The Heider balance and the looking-glass self

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in cooperation with

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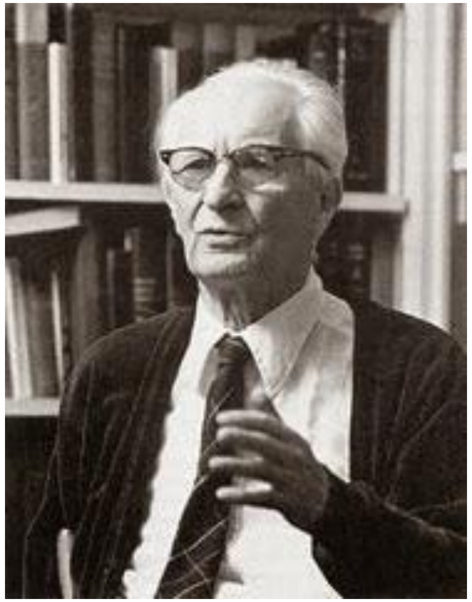
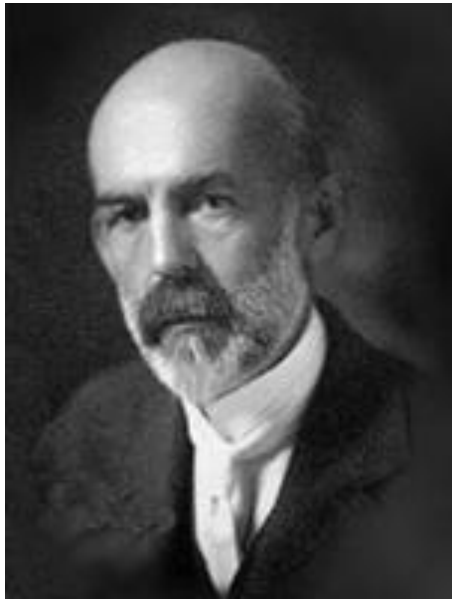
the Heider (structural) balance:



- world divided into two groups, internally friendly and mutually hostile



The looking-glass self:

- I am not who I think I am
- I am not who you think I am
- I am who I think you think I am

outline

Networks of social relations

How to reach the balance:

- why?
- from triads to network
- an example
- discrete and continuous dynamics
- jammed states
- asymmetric relations

