

# The norm game - - how a norm fails

KRZYSZTOF KUŁAKOWSKI and ANTONI DYDEJCZYK



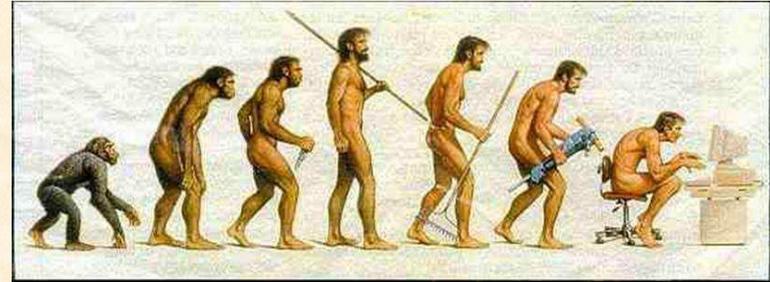
AGH UNIVERSITY OF SCIENCE  
AND TECHNOLOGY

Kraków, Poland



1. Some mean field
2. Some simulations
3. Some examples

# Three mechanisms of an evolutionary change



1. More effective individuals are more likely to survive and reproduce.
2. The players **learn by trial and error**, keeping effective strategies and altering ones that turn out poorly.
3. The players observe each other, and those with poor performance tend to imitate the strategies of those they see doing better.



[Robert Axelrod]

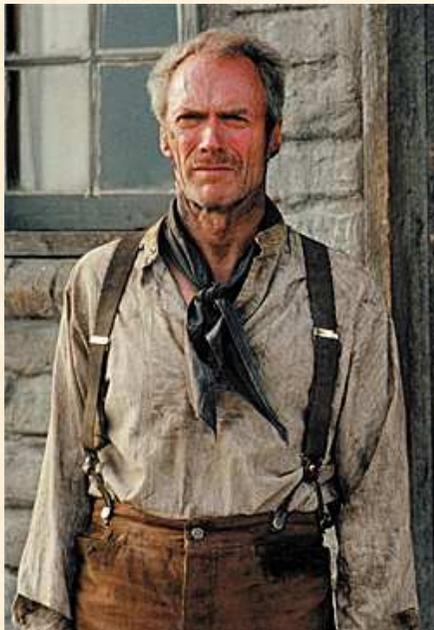
## norms appear with social systems

„The first need of  
any social system  
is to create  
incentives  
to make people do  
more work than  
that required by  
their immediate  
wants.”

[*Elias J. Bickerman*]



## norms are restored by punishment



„Don't you be cutting up or otherwise harming any whores, or I'll come back and kill all you sons-a-bitches.”  
[Eastwood as Munny, in *Unforgiven*, 1992]

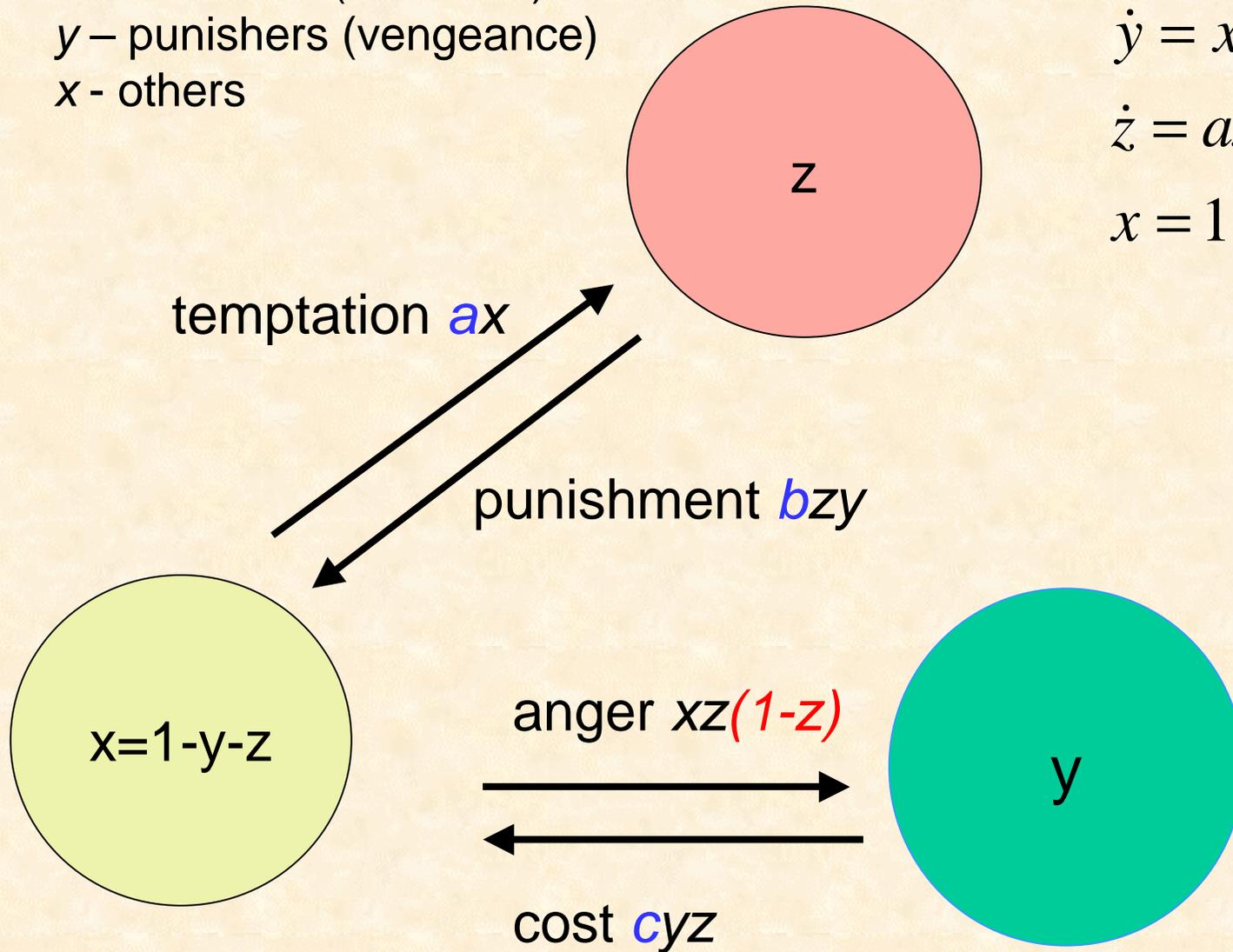
# Defecting and punishing mutually exclude

