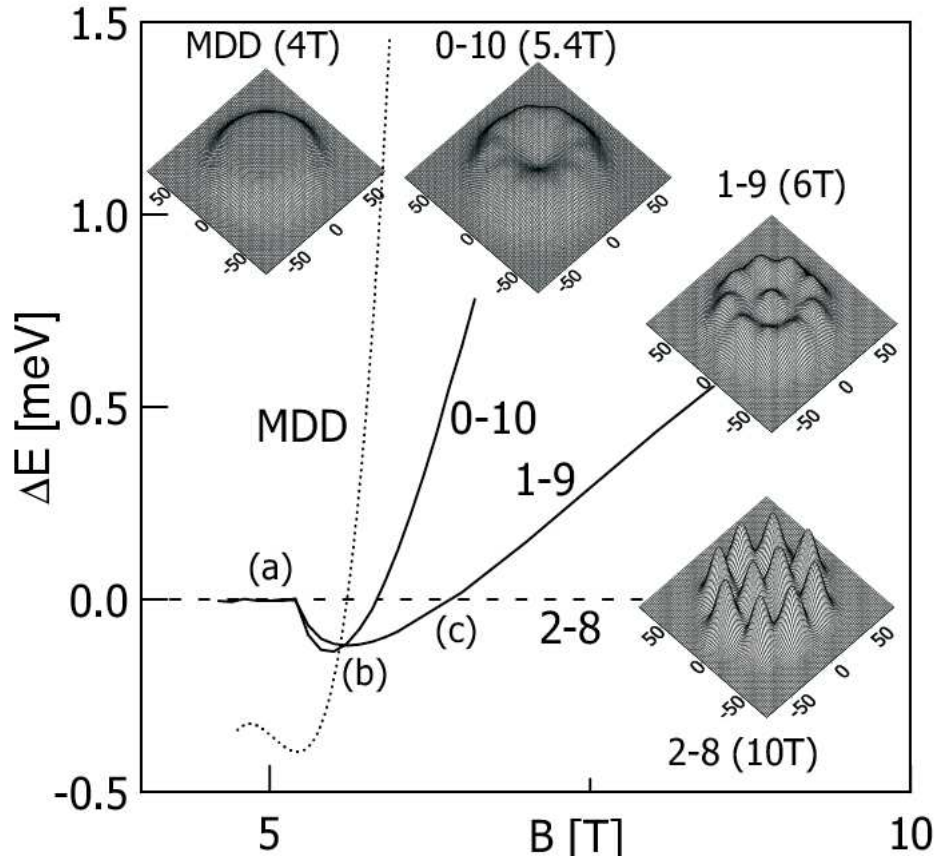

Figure 6: Energy separations ΔE between ground states of different phases of 10-electron system in circular QD with parabolic confinement and the ground-state energy of isomer $2-8$ (horizontal dashed line) as functions of magnetic field B. Inset: electron density on the $x-y$ plane.

