ATHERMAL PHASE TRANSITION IN THE ISING MODEL

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outline

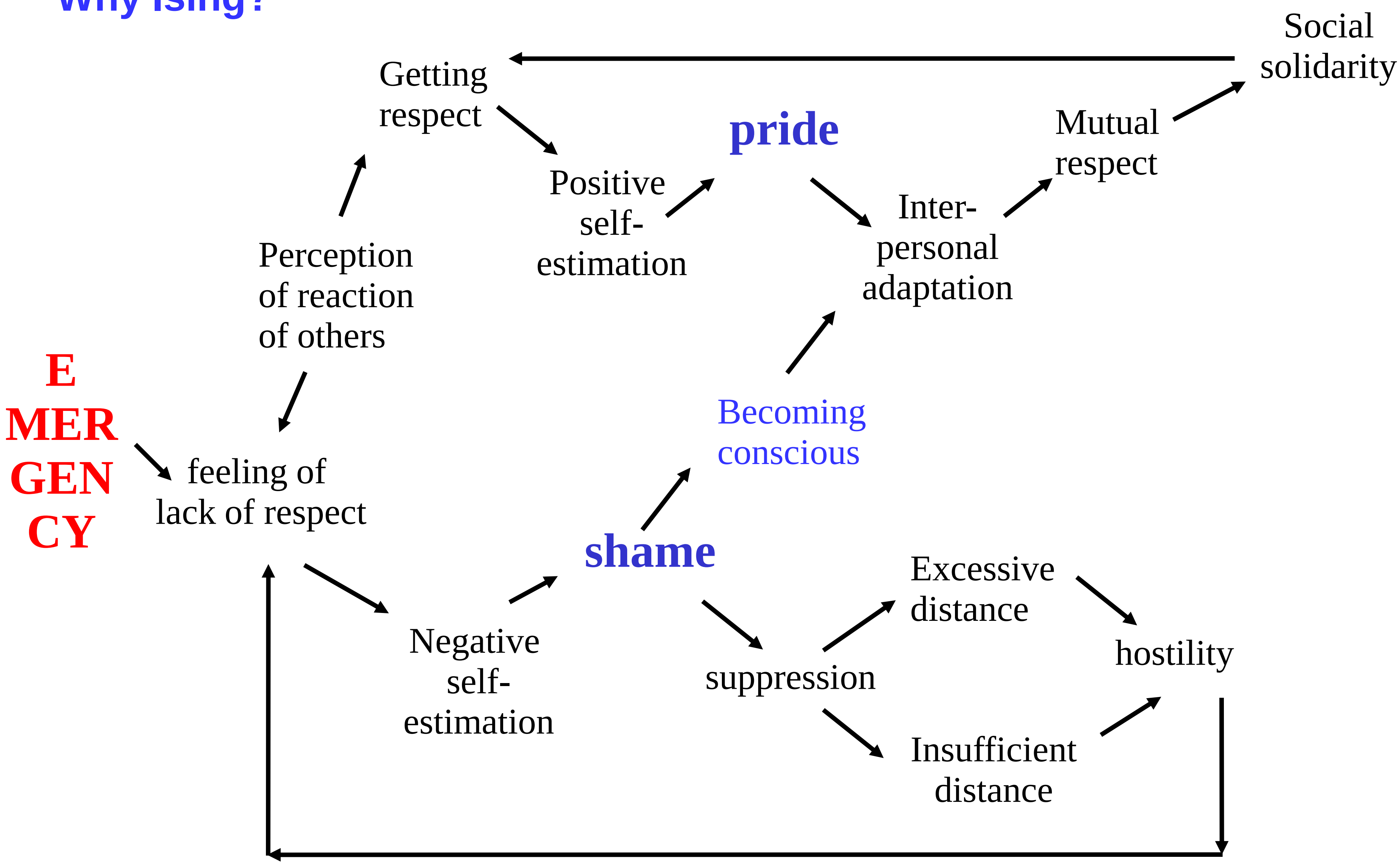
Sociophysical motivation

Rates of spin flips in the Ising model

Mean field with rates

An example: selfishness in crowd

Why Ising?



Scheff model of adaptation and solidarity

(J.H.Turner, J.E.Stets, *The Sociology of Emotions*, CUP 2005)

Why athermal?

