

# More stochastic repulsion in culture dissemination

Małgorzata Krawczyk, Krzysztof Kułakowski

*AGH University of Science and Technology,  
Faculty of Physics and Applied Computer Science,  
al. Mickiewicza 30, Kraków 30-059, Poland*



## *outline*

1. Bounded confidence and repulsion
2. The Axelrod model of culture dissemination
3. What some other authors have done
4. Results when there is no repulsion
5. New here: more stochastic repulsion
6. New here: the role of an initial state
7. Results of initial preparation
8. Results of repulsion
9. Conclusions

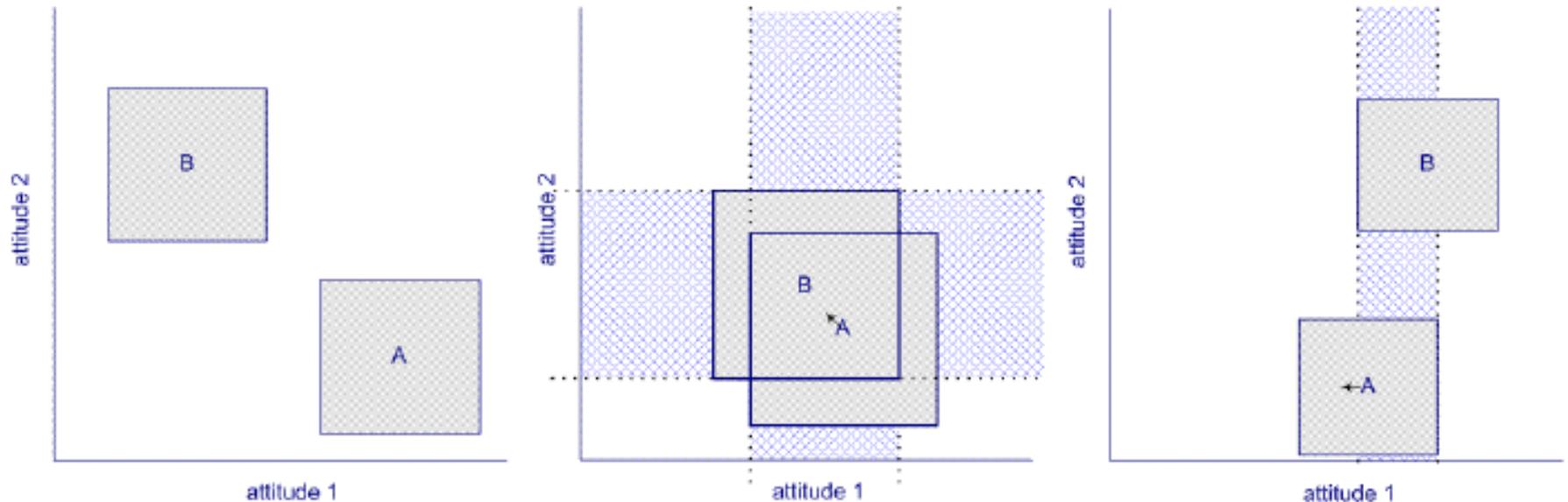


*„square  
lattice”*

# *bounded confidence and repulsion*



Fig. 1 illustrates the different types of interactions.



**Fig. 1.** Situations of interaction. Left: no influence. Middle: B attracts A. Right: B rejects A on attitude 1 and does not influence A on its attitude 2. The centres of squares  $A$  and  $B$  are opinion vectors  $(a_1, a_2)$  and  $(b_1, b_2)$ . The size of the squares is the uncertainty  $u$ . The arrow shows the direction of change of  $A$ .

[S. Huet and G. Deffuant, *Bounded Confidence with Rejection: Clusters or Scattered Opinions?* 5-th Conference of the European Social Simulation Association, Brescia, Italy, September 1-5, 2008]

F=5 q=7

12335	34224	23455
12335	63114	44264
31421	24542	37423

2 active bonds

12335	34224	23455
12335	63111	44264
31421	24542	37423

frozen absorbing state



## ***The algorithm of the evolution in the Axelrod model***

The model we study is defined by considering  $N$  agents as the sites of a lattice. The state of agent  $i$  is a vector of  $F$  components (cultural features)  $(\sigma_{i1}, \sigma_{i2}, \dots, \sigma_{iF})$ . Each  $\sigma_{if}$  is one of the  $q$  integer values (cultural traits)  $1, \dots, q$ , initially assigned independently and with equal probability  $1/q$ . The time-discrete dynamics is defined as iterating the following steps:

1. Select at random a pair of neighboring lattice sites  $(i, j)$ .
2. Calculate the *overlap* (number of shared features)  
$$l(i, j) = \sum_{f=1}^F \delta_{\sigma_{if}, \sigma_{jf}}.$$
3. If  $0 < l(i, j) < F$ , the bond is said to be *active* and sites  $i$  and  $j$  interact with probability  $l(i, j)/F$ . In case of interaction, choose  $g$  randomly such that  $\sigma_{ig} \neq \sigma_{jg}$  and set  $\sigma_{ig} = \sigma_{jg}$ .

## *What some other authors have done*

- Axelrod: model formulation [J Confl Res 1997]
- Castellano ea: non-equilibrium phase transition with  $q$ ,  
scaling of  $n(s)$  at the transition  $q^*$ , languages? [PRL 2000]
- Klemm ea: ordering stabilized by noise, disordered phase metastable [PRE 2003a]
- Klemm ea: SF networks -  $q^* \propto N^{0.39}$ ; WS networks -  $q^* \propto p^{0.39}$  [PRE 2003b]
- Gonzalez-Avella: disorder stabilized by uniform interaction, like media [PRE 2005]
- Centola ea: breaking bonds between entirely different sites,  
non-stationary phase, anomie? [J Confl Res 2007]
- Radillo-Diaz ea: threshold-dependent **repulsion** [PRE 2009]

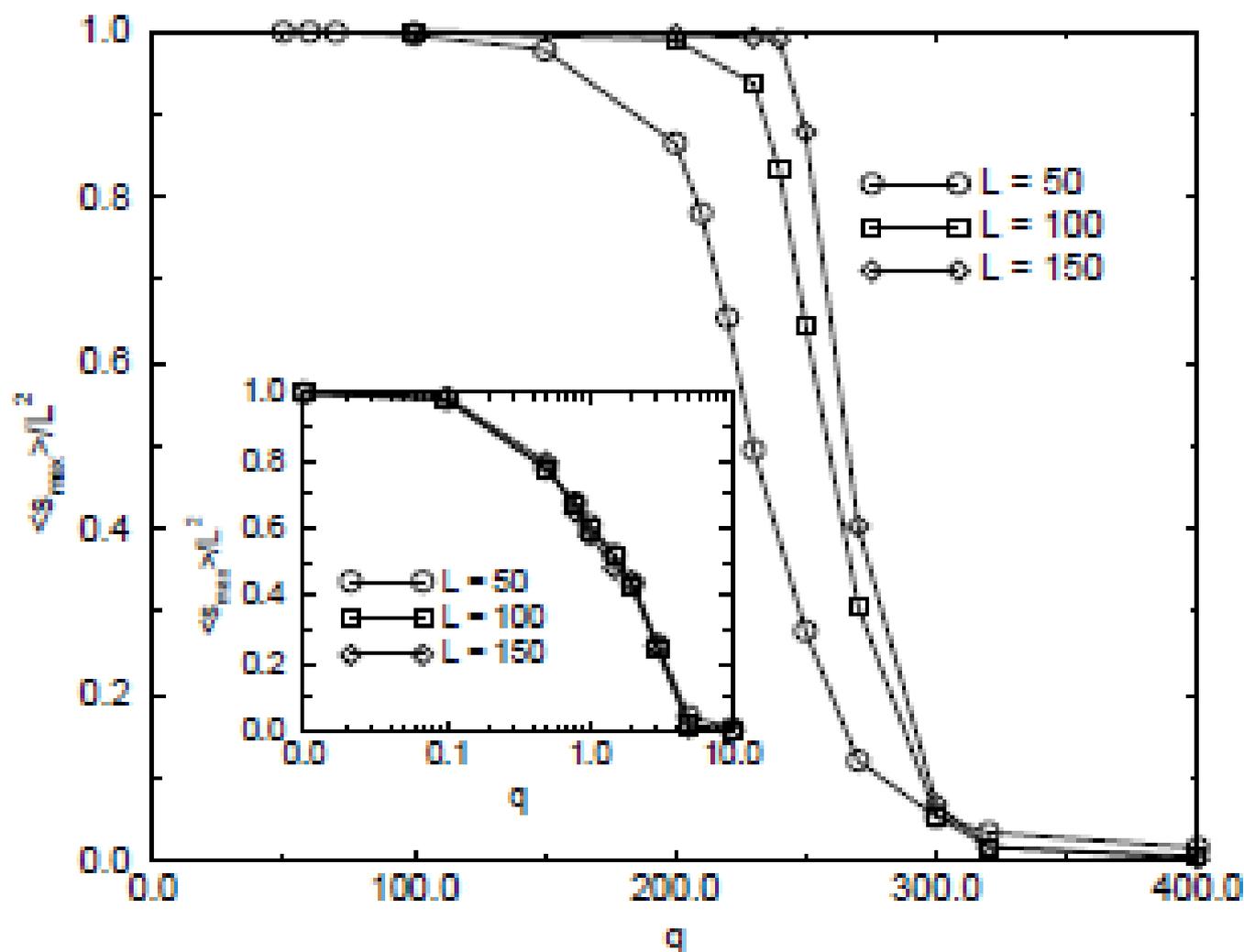
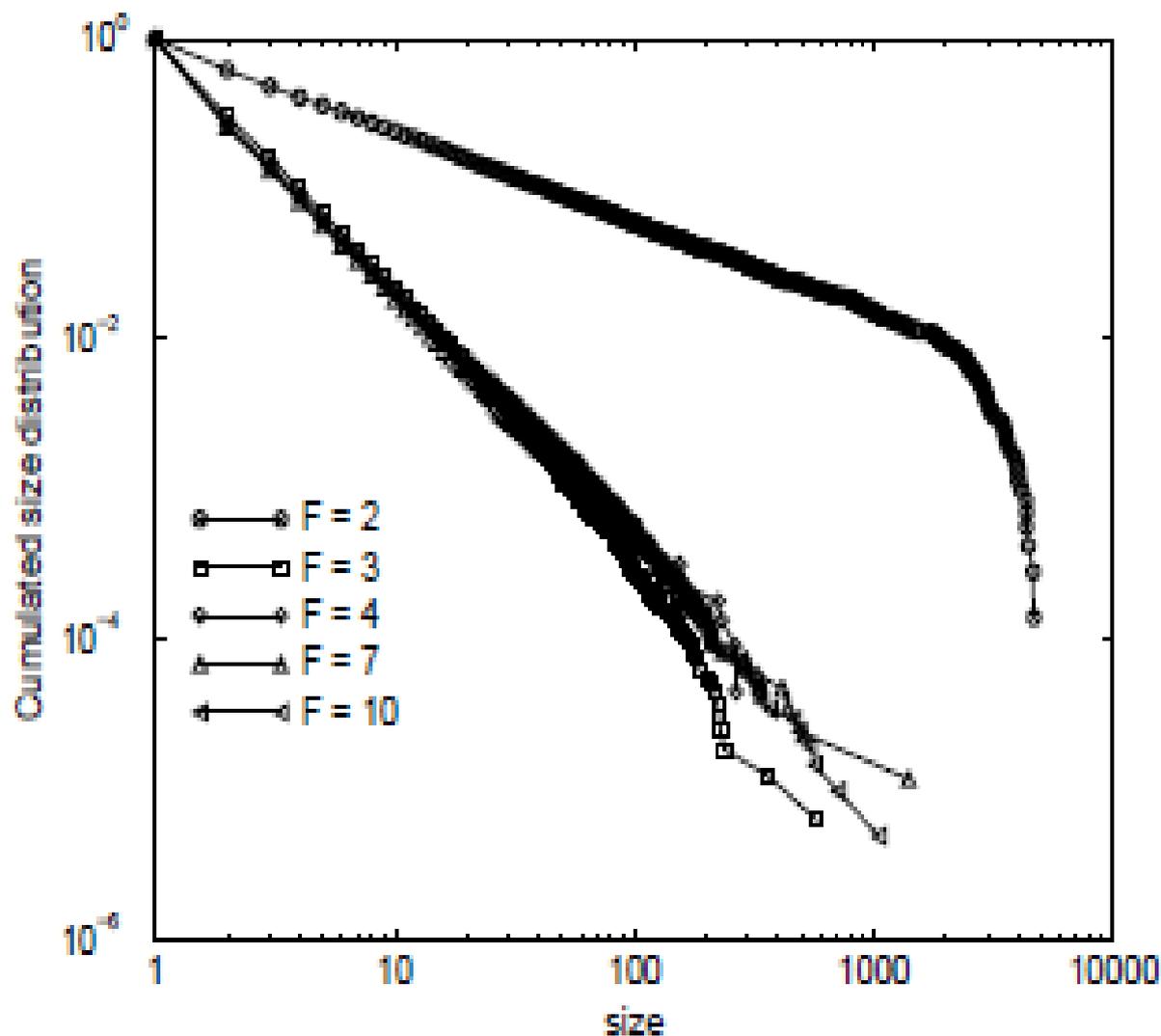


FIG. 1. Behavior of  $s_{\max}/L^2$  vs  $q$  for three different system sizes and  $F = 10$ . In the inset the same quantity is reported for  $F = 2$ .



$F=2$   $a=1.6$   
 $F>2$   $a=2.6$

FIG. 2. Cumulated distribution of region sizes for  $q \approx q_c$ ,  $L = 100$  and several values of  $F$ .

## ***Looking for applications: LANGUAGES?...***

„...it is tempting to compare the picture derived in this Letter with some recent analysis on the diversity of languages [5], which to some extent can be considered as an indicator of cultural homogeneity. Gomes and coauthors, by analyzing the statistics of more than 6700 languages, have found that the number  $L$  of linguistically homogeneous regions inhabited by  $N$  individuals scales as  $L \sim N^{-a}$ , with  $a \simeq 1.5$  for populations smaller than  $6 \times 10^6$  individuals. (...) These findings can be compatible with our continuous phase transition scenario ( $a \simeq 1.6$ ) assuming the equivalence between language and cultural features. **However it remains unclear why language spreading is self-organized close to the transition point or why the value  $F = 2$  should be relevant for the process.**”

[Castellano et al. PRL 85 (2000) 3536]

***...or MUSIC? We hear more than one piece.***

***...or RELIGIONS?***

Some data (International Data Base) suggest that numbers of adherents of different religions fit to the (cumulative) Pareto law with the exponent 0.5, but other data (World Christian Encyclopedia) do not confirm this. **Anyway, the above doubts remain.**

[Ausloos et al., Europhys. Lett. 77 (2007) 38002]

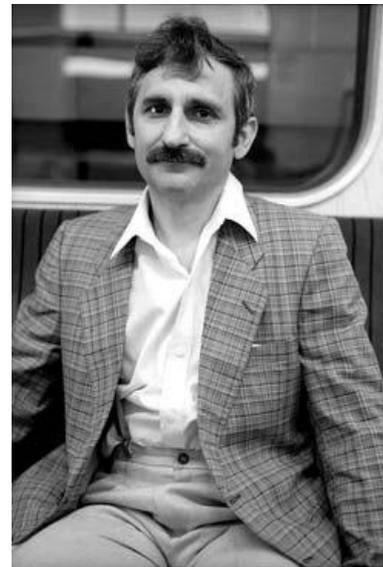
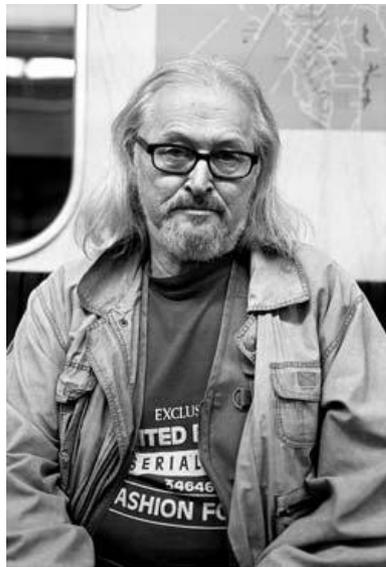
## ***FADS AND FASHIONS***

When people want to be different from others, fads will come and go. When some want to be different but others want to copy them, the result is fashion: a never-ending chase of followers running after leaders.

[R. Axelrod, *The Complexity of Cooperation*, 1997]

In this interpretation of the Axelrod model **culture** is equivalent to a set of features, as fashion in clothing, way of talking, expressed opinions etc. In any society, these characteristics are continuously modified, as they serve also as means to reconstruct social identities. In this way, large values of  $F$  and  $q$  as well as the idea of **repulsion** are justified.

# If so large $F$ and $q$ are relevant, what is their meaning ?



Tomasz Kizny: "Pasażerowie. Warszawa"

<http://fototapeta.art.pl/fti-metro.html>

Copyright © 1997-2010 Marek Grygiel / Copyright for www edition © 1997-2010 Zeta-Media Inc.