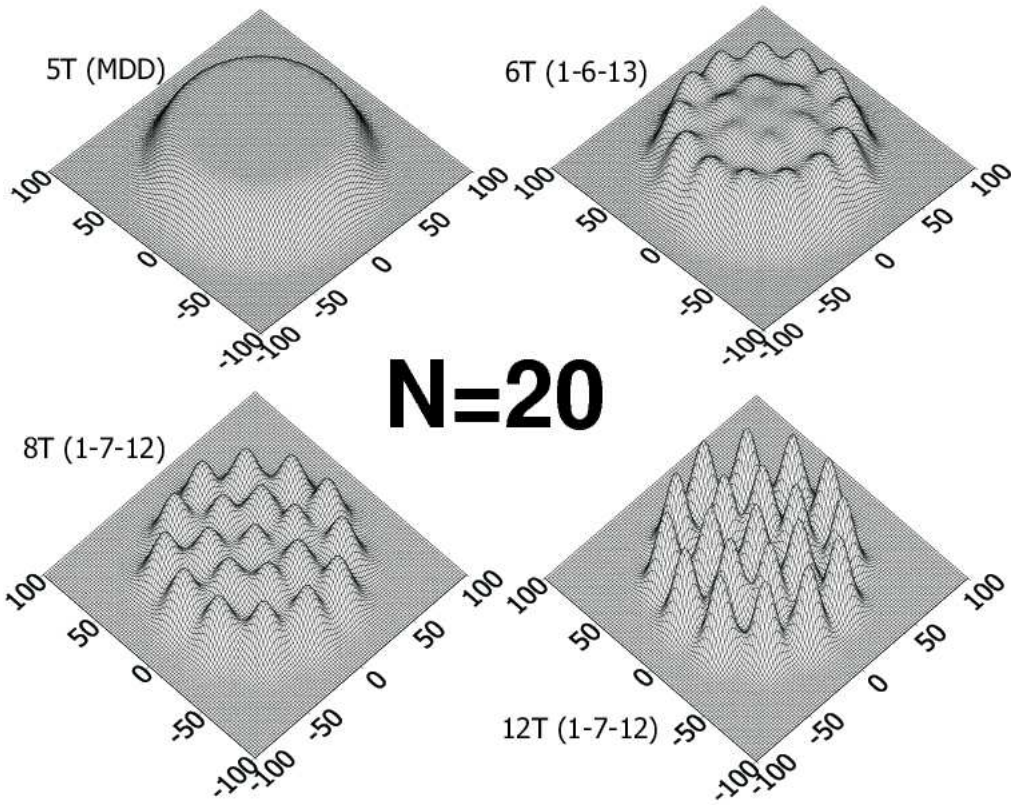

Figure 7: Electron density distribution on $x - y$ plane for 20 electrons in in circular QD with parabolic confinement for different magnetic fields. Length is measured in nm.

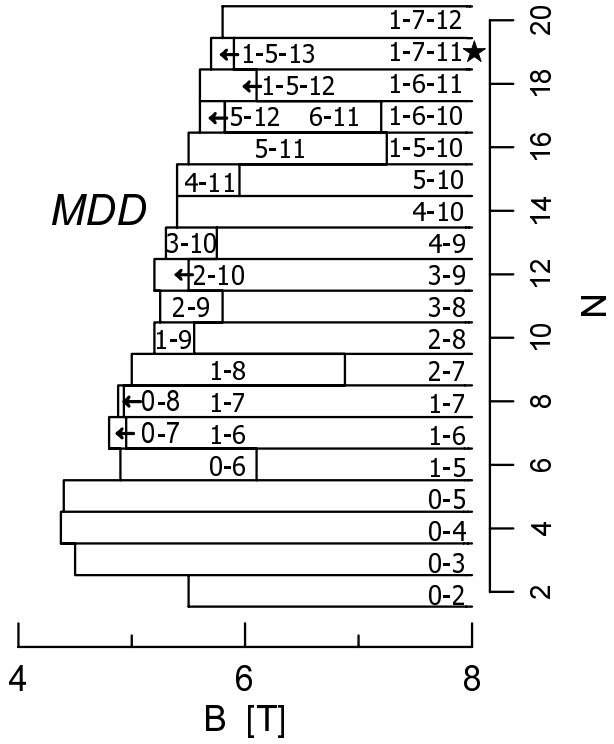

Figure 8: Phase diagrams for N-electron Wigner molecules as functions of magnetic field B.

Comment

In the case of confinement potential with cylindrical symmetry, the rotation in $x-y$ plane on arbitrary angle does not change the energy of electron system.

$\Longrightarrow$ We deal with **infinitely fold degeneracy** of N-electron ground state with respect to the rotations.

$\Longrightarrow$ The electron density distribution possesses the rotational symmetry (in laboratory frame).

The breaking of rotational symmetry (island-like distribution), obtained in calculations, occurs in **relative interelectron coordinates**.

In experiments, the phase transitions in Wigner molecules are observed as rapid changes of ground-state energy in N-electron system confined in the QD at high magnetic field.

