A LINE GRAPH
as
a model of a social network

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SOCIONICAL

FENS
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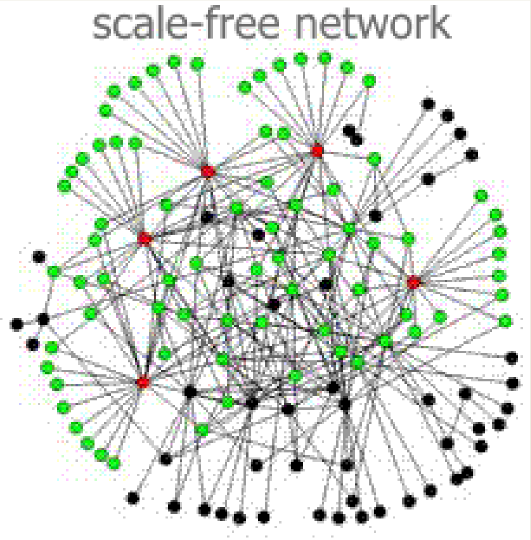
5 Ogólnopolskie Sympozjum
Fizyka w Ekonomii i Naukach Społecznych

outline

- ideas, definitions, milestones

- line graphs

- LiveJournal data

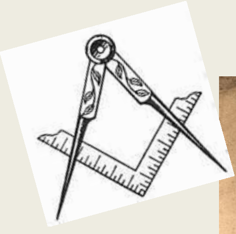


R. Albert ea, Nature 406, 378-382(27 July 2000)

Floor-level networking is often staged to look like friendly socializing. Nevertheless, it has an instrumental goal: useful contacts. Every networker knows this, so the instrumentality is not latent. It is not completely manifest, either. I presume that there is a mutual implicit contract of not breeching the situation by stating the instrumental goal aloud. The getting of contacts is a conscious but tacit function of socializing.

Juha Klemelä,
Turku University

Managing Mixed Emotions in the Layered Ritual Reality of Networking Events,
XVII ISA World Congress of Sociology, Gothenburg, 11-17 July 2010



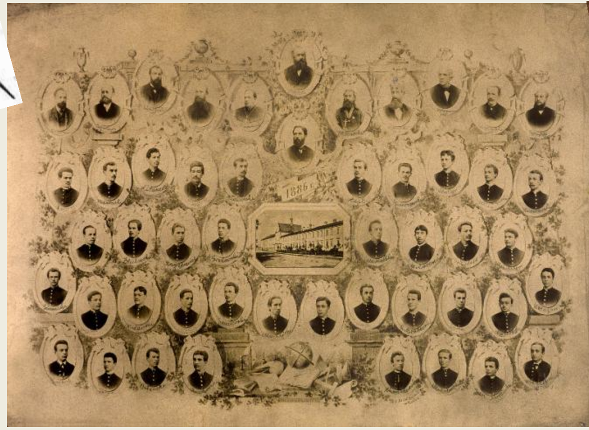
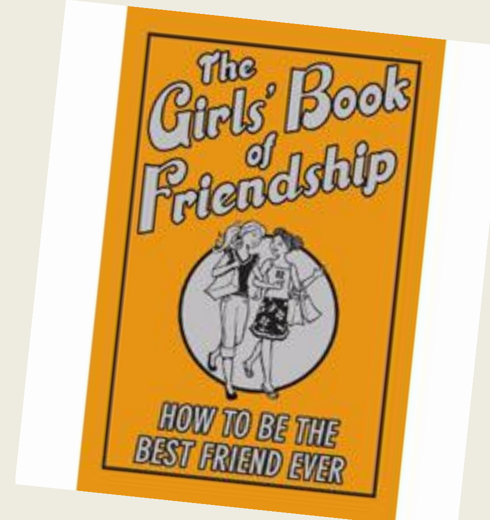
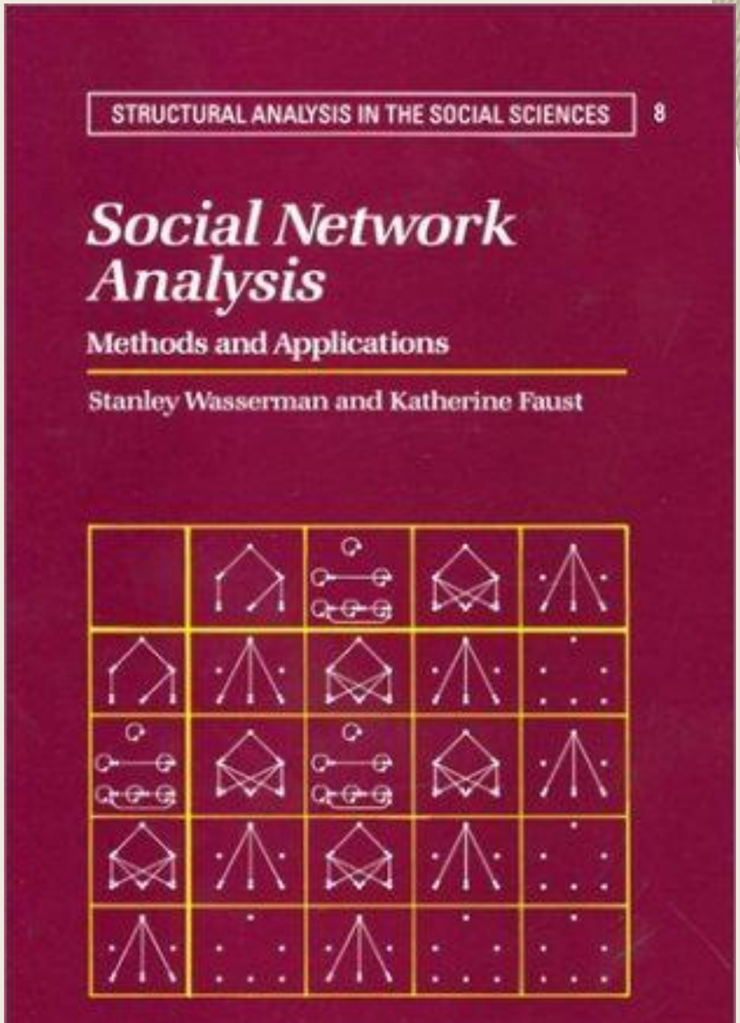
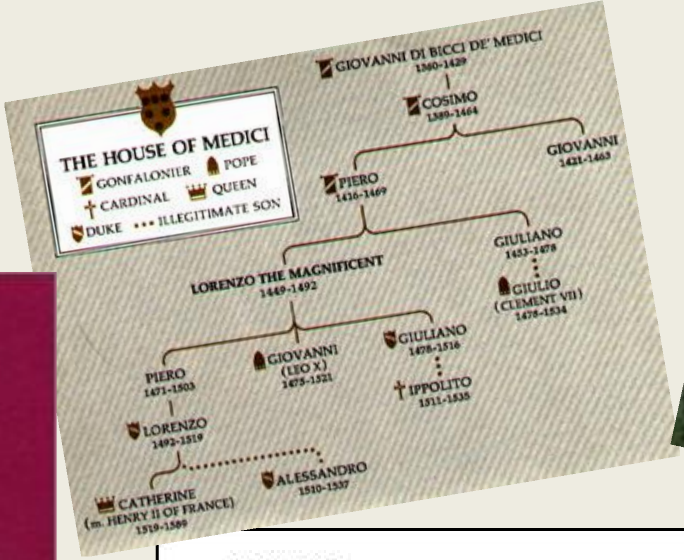


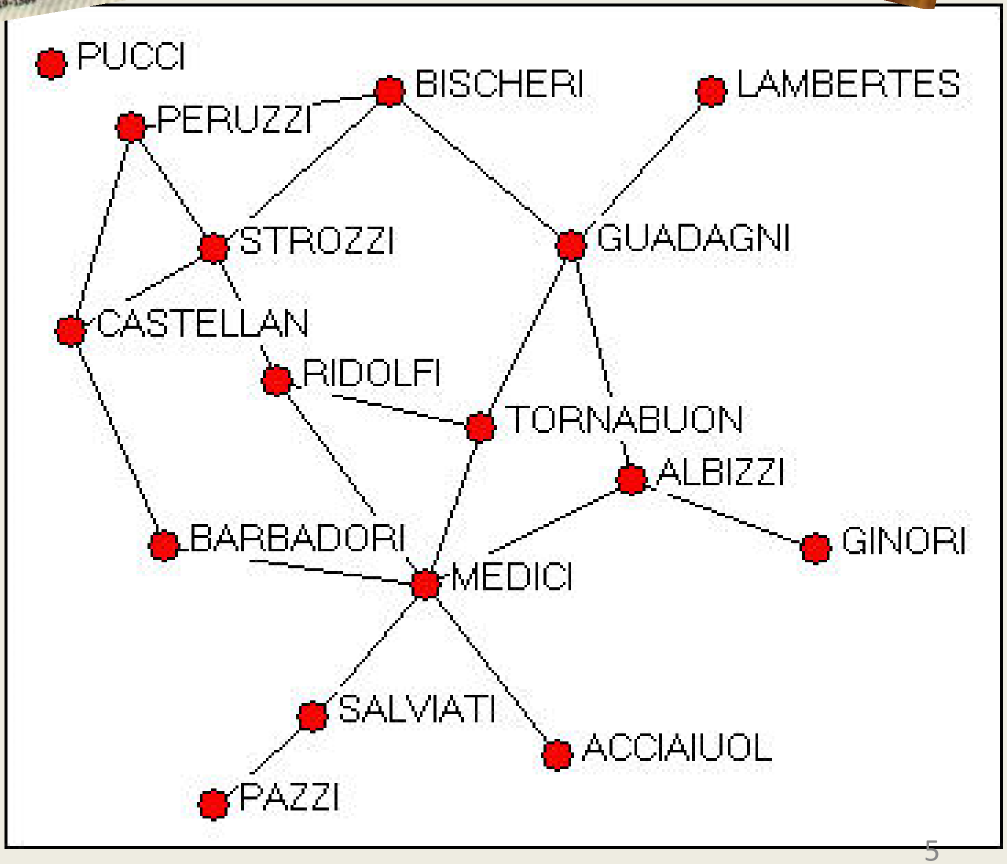

PHOTO: PAUL HUDSON/FSTOP/GETTY IMAGES


The Girls' Book of Friendship
HOW TO BE THE BEST FRIEND EVER


facebook
Facebook helps you connect and share with the people in your life.