$z$  – defectors (boldness)  
 $y$  – punishers (vengeance)  
 $x$  - others

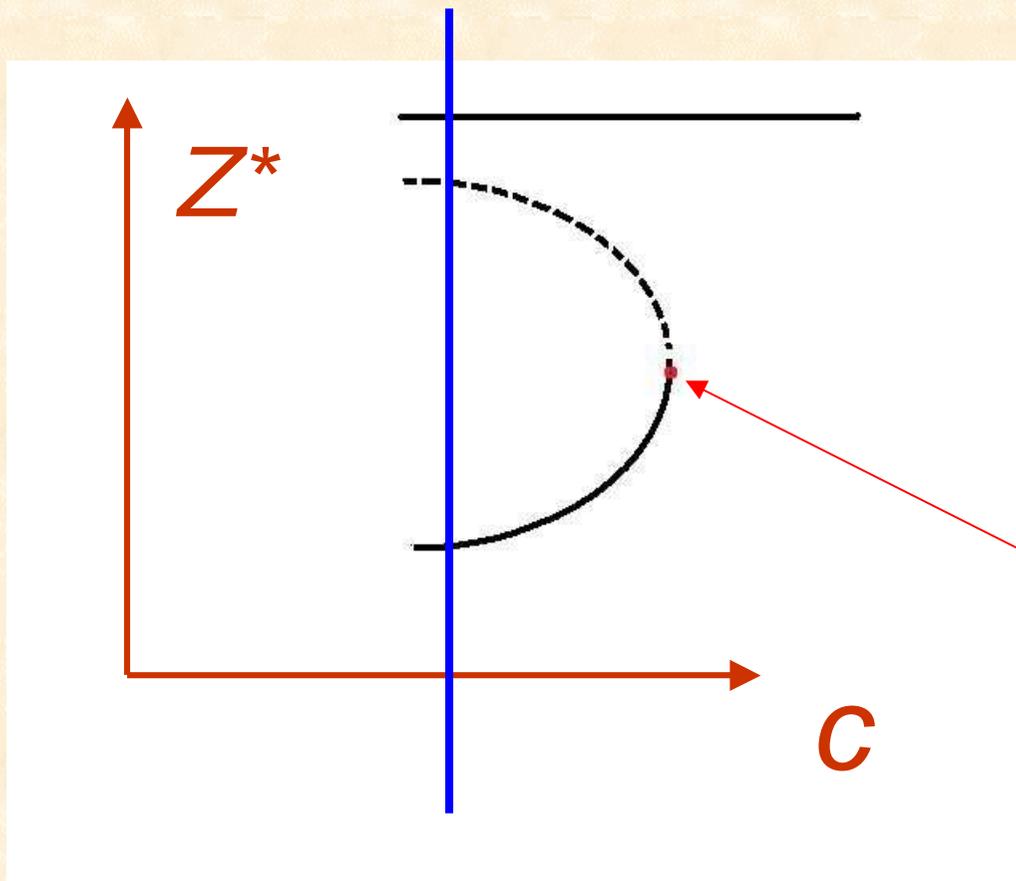
$$\dot{y} = xz(1 - z) - czy$$

$$\dot{z} = ax - bzy$$

$$x = 1 - y - z$$



## Stationary solution: fixed points



$$z^* = 1$$

$$z^* = \frac{1}{2} \left( 1 \pm \sqrt{1 - 4 \frac{ac}{b}} \right)$$

**saddle-node bifurcation  
 at  $4ac=b$**

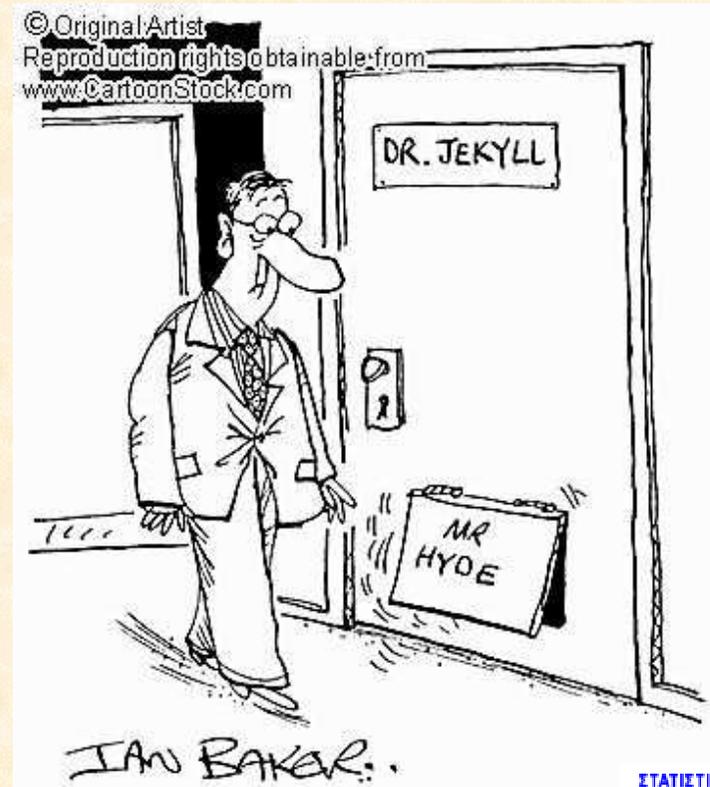
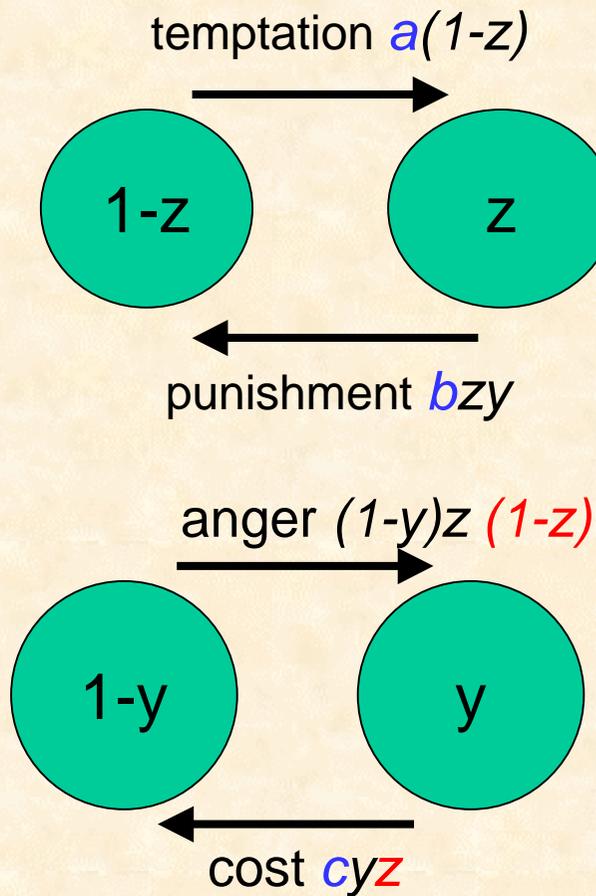
$$x^* = \frac{1 - z^*}{1 + \frac{a}{bz^*}} \quad y^* = \frac{1 - z^*}{1 + \frac{bz^*}{a}}$$

# Defecting and punishing uncorrelated

$z$  – defectors (boldness)  
 $y$  – punishers (vengeance)