## Order stabilized by noise

we have verified the metastability of disordered configurations for  $d = 2$  with nearest neighbor interaction as well. The absorbing states are subject to *single feature perturbations*, defined as randomly choosing  $i \in \{1, \dots, N\}$ ,  $f \in \{1, \dots, F\}$  and  $s \in \{1, \dots, q\}$  and setting  $\sigma_{if} = s$ . Then the simulations are designed as follows: (I) Draw a random initial configuration. (II) Run the dynamics by iterating steps (1), (2) and (3), until an absorbing state is reached. (III) Perform a single feature perturbation of the absorbing state and resume at (II). We find that by these cycles of relaxation (step II) and perturbation (step III) the system is driven to complete order, where  $\sigma_{if} = \sigma_{jf}, \forall i, j$  and  $f$ . Thus, as in the one-dimensional case, only the completely ordered configurations are stable, all other absorbing configurations are merely metastable.

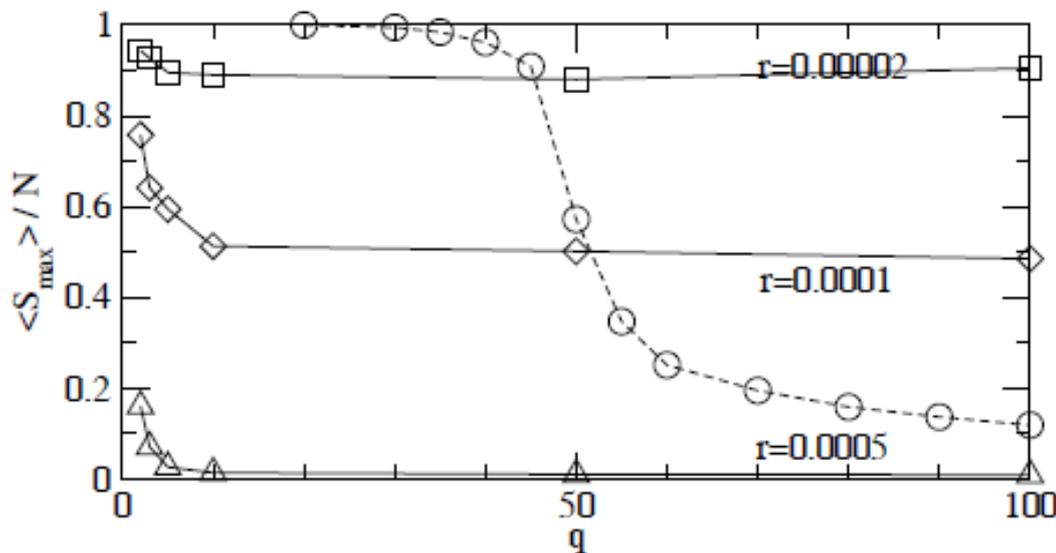
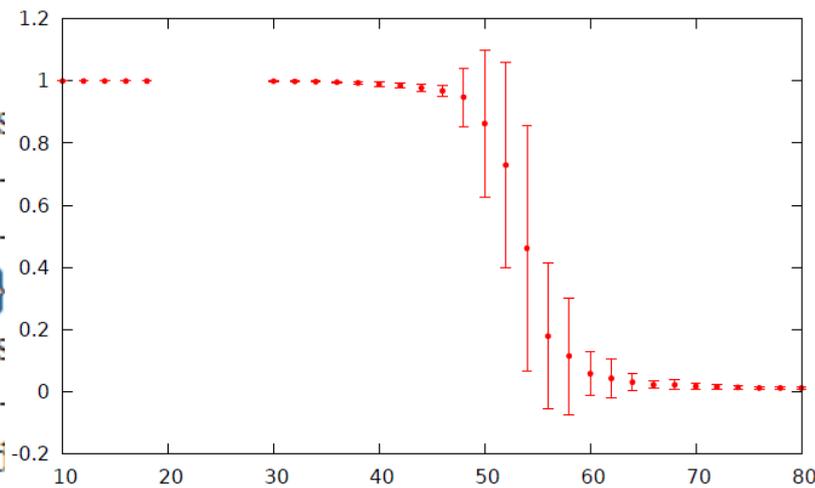


FIG. 2.  $\langle S_{\max} \rangle$  as a function of  $q$  for different values of  $r$ . The dashed line is for  $r = 0$  with a transition at  $q = q_c \approx 50$ . The simulations used  $N = 50^2$  sites with  $F = 10$  features. Values  $\langle S_{\max} \rangle$  in Figures 2, 3 4, and 5 are averages over 500 configurations. Measurements were taken after a relaxation time of  $100N^2$  time steps.

[Klemm ea, PRE 67 (2003) 045101]

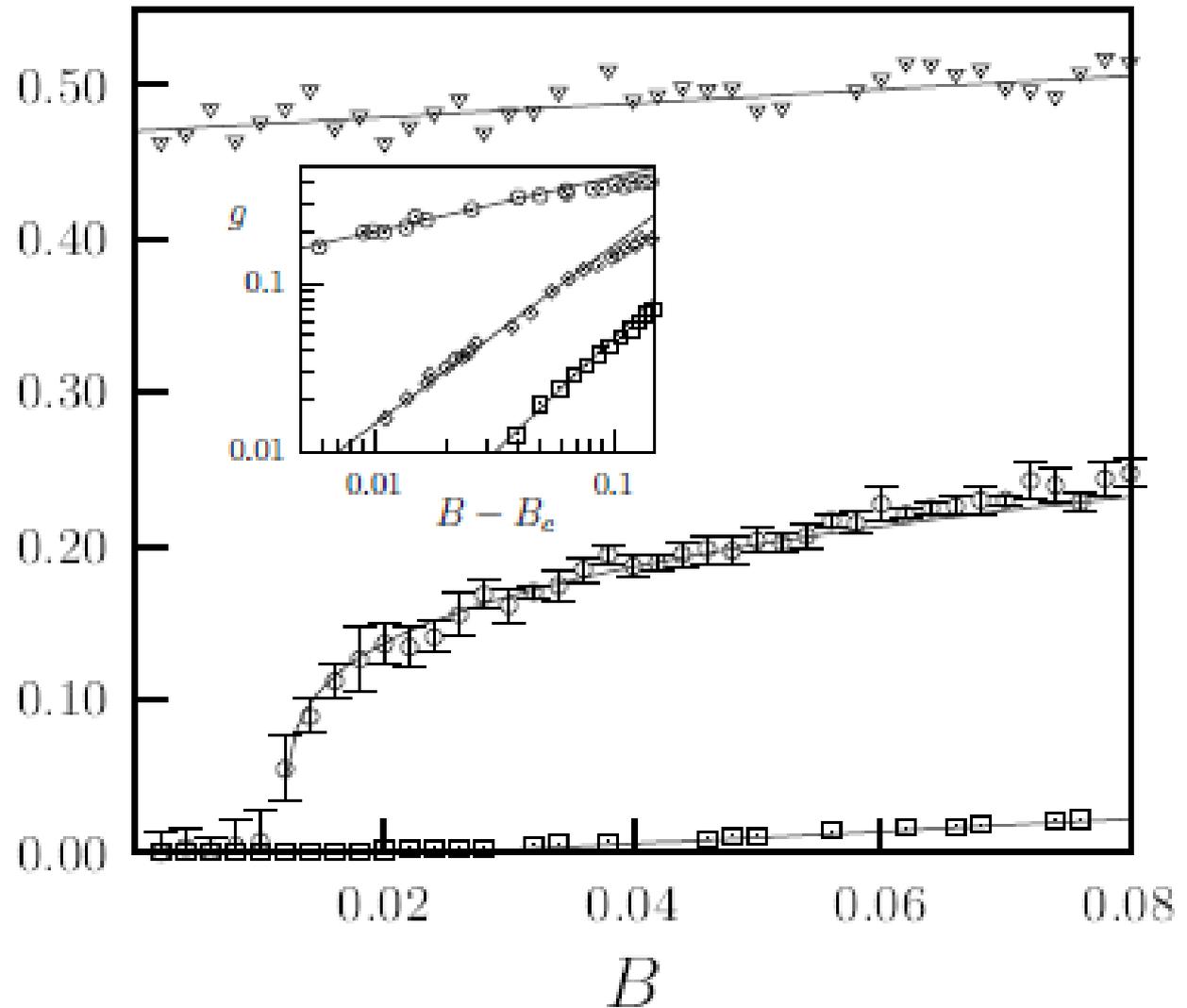
„Influence of media”

[Gonzalez-Avella ea PRE 72 (2005) 065102]



$g$

$$g \equiv \frac{\#(\text{groups})}{N}$$



$B$  – probability of interaction with a non-evolving site

## Threshold-dependent repulsion

(1) the lattice is swept in an orderly fashion. For each of the agents considered, one of its four nearest neighbors is randomly chosen. These two individuals make up the interacting pair.

(2) Once the interacting pair has been chosen, their similarity is determined by calculating their overlap  $w$  (proportion of features shared, i.e.,  $A/F$ , where  $A$  is the number of features shared, and  $F$  is the total number of features);

(3) the overlap  $w$  is then compared with the degree of repulsion  $\gamma$ . The case  $w < \gamma$  will correspond to repulsive behavior between the agents, while  $w \geq \gamma$  will correspond to attractive behavior.



$\gamma = \text{threshold}$

attraction :  $A \rightarrow A+1$

repulsion:  $A \rightarrow A-1$

# Threshold-dependent repulsion

$\gamma = \text{threshold}$

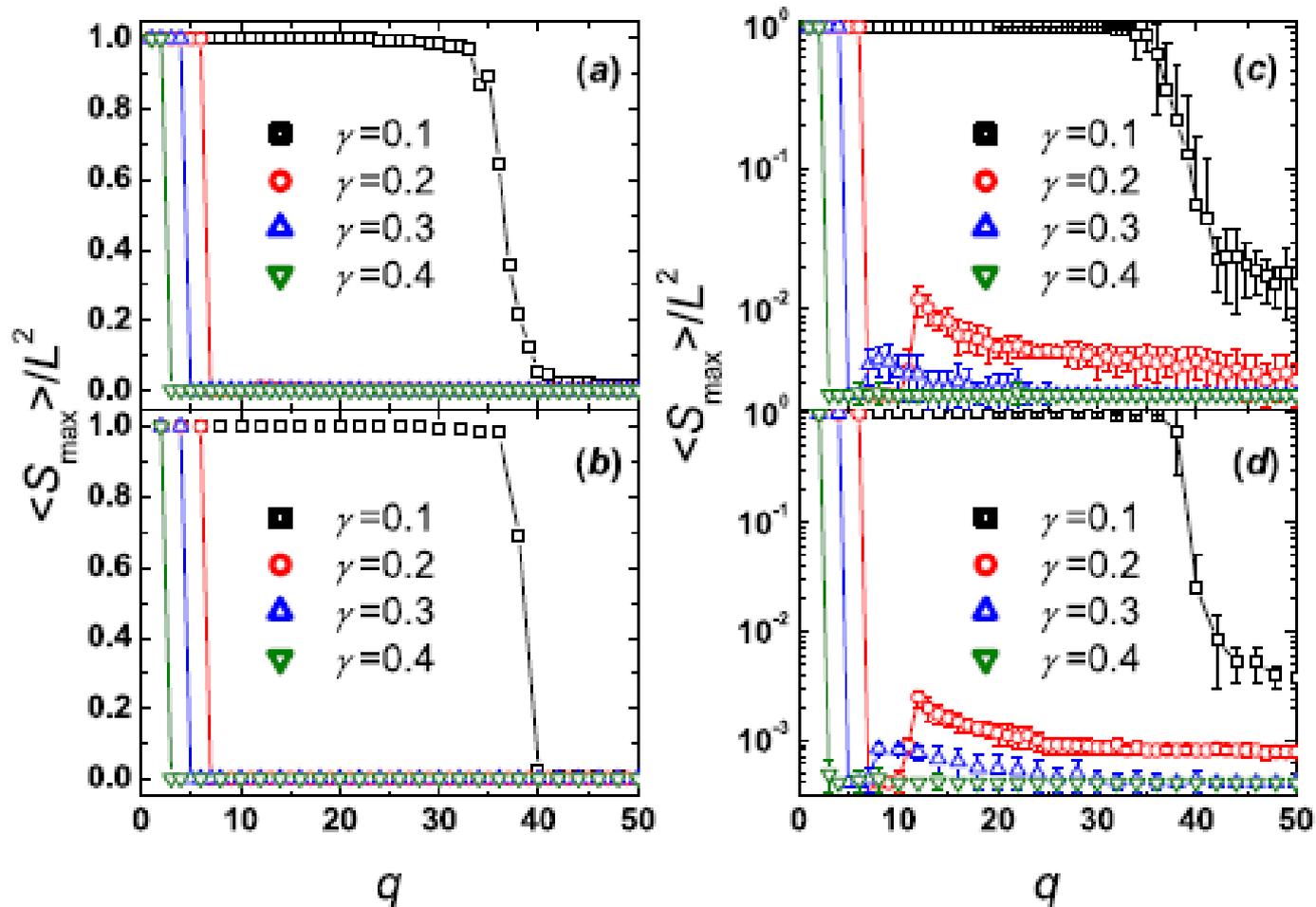


FIG. 2. (Color online) Normalized  $\langle S_{\max} \rangle$  versus  $q$  for a system with  $F=20$  and (a)  $L=20$ , (b)  $L=50$ . (c) and (d), respectively, correspond to (a) and (b) but with the vertical axis in logarithmic scale.

*new here: more stochastic repulsion*

For a selected pair of nodes (i,k) a position  $f \in \{1, \dots, F\}$  is randomly chosen.

$\sigma_{if} = \sigma_{kf}$  : attraction,  $A \rightarrow A+1$ , prob.  $A/F$

$\sigma_{if} \neq \sigma_{kf}$  : repulsion,  $A \rightarrow A-1$ , prob.  $1-A/F$



where  $A$  is  
the number  
of shared  
features

*new here: initial rate  $d$  of active bonds*



$\neq$



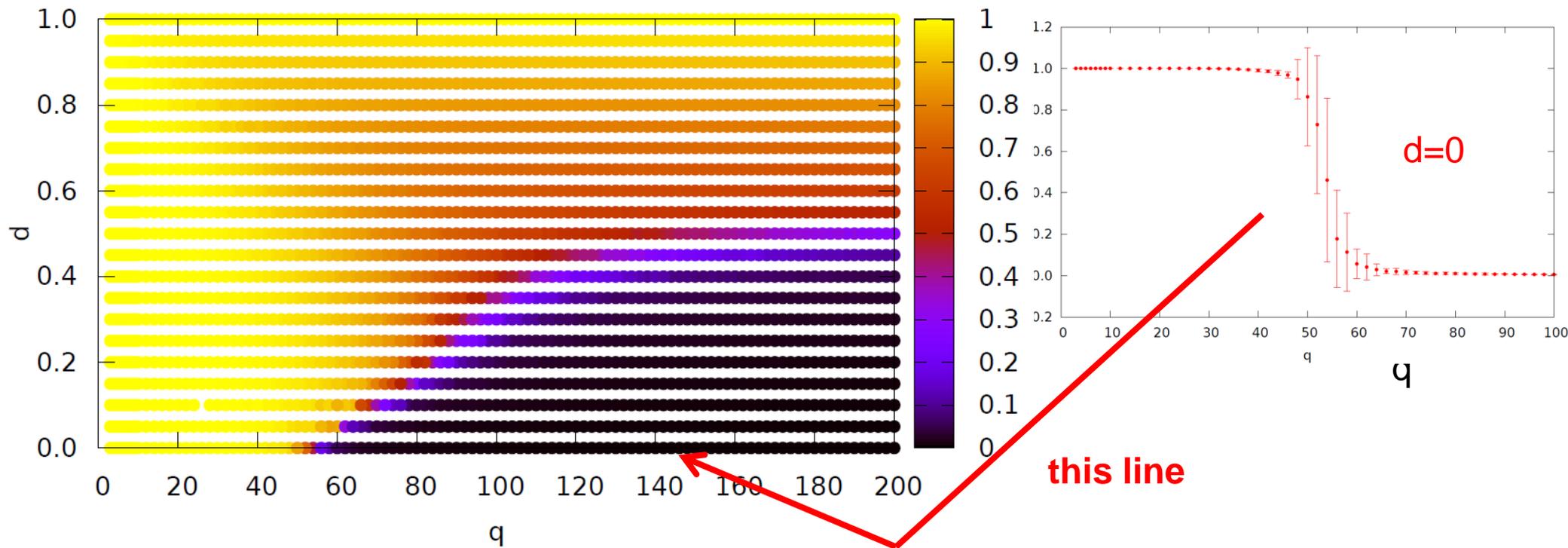
For each  $k=1, \dots, F$  find the value of  $m=1, \dots, q$  which appears most frequently. Set this value at position  $k$  for  $d$  randomly selected nodes.