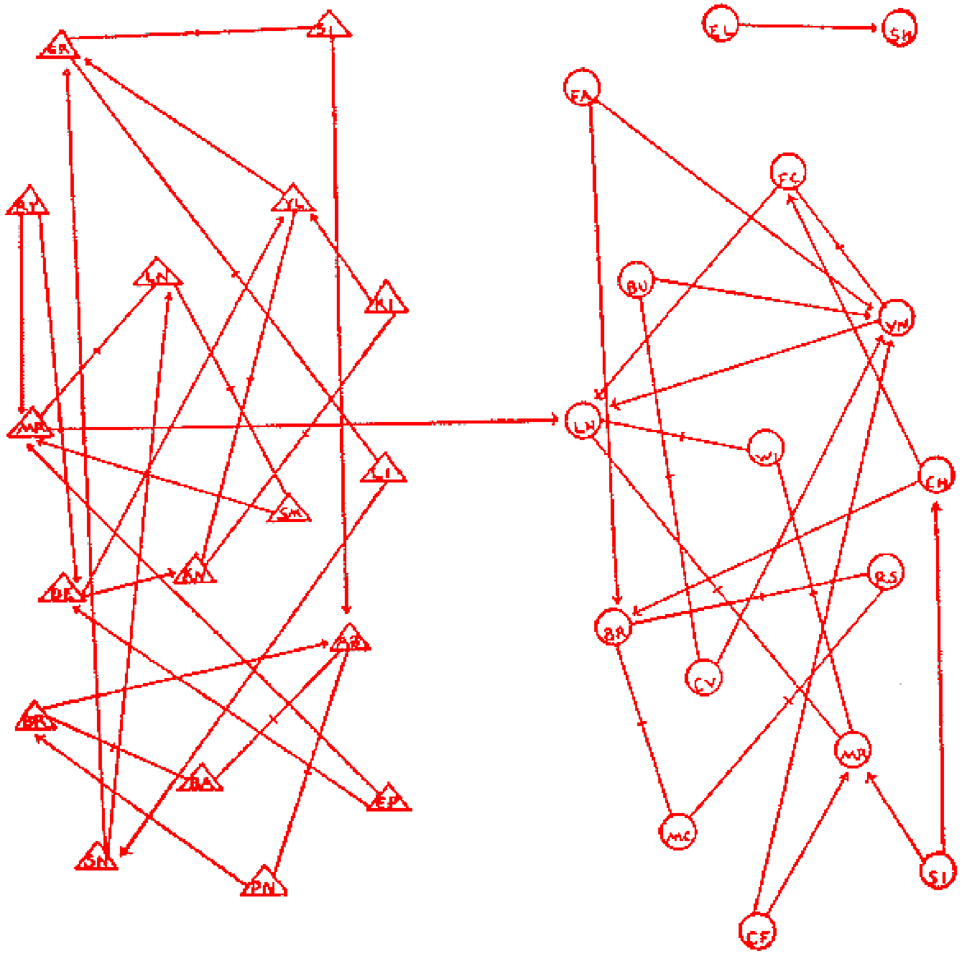
Self-evaluation

Some stable configurations and an algorithm to classify them

More configurations and why they are so similar?

A tentative application

Networks of social relations



1928-1933: first sociograms

- Sing Sing prison,
- a reformatory for delinquent girls,
- public and private Brooklyn schools.

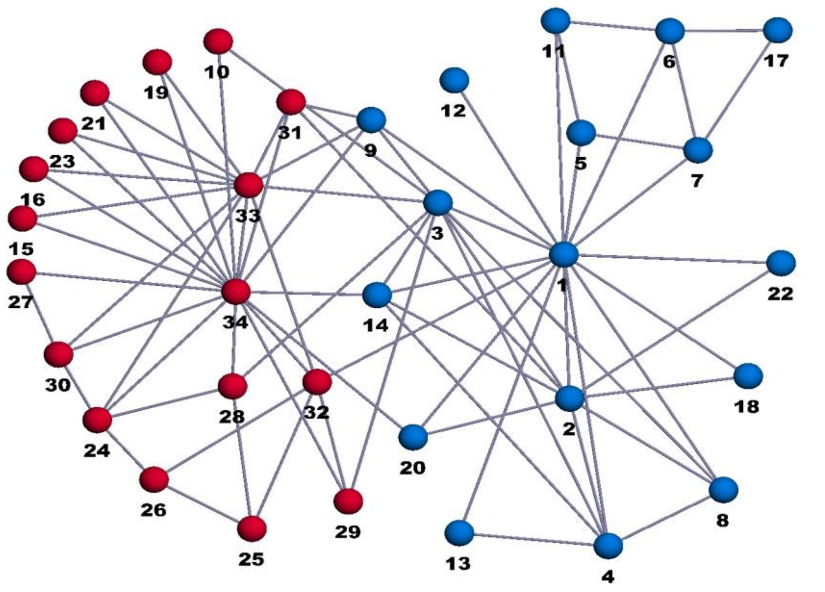
https://archive.org/stream/whoshallsurviven00jlmo#page/38/mode/2up
www.slate.com/articles/technology/future_tense/2014/10/j_l_moreno_a_psychologist_s_30s_experiments_invented_social_networking.html

Networks of social relations: an example

Zachary karate club – first 10 rows/columns

0	1	1	1	1	1	1	1	1	0
1	0	1	1	0	0	0	1	0	0
1	1	0	1	0	0	0	1	1	1
1	1	1	0	0	0	0	1	0	0
1	0	0	0	0	0	1	0	0	0
1	0	0	0	0	0	1	0	0	0
1	0	0	0	1	1	0	0	0	0
1	1	1	1	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0

1 means that „ the two individuals being represented consistently interacted in contexts outside those of karate classes, workouts, and club meetings."



„Students, teachers and administrators of a college karate club that suffered a strong conflict that led to the club excision in two groups. Node colors in the graph describe the final faction of each individual. „