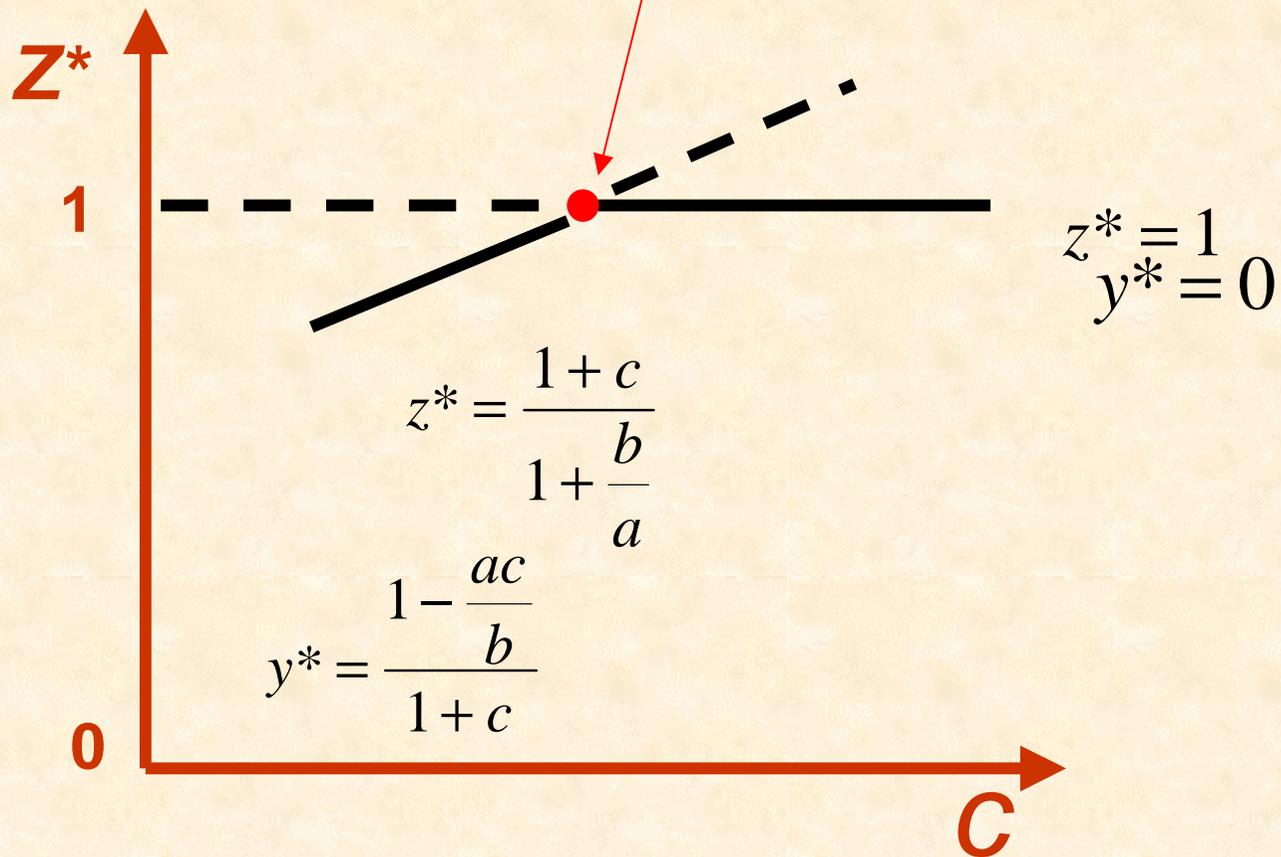
$$\dot{z} = a(1-z) - bzy$$

$$\dot{y} = -czy + (1-y)z(1-z)$$



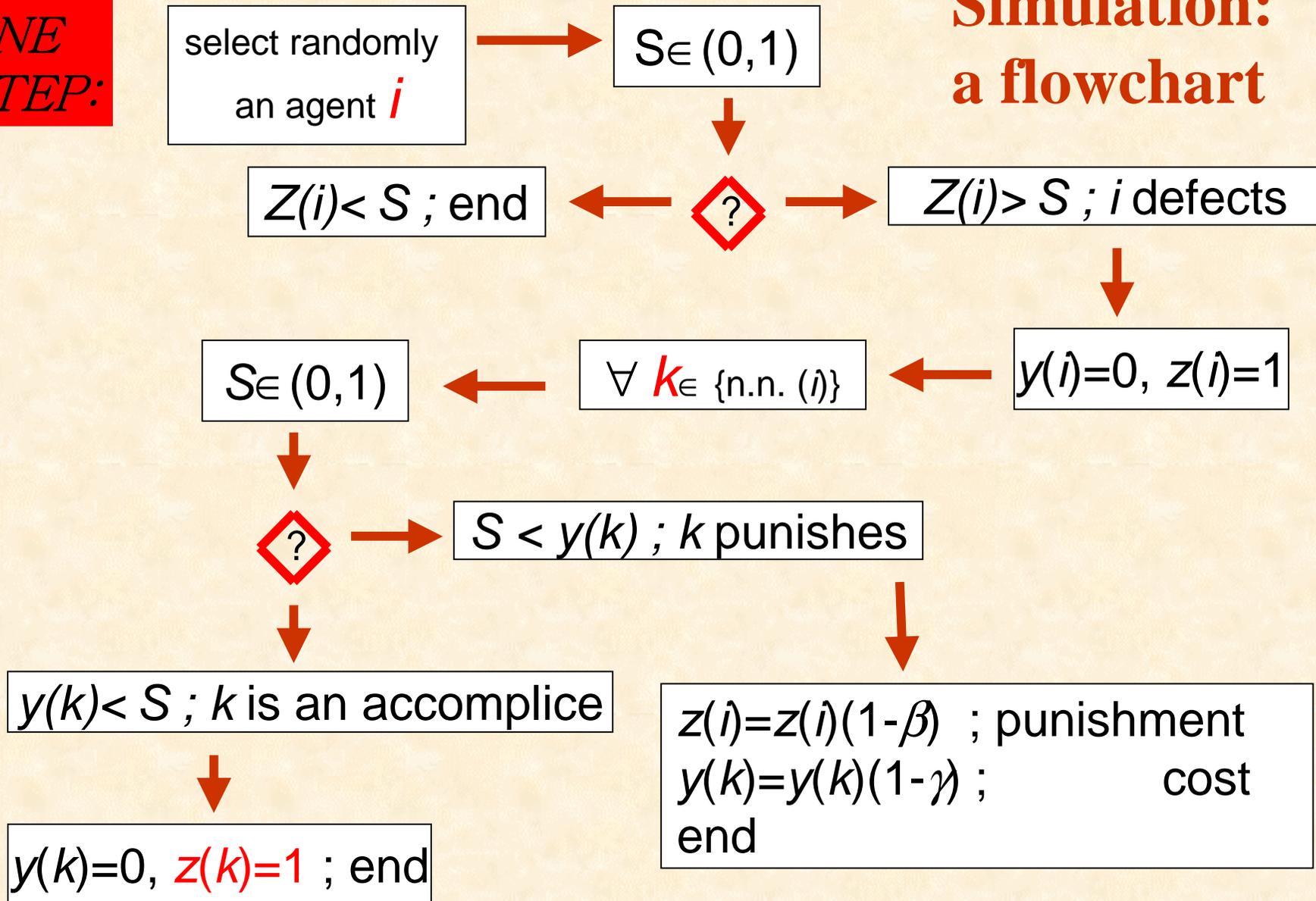
## Stationary solution: fixed points

transcritical bifurcation  
at  $ac=b$

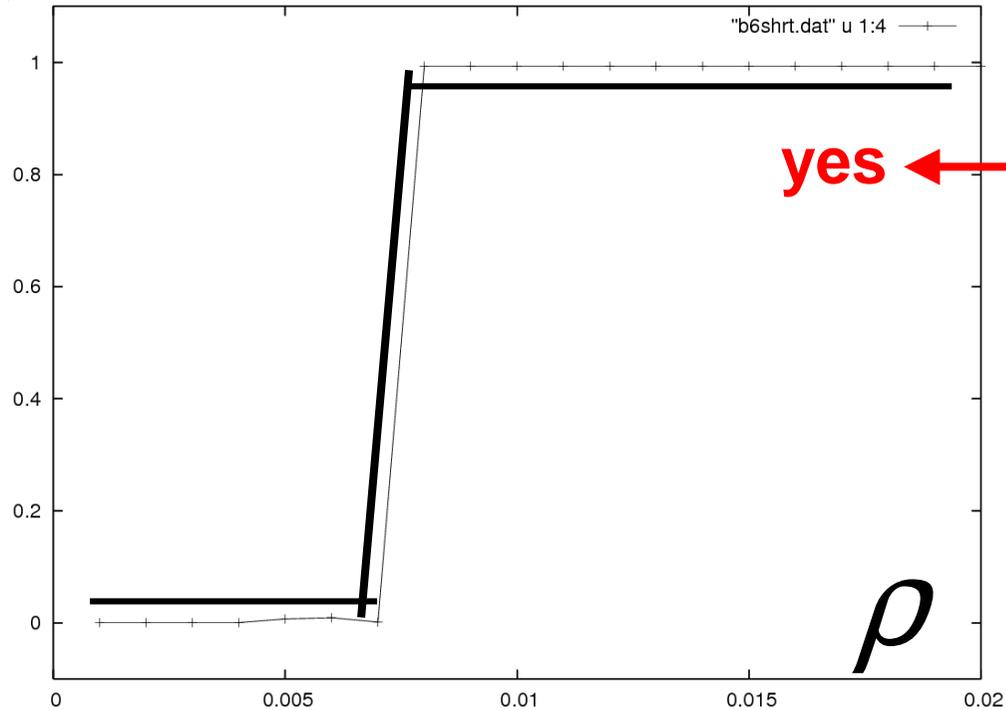


**ONE  
STEP:**

## Simulation: a flowchart



$\langle Z \rangle$



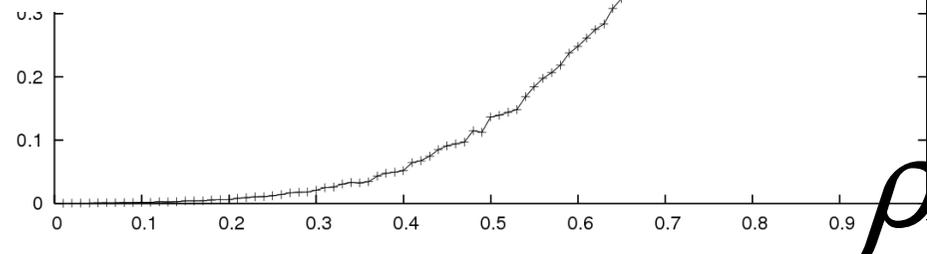
yes

$k$  did not punish



$z(k)=1$

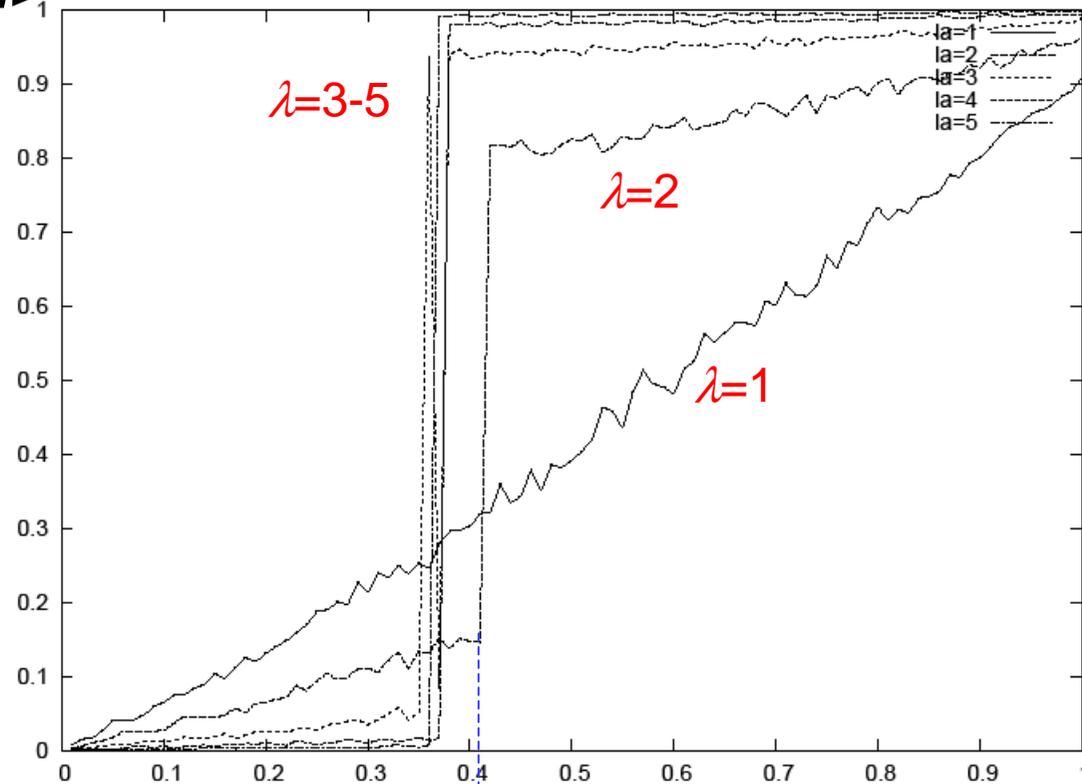
no



$\rho$  - initial boldness