## How to prepare an initial state?

- Option A1: for each position  $k$ , the set of  $d$  nodes is determined separately and randomly
- Option A2: the set of  $d$  nodes is determined separately as a squared cluster, different for each position  $k$
- Option B1: the set of  $d$  nodes is determined randomly, but it is the same for each position  $k$
- Option B2: the set of  $d$  nodes is determined as a squared cluster, the same for each position  $k$

**Numerical results:**

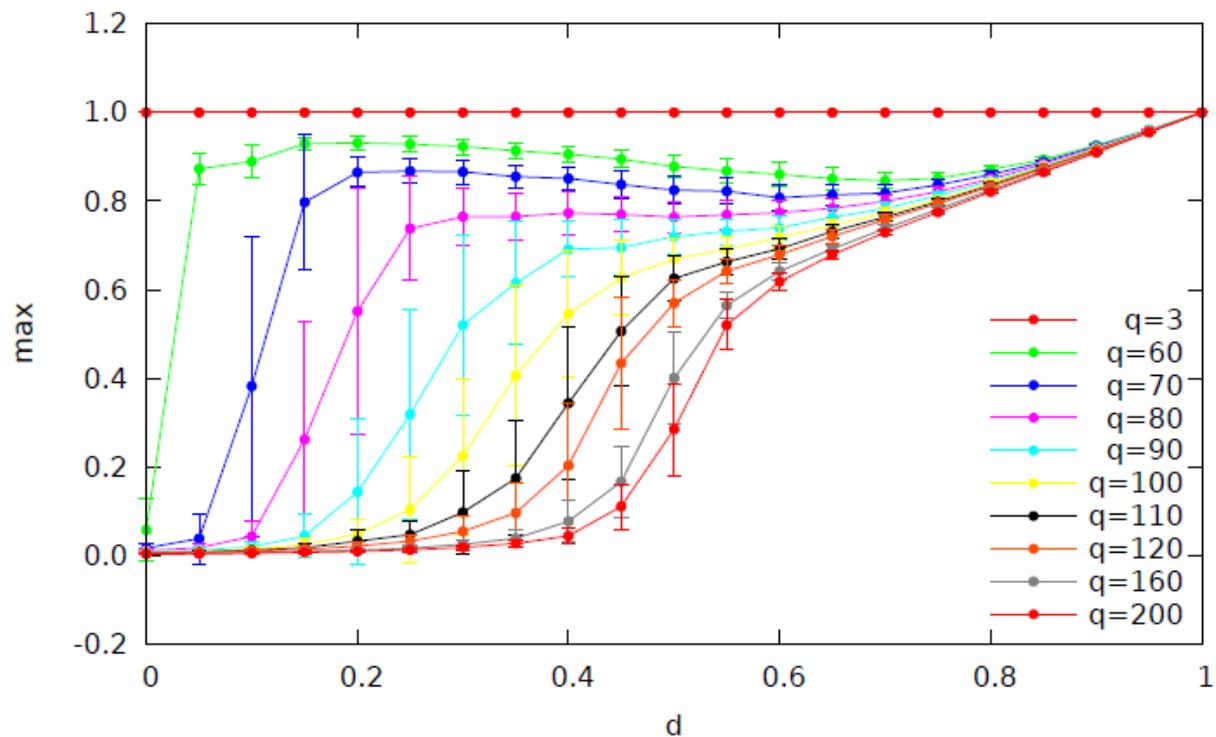
**NO REPULSION**



no repulsion

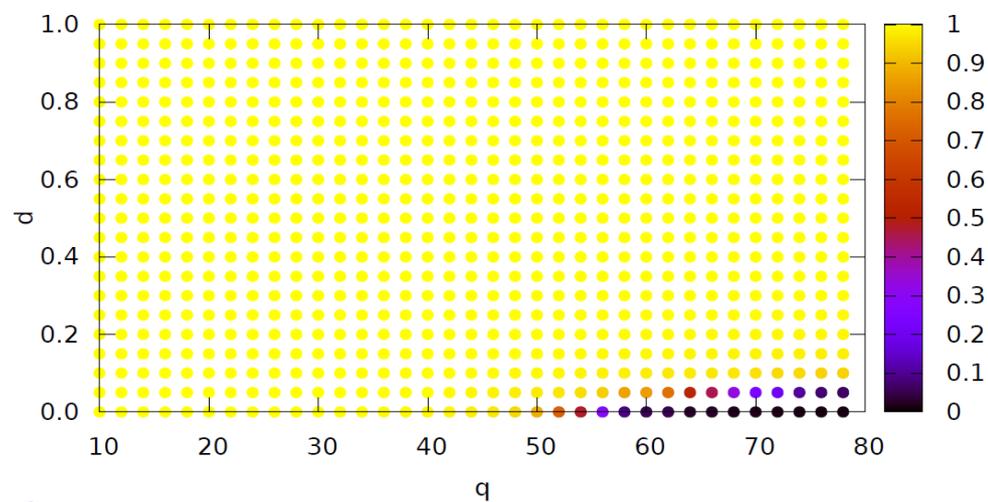
$F=10, L=50$

Option B1

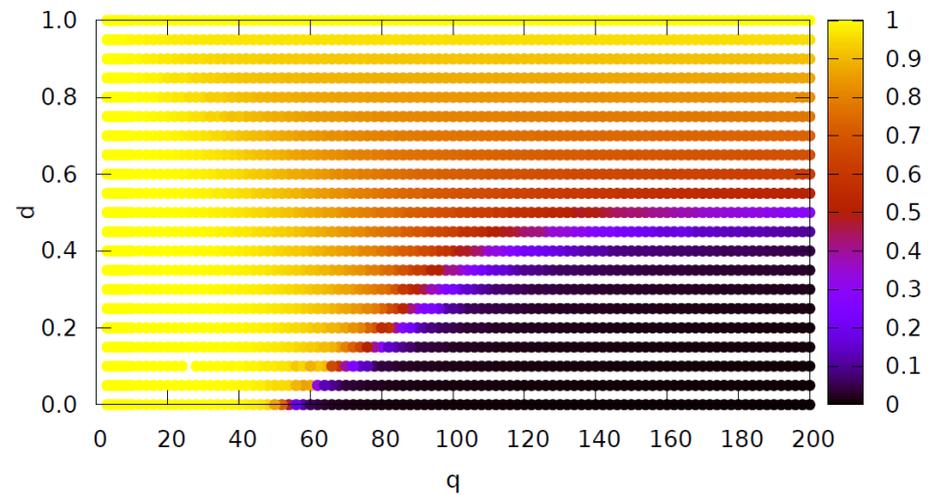


# no repulsion, $L=50$ , $F=10$

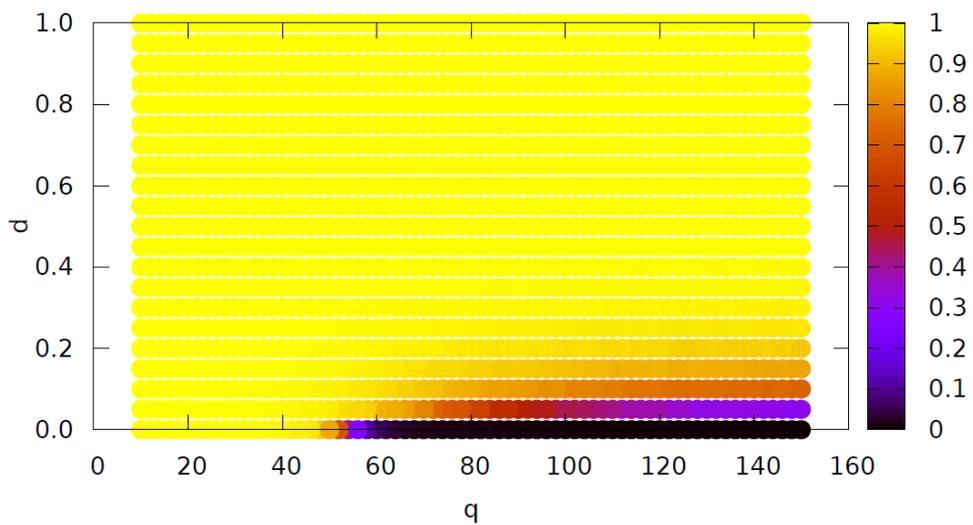
## A1



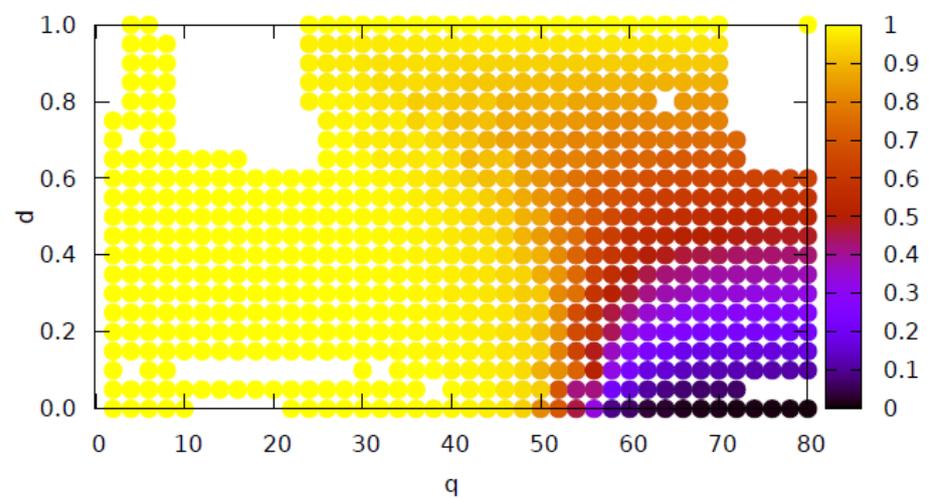
## B1



## A2

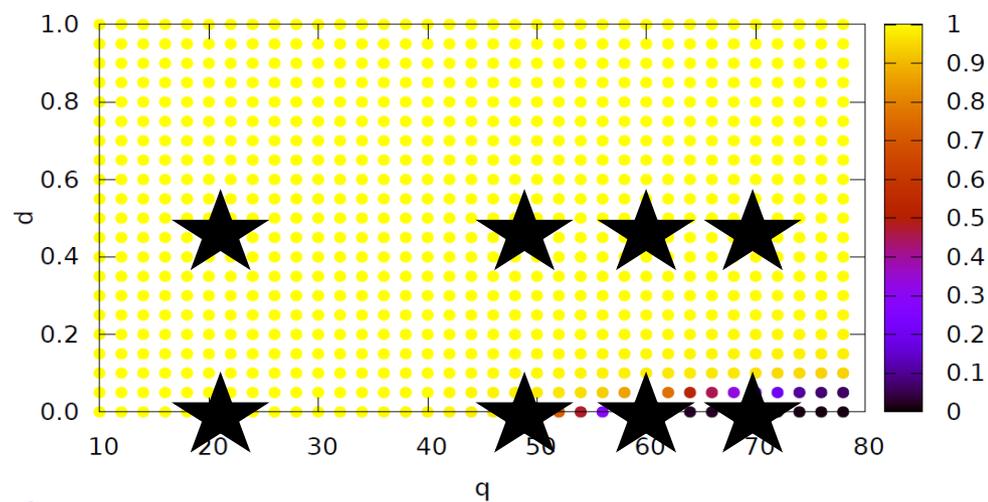


## B2

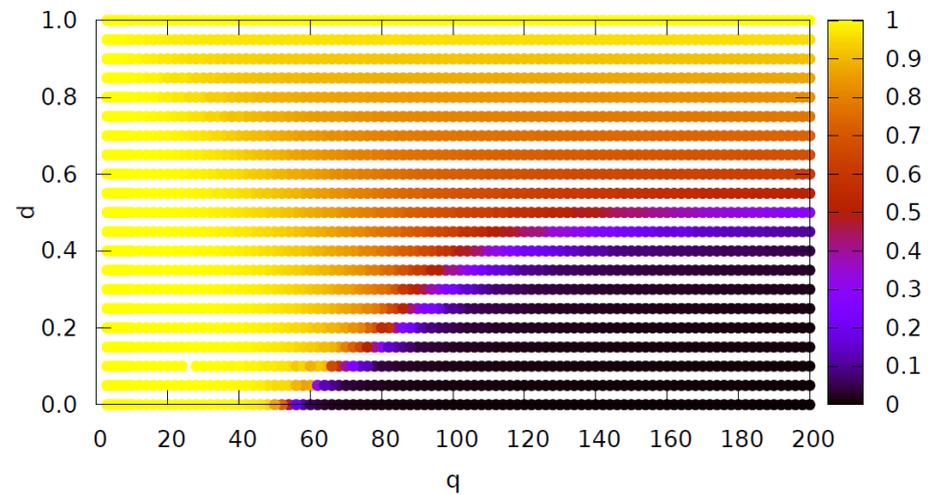


# no repulsion, $L=50$ , $F=10$

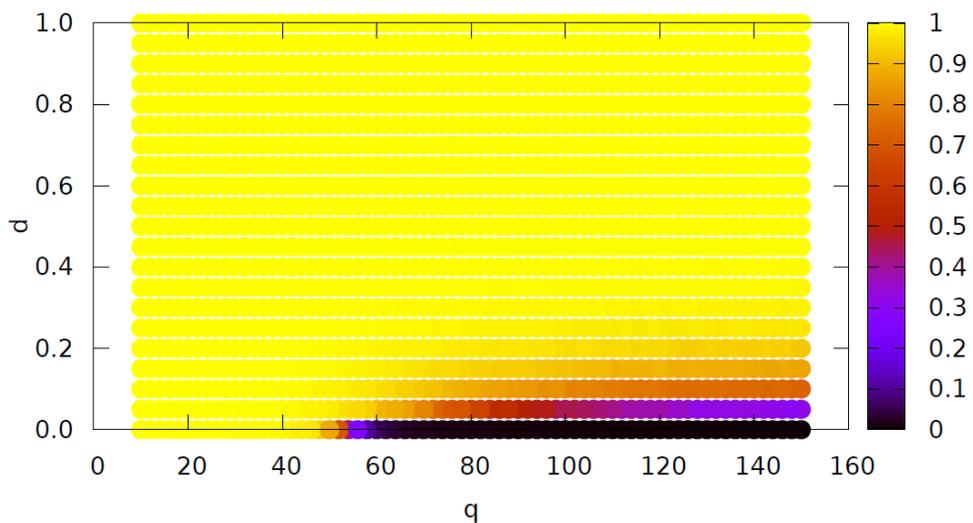
## A1



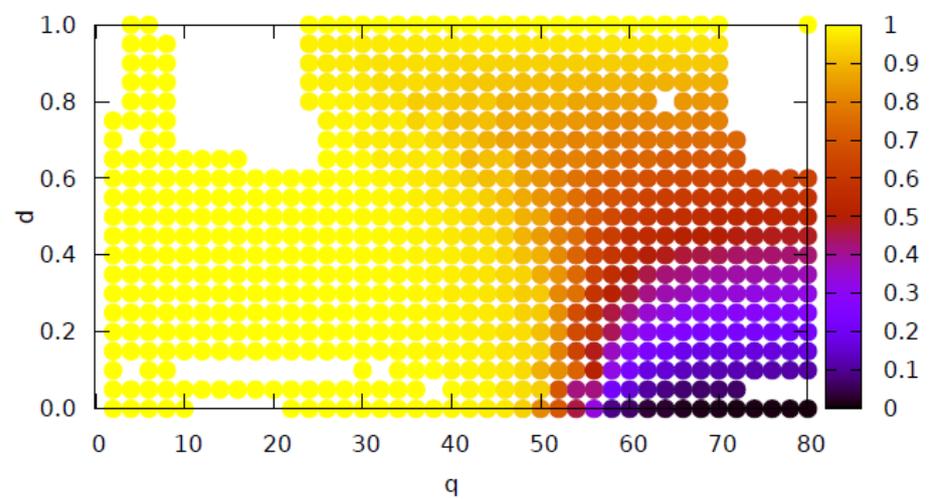
## B1



## A2



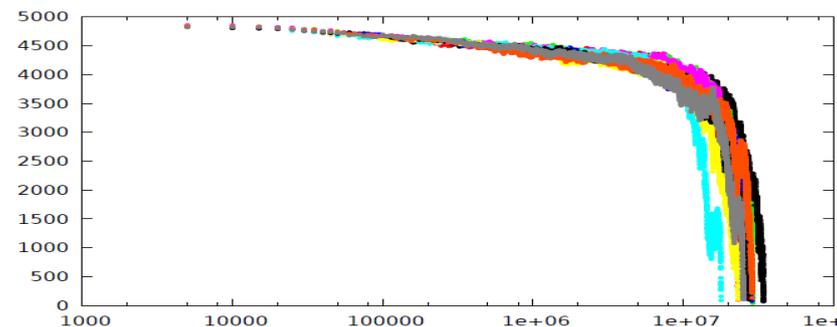
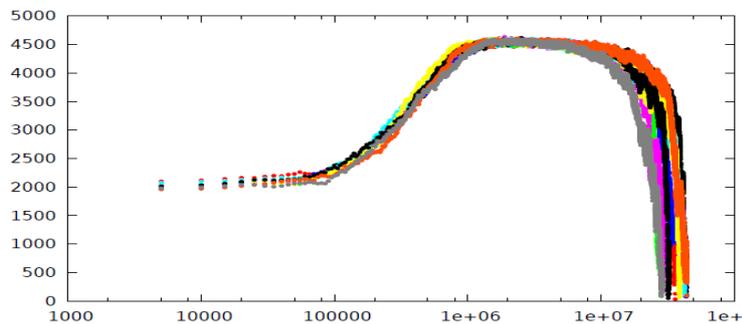
## B2



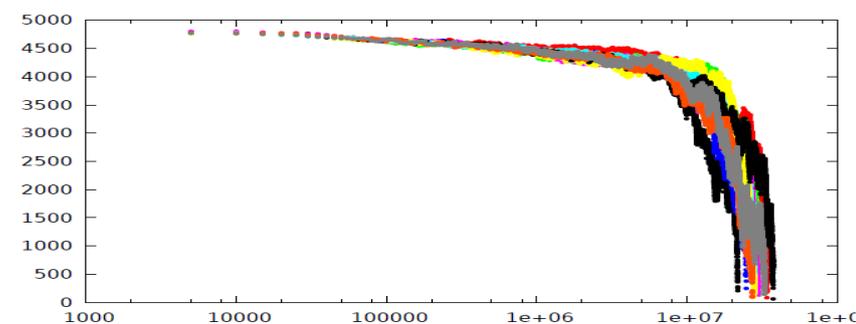
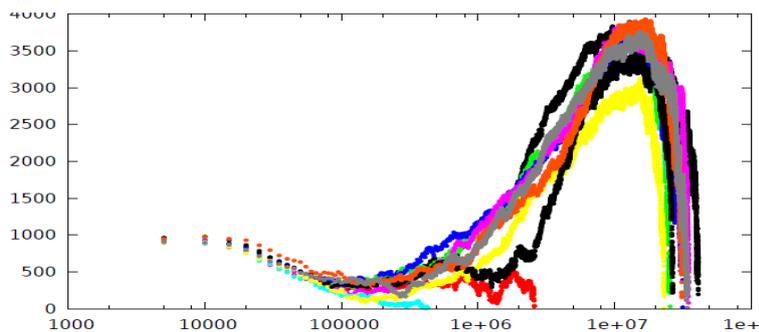
# # active bonds vs time. No repulsion.