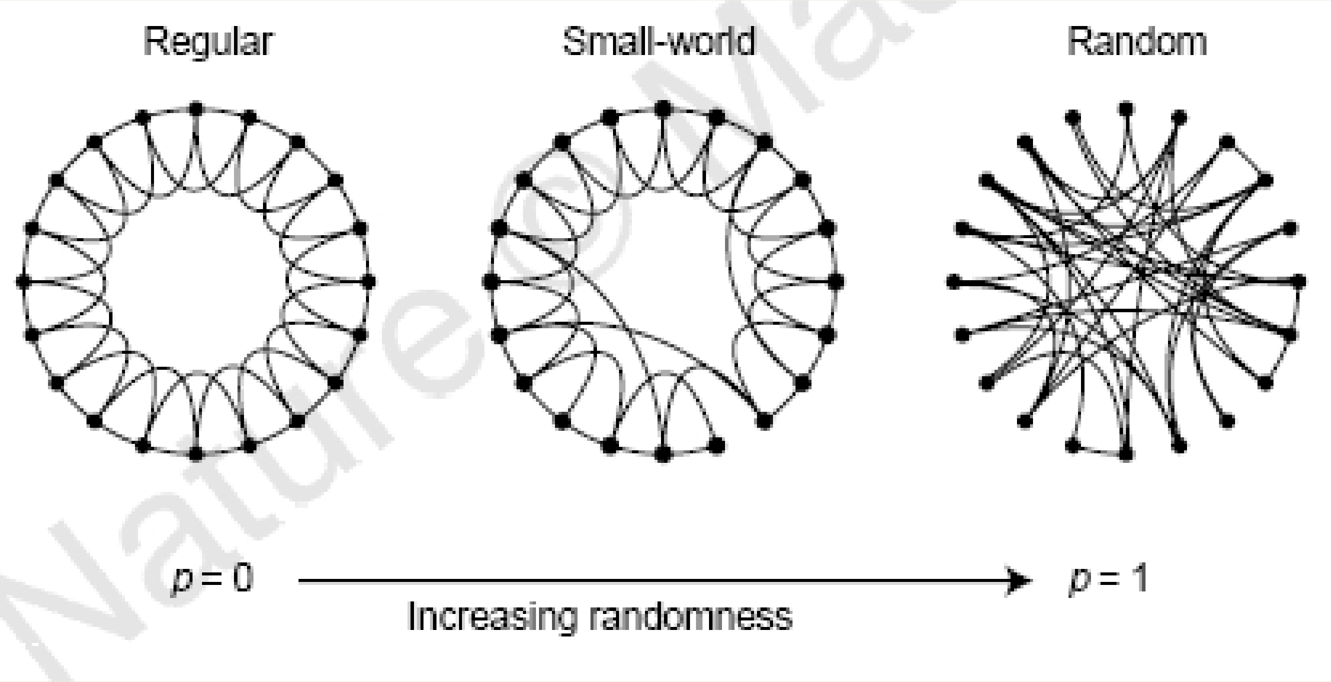




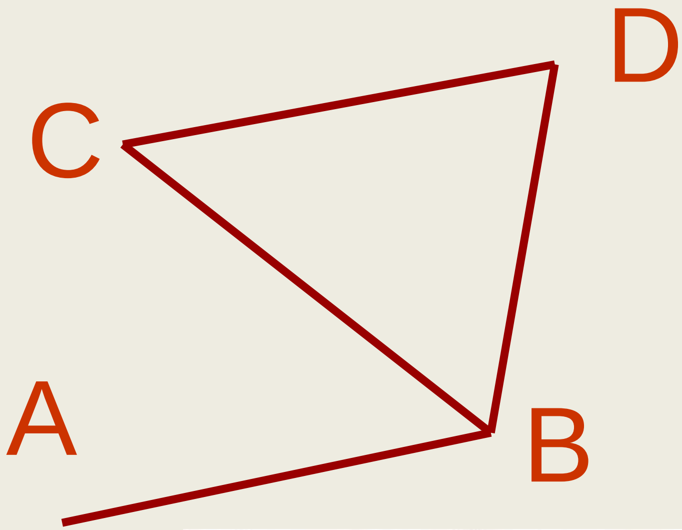
Cambridge UP, 1994

b. 1971
B.Sc. in physics
Ph.D. in theoretical and applied mechanics

b. 1959
B. Sc. in mathematics
PhD in applied mathematics



D J Watts, S H Strogatz, Collective dynamics of 'small-world' networks, *Nature* **393** (1998) 6684



Connectivity matrix („sociomatrix”)

$$\begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{matrix}$$

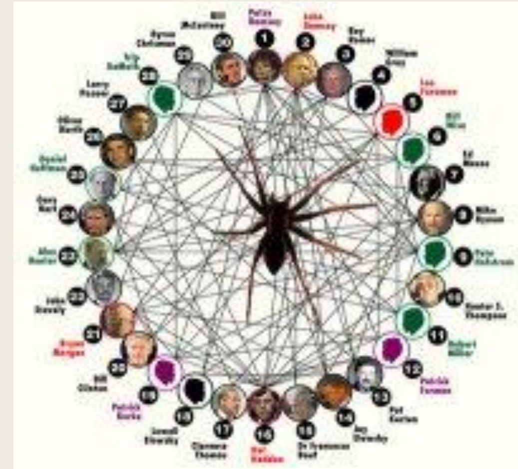
Clustering coefficient

$$C = \frac{1}{N} \sum_{i=1}^{N} \frac{2L_i}{k_i(k_i - 1)}$$

Mean free path („diameter”)

$$L(N) = \frac{1}{N(N-1)} \sum_{i,j=1}^{N} shortest\_path(i,j)$$

Small-world effect $\quad L(N) \propto \ln(N)$



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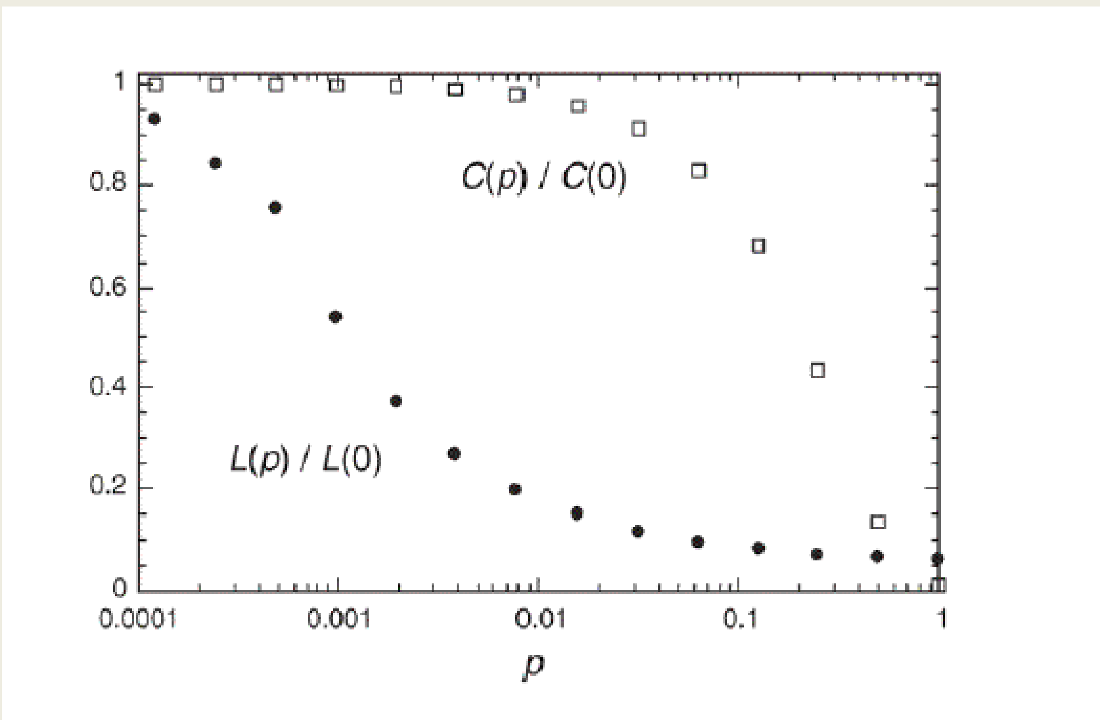
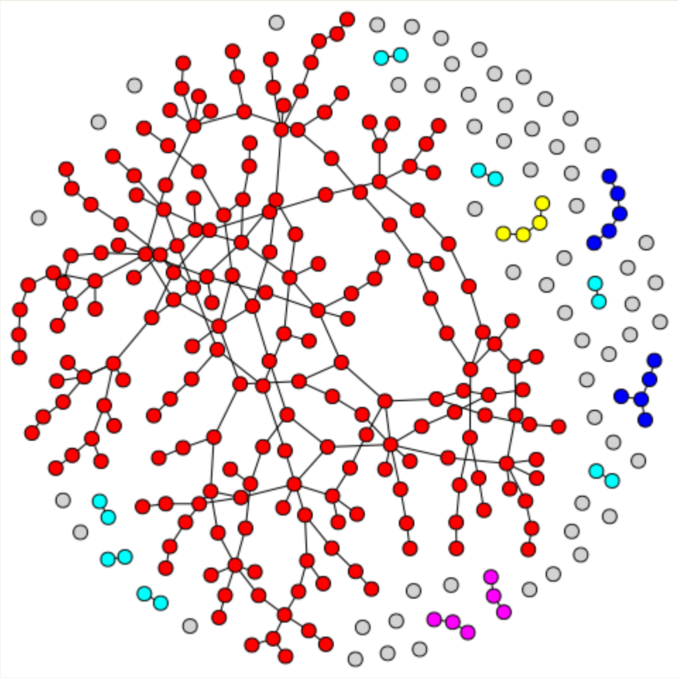


D J Watts, S H Strogatz, *Nature* **393** (1998) 6684

Erdös-Rényi networks



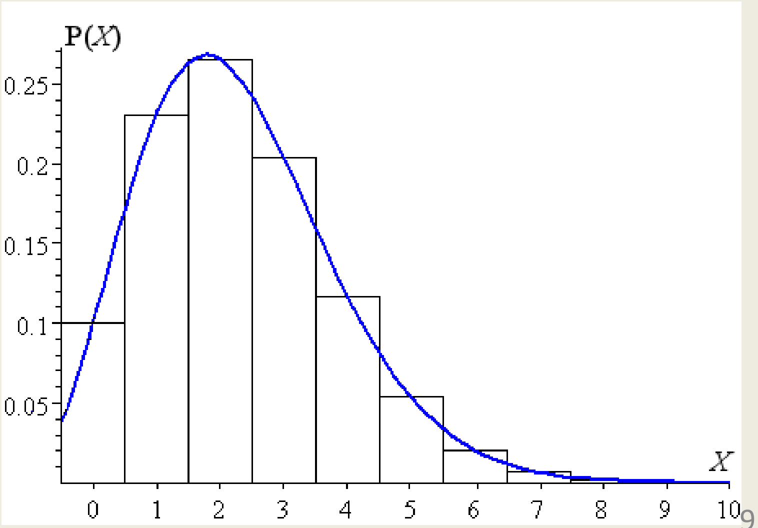
igraph.sourceforge.net/screenshots2.html