§: „ $k$  did not denounce =  $k$  is guilty”  
⇒ the bistability

$\langle z \rangle$



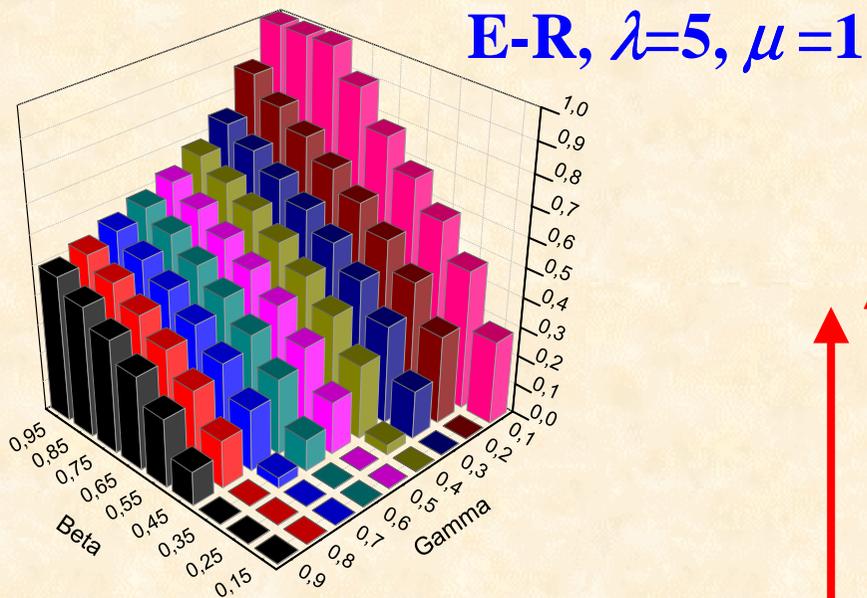
$\rho$

Initial distribution :  
 $0.9\rho < z(i) < \rho$   
 $y(i) = (1-z(i))/\mu$

$\rho_c$  for  $\lambda=2$

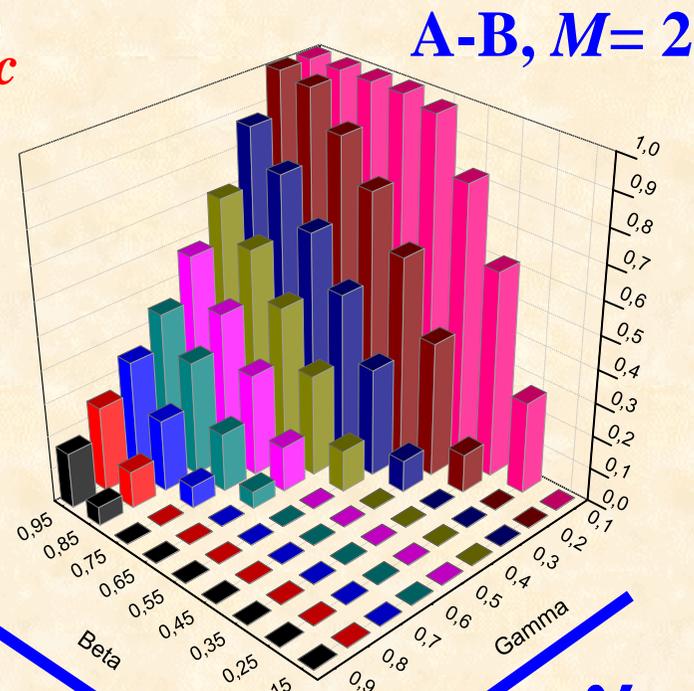
Equilibrium value of the mean boldness  $\langle z \rangle$  at the stationary state. A sharp transition is observed except the case  $\lambda=1$ , where  $\exp(-1) \approx 0.36$  nodes cannot be punished.