2.2 Wigner molecules in anisotropic QD's

Parabolic confinement potential with elliptic symmetry

$$U_{conf}(x,y) = \frac{m_e}{2}(\omega_x^2 x^2 + \omega_y^2 y^2) \ , \tag{5}$$

ω_x, ω_y = confinement frequencies for x and y directions
$\omega_y > \omega_x$

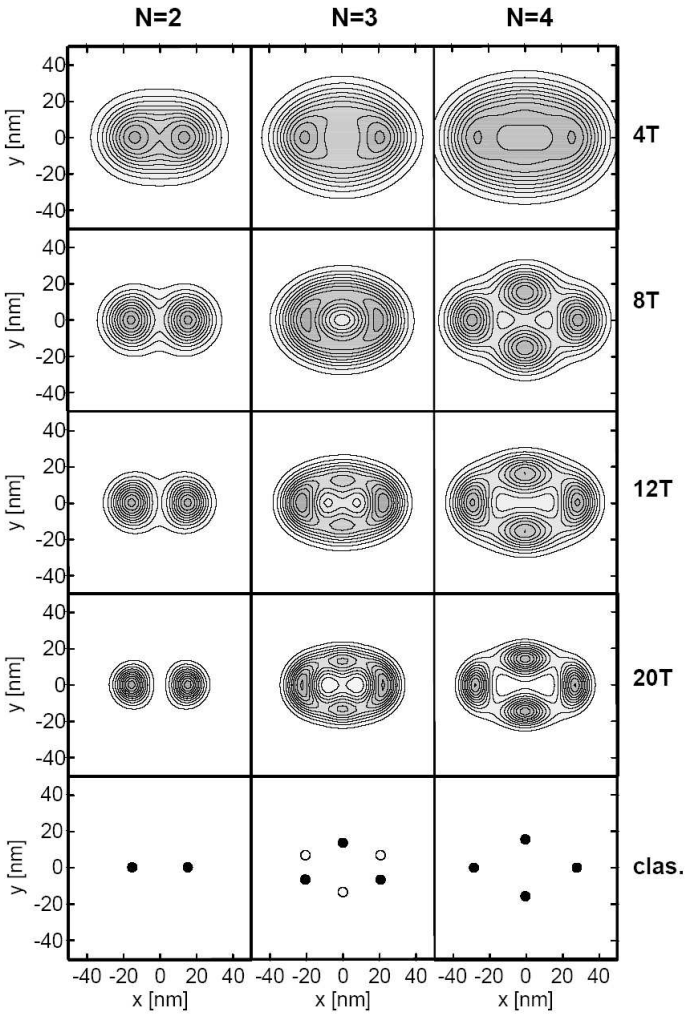

Figure 9: Contours of charge density for two-, three-, and four-electron systems in elliptic parabolic QD ($\hbar\omega_x = 3$ meV, $\hbar\omega_y = 4$ meV) for different magnetic fields. The lowest panel displays the corresponding classical equilibrium configurations.