-Take N nodes
-Connect each pair with probability p

Mean degree
$<k> = (N-1)p$

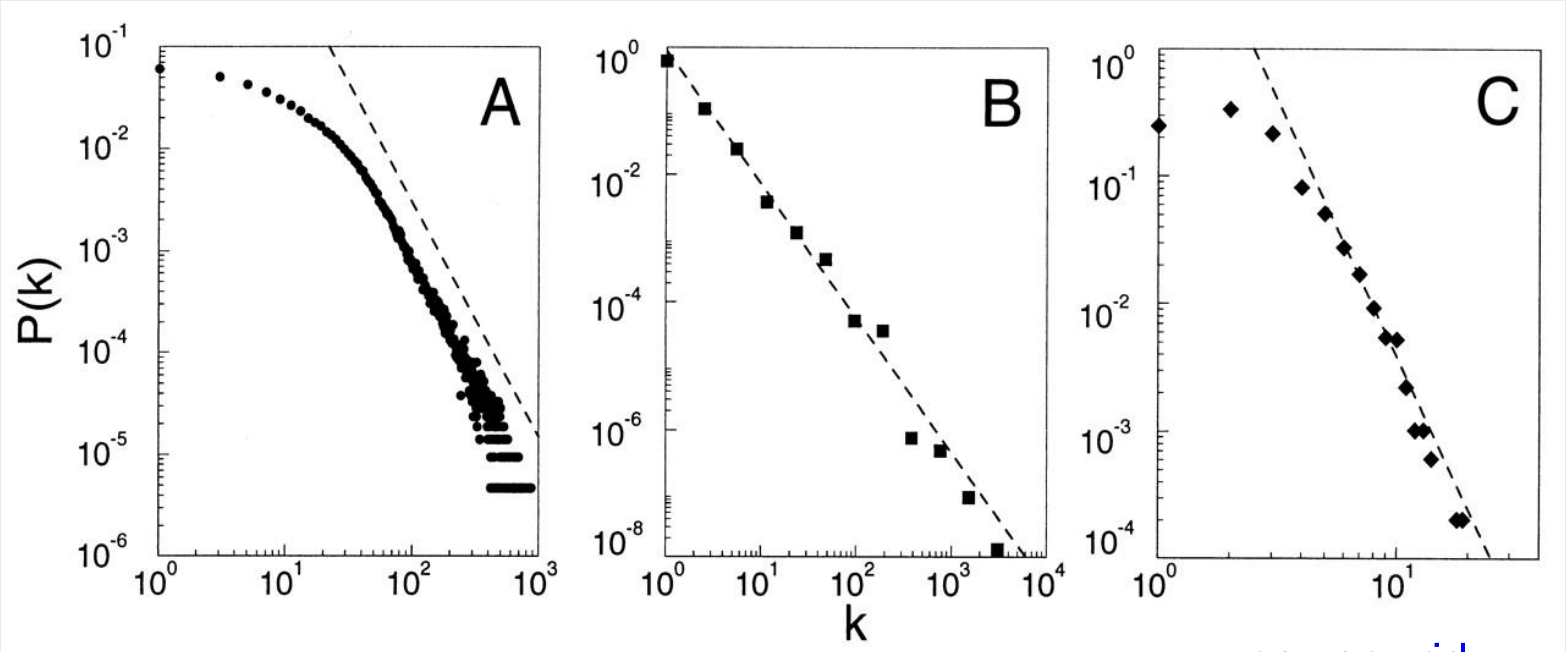
Degree distribution

$$P(k) = e^{-<k>} \frac{<k>^k}{k!}$$



PhD in complex networks 2001

b. 1967
MSc in physics and engineering
PhD in physics



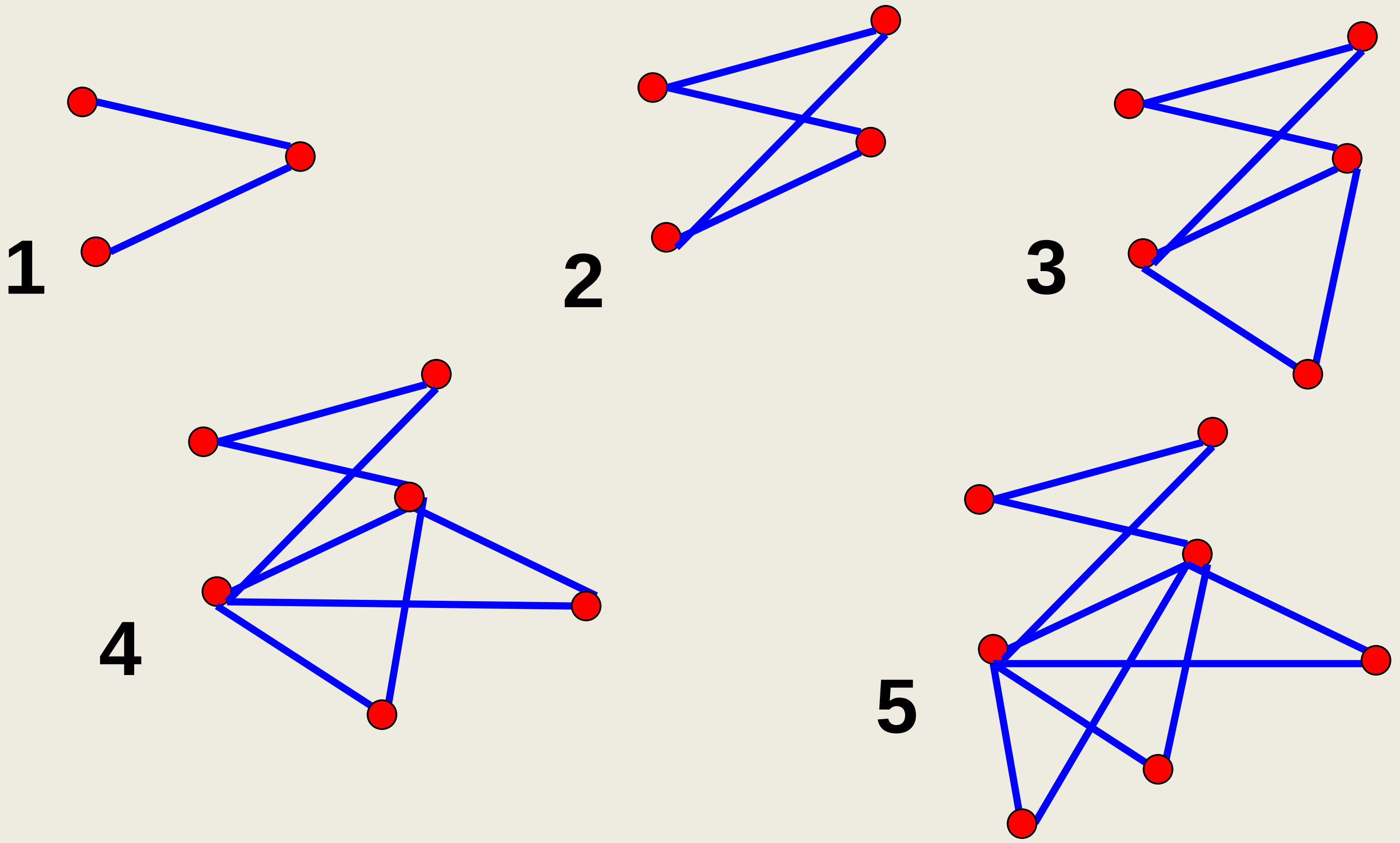
actors

www

power grid

R Albert, A-L Barabasi, Emergence of Scaling in Random Networks, *Science* 286 (1999) 509

Growing networks – construction for *M* =2



Nodes to attach a new node are selected with probability:

- constant (exponential graphs)
- proportional to their degree (scale-free graphs)

networks	N	#	d	C	γ
actors	**45x10^4**	**25x10^6**	**3.48**	**0.78**	**2.3**
www Altavista	2x10^9	2x10^10	16.18	-	2.1/2.7
coauthorship - math	25x10^4	50x10^4	7.57	0.34	-
coauthorship - phys	5^10^4	25x10^4	6.19	0.56	-
Phone calls	47x10^6	8x10^7	-	-	2.1
Web chains in water	92	997	1.9	0.087	-
Protein interactions	2115	2240	6.8	0.071	2.4
Sexual contacts	2810	-	-	-	3.2
Words in sentences	46x10^4	17x10^6	-	0.44	2.7

M E J Newman, SIAM Review 45, 167 (2003)