# A1

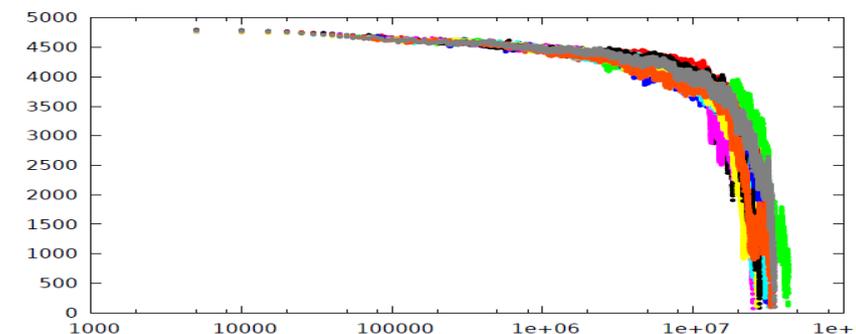
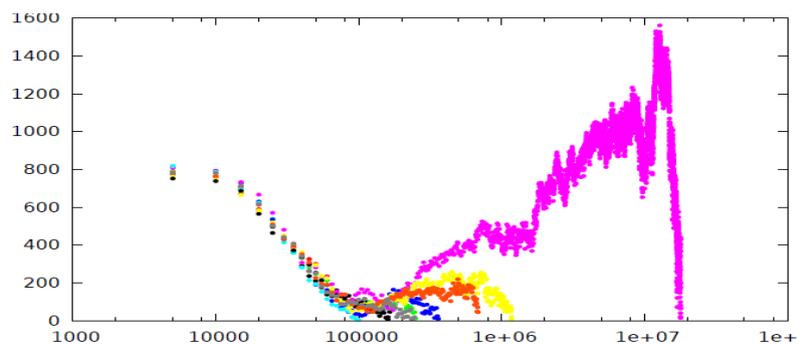
q=20



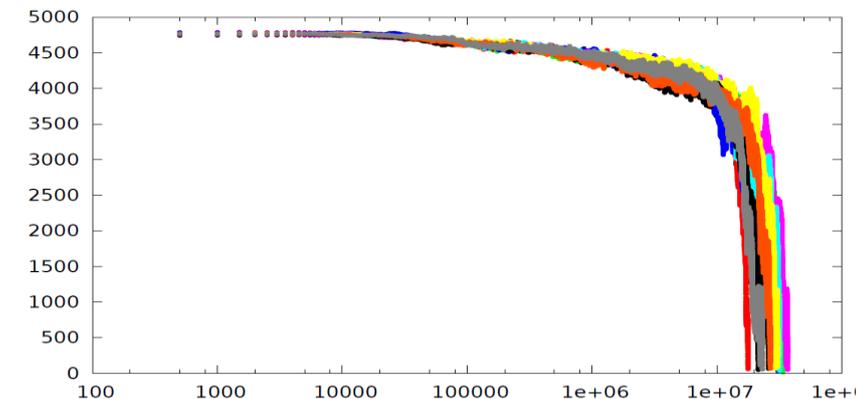
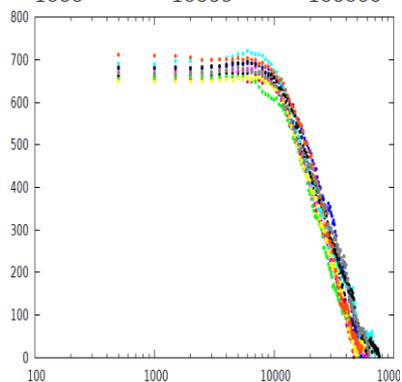
q=50



q=60



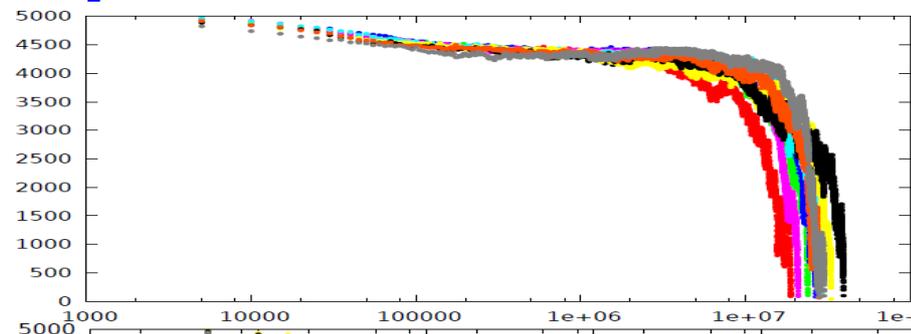
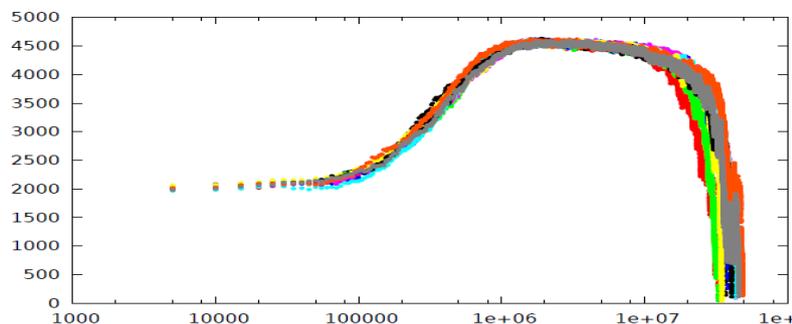
q=70



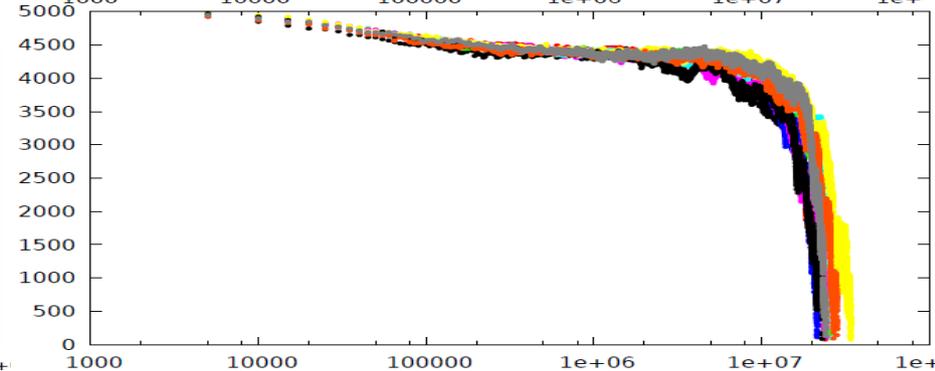
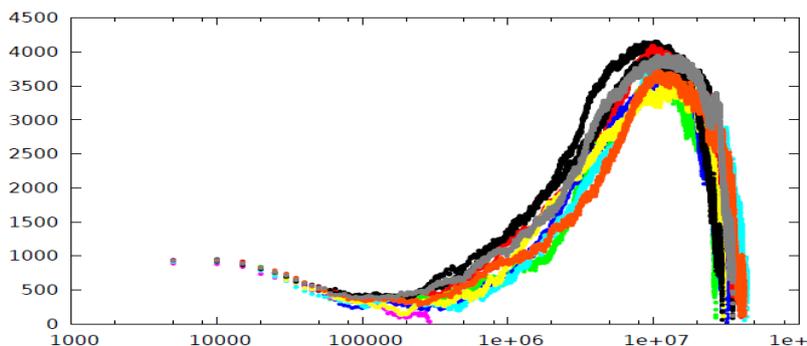
# # active bonds vs time. No repulsion.

# A2

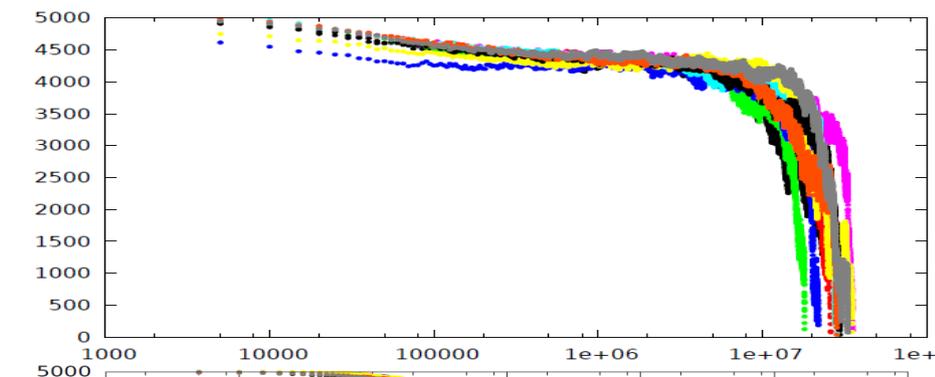
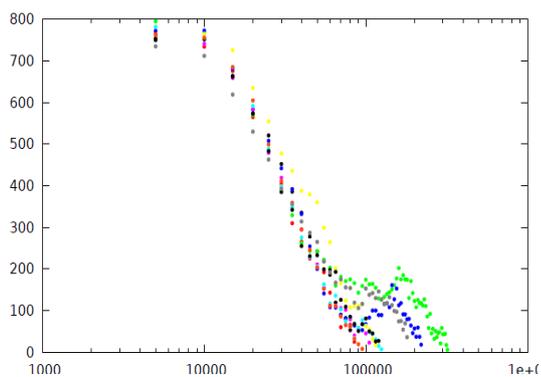
q=20



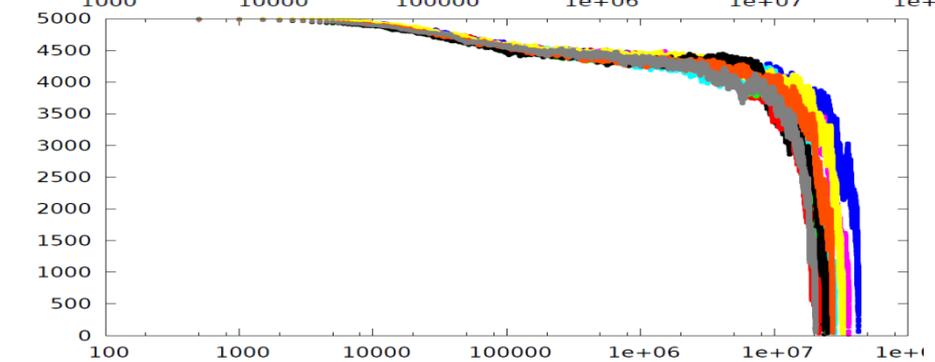
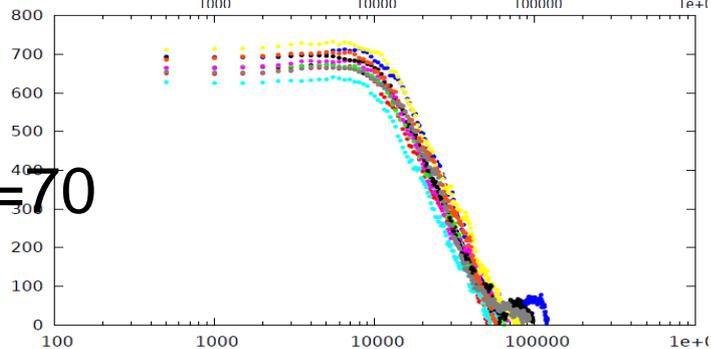
q=50



q=60



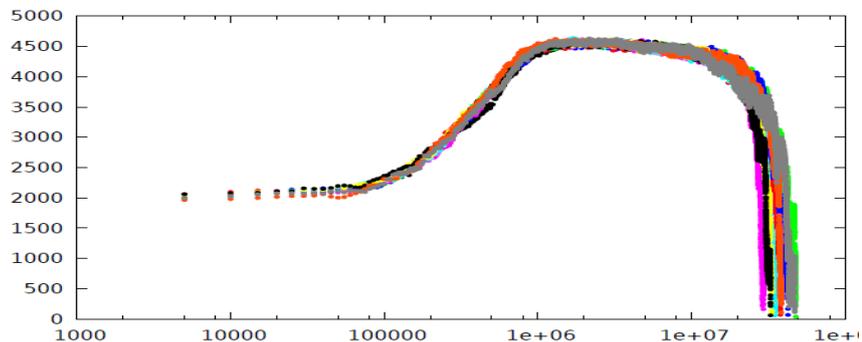
q=70



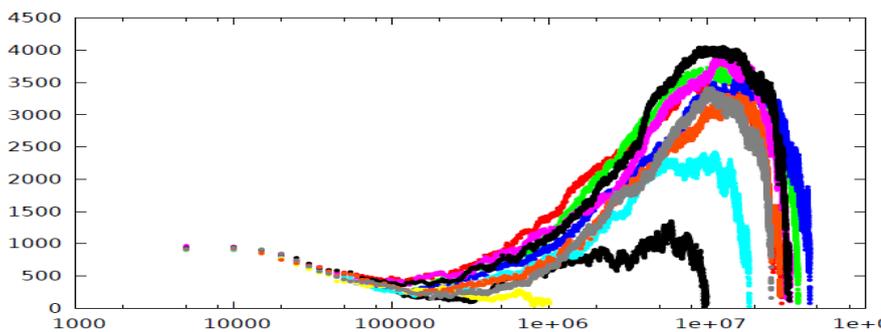
# # active bonds vs time. No repulsion.