Individuals are influenced by the group’s temperature. When a group's social temperature is high, very little provocation is necessary to induce an individual to change opinion. At low group temperatures, individuals appear more phlegmatic or stubborn and much greater provocation is required to induce a change in opinion. High social temperatures amplify the slightest excuse for chanqe, while low temperatures diminish the arguments for change. Note that each individual's opinion strength is unaltered, but as the group's temperature changes, so does an individual's decision making abilities. Although we are only appealing to analogies at this point, as shown shortly, temperature in sociology is a completely defined mathematical concept with no ambiguity. Any apparent ambiguity comes from the difficulty of describing a mathematical concept with words.

[D. B. Bahr and E. Passerini, J. Math. Soc. 23 (1998) 1]

The parameter T may be interpreted as a „social temperature" describing a degree of randomness in the behaviour of individuals, but also their average volatility (cf. [24]).

[K. Kacperski and J. Hołyst, Physica A 269 (1999) 511]

Why athermal?

…we add an internal factor to Sznajd model, which determines the probability of one's acceptance of other's opinion. We define this factor as social temperature, and we study the relationship between this temperature and time.

[M. He et al., IJMPC 15 (2004) 997]

Introducing a social permeability 1/T where T is the equivalent of the physical temperature, a global dissatisfaction function F, indeed a free energy, can be calculated.

[S. Galam, IJMPC 19 (2008) 409]

The parameter β simply measures how fast the entropy of the equilibrium macro-state increases versus global cooperation variations. Its formal role is that of inverse thermodynamical temperature.

[M. Floria et al., PRE 79 (2009) 026106]

We can define however a temperature-like parameter T in such a simple way that it would measure the mean rate of microstate change at equilibrium. To make it system size independent let us normalize it to one: we define T as the average number of elementary changes of the microstate in a single Monte Carlo step divided by the size of the system. Thus T = 0 indicates no changes at all and T = 1 means that the microstate is changing at every possible instant of time.

[G. Kondrat, arXiv:0912.1466]

Ising standards:

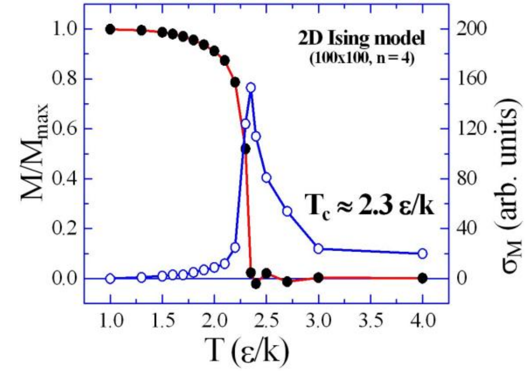
- interaction *vs* thermal noise
- ergodic hypothesis
- up-down symmetry

Social world:

- no energy
- no temperature
- no equilibrium
- no symmetry

Rates of spin flips in the Ising model –
Metropolis algorithm :

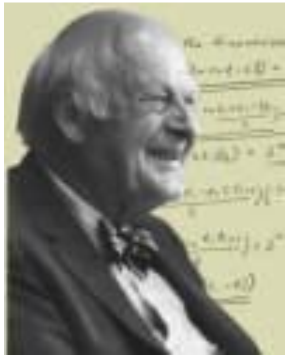
- select a spin
- if its flip lowers energy, do it
- if not, do it with probability *exp*(-ΔE/kT)



↑ from web-page of Dr. Kurt Gloos, Turku University

The rates *U*=1, *W*=*exp*(-Δ*E*/*kT*)

- satisfy the detailed balance conditions
- allow to reproduce the exact solution.



Our reinterpretation:

The rates do not necessarily represent a thermal noise.

Any process can produce the rates.

In particular, the dependence on *W* reproduces the order-disorder phase transition.

For social processes *E, T, E/T* do not exist.

Interaction enters *via* the dependence of the rates
on the state of neighborhood.

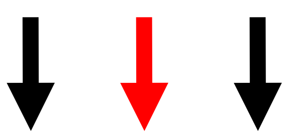
Consider a regular lattice; each spin has z neighbours.

If a spin is up with n neighbours up, its flip rate = $w(n)$;
If a spin is down with n neighbours down, its flip rate = $v(n)$.