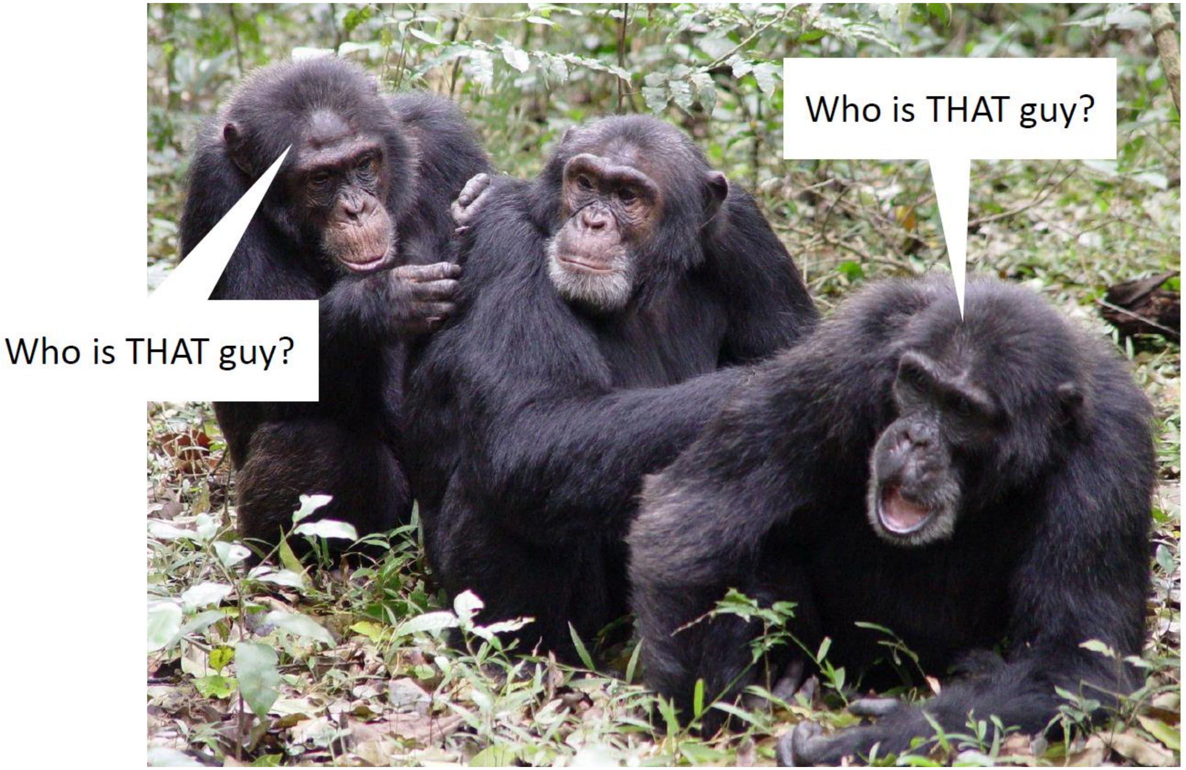
[webpage of José Javier Ramasco, ifisc.uib-csic.es/~jramasco/]

W. Zachary, *An information flow model for conflict and fis* *in small groups*, J. Anthrop. Res, 33 (1977) 452.

vlado.fmf.uni-lj.si/pub/networks/data/ucinet/zachary.dat

[chelmno-kyokushin-kan.pl.tl/O-klubie.htm]

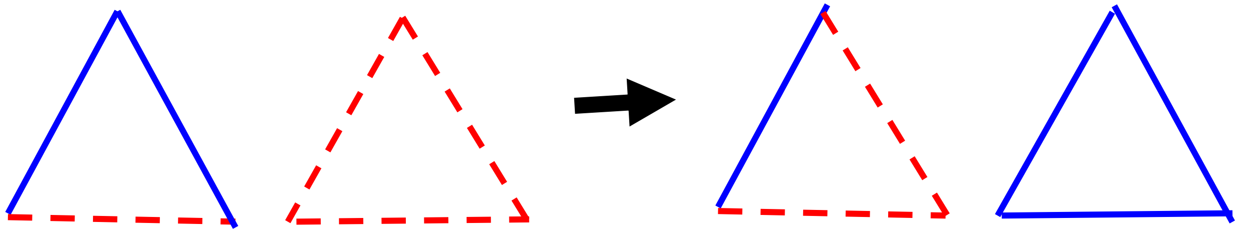
Towards structural balance – why?



www.yalescientific.org/2008/11/stress-hormones-and-social-rank/

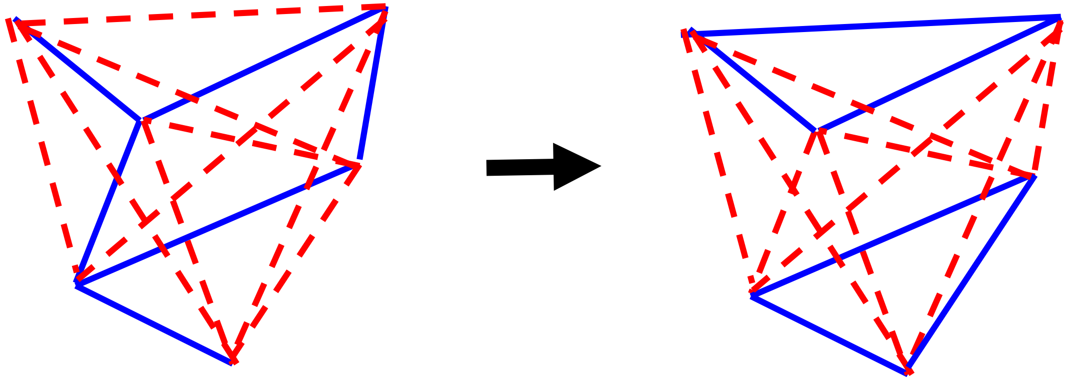
Cognitive dissonance
about friends and enemies
is removed if:

- a friend of my friend is my friend,
- a friend of my enemy is my enemy,
- an enemy of my friend is my enemy,
- an enemy of my enemy is my friend.