Assortativity

$$r = \frac{\sum_{jk} jk(e_{jk} - q_j q_k)}{\sigma_q^2}$$

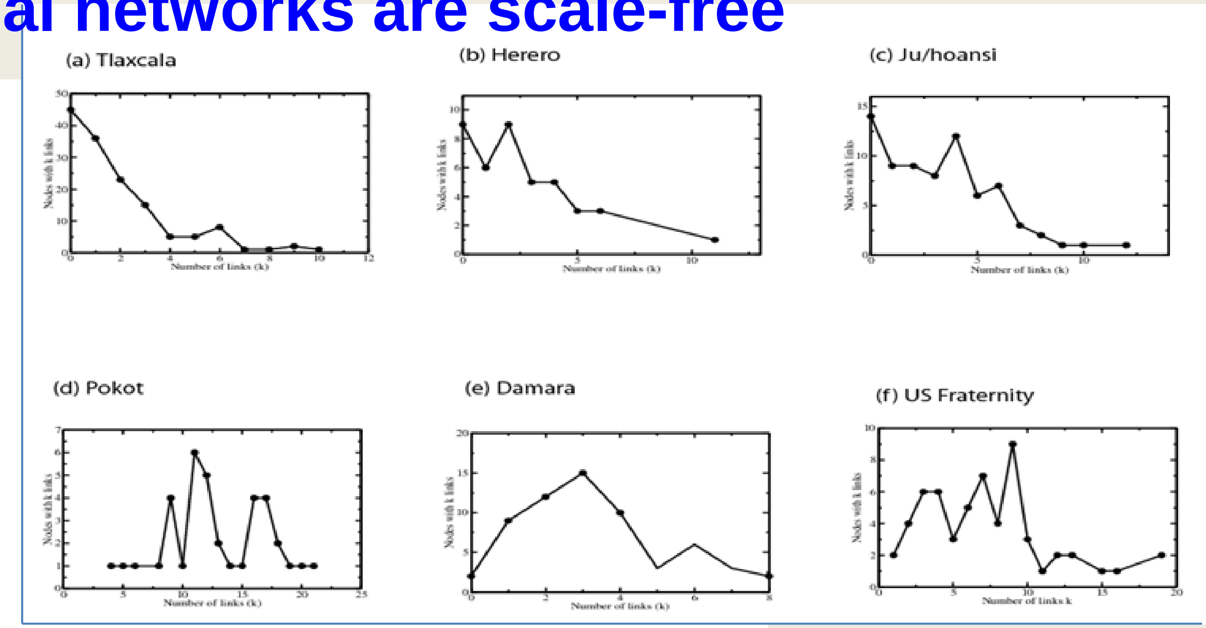
where $q_k = \frac{(k+1)p_{k+1}}{<k>}$

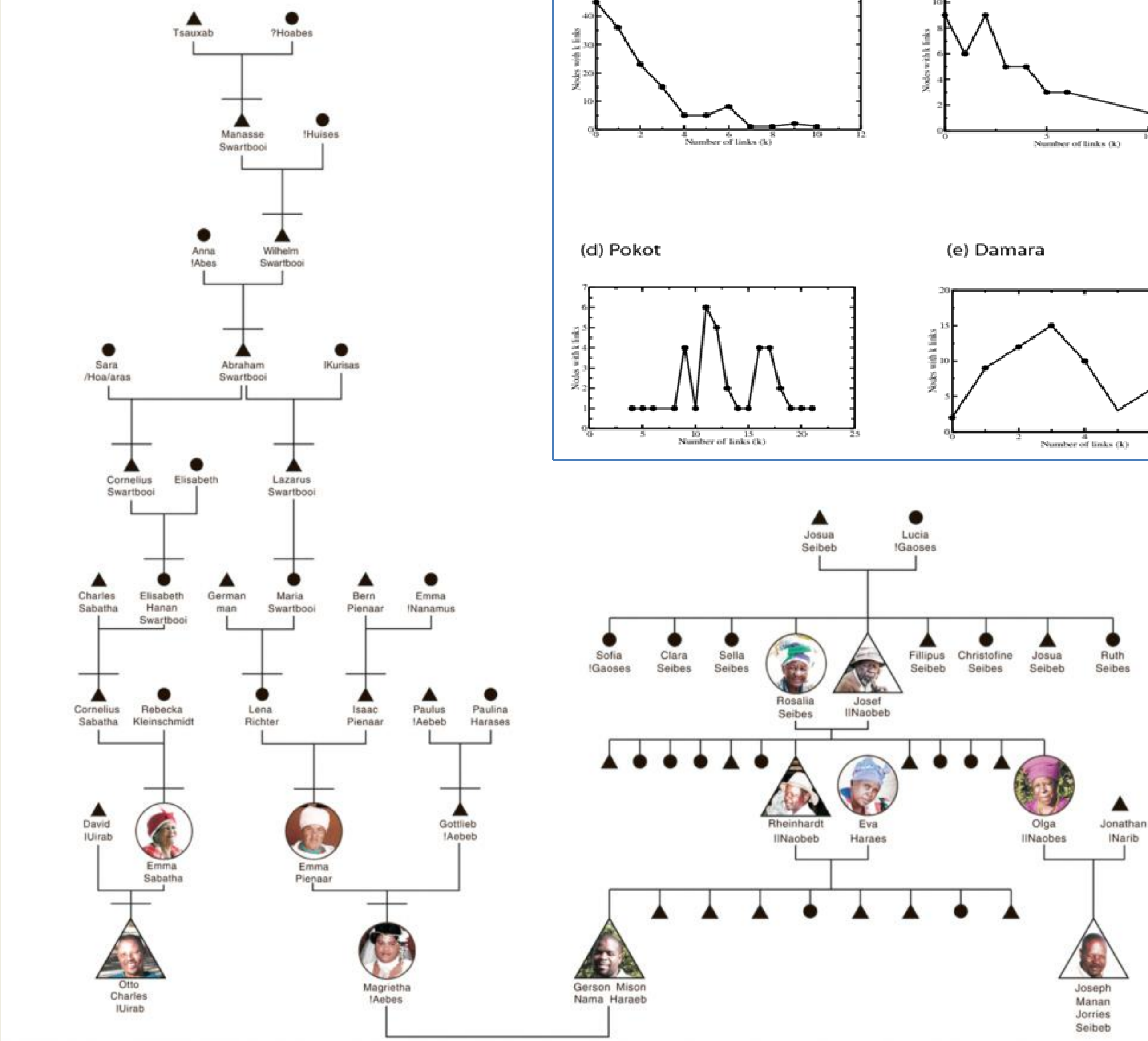
e_{jk} – fraction of links between nodes *j*,*k*

	network	type	size n	assortativity r	error σ_r
social	physics coauthorship	undirected	52 909	0.363	0.002
	biology coauthorship	undirected	1 520 251	0.127	0.0004
	mathematics coauthorship	undirected	253 339	0.120	0.002
	film actor collaborations	undirected	449 913	0.208	0.0002
	company directors	undirected	7 673	0.276	0.004
	student relationships	undirected	573	−0.029	0.037
	email address books	directed	16 881	0.092	0.004
technological	power grid	undirected	4 941	−0.003	0.013
	Internet	undirected	10 697	−0.189	0.002
	World-Wide Web	directed	269 504	−0.067	0.0002
	software dependencies	directed	3 162	−0.016	0.020
biological	protein interactions	undirected	2 115	−0.156	0.010
	metabolic network	undirected	765	−0.240	0.007
	neural network	directed	307	−0.226	0.016
	marine food web	directed	134	−0.263	0.037
	freshwater food web	directed	92	−0.326	0.031

MEJ Newman, PRE 67 (2003) 026126

Why not ALL social networks are scale-free





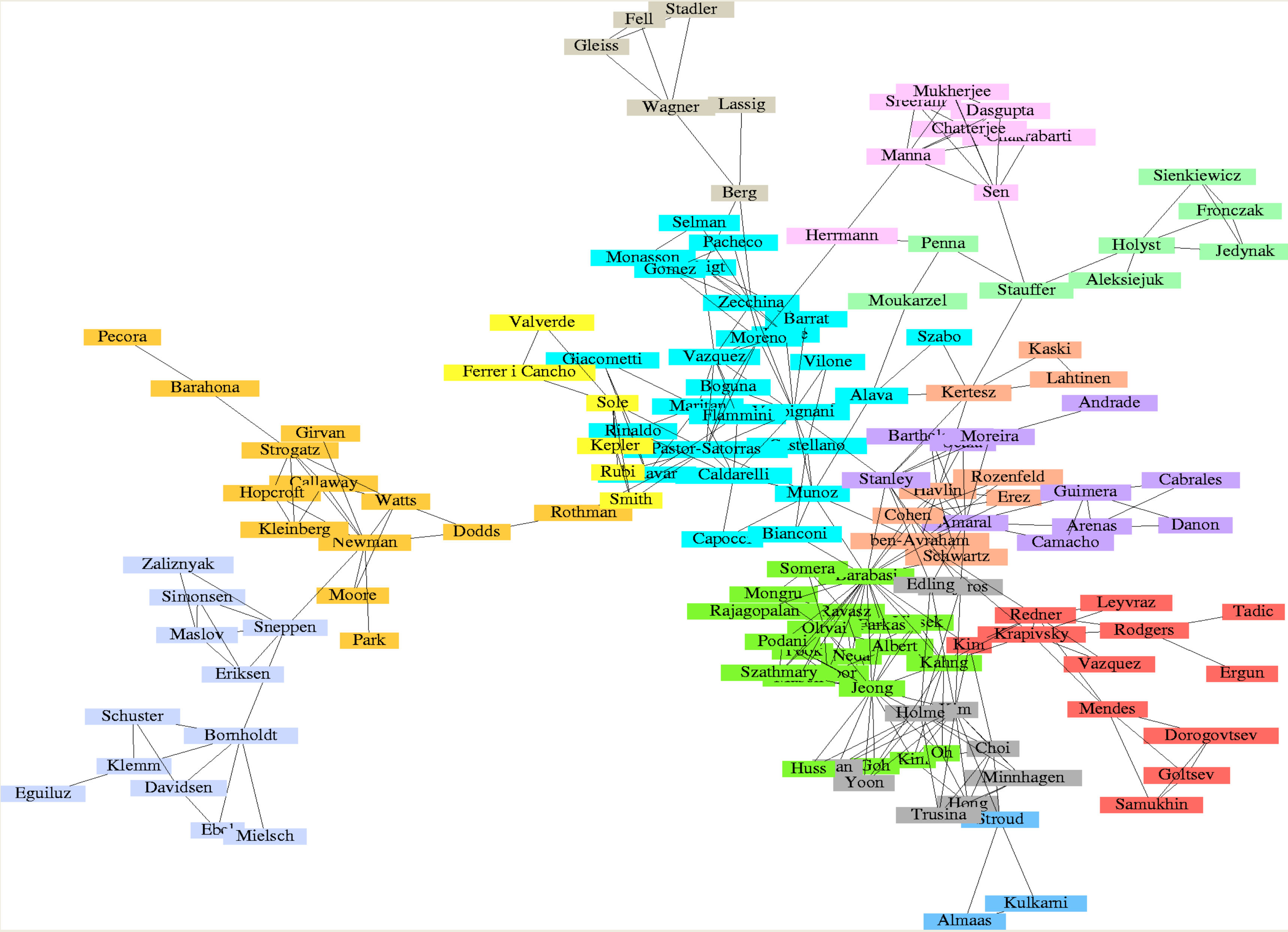
Michael Schnegg, http://www.fransfontein.org/; IJMPC 17 (2006) 1067

Why social networks are different from other types of networks

In this paper we have argued that social and non-social networks differ in two important ways. First, they show distinctly different patterns of correlation between the degrees of adjacent vertices, with degrees being positively correlated (assortative mixing) in most social networks and negatively correlated (disassortative mixing) in most non-social networks. Second, social networks show high levels of clustering or network transitivity, whereas clustering in many non-social networks is no higher than one would expect on the basis of pure chance, given the observed degree distribution.

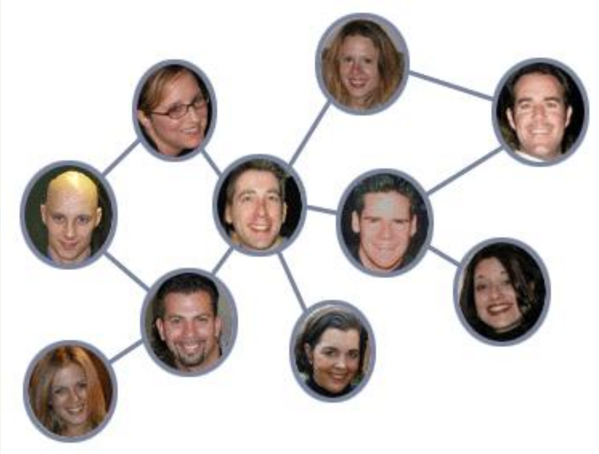
We have shown that both of these differences can be explained by the same hypothesis, that social networks are divided into communities, and non-social networks are not.