$\rho_c$

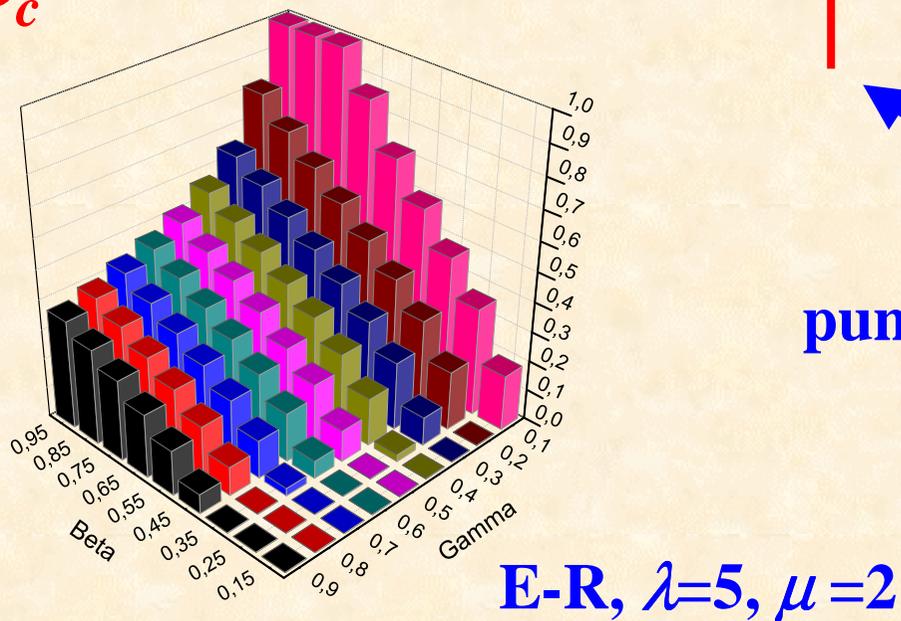


$\rho_c$  – the separatrice

$\rho_c$



$\rho_c$



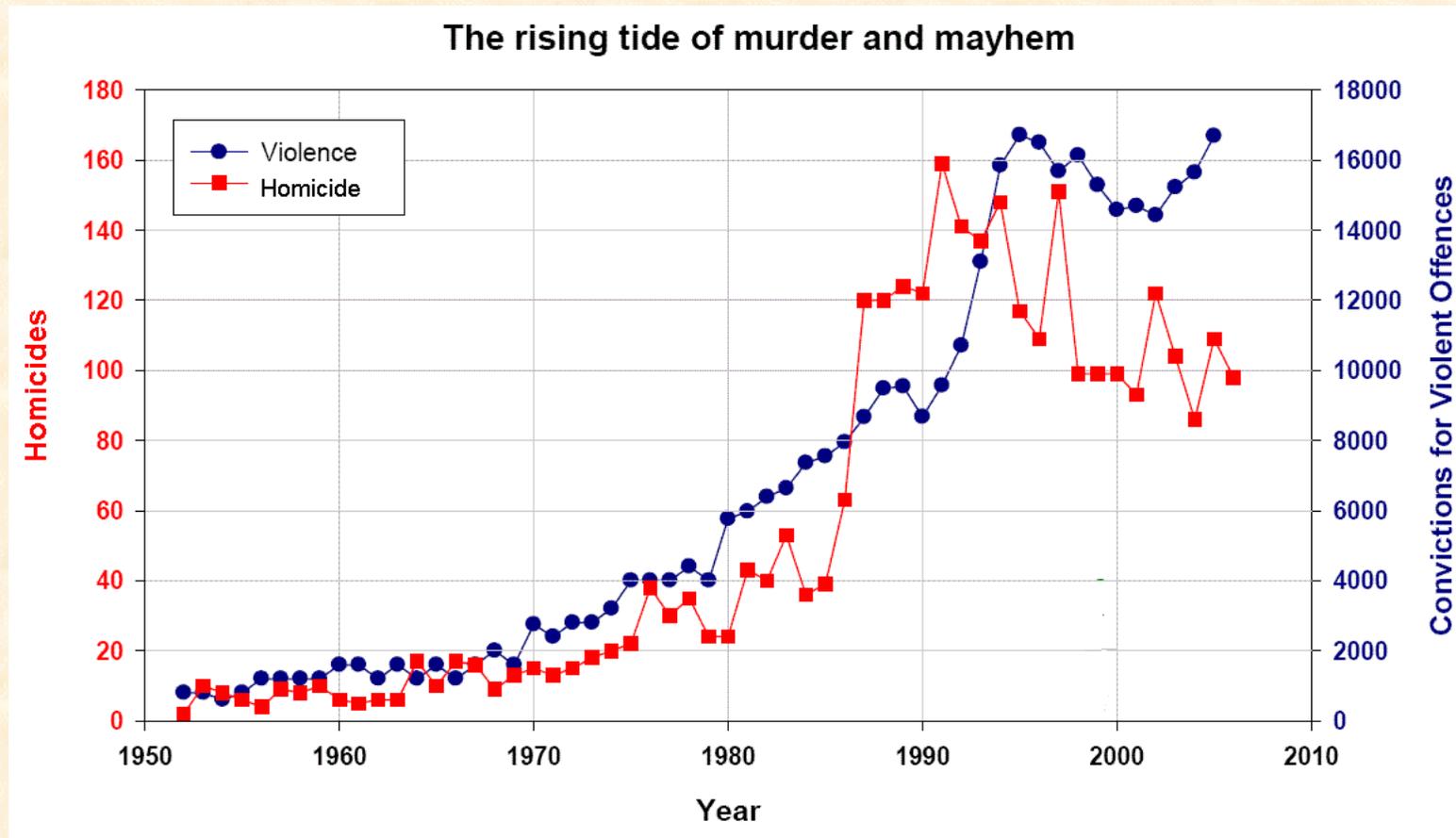
$\beta$   
punishment

$\gamma$   
cost

# model results

1. Analytical considerations indicate, that the saddle-node bifurcation and the bistability appear if the roles of defectors and punishers are separated, Otherwise the bifurcation is transcritical and the boldness  $z^*$  varies continuously at the bifurcation point.
2. The bistability of the stationary phase appears also as a result of the simulation on a directed Erdős-Rényi network. There is no saddle-node bifurcation. If the non-punishers do not become defectors, the bistability vanishes.