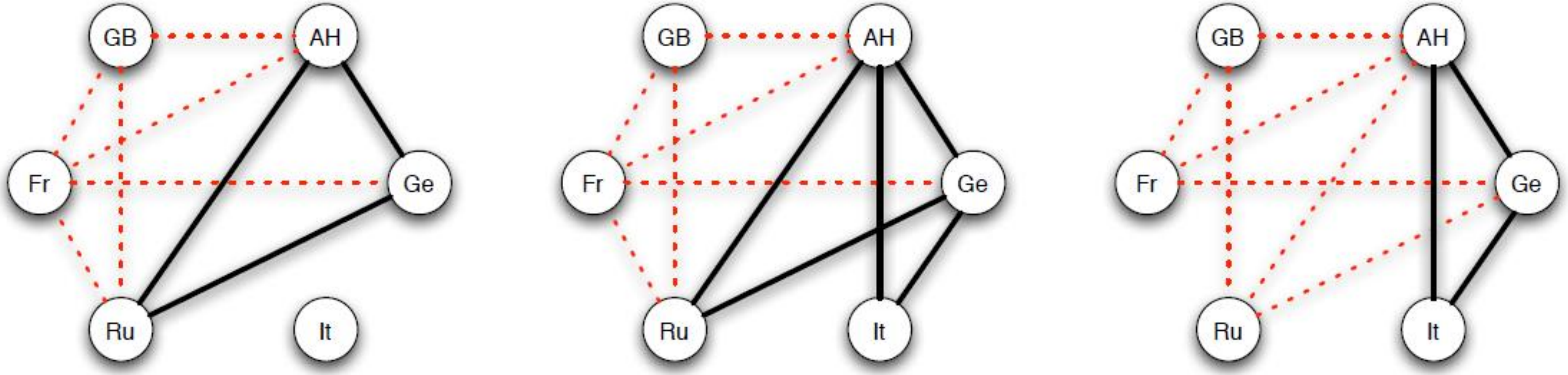
Towards structural balance

Cartwright – Harary Theorem: If a labeled complete graph is balanced, then either all relations are friendly, or else the nodes can be divided into two groups, *X* and *Y*, such that every pair of people in *X* like each other, every pair of people in *Y* like each other, and everyone in *X* is the enemy of everyone in *Y*.

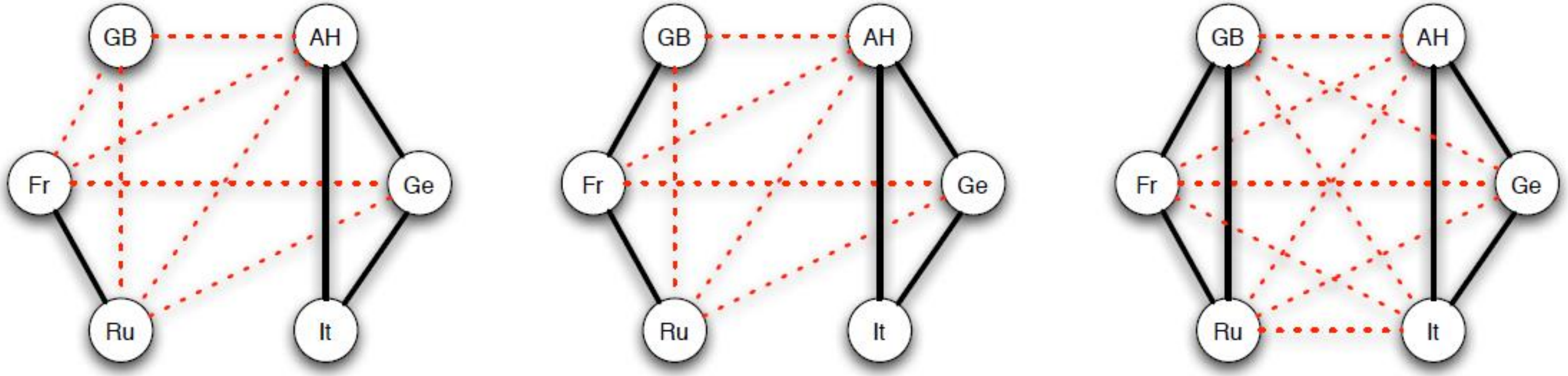


twitter.com/TurkeyPurge/status/1066773251437690880

[D. Cartwright and F. Harary, *Structural balance; a generalization of Heider's theory*, Psychol. Rev. 63 (1956) 277;
D. Easley, J. Kleinberg, *Networks, Crowds, and Markets*, Cambridge UP, 2010]