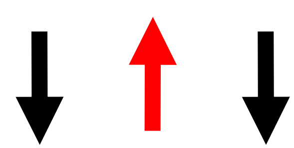
A special case: up-down symmetry,

then $v(n)=w(n)$.

Ordering interaction: $w(n)$ decreases:



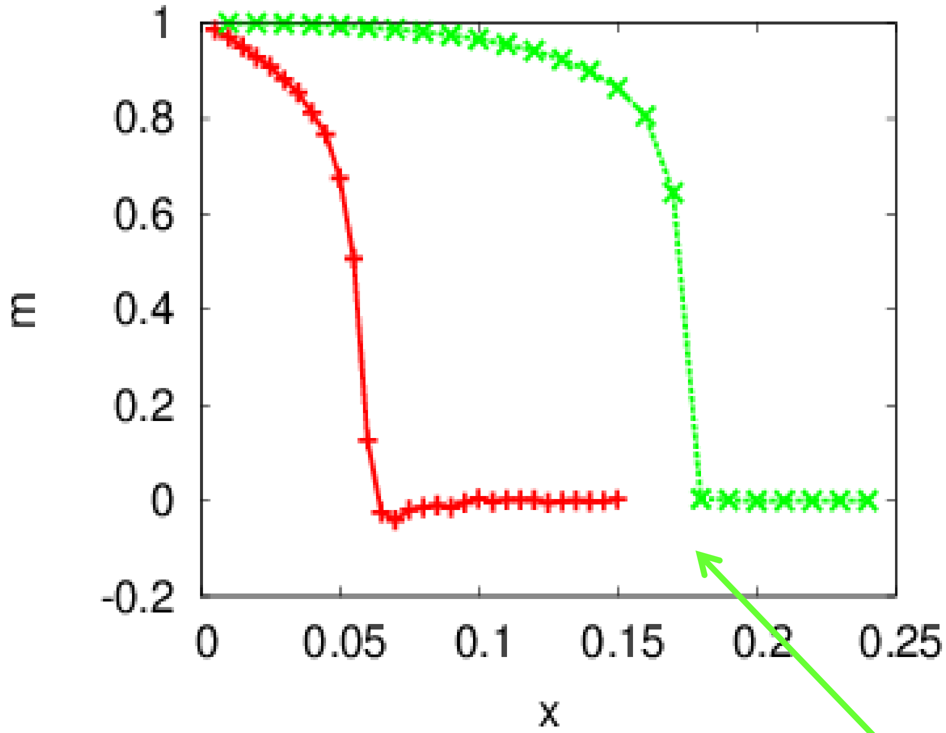
slow



fast

$w(2) < w(0)$

Numerical results for two symmetric cases



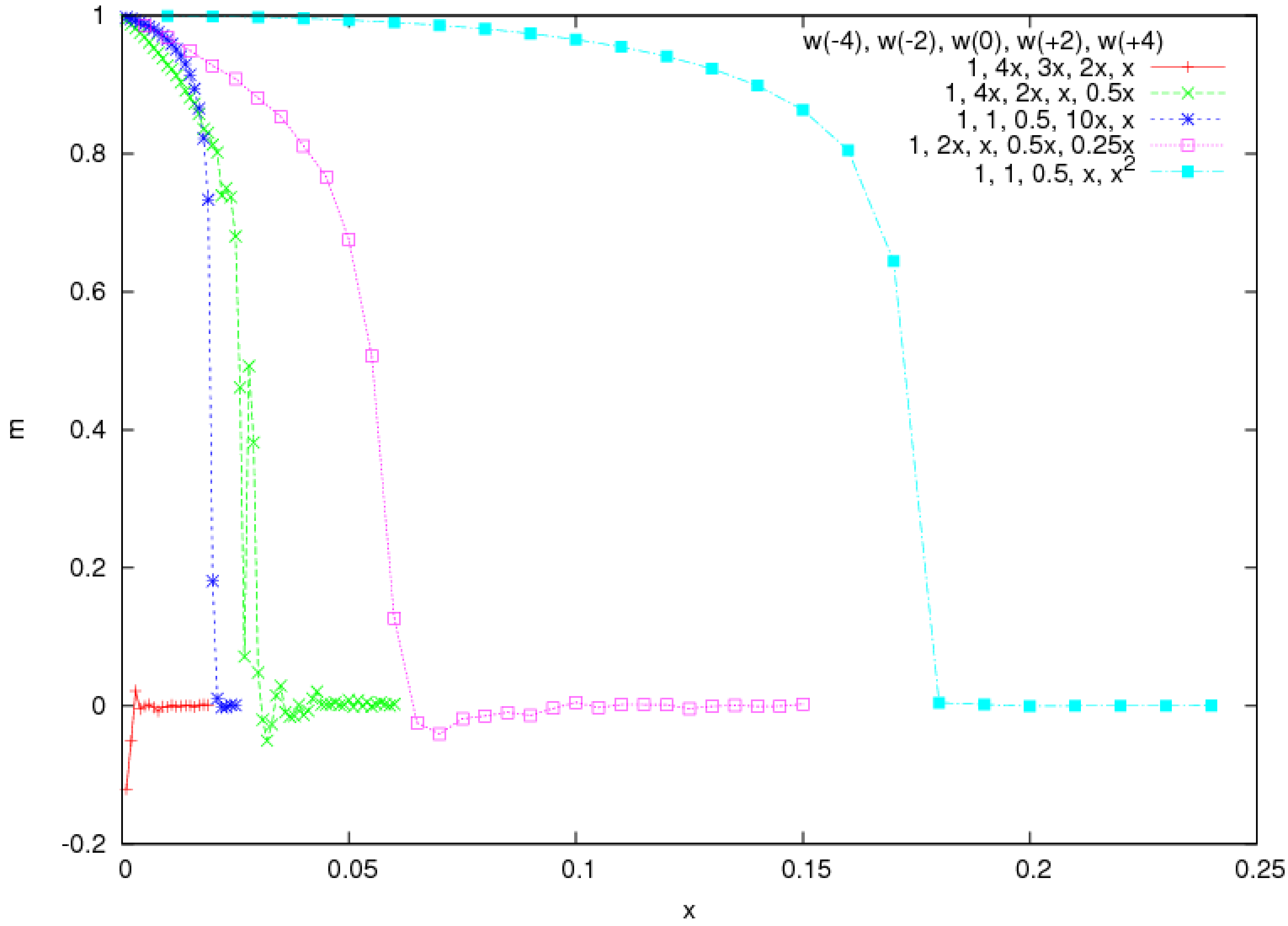
green ≡ Metropolis:

$w(0)=w(1)=1$, $w(2)=1/2$, $w(3)=x$, $w(4)=x^2$, where $x = \exp(-4J/kT)$

(exact $x=3-2\sqrt{2} \cong 0.172$)

red ≡ $W(0)=1$, $w(1)=2x$, $w(2)=x$, $w(3)=x/2$, $w(4)=x/4$

more numerical results



Mean field with rates

Let *a* = *p*(↑), *b* = *p*(↓), *a*+*b* = 1, *a*-*b* = *m*. The model equation of motion is