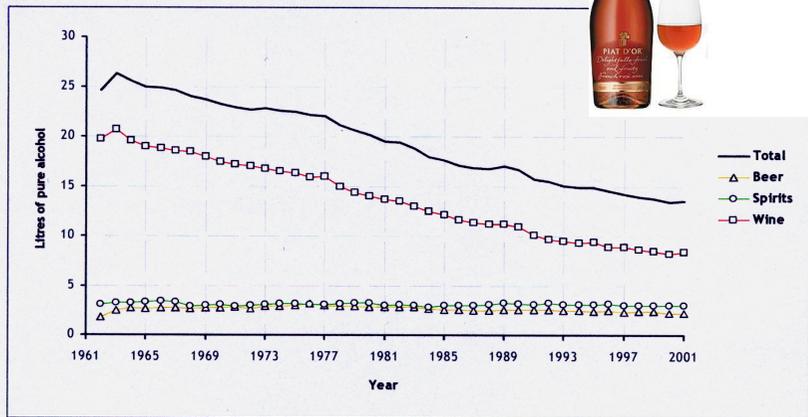
# A change: murders in New Zealand



Since 1984, successive governments have engaged in major macroeconomic restructuring, transforming New Zealand from a highly protectionist and regulated economy to a liberalised free-trade economy. These changes are commonly known as Rogernomics and Ruthanasia after Finance Ministers Roger Douglas and Ruth Richardson. [Wikipedia]

## FRANCE

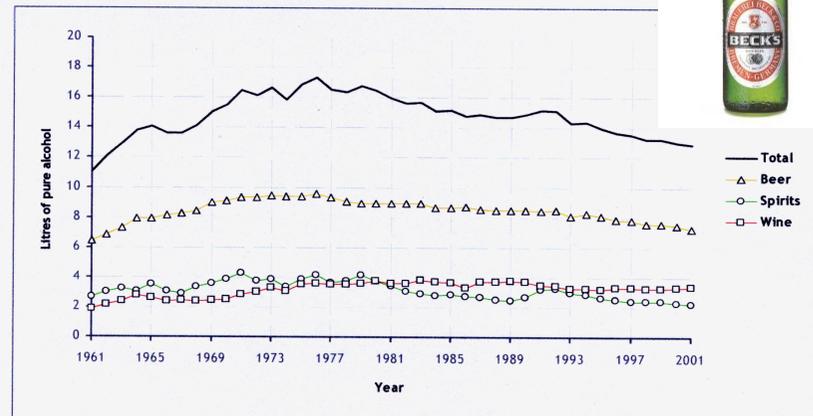
Recorded adult per capita consumption (age 15+)



Sources: FAO (Food and Agriculture Organization of the United Nations), World Drink Trends 2003

## GERMANY

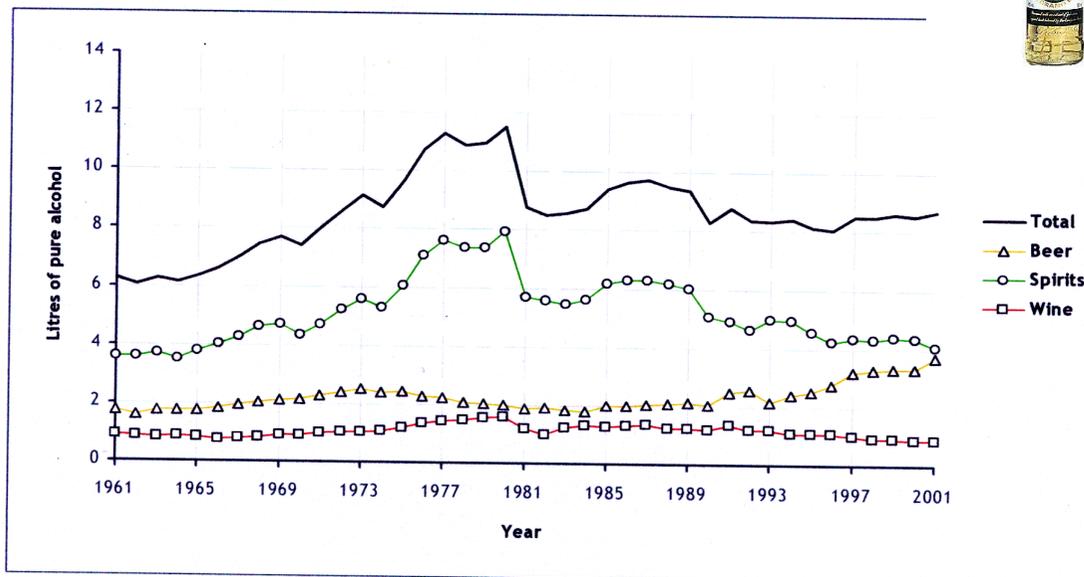
Recorded adult per capita consumption (age 15+)