MEJ Newman, J Park, PRE 68 (2003) 036122



MEJ Newman, J Park, PRE 68 (2003) 036122

It is used to assume



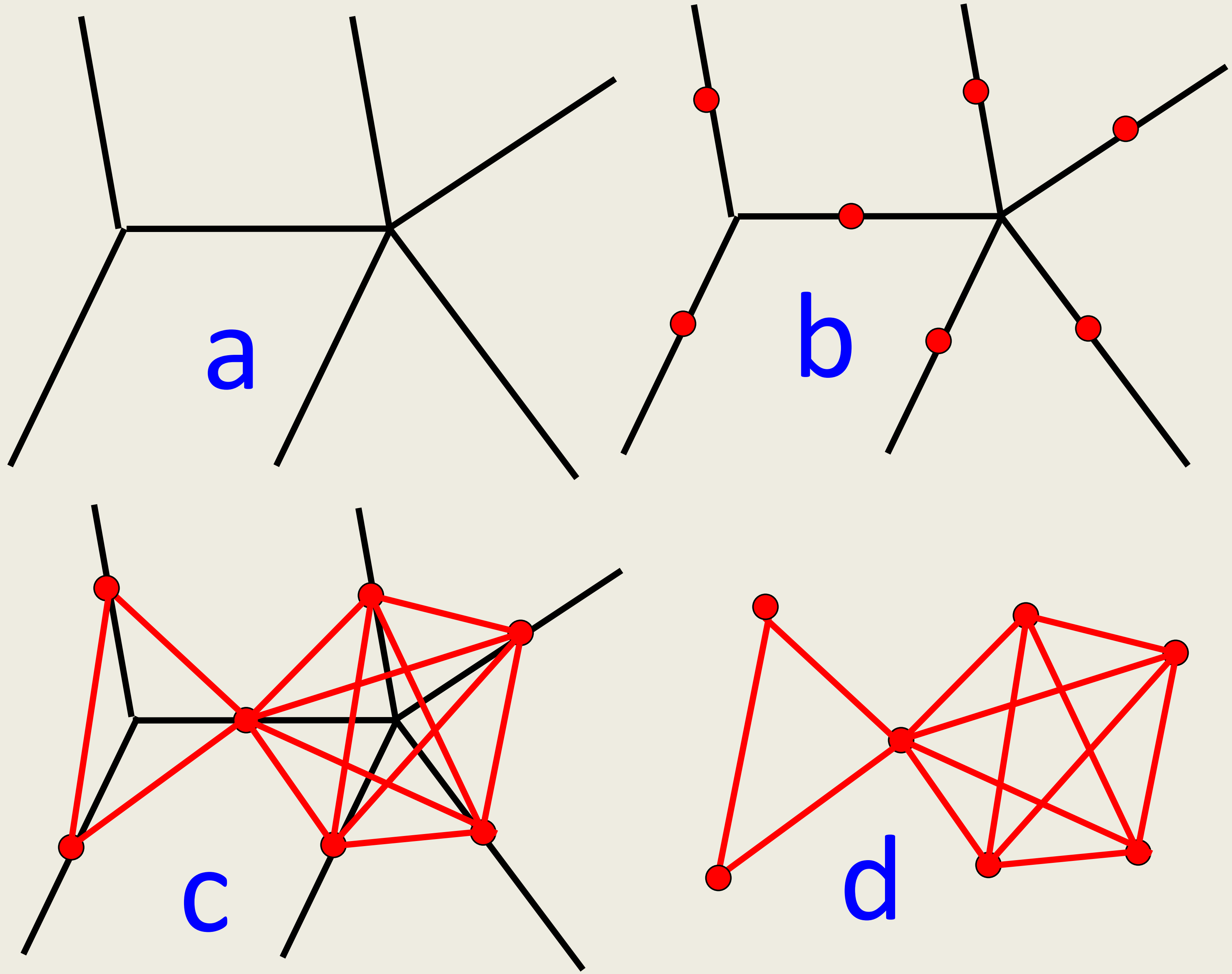
but it could be

Role of line graphs here

In the model proposed here, a social network is the line graph of an initial network of families, communities, interest groups, school classes and small companies. These groups play the role of nodes, and individuals are represented by links between these nodes.

M J Krawczyk et al, arXiv:1010.2460

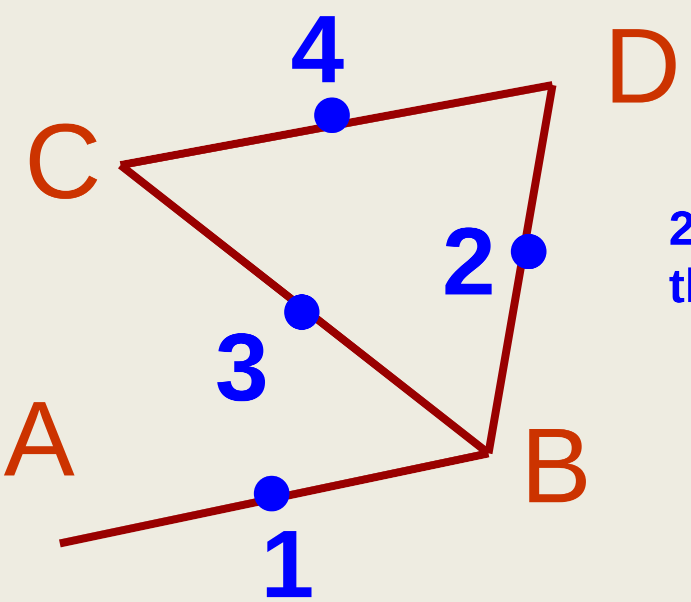
line graph – the construction



Algorithm

1. assign numbers to the elements of the connectivity matrix above the diagonal. Make the matrix symmetric.

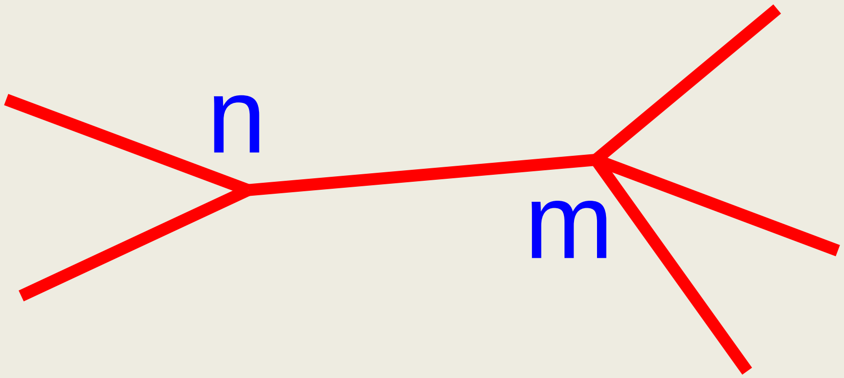
0	1	0	0
1	0	2	3
0	2	0	4
0	3	4	0



2. If *i,j* are in the same row or column, then the element *C(i,j)* of the transformed matrix is 1

0	1	1	0
1	0	1	1
1	1	0	1
0	1	1	0

Degree distribution of a line graph

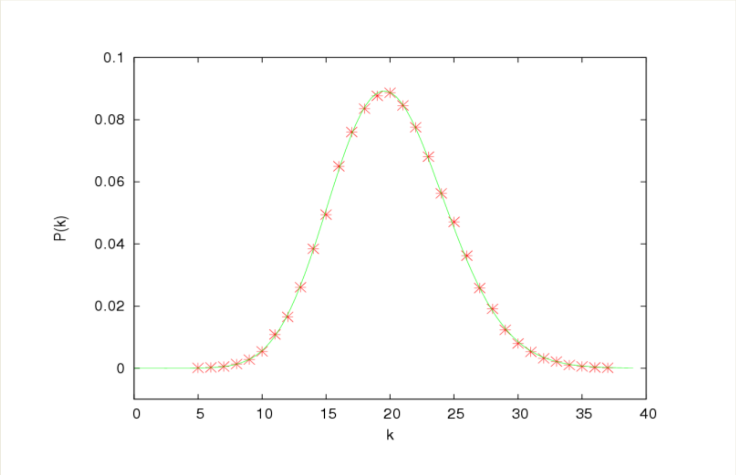


$$P_t(k) = \frac{\sum_{n=1}^{\infty} nP(n) \sum_{m=1}^{\infty} mP(m)\delta_{k,m+n-2}}{\sum_{n=1}^{\infty} nP(n) \sum_{m=1}^{\infty} mP(m)} =$$

$$= \frac{1}{\lambda^2} \sum_{n=1}^{k+1} nP(n)(k-n+2)P(k-n+2)$$

A. Mańka-Krasoń et al, Comp. Phys. Comm. 110 (2010) 118

Degree distribution *P(k)*

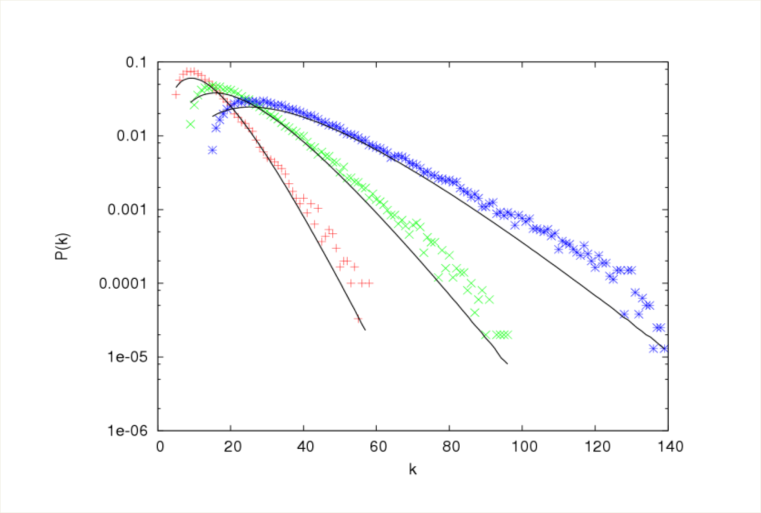


Erdös-Rényi networks

$$P_t(k) = \lambda^k e^{-2\lambda} \sum_{n=1}^{k+1} \frac{1}{(n-1)!\,(k-n+1)!} =$$