$$\frac{da}{dt} = \sum_{n=0}^{z} \binom{z}{n} a^n b^{z-n} [-aw(n) + bv(z-n)]$$

Number of possibilities

n neighbours up *and* *z*-*n* neighbours down

central spin up

The rate, when its *n* neighbours up

central spin down

The rate, when its *z*-*n* neighbours down

A thermodynamical test: $w(n) = \gamma^{4-2n}$, where $\gamma = \exp(J/kT)$. Then

$$\frac{da}{dt} = -a(a/\gamma + b\gamma)^z + b(a\gamma + b/\gamma)^z$$

For z=4 (square lattice), the stationary solution is $m = 0$ or

$$m = \frac{\pm\sqrt{-5+4y+6y^2-4y^3-y^4+2\sqrt{(y-1)^4(y+1)^2(5+2y+y^2)}}}{(y-1)^2}$$

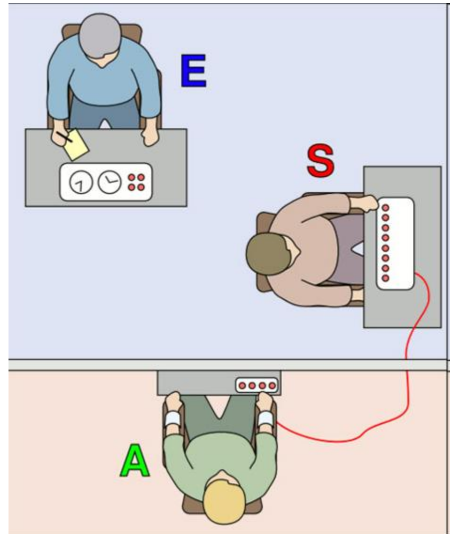
where $y \equiv \gamma^2$.