Sources: FAO (Food and Agriculture Organization of the United Nations), World Drink Trends 2003

## POLAND

Recorded adult per capita consumption (age 15+)



Sources: FAO (Food and Agriculture Organization of the United Nations), World Drink Trends 2003

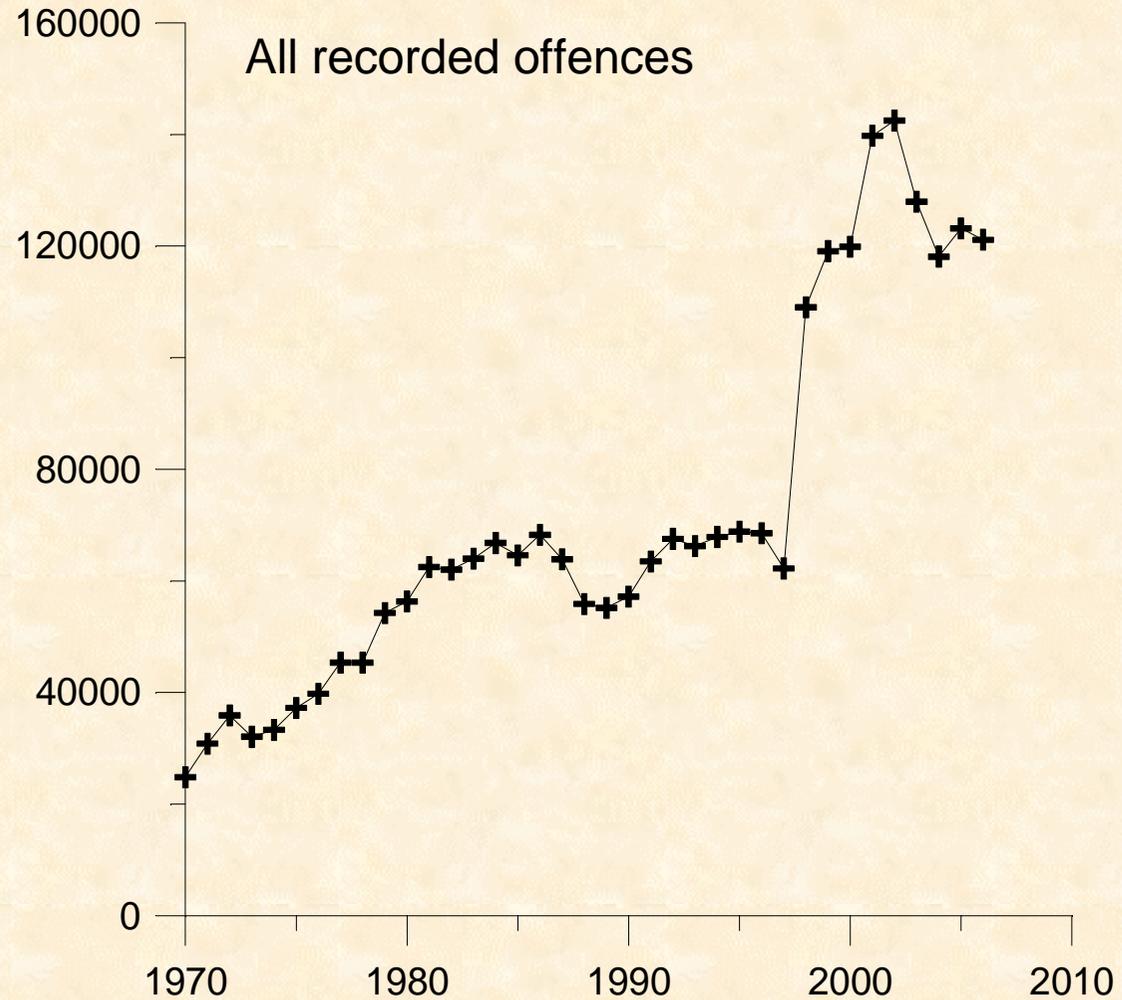
A change:  
drinking  
vodka  
in Poland

# A change: crime in Northern Ireland



Since mid-1997, the main paramilitary group, the Provisional IRA, has observed a ceasefire.

[Wikipedia]