# B1

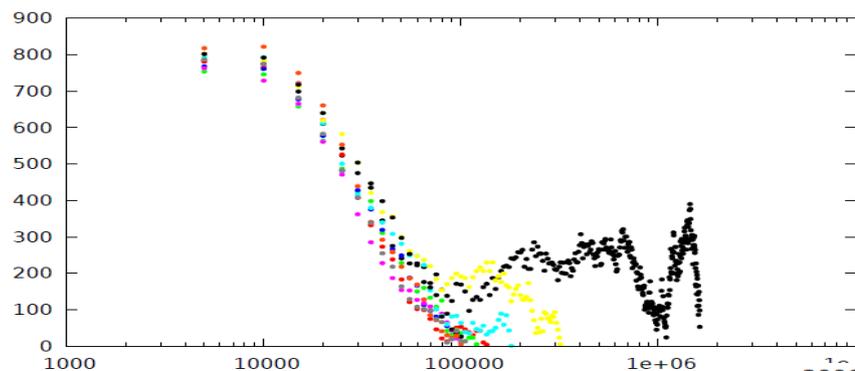
q=20



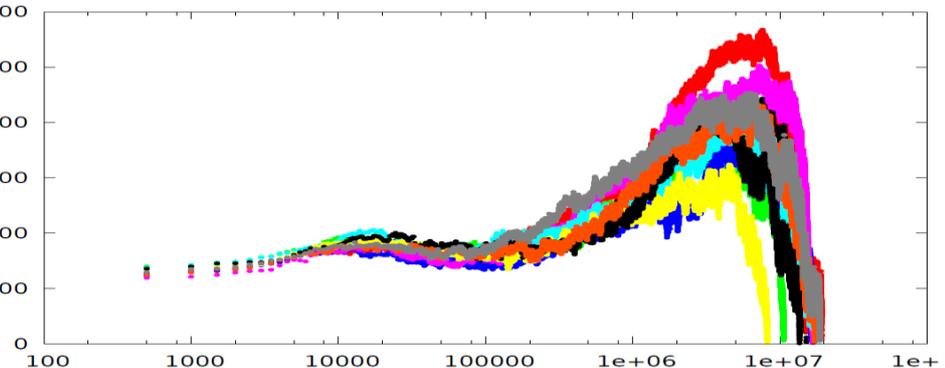
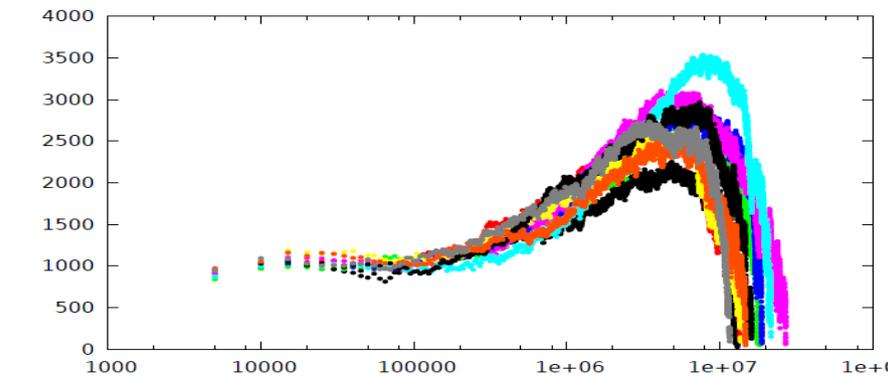
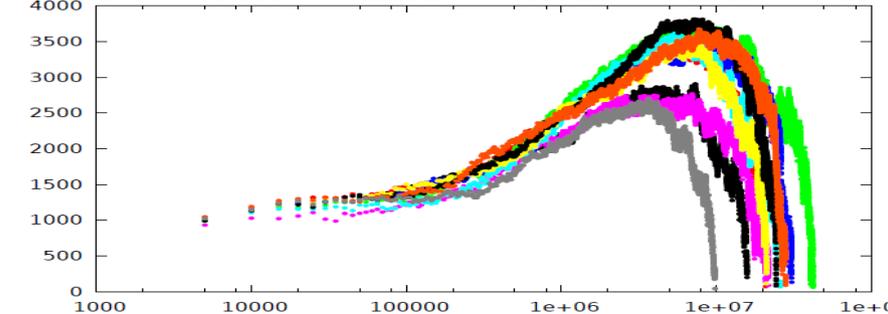
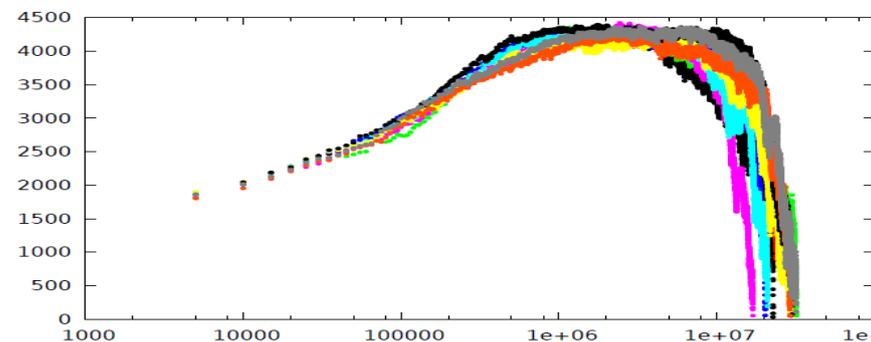
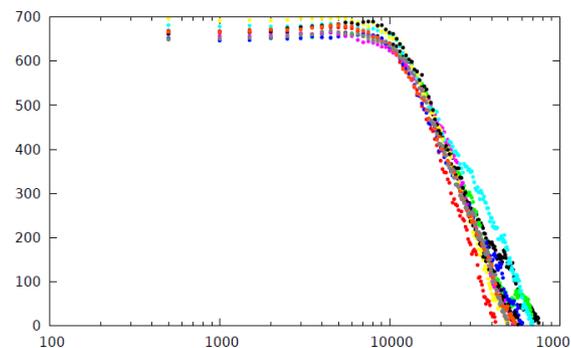
q=50



q=60



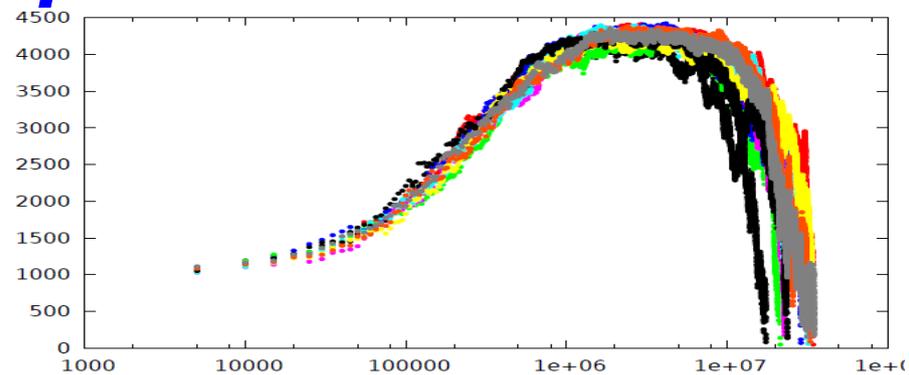
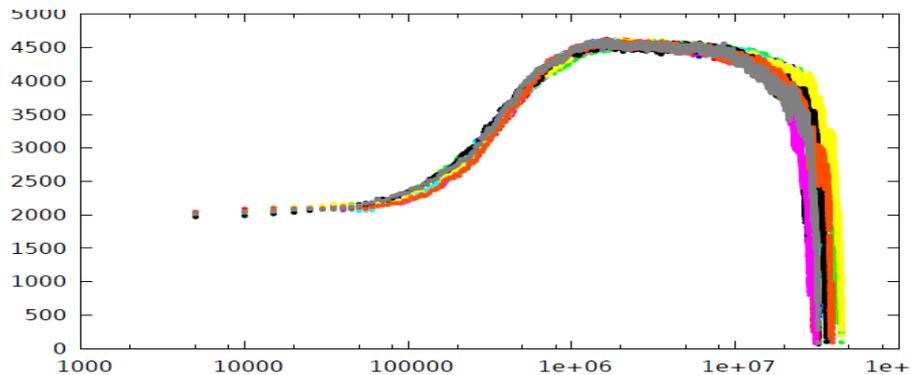
q=70



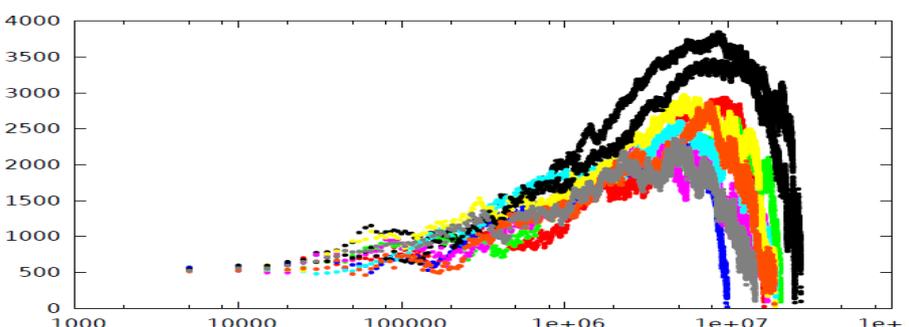
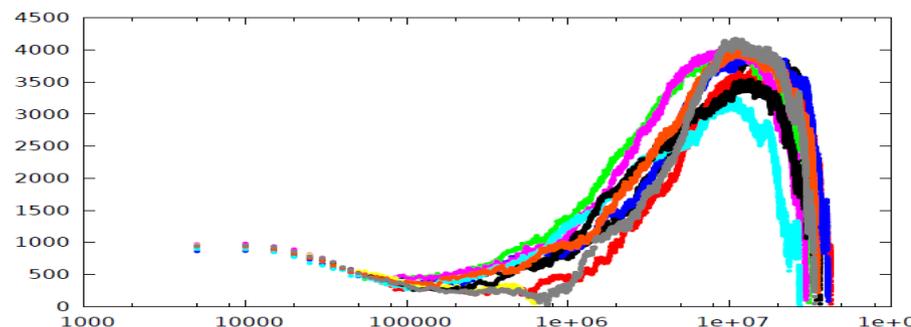
# # active bonds vs time. No repulsion.

# B2

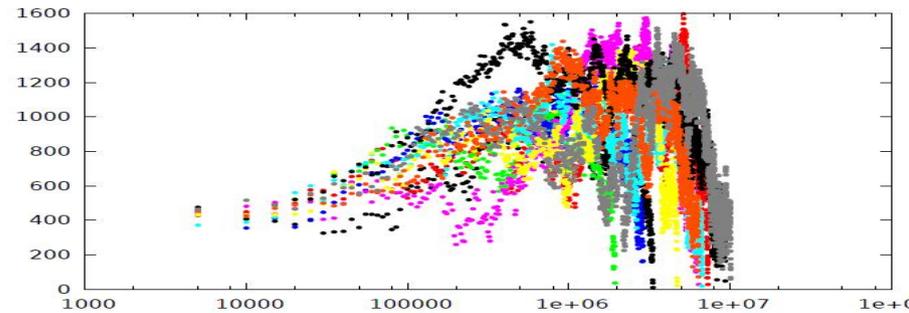
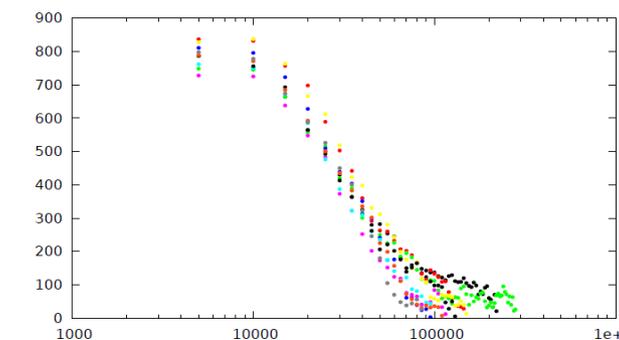
q=20



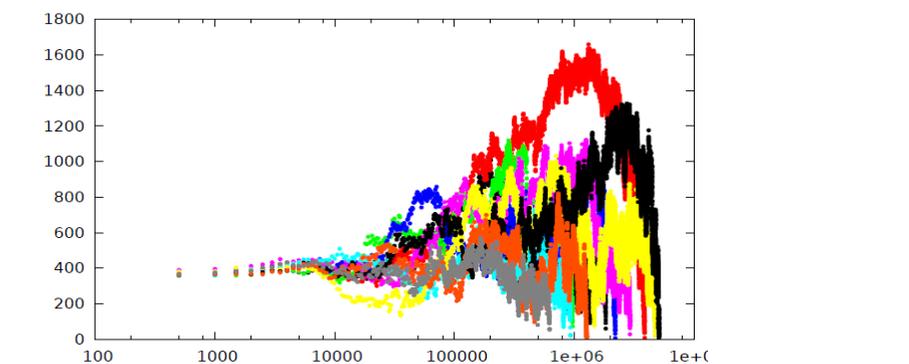
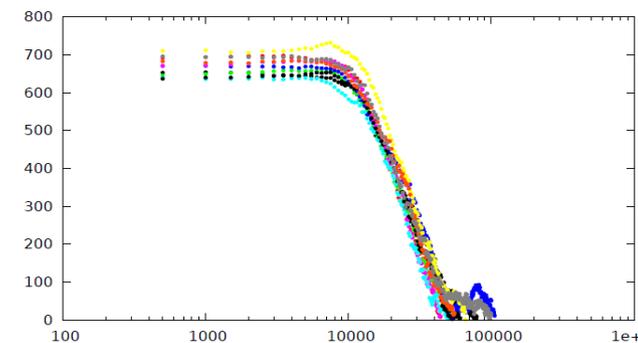
q=50



q=60



q=70



**NO REPULSION – summary :**

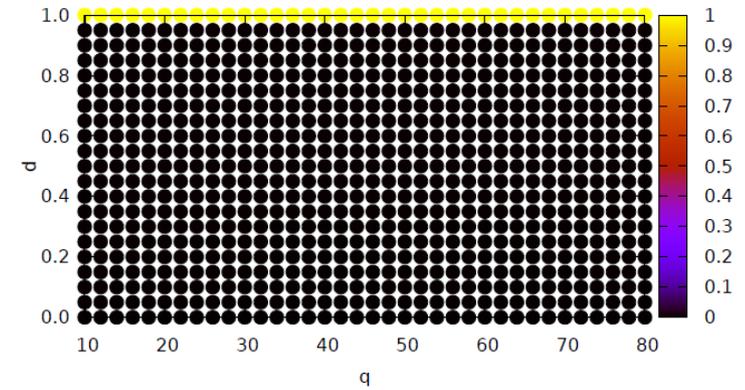
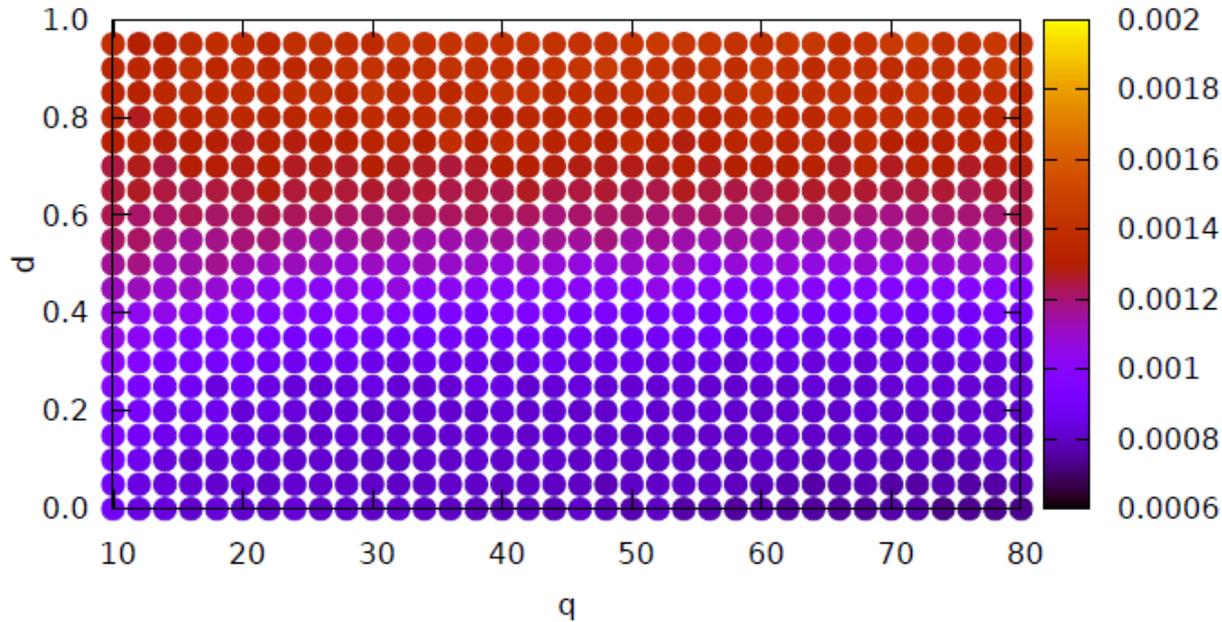
**The method of preparation of initial states important**