At the critical point, the solution m = 0 loses stability .

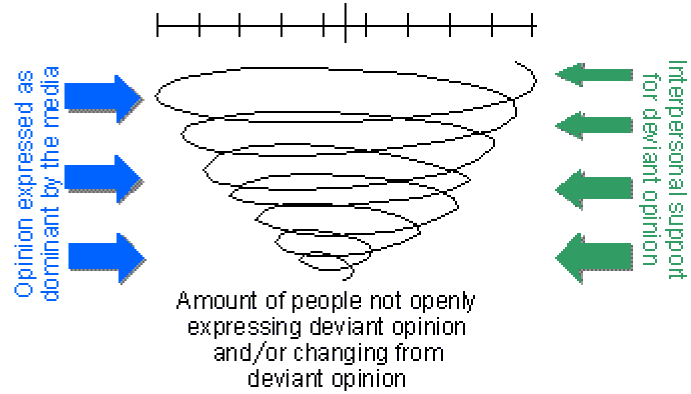
Bragg- Williams	kT_C/J = 4.0		$\beta_c J = 1/z$
this approach		3.92	$\tanh \beta_C J = 1/z$
Bethe-Peierls		2.89	$\tanh \beta_C J = 1/(z-1)$
Onsager (exact, *z*=4)		2.27	$\sinh 2\beta_C J = 1$

Towards application: social norms

Vinnytsia 1942

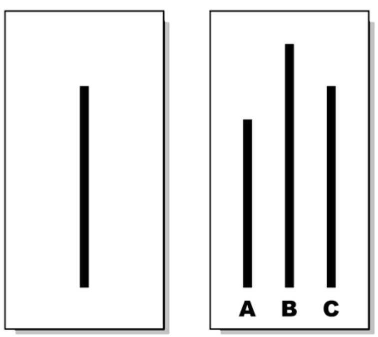


Milgram 1961



Noelle-Neumann's Spiral of Silence

Noelle-Neumann 1976



Asch 1950

Zimbardo 1971

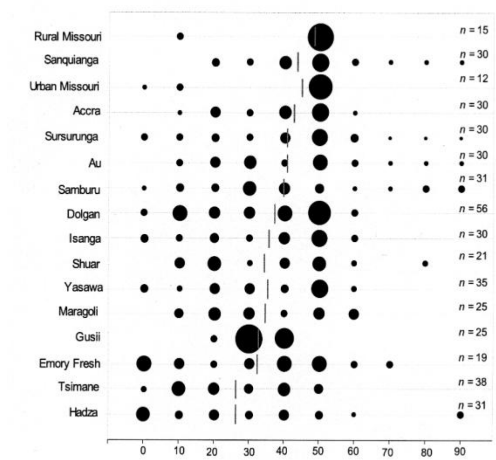


Figure 2. The distribution of offers in the Dictator Game.
Reading horizontally for each of the populations listed along the left vertical axis, the area of each bubble represents the fraction of our sample who made that offer, so each horizontal set of bubbles provides the complete distribution of offers for each population. The blue slash gives the mean offer for each population. The *n* values on the right side provide the number of pairs.