(a) *Three Emperor's League 1872–81* (b) *Triple Alliance 1882* (c) *German-Russian League 1890*

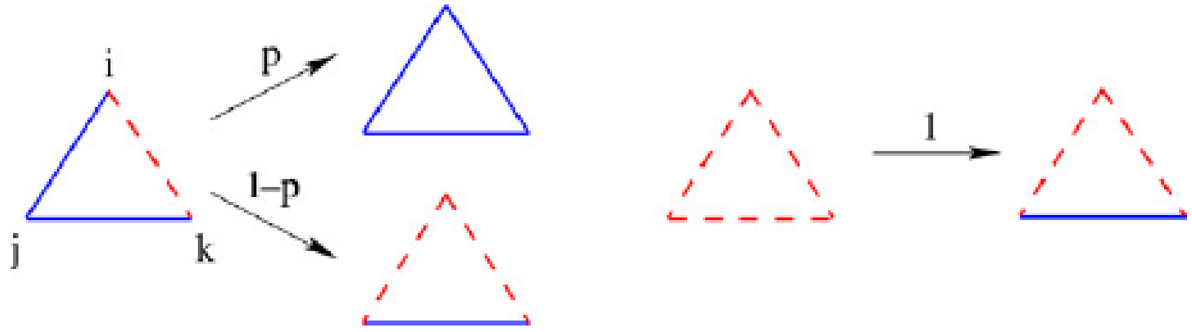


(d) *French-Russian Alliance 1891–94* (e) *Entente Cordiale 1904* (f) *British Russian Alliance 1907*

[T. Antal, P. Krapivsky. S. Redner, *Social balance of networks: the dynamics of friendship and enmity*, Physica D 224 (2006) 130]

Towards structural balance : discrete MC algorithms

A. Local Triad Dynamics



B. Constrained Triad Dynamics $\equiv$ Metropolis algorithm, where

$$U = -\sum_{k=1}^{N} x(ik)x(kj)x(ji)$$

1. Find an unbalanced triad
2. Change a random link
3. If *U* increases, withdraw the change
4. If *U* is not changed, withdraw the change with *p*=1/2

[T. Antal, P. Krapivsky. S. Redner, *Dynamics of social balance on networks*, Phys. Rev. E 72 (2005) 036121]

Discrete MC algorithms: jammed states

In the Constrained Triad Dynamics, some configurations remain **unbalanced and stable.**

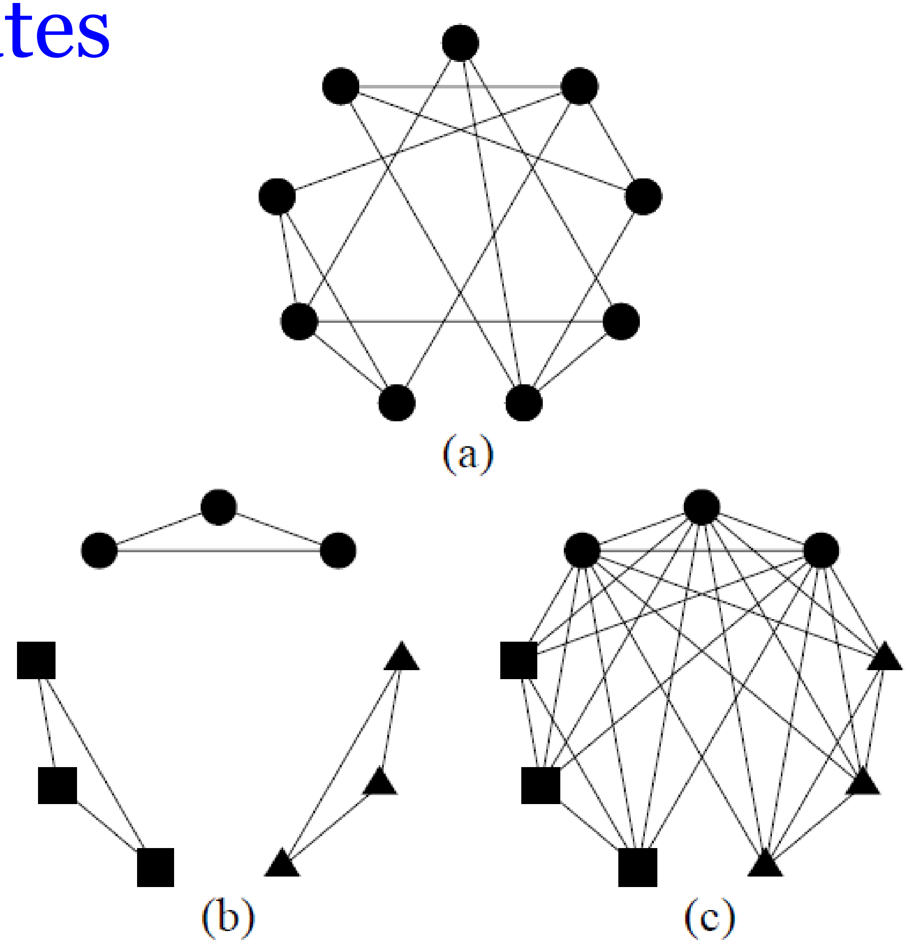
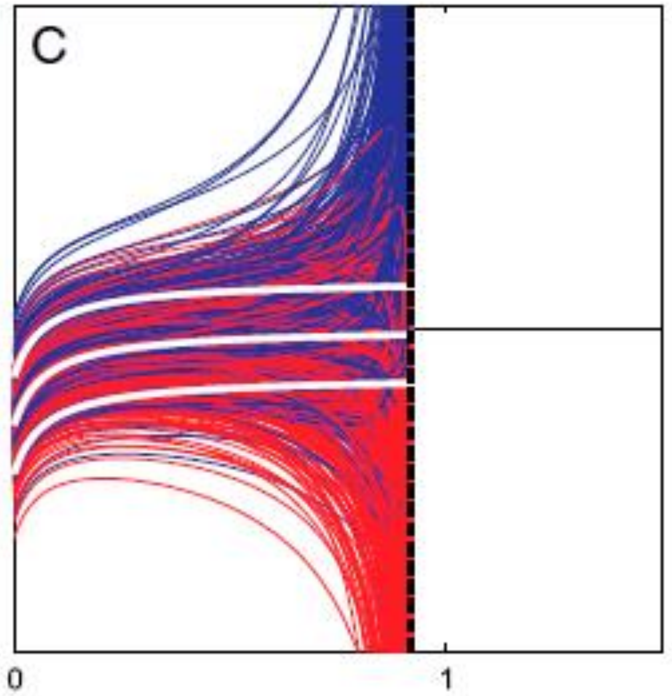


FIG. 7: Examples of jammed configurations for $N = 9$ (only friendly links are displayed). (a) A jammed configuration that appeared in simulations. (b) A jammed state consisting of three mutually antagonistic cliques. (c) A jammed state derived from (b) in which the top clique from (b) is friendly toward the remaining two cliques.

[T. Antal, P. Krapivsky. S. Redner, *Dynamics of social balance on networks*, Phys. Rev. E 72 (2005) 036121]

Towards structural balance : continuous dynamics

$$\frac{dx_{ij}}{dt} = \sum_{k\neq i,\neq j}^{N-2} x_{ik} x_{kj} = -\frac{\partial U}{\partial x_{ij}}$$



or just the matrix Riccati equation

$$\frac{dX}{dt} = X^2$$

with the solution $X(t) = X(0)[I - X(0)t]^{-1}$

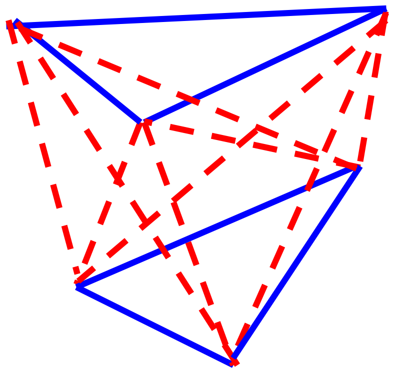
[SA Marvel, J Kleinberg, RD Kleinberg, SH Strogatz, PNAS 108 (2011) 1771]

Towards structural balance : continuous dynamics

$$\frac{dx_{ij}}{dt} = (1 - x_{ij}{}^2) \sum_{k \neq i, \neq j}^{N-2} x_{ik} x_{kj}$$

with the asymptotic solutions $x_{ij} = \pm 1$

such that $$x_{ij} = sign\left(\sum_{k \neq i, \neq j}^{N-2} x_{ik} x_{kj} \right)$$



which appears to be balanced in most cases.

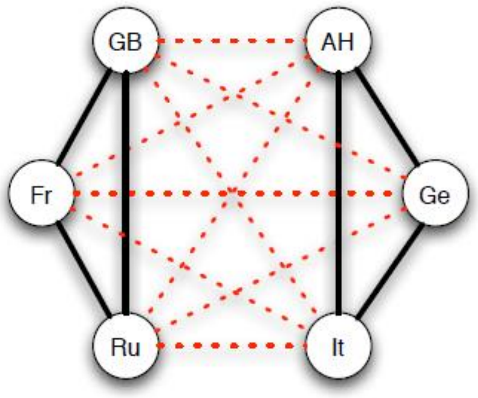
*[KK, P Gawroński, P Gronek, Int. J. Mod. Phys. C **16** (2005) 707]*

Continuous dynamics: jammed states

The condition $x_{ij} = \pm 1 = sign \sum_{k \neq i, \neq j}^{N-2} x_{ik} x_{kj}$ provides the stability.

$$A_{ij} \overset{df}{=} \sum_{k \neq i, \neq j}^{N-2} x_{ik} x_{kj}$$

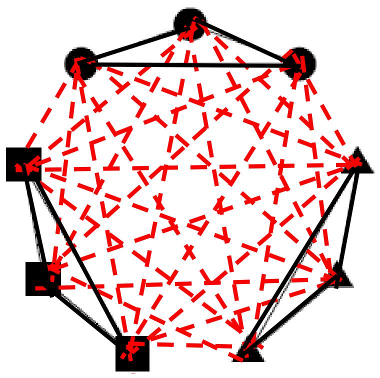
Example 1:



A(—)=+4

A(----)=-4

Example 2:



A(—)=+7

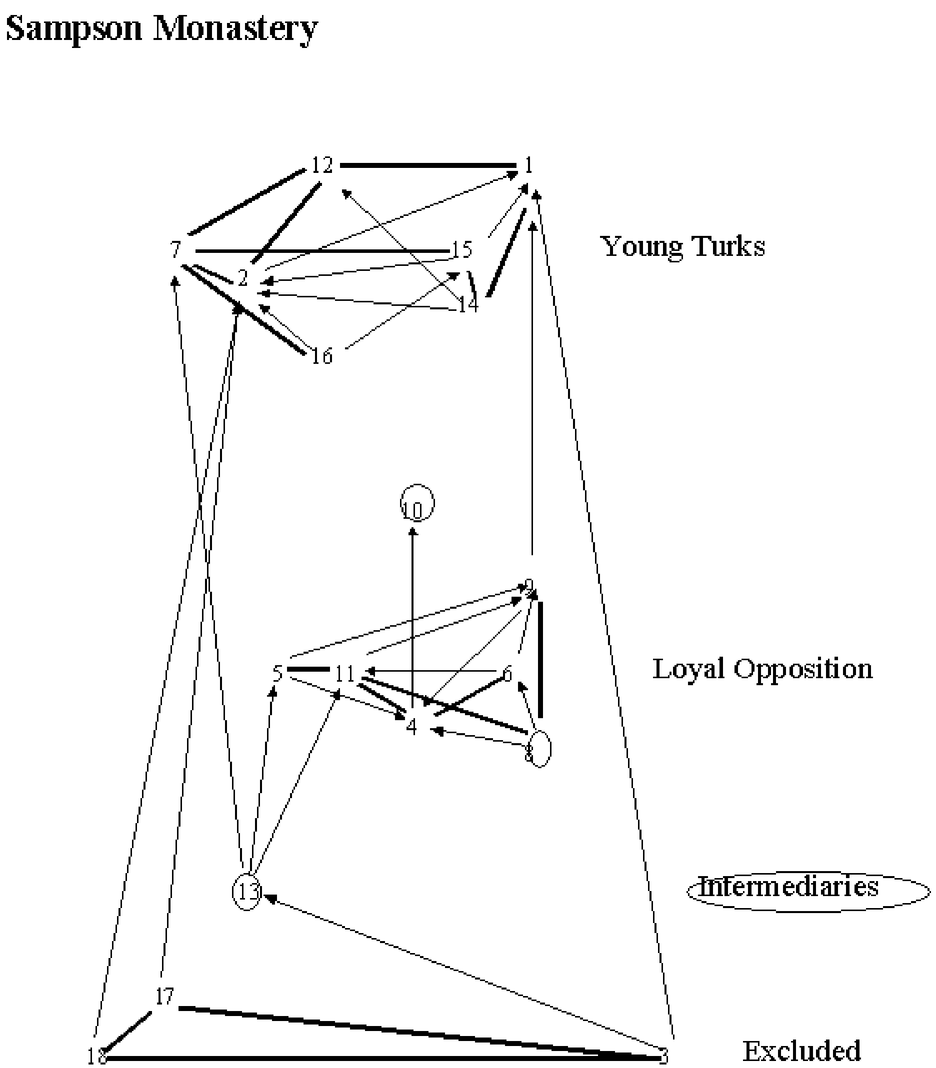
A(----)=-1

unreciprocated relations: an example

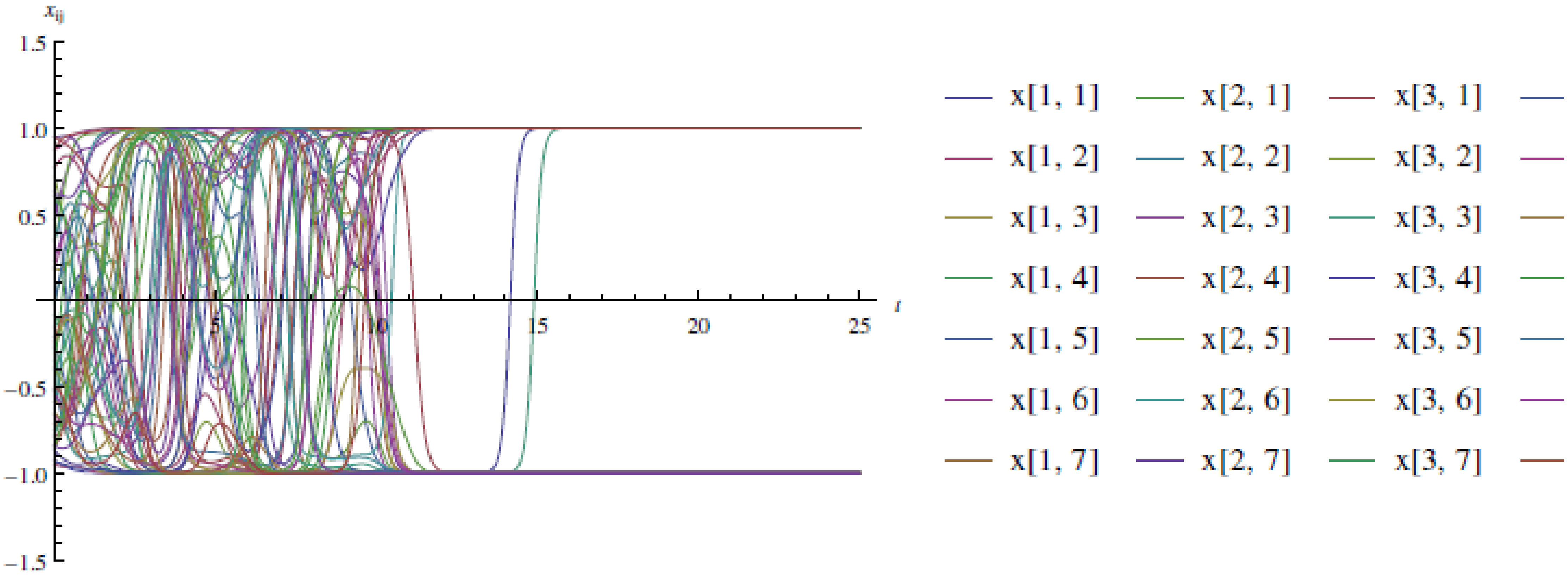
Sampson data - first 10 rows/columns

0	0	2	0	3	0	0	0	0	0
3	0	0	0	0	0	2	0	0	0
2	3	0	0	0	0	0	0	0	0
0	0	0	0	3	1	0	0	0	2
0	0	0	3	0	0	0	0	0	0
1	0	0	3	0	0	0	0	2	0
0	2	0	0	0	0	0	1	0	0
3	2	0	0	0	0	0	0	1	0
0	0	0	0	2	0	0	3	0	0
0	0	0	3	0	0	0	1	0	0

Who likes whom? Each member ranked only his top three choices on that tie.



[S. Sampson, *Crisis in a cloister*, Unpublished doctoral dissertation, Cornell University, 1969; eclectic.ss.uci.edu/~drwhite/socnetweb/courses/sampson.html] the same dynamics $$\frac{dx_{ij}}{dt} = (1 - x_{ij}{}^2) \sum_{k \neq i, \neq j}^{N-2} x_{ik} x_{kj}$$ for asymmetric relations $x_{ij} \neq x_{ji}$



lead to $x_{ik} = \pm 1$ in the stationary states, with the same stability conditions

SELF-EVALUATION

"If a boy [...] has any success, [...] he gloats over it [...]. He is eager to call in his friends [...], saying to them, 'See what I am doing! Is it not remarkable?' feeling elated when it is praised, and resentful or humiliated when fault is found with it."

each to each a looking glass
reflects the other that doth pass

A SELF-EVALUATION INDEX

where only „significant others" are counted

$$y_i = \frac{1}{2}\sum_{k \neq i}^{N-1}(1 + x_{ik})x_{ki}$$

$y_i \geq 0 \rightarrow$ self-evaluation positive or zero

$y_i < 0 \rightarrow$