$$= e^{-2\lambda} \frac{(2\lambda)^k}{k!}$$

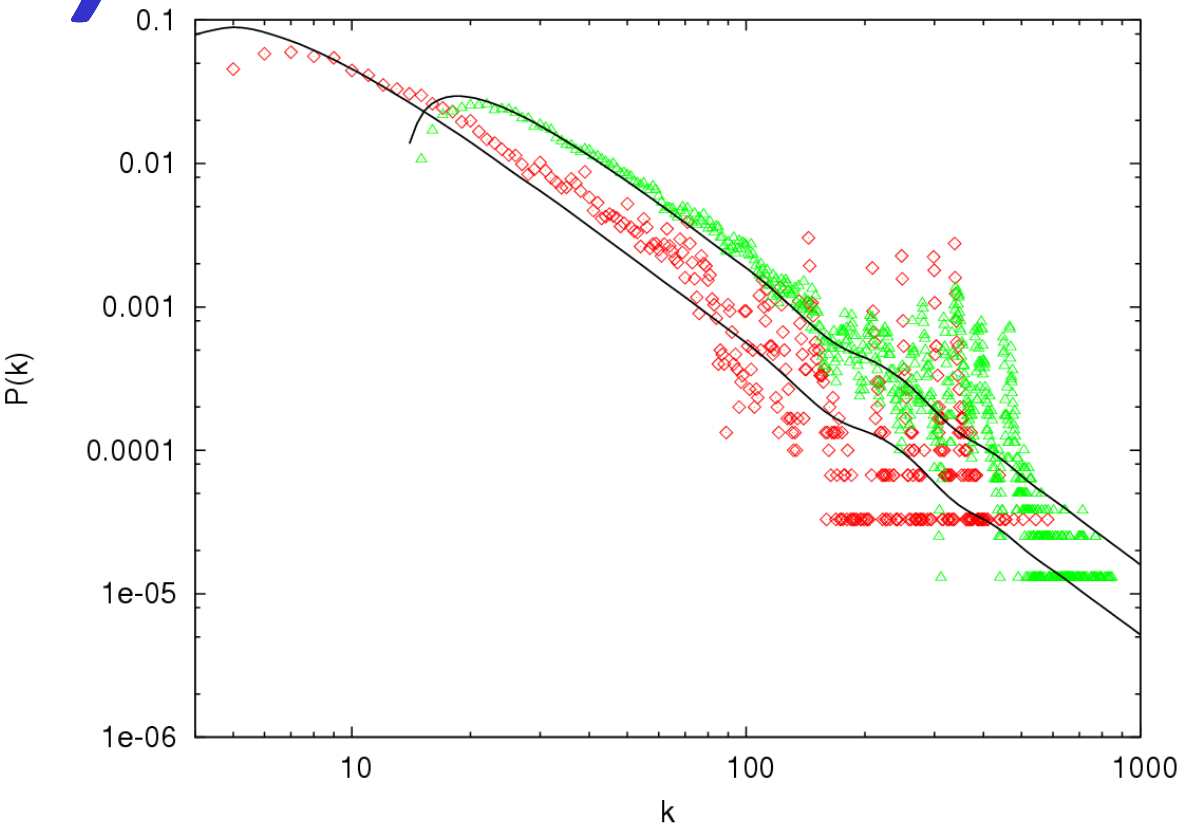
exponential networks



$$P_t(k) = \frac{(1-c)^4}{6} (k+1)(k+2)(k+3)c^k$$

The degree distribution of a line graph on a scale-free network

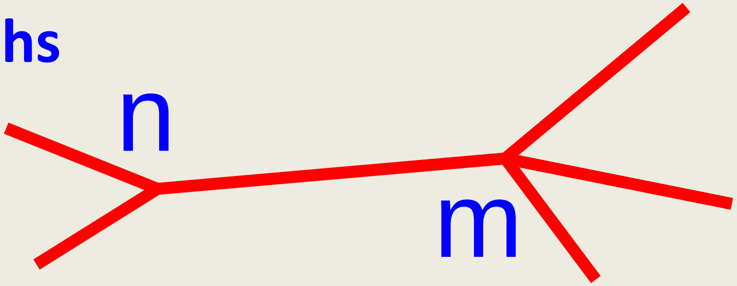
$P_t(k)$



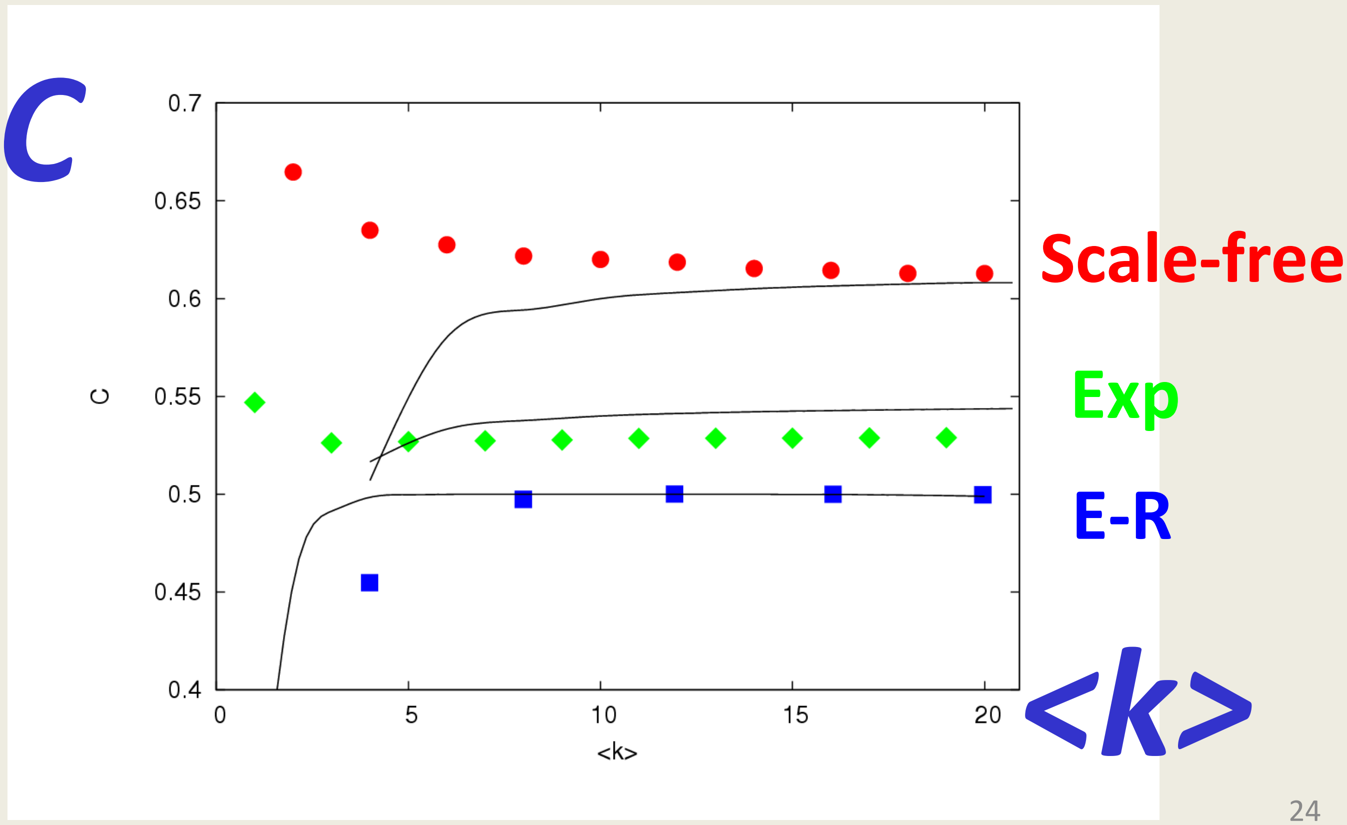
k

$<k>=6,16$

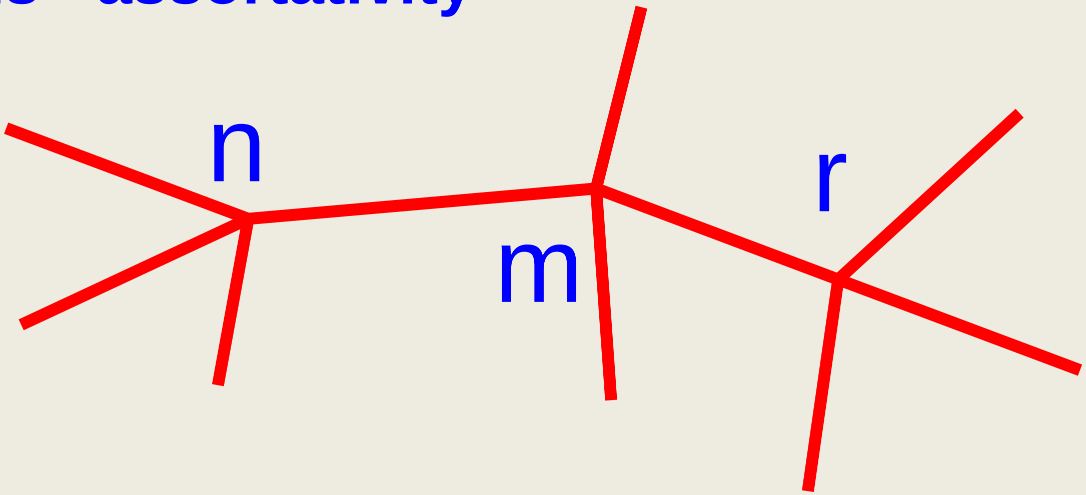
Clustering coefficient in line graphs



$$C = \frac{1}{\lambda^2}\sum_{n=1}^{\infty} nP(n)\sum_{m=1}^{\infty} mP(m)\frac{(n-1)(n-2)+(m-1)(m-2)}{(n+m-2)(n+m-3)}$$



Line graphs - assortativity



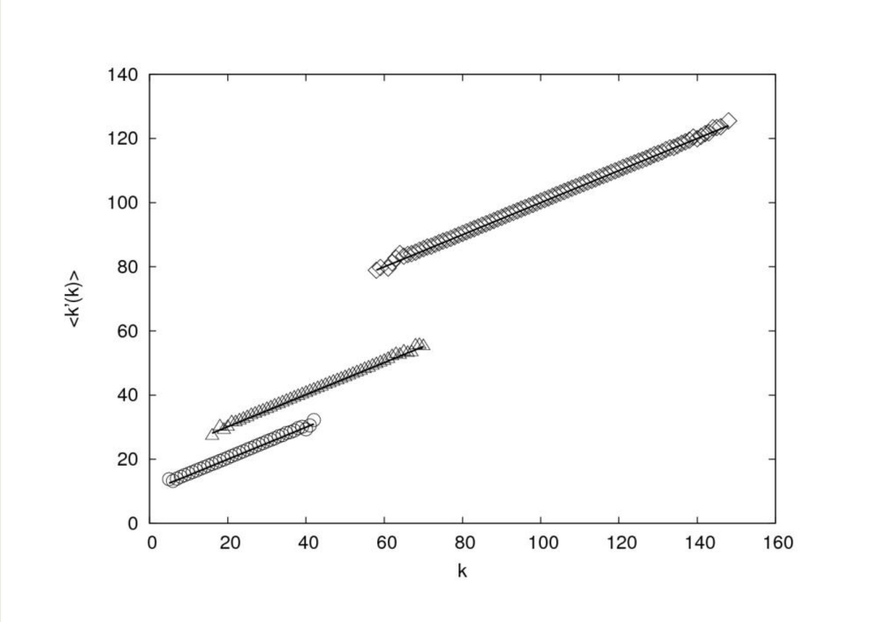
$$< k'(k) >= \frac{\sum_n nP(n)\sum_m mP(m)\sum_r rP(r)(n+m-2)\delta_{k,m+r-2}}{\sum_n nP(n)\sum_m mP(m)\sum_r rP(r)\delta_{k,m+r-2}}$$