Henrich 2005

an application: selfishness in crowd

most simple case: a queue, $z = 2$ (a transition only in mean field model), symmetric
Two strategies: selfish (blind pushing) and cooperative (flocking)
Three parameters: $w(n)$, $n = 0,1,2$

$$\frac{da}{dt} = a^2[-aw(2)+bw(0)] + 2ab[-aw(1)+bw(1)] + b^2[-aw(0)+bw(2)]$$

Stationary solution and relaxation

supercritical fork bifurcation

$m = 0$ or $m = \pm\sqrt{\dfrac{\Phi - 3}{\Phi + 1}}$, where $\Phi \equiv \dfrac{w(0) - 2w(1)}{w(2)}$

The disordered solution loses its stability at $\Phi = 3$. For $3-\Phi$ small and positive

$m(t) \cong m(0)\exp\left(-t/\tau\right)$ where

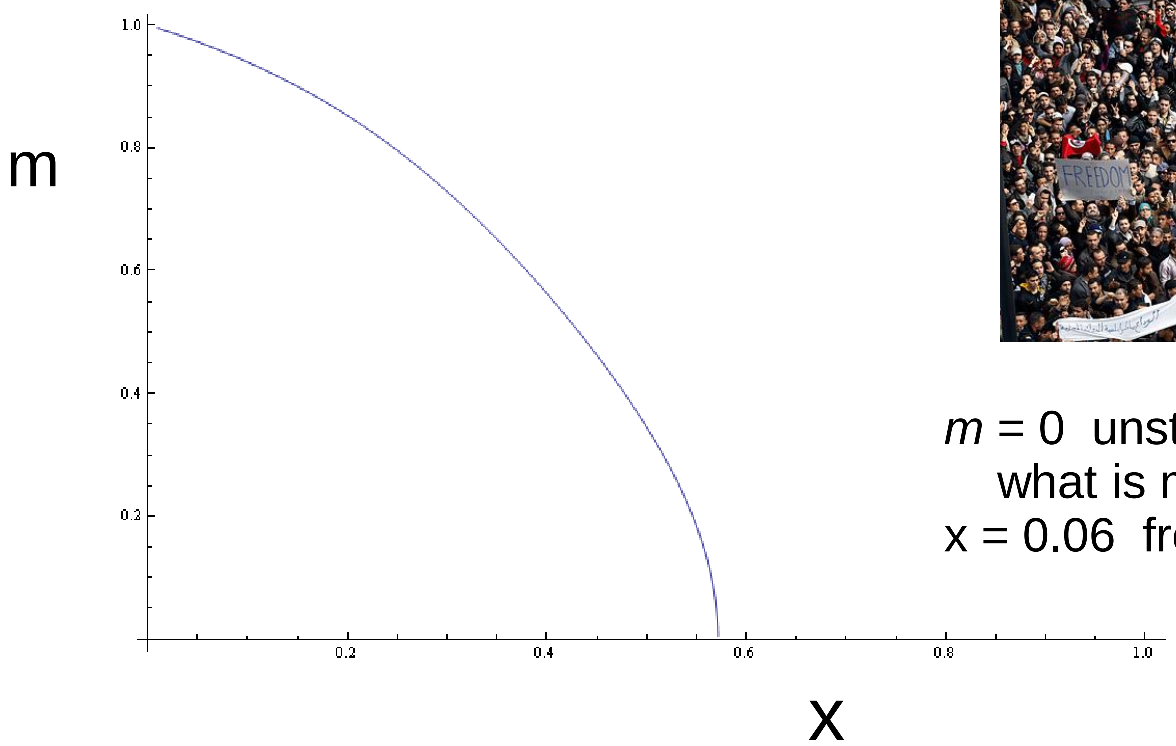
$$\tau \equiv \frac{2}{3w(2) + 2w(1) - w(0)} = \frac{2}{(3-\Phi)w(2)}$$

critical slowing down

More realistic: a two-dimensional crowd, $z = 4$

Model case: $w(0)=1$, $w(1)=2x$, $w(2)=x$, $w(3)=x/2$, $w(4)=x/4$

Stationary solution: $m = 0$ or $$m = \pm\sqrt{\frac{4+5x-2\sqrt{16-56x+85x^2}}{15x-4}}$$



$m = 0$ unstable for $x > 0.571$, what is much larger than $x = 0.06$ from the simulations *final remarks*

In sociologically motivated problems the analogy of energy and temperature is not necessary.

The rates of social processes can be measured by polls. The number of parameters is not small.

In the mean field theory, the correlations are diminished. The ordered phase is overestimated.

Monte-Carlo simulations should provide proper results.

K. Malarz, R. Korff and K. Kułakowski, Norm breaking in crowd – athermal phase transition, IJMPC (2011), in print (arXiv:1102.2832)


KIITOS = THANK YOU