**Initial states important**

**Numerical results:**

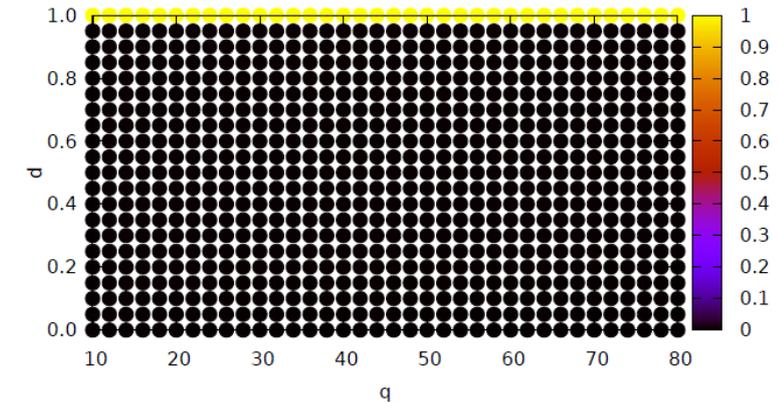
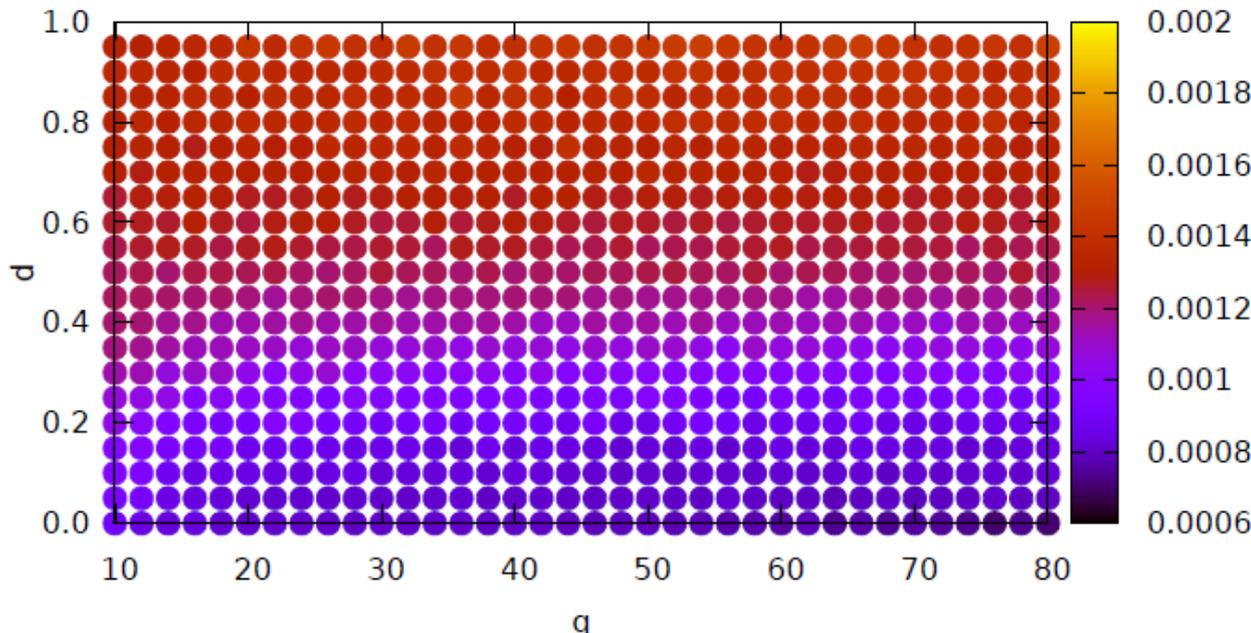
**REPULSION**

**A1:** for each position  $k$ , the set of  $d$  nodes is determined separately and randomly



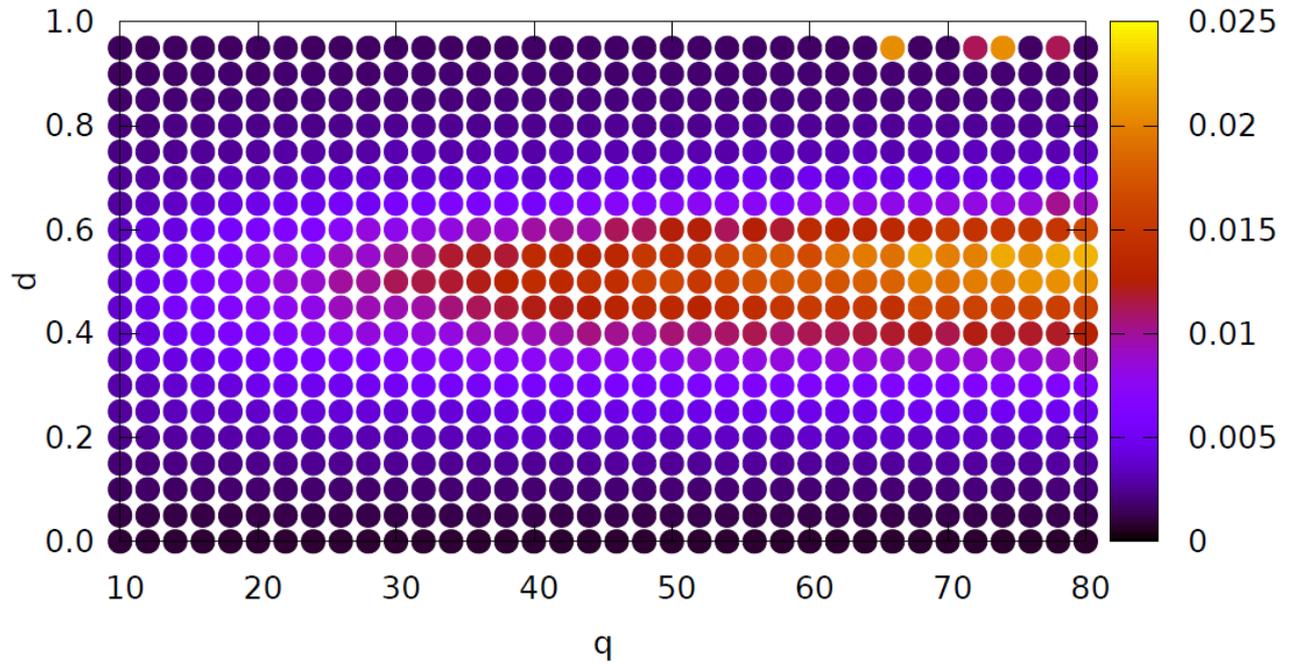
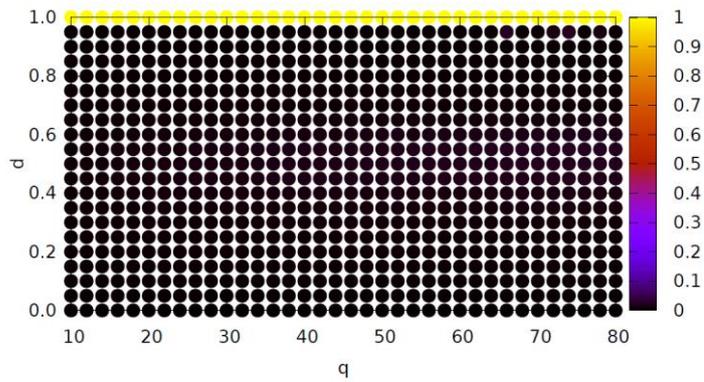
$$L=50, F=10$$

**A2:** for each position  $k$ , the set of  $d$  nodes is determined separately as a squared cluster

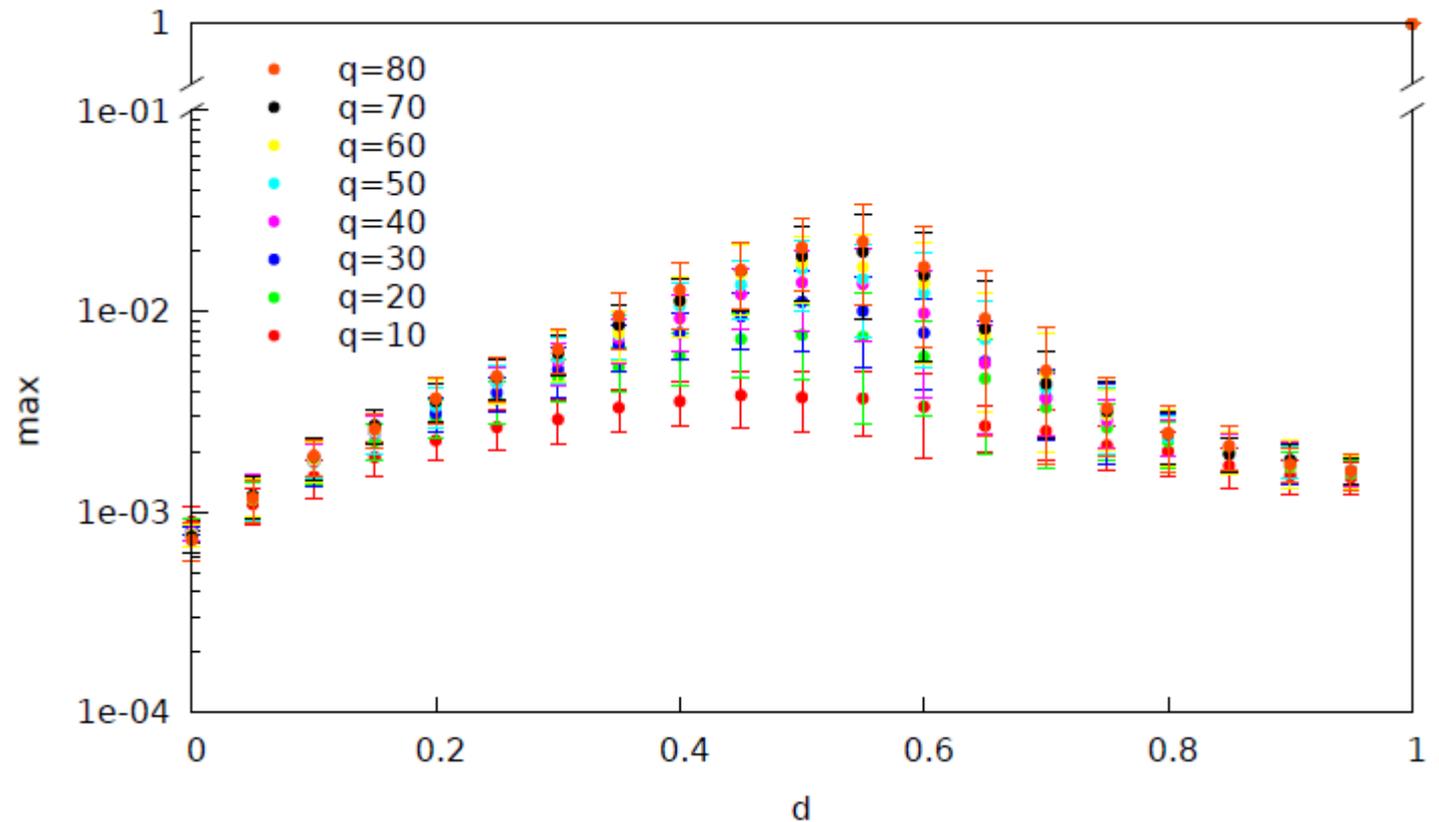


$$L=50, F=10$$

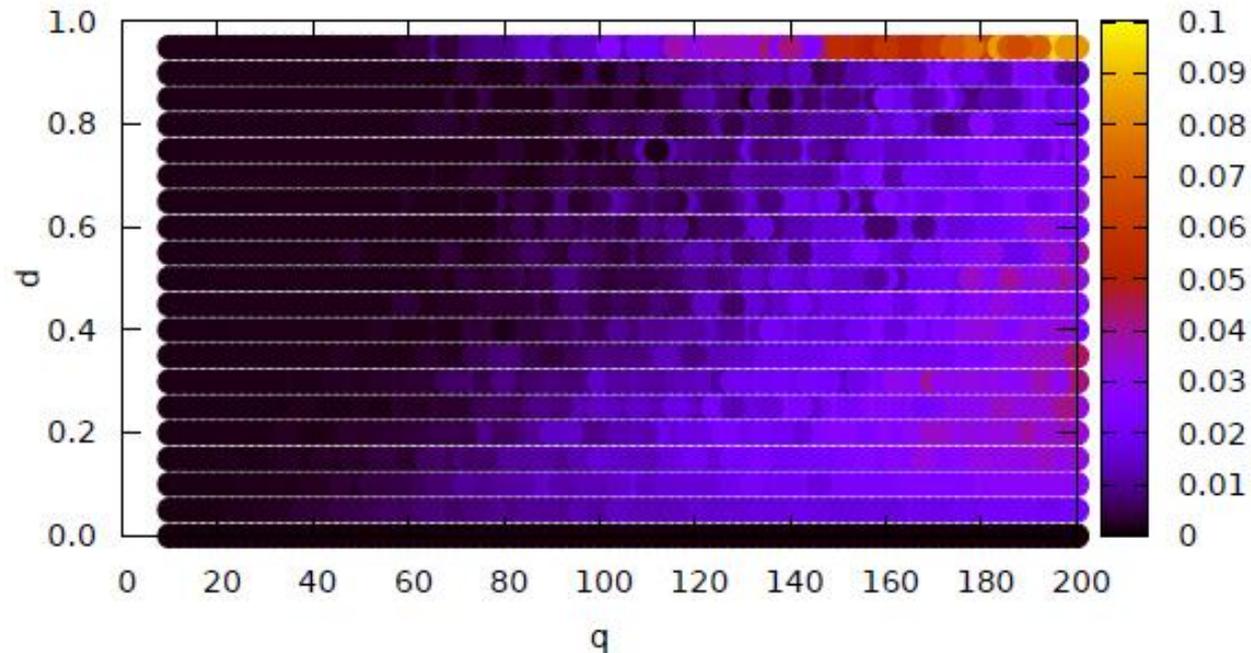
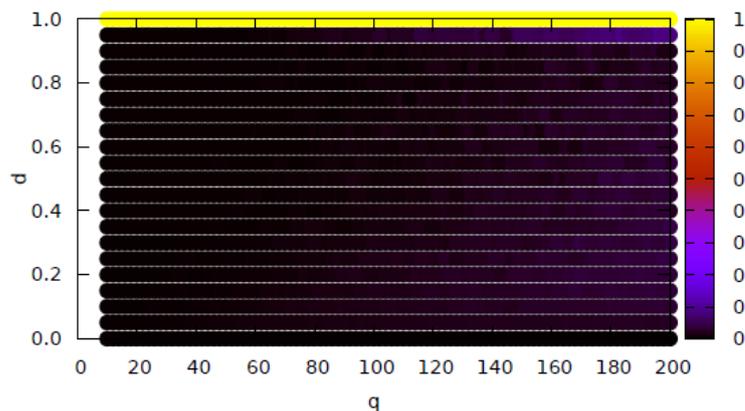
$L=50, F=10$



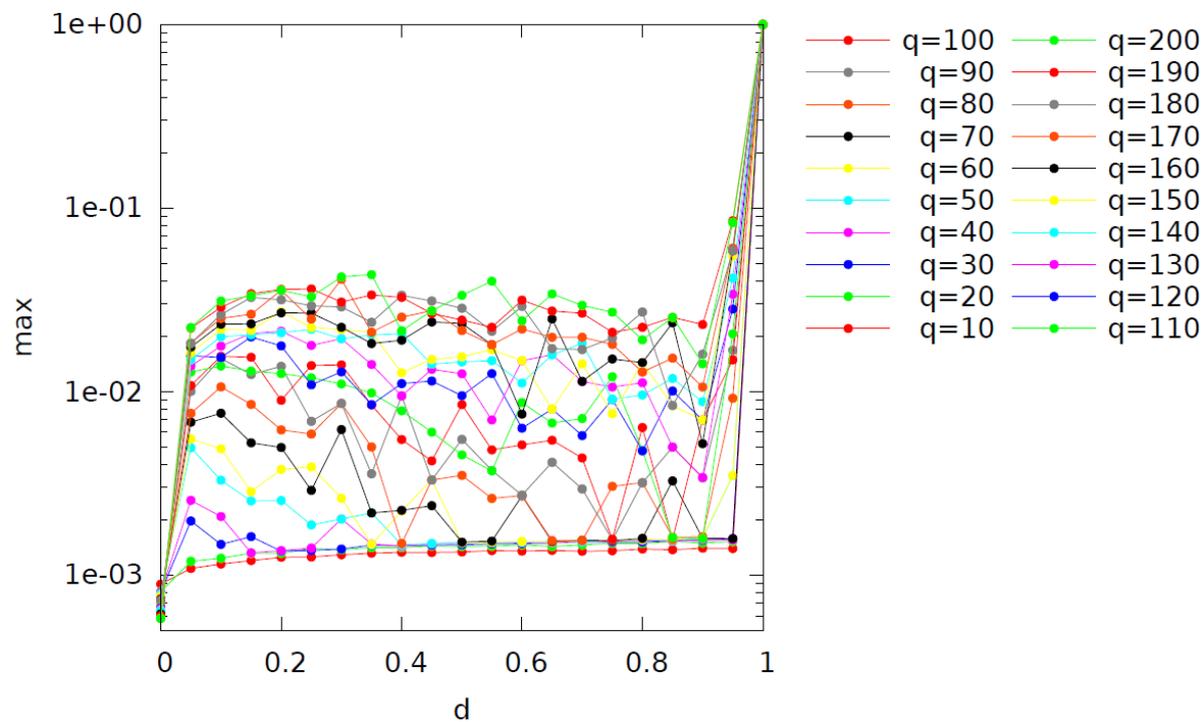
**B1:**  
the set of  $d$   
nodes is  
determined  
randomly, but  
it is the same  
for each  
position  $k$