A. Mańka-Krasoń et al, APPB Proc Suppl 3 (2010) 259

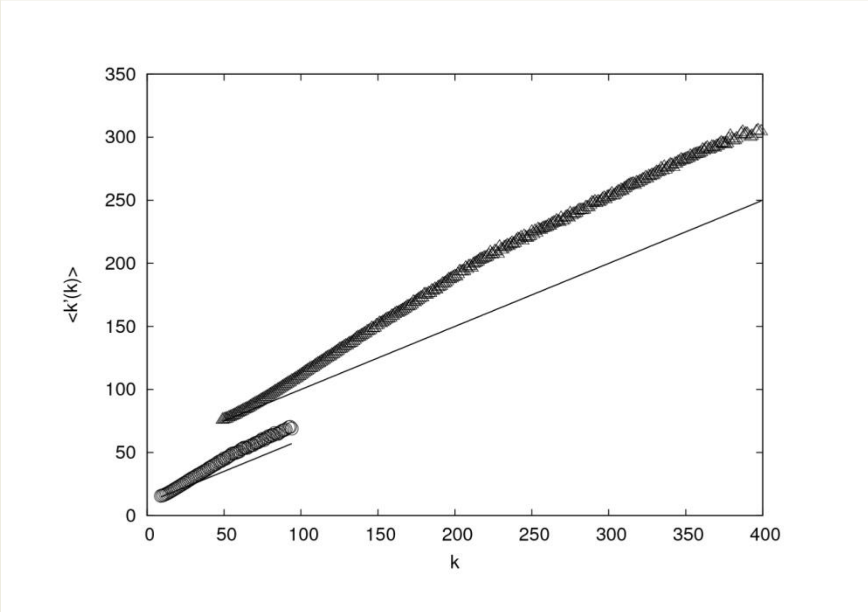
Assortativity <*k'(k)*>

Erdös-Rényi networks

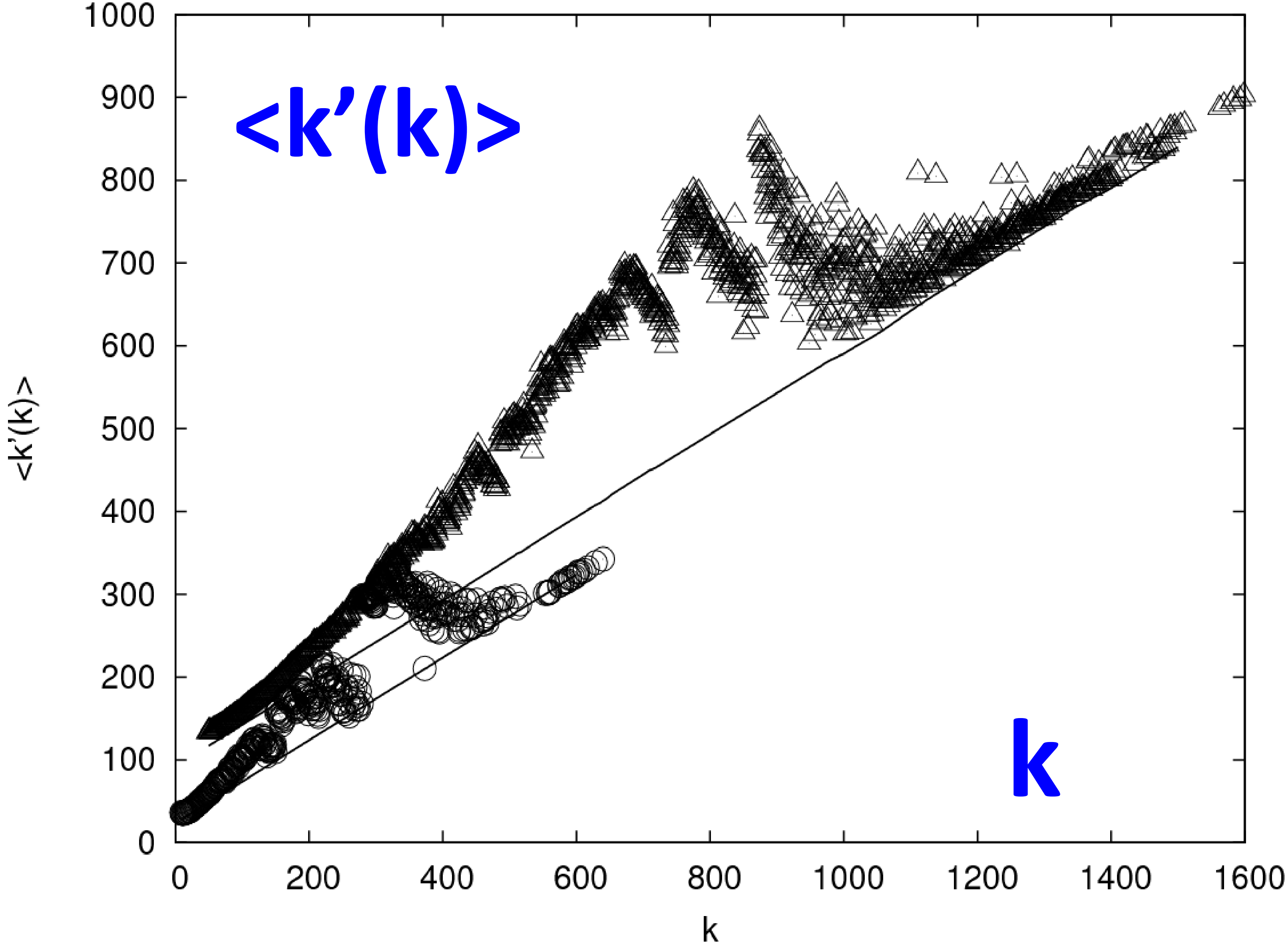
$$< k'(k) >= \lambda + \frac{2^{k-1}k}{2^k - 1} \approx \lambda + \frac{k}{2}$$



exponential networks



Assortativity of a line graph on a scale-free network



LiveJournal

LiveJournal is a remarkably popular platform for personal blog management, populated with over 8 million blogs and over 1 million of communities. LiveJournal was among the first of such platforms available online and it still remains one of the most active and popular. Its users manage personal blogs where they share their daily experiences, political views or discuss news events. Users can also comment on posts of other users.

We defined the network nodes to correspond to personal blogs. Directional links connecting these nodes represent the record that a particular user (owning one blog) monitors another blog (owned by another user).

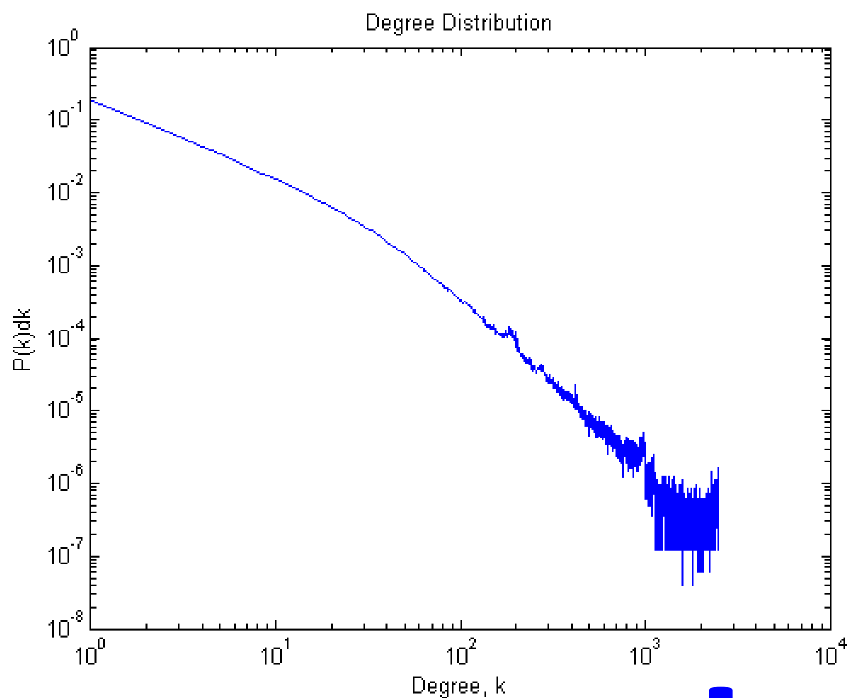
M J Krawczyk et al, arXiv:1010.2460

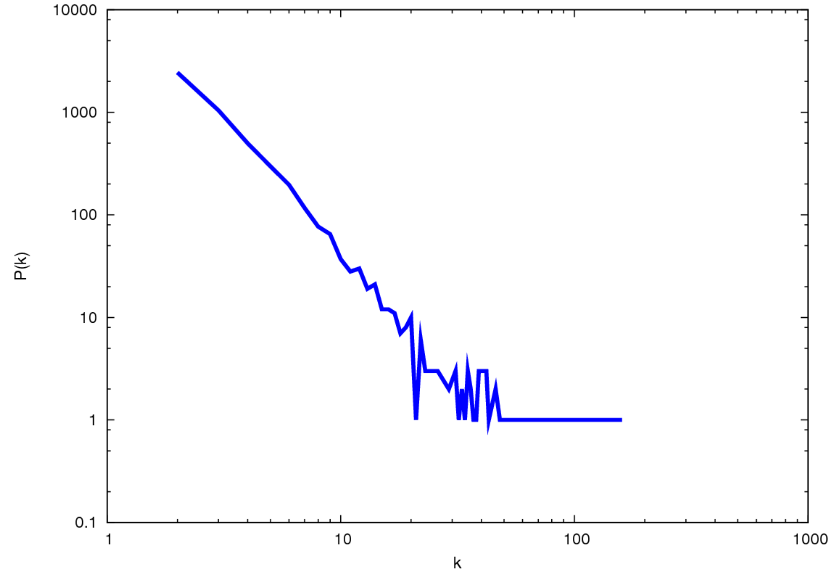
LiveJournal: data

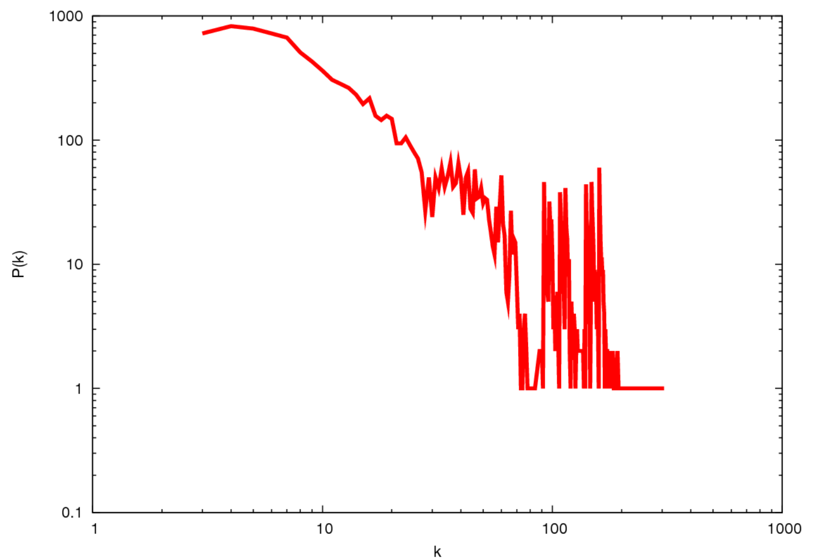
$N=8.1*10^6$; $\#=125*10^6$

simulations: $N=9*10^3$

P(k)