<http://epp.eurostat.ec.europa.eu/>

# A change: divorces in USA

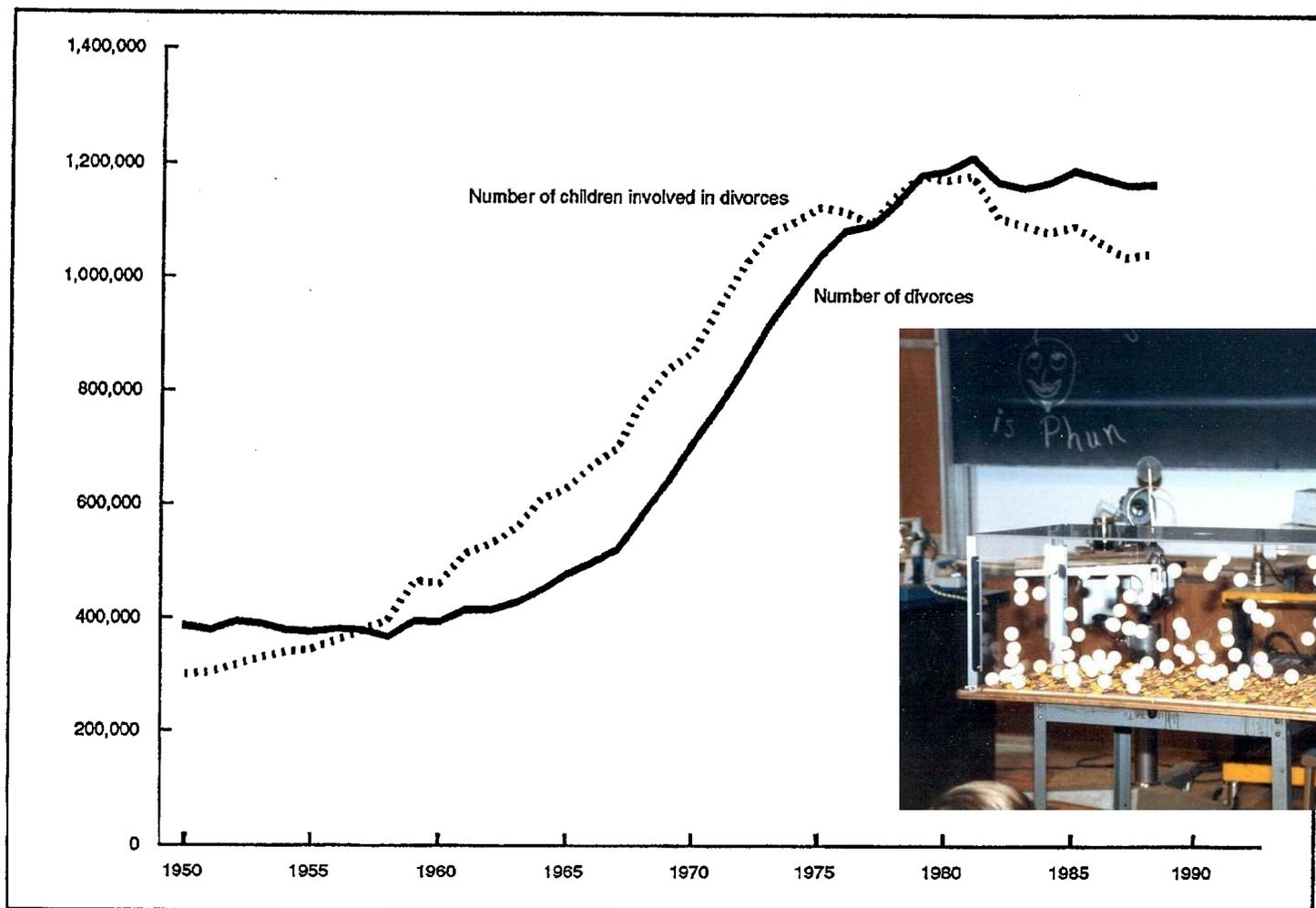
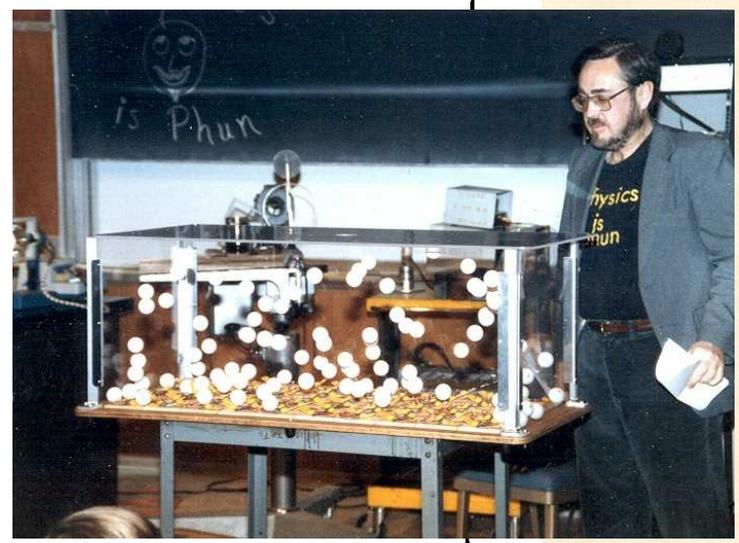
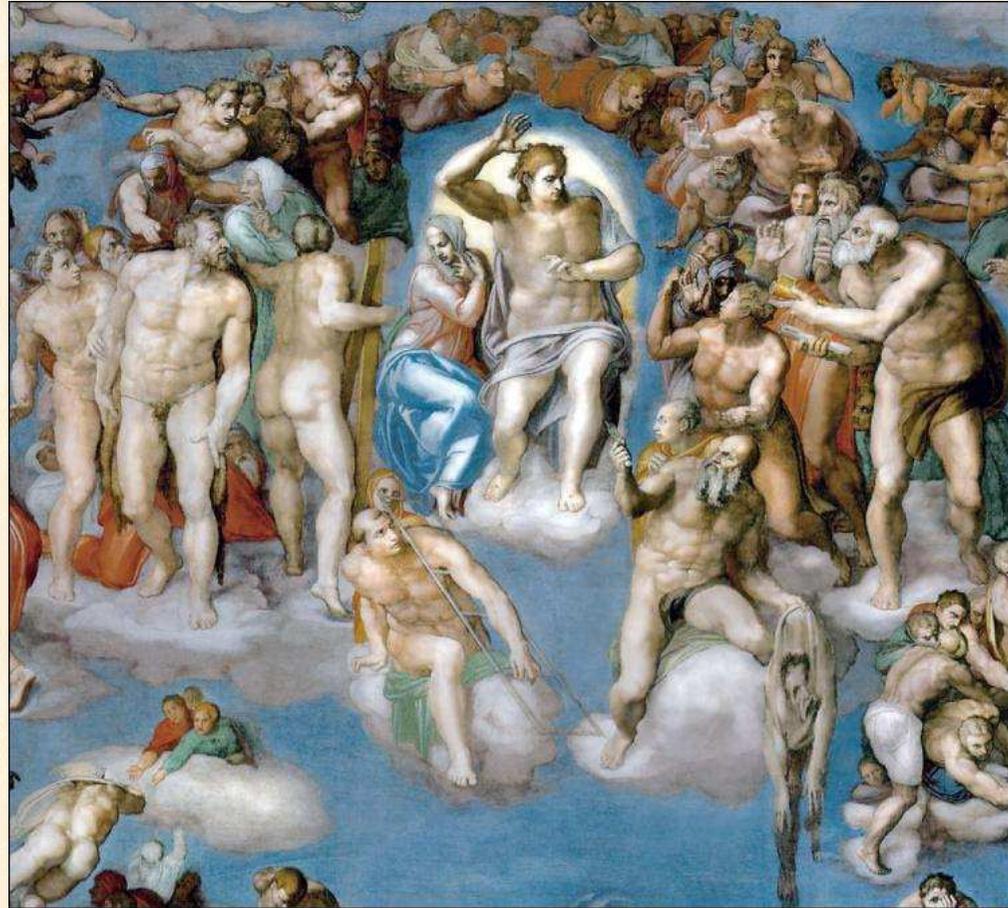


Figure 1. Estimated number of divorces and children involved in divorces: United States, 1950–88



*And he shall set the sheep on his right hand, but the goats on the left*  
[King James Bible, Matthew 25:33]

**How a norm  
cannot be  
restored:**



**...- this division is not recommended BEFORE the end of time.**

# final comments

Our examples suggest that indeed a separation of roles can be relevant. One cannot:

- be a murderer and punish crime,
- be a teetotaler and punish abstaining from drinking,
- use his gun from IRA to rob banks and punish this,
- divorce and punish divorcing.

Once a group of defectors appears, punishment is no more evolutionarily stable.

It is less clear if non-punishers could be classified as defectors. This is true with crimes, but what about non-drinking vodka? In the last case a sexual promiscuity seems to be a better group trait than the formal divorce.

**a norm  $\equiv$  to punish ?**

**Ευχαριστώ**