$L=50, F=10$

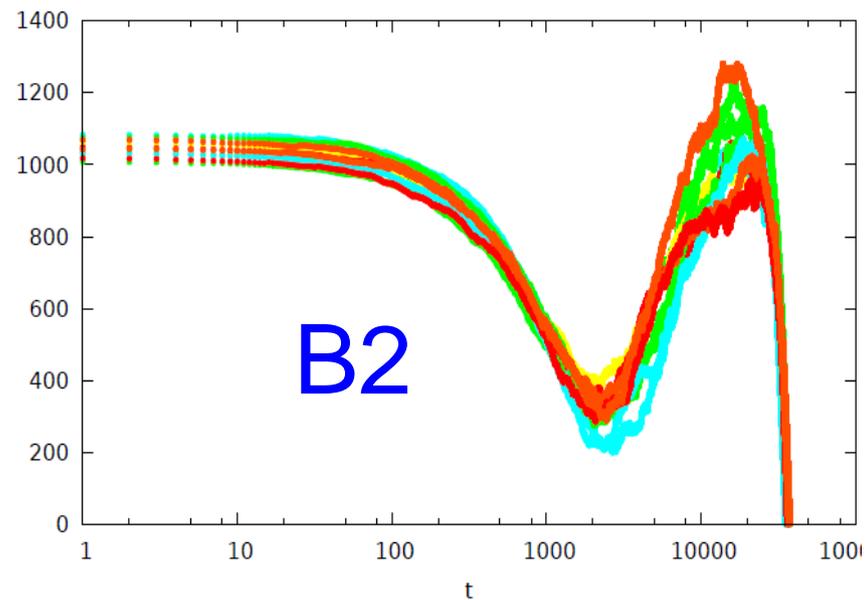
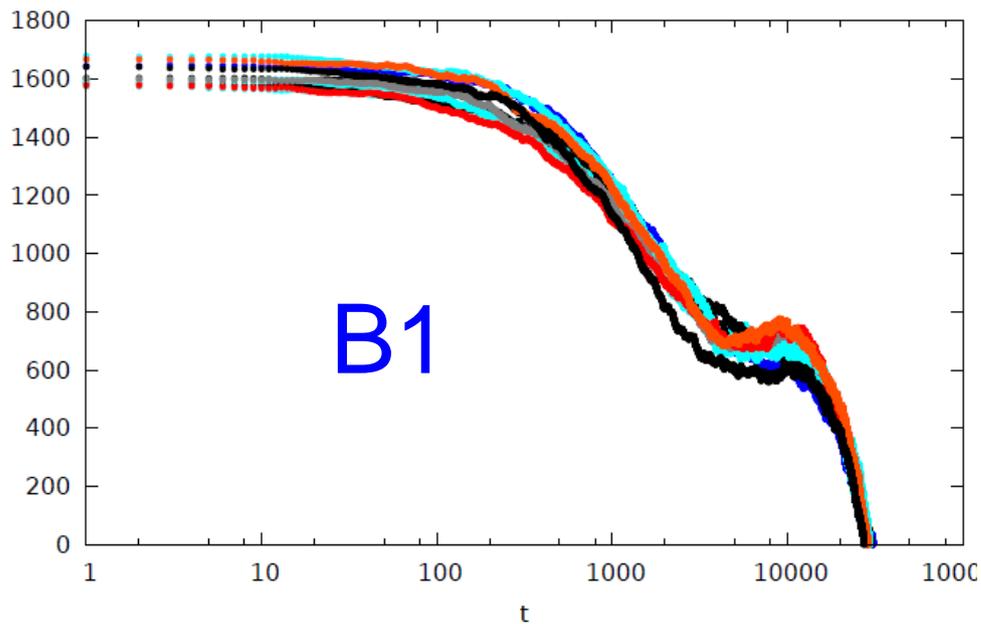
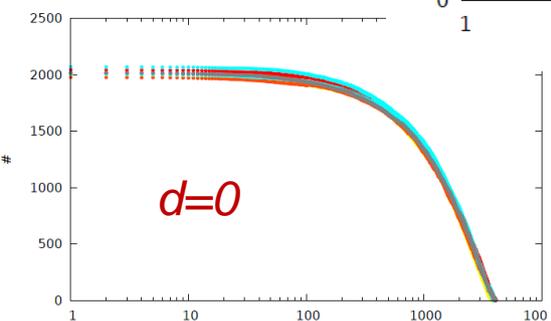
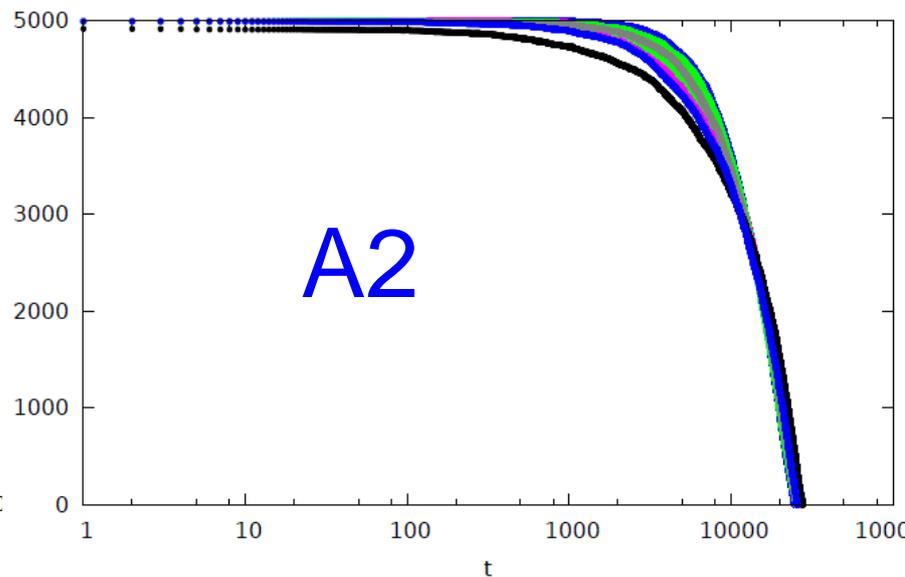
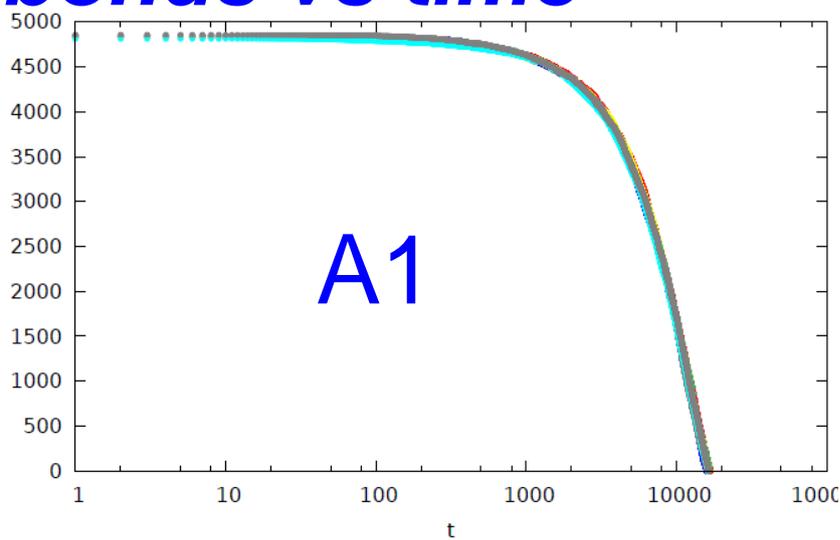


**B2:**  
the set of  $d$  nodes is  
determined as a  
squared cluster,  
the same for each  
position  $k$



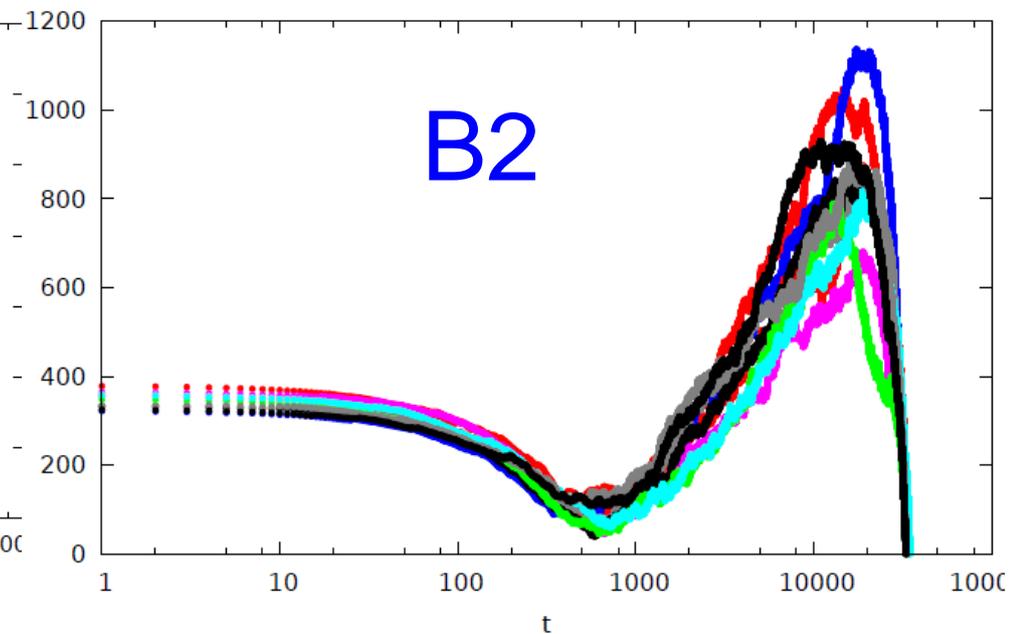
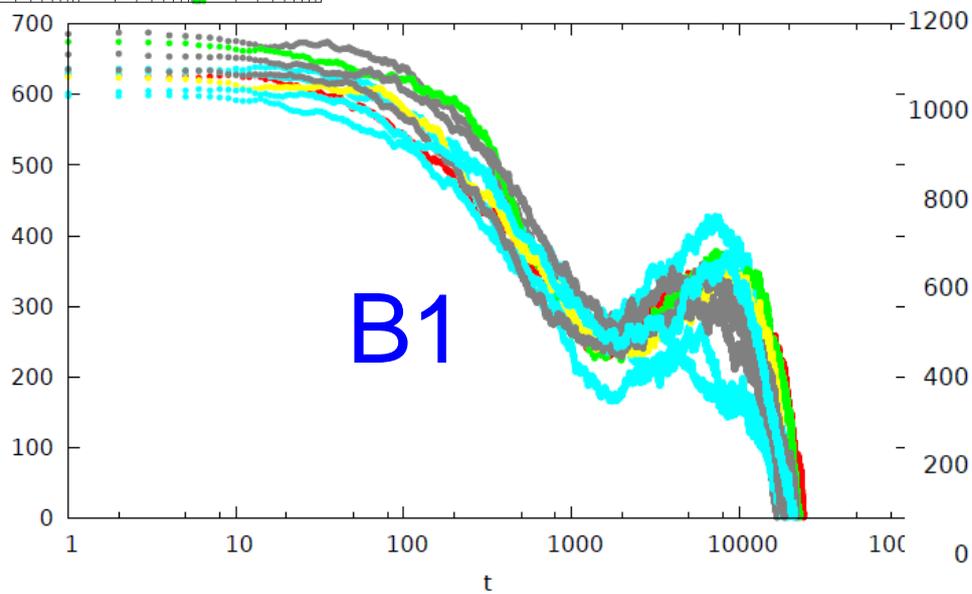
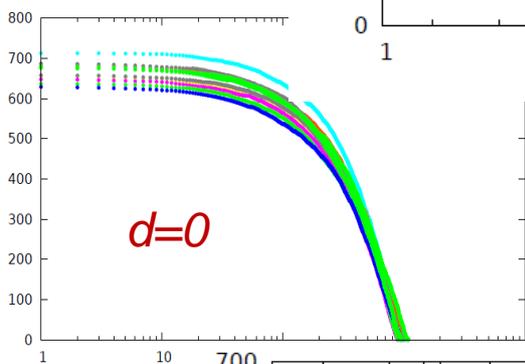
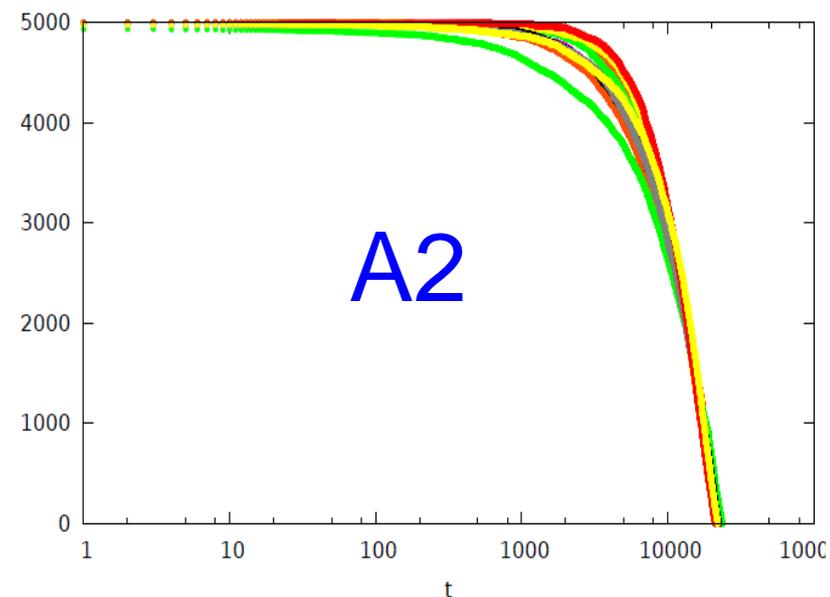
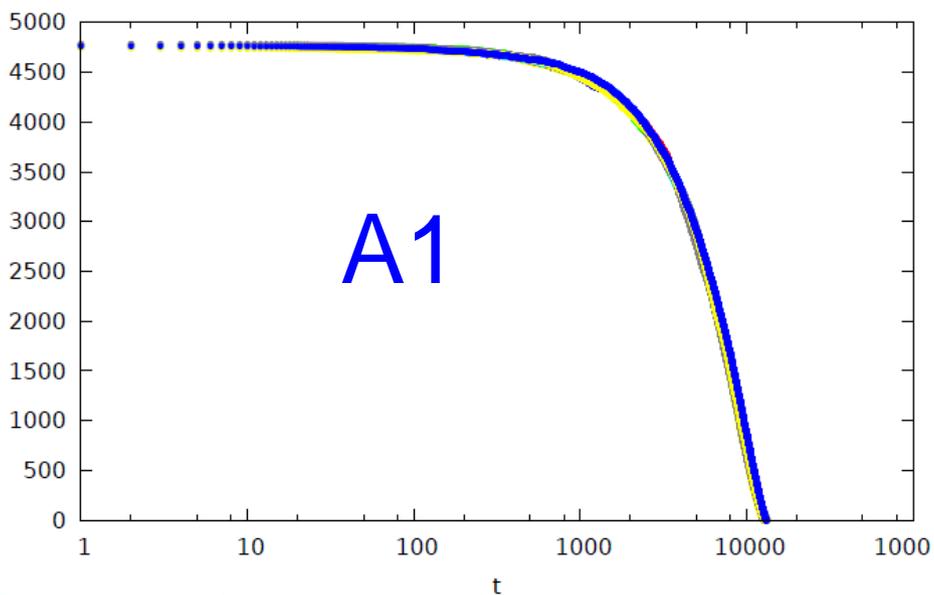
# # active bonds vs time