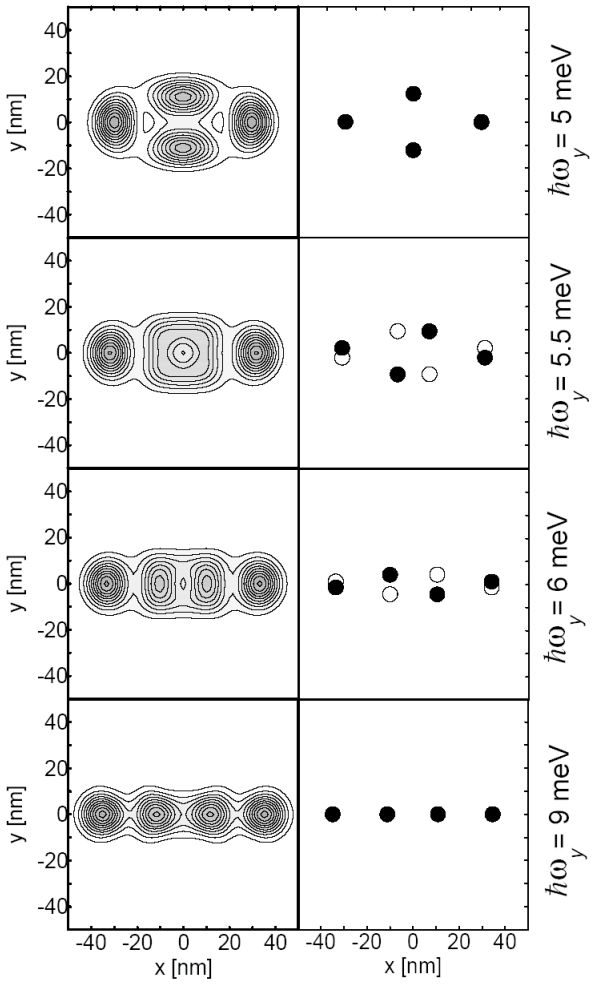

Figure 10: Left panels: Charge density contours for four electrons in elliptic parabolic QD with $\hbar\omega_x = 3$ meV and several values of $\hbar\omega_y$ at $B = 20$T.Right panels: Corresponding classical equilibrium configurations. Full and empty circles correspond to two degenerate configurations.

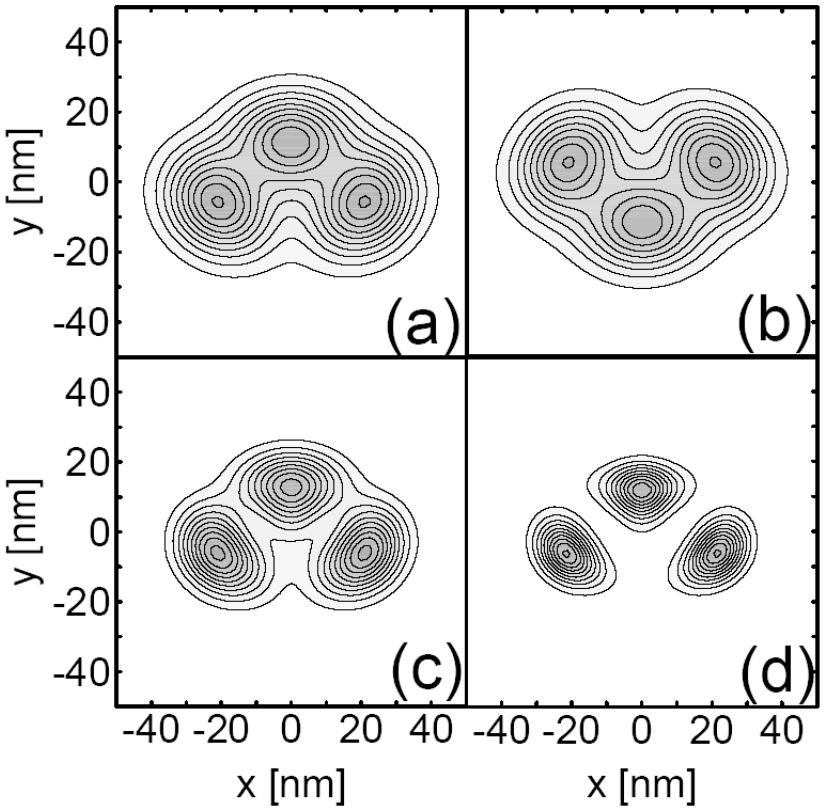

Figure 11: Charge density contours for three electrons in elliptic parabolic QD with $\hbar\omega_x = 3$ meV and $\hbar\omega_y = 4$ meV at $B = 6.3$ T (a,b), 14.125 T (c) i 25 T (d).

Conclusion

If the confinement potential does not possess the rotational symmetry (which occurs in the majority of real QD's), the island-like electron distribution is formed in the laboratory frame, i.e., can be directly observed experimentally.

$\Longrightarrow$ **wave-function mapping**

3 Applications of QD's

(1) single-electron transistor

(2) very sensitive electrometer

(2) quantum computer