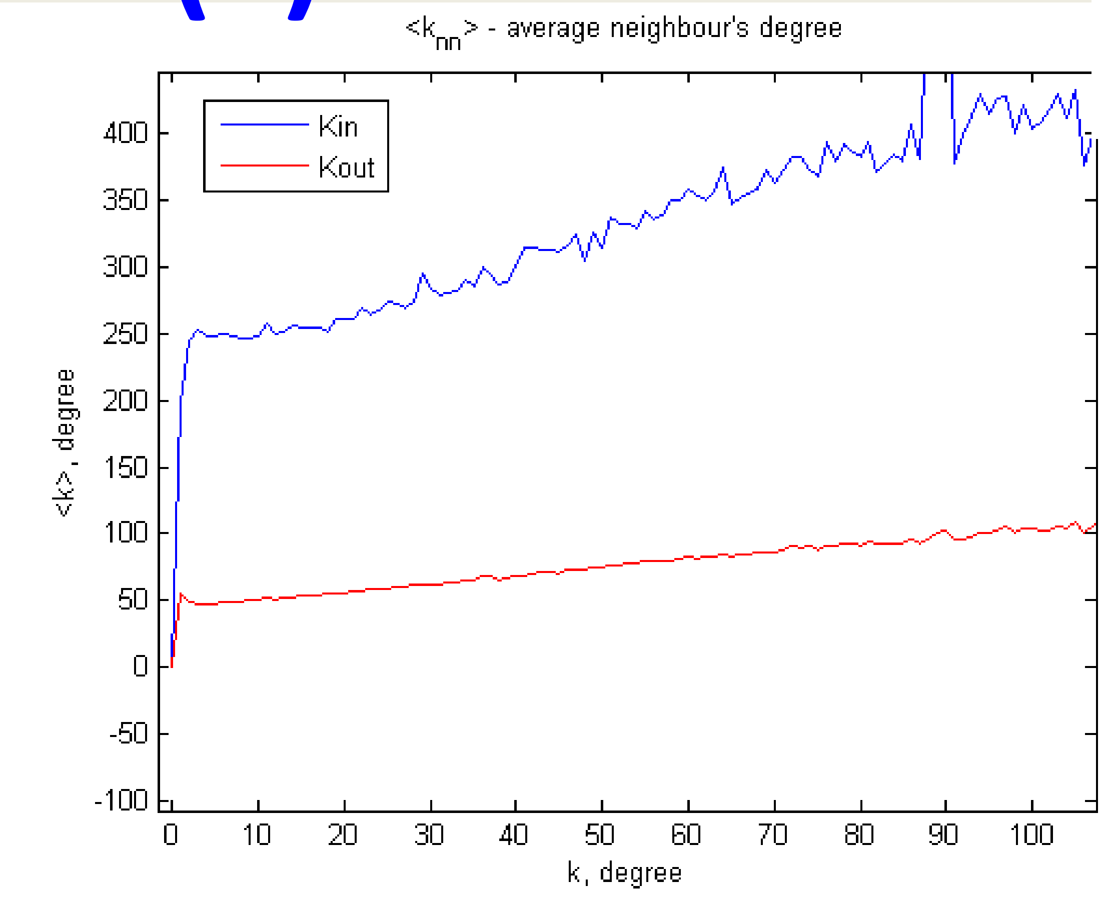


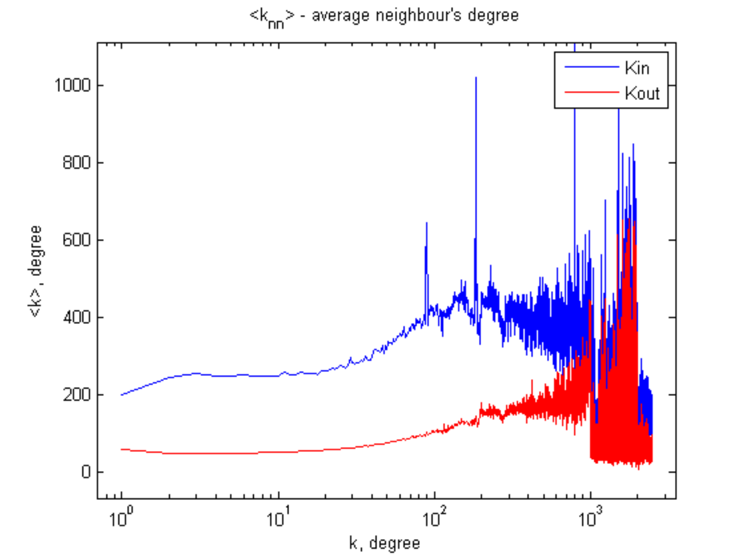
k

LiveJournal:

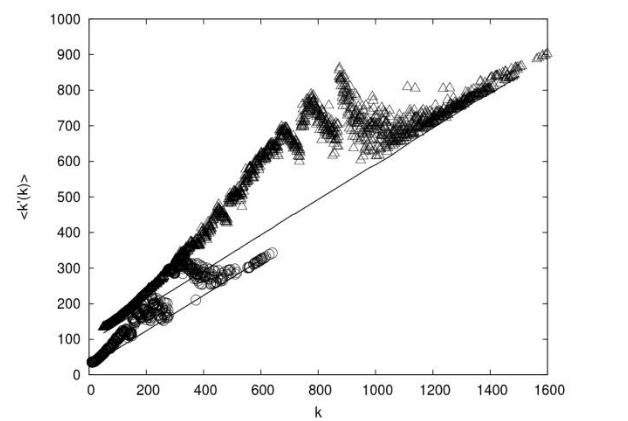
assortativity

<k'(k)>





simulations

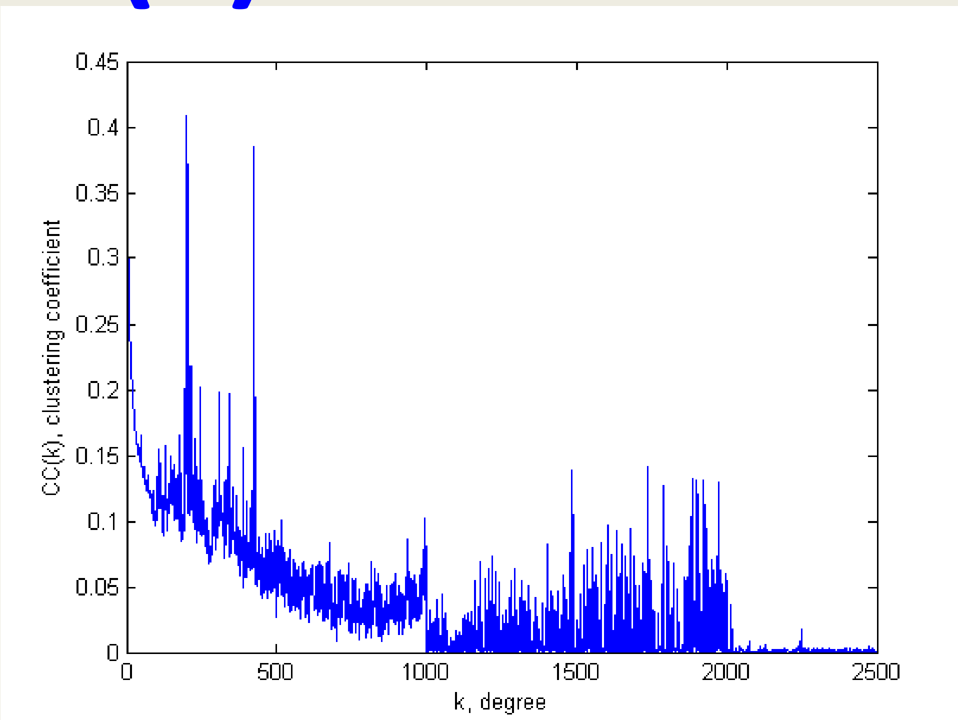


k

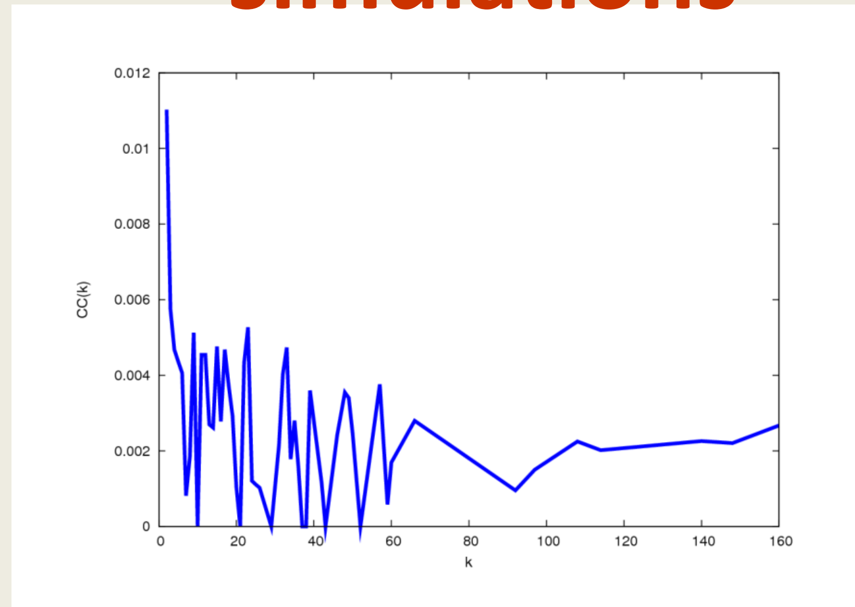
simulations

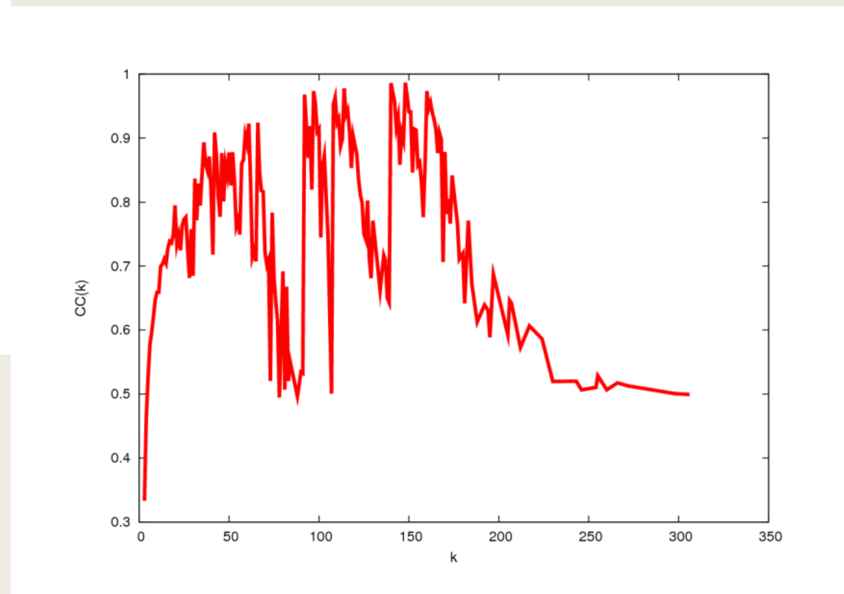
LiveJournal

C(k)



k





conclusions

The degree distribution of a line graph is close to the degree distribution of its initial network. Line graphs are clusterized and assortative. The degree-dependent clustering coefficient *C(k)* indicates the presence of cliques.

We have shown that LiveJournal, where *P(k)* is scale-free, displays qualitatively the same features.

Thank you