$L=50$   
 $F=10$   
 $q=20$   
 $d=0.5$



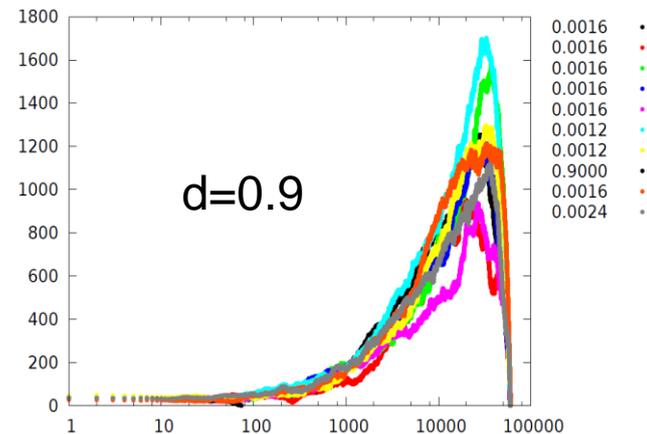
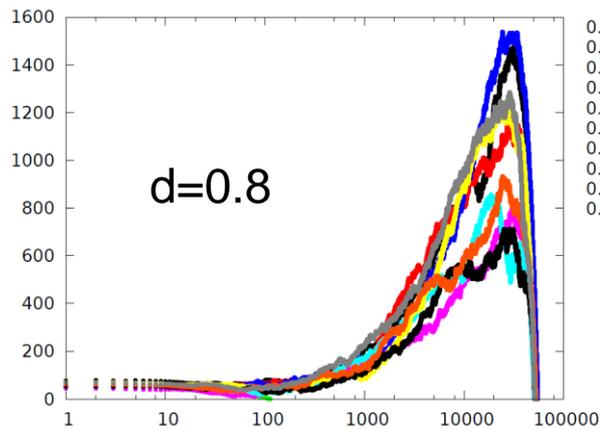
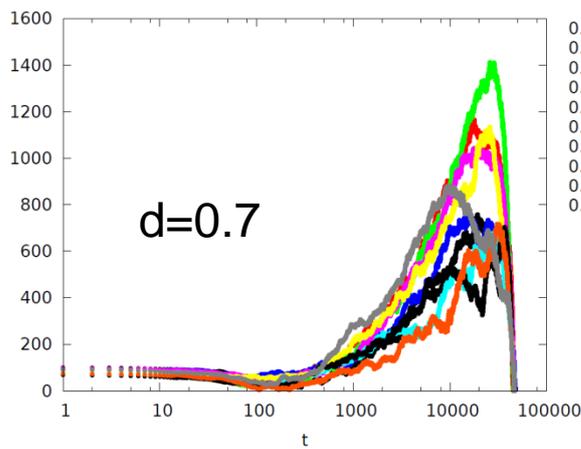
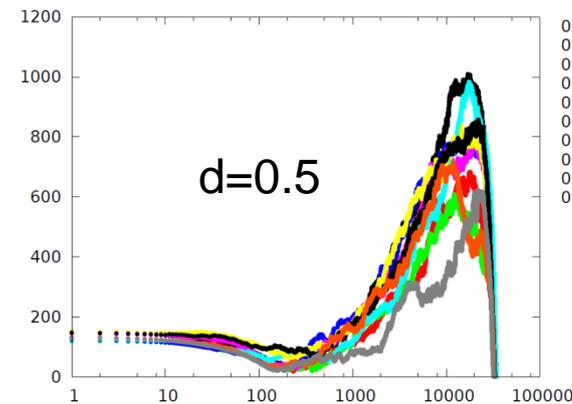
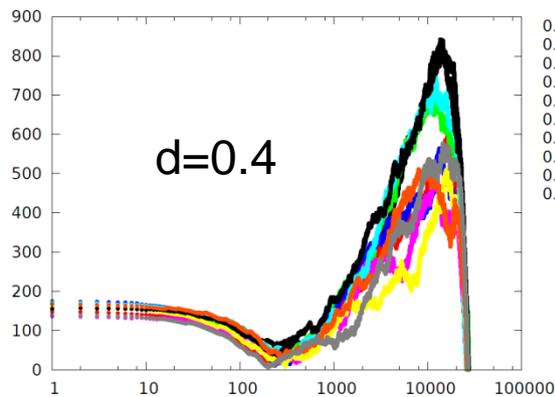
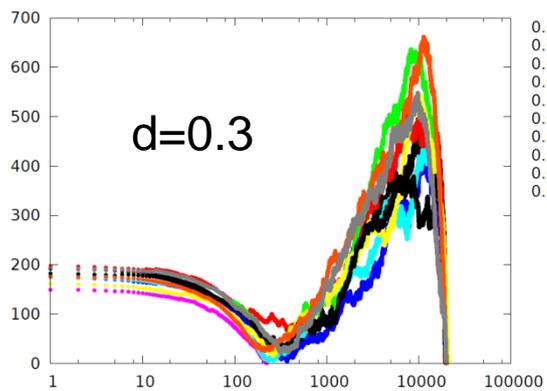
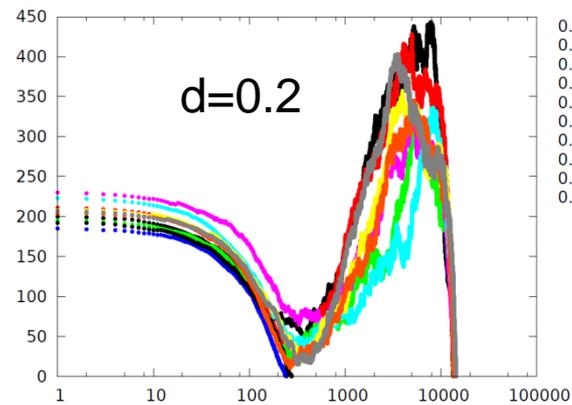
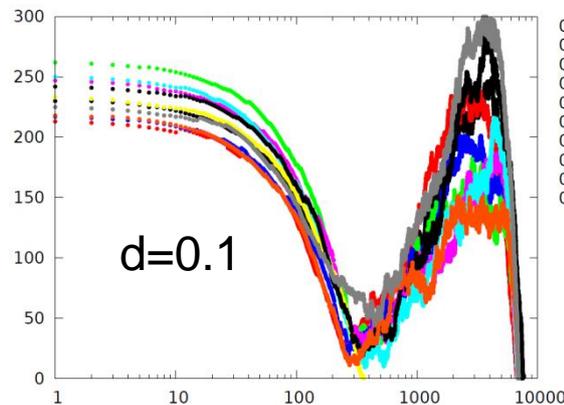
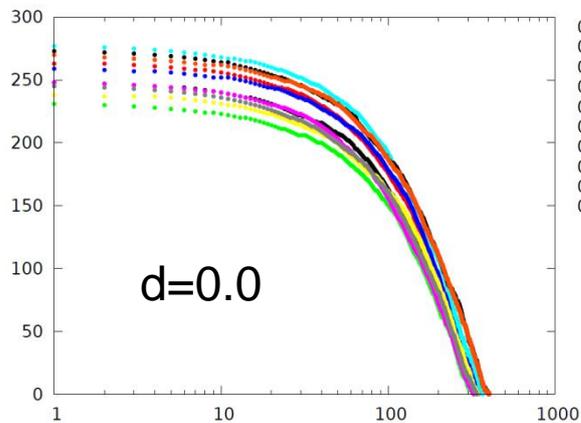
# # active bonds vs time

$L=50$   
 $F=10$   
 $q=70$   
 $d=0.5$

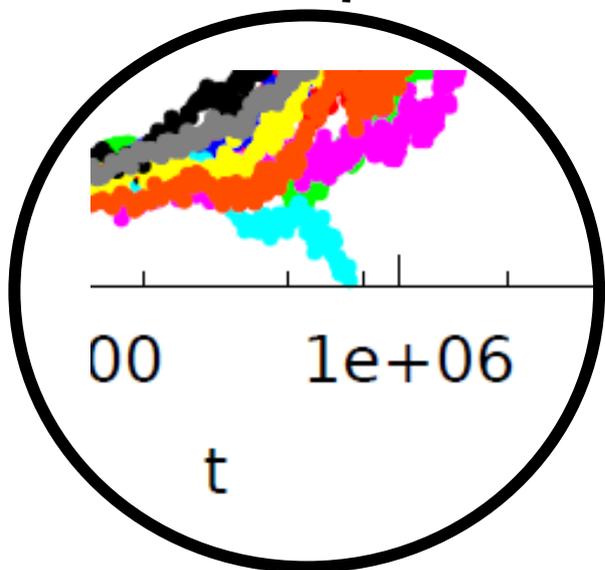
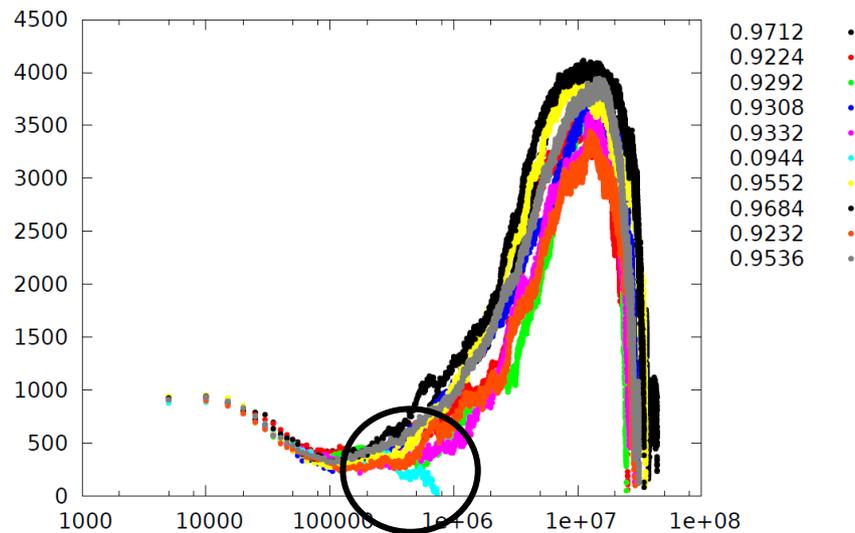


# # active bonds vs time

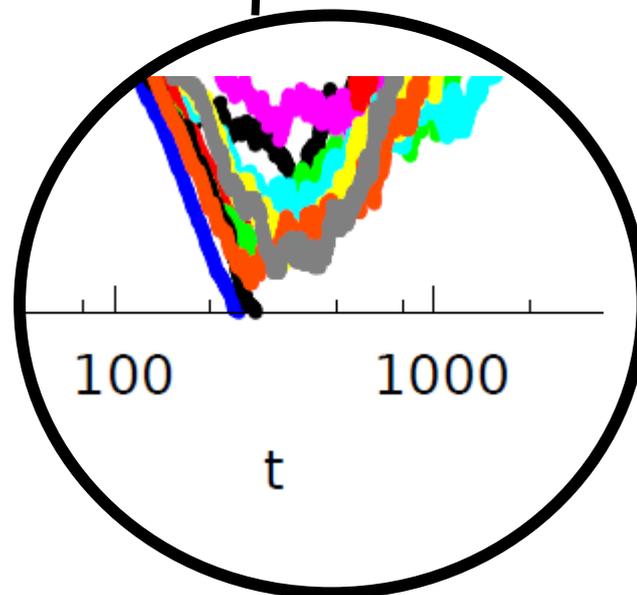
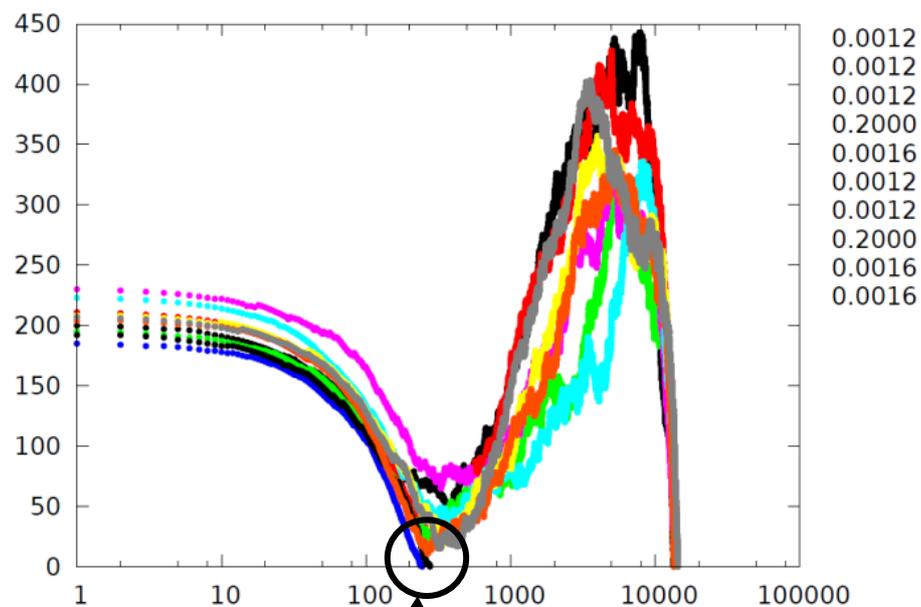
$L=50, F=10, q=200, d=0.0, \dots, 0.9$



No repulsion,  $B1$ ,  $q=50$ ,  $d=0.0$



Repulsion,  $B1$ ,  $q=200$ ,  $d=0.2$



p  
r  
e  
d  
i  
c  
t  
i  
o  
n

n  
o  
i  
t  
a  
n  
a  
l  
p  
r  
o  
c  
e  
d  
u  
r  
e

To assess the usefulness of a modelling technique one has to look at the strength of three stages in the use of a model:

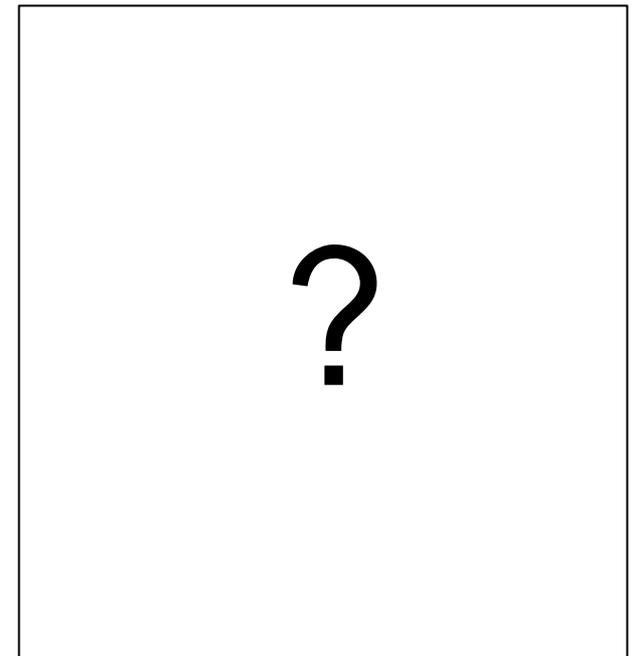
-(**encoding**) the map from the known or hypothesised facts and processes into the model set-up,

- (**inference**) the deduction of results from the set-up to the outcomes, and finally

-(**decoding**) the mapping of the results back to the phenomena of concern.

Roughly, the usefulness of a model is the reliability of the whole modelling chain: encoding + inference + decoding.

[Bruce Edmonds and Mario Paolucci, 2012  
[jasss.soc.surrey.ac.uk/15/2/reviews/6.html](http://jasss.soc.surrey.ac.uk/15/2/reviews/6.html)]





## In terms of fashion:

The maximal (although small) cluster is formed only if a set of agents is initially endowed with the leading pattern of fashion, i.e. with the leading (most frequent) features. On the contrary, the maximal cluster is not formed if the same leading features are distributed randomly between agents. To make it short, it is not the leading features what make fashion, but the leading agents endowed with these features.

Taking this result directly, we are tempted to interpret it in a ‘personalized’ way: We could imagine, that some amount of agents endowed with the leading features seem to others to be more ideal and worth imitation.



p  
r  
e  
d  
i  
c  
t  
i  
o  
n



In a society, imitation is complemented by repulsion.

When repulsion is added to the Axelrod model, the outcome is a small cluster of units in the same state.

The formation of this kind of cluster depends on the system history, and some metastable states are possible.

n  
o  
i  
t  
a  
n  
a  
l  
p  
x  
e



In each society, we observe some 'types'. The concentration of a given type is low.

p  
r  
e  
d  
i  
c  
t  
i  
o  
n



Thus fashion on the one hand signifies union with those in the same class, the uniformity of a circle characterized by it, and, *uno actu*, the exclusion of all other groups. [G. Simmel, Int. Quarterly, 1904] (see also Veblen, Bourdieu,...)



n  
o  
i  
t  
a  
n  
a  
l  
p  
x  
e



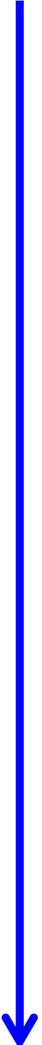
In a society, imitation is complemented by repulsion.

When repulsion is added to the Axelrod model, the outcome is a small cluster of units in the same state.

The formation of this kind of cluster depends on the system history, and some metastable states are possible.

In each society, we observe some 'types'. The concentration of a given type is low.

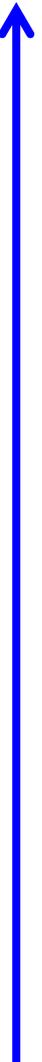
p  
r  
e  
d  
i  
c  
t  
i  
o  
n



Thus fashion on the one hand signifies union with those in the same class, the uniformity of a circle characterized by it, and, *uno actu*, the exclusion of all other groups. [G. Simmel, Int. Quarterly, 1904] (see also Veblen, Bourdieu,...)



n  
o  
i  
t  
a  
n  
a  
l  
p  
x  
e



In a society, imitation is complemented by repulsion.

When repulsion is added to the Axelrod model, the outcome is a small cluster of units in the same state.

The formation of this kind of cluster depends on the system history, and some metastable states are possible.

In each society, we observe some 'types'. The concentration of a given type is low.

The model does NOT capture the dynamic character of fashion.

p  
r  
e  
d  
i  
c  
t  
i  
o  
n



Thus fashion on the one hand signifies union with those in the same class, the uniformity of a circle characterized by it, and, *uno actu*, the exclusion of all other groups. [G. Simmel, Int. Quarterly, 1904] (see also Veblen, Bourdieu,...)



n  
o  
i  
t  
a  
n  
a  
l  
p  
x  
e



In a society, imitation is complemented by repulsion.

When repulsion is added to the Axelrod model, the outcome is a small cluster of units in the same state.

The formation of this kind of cluster depends on the system history, and some metastable states are possible.

In each society, we observe some 'types'. The concentration of a given type is low.

The model does NOT capture the dynamic character of fashion.

**THANK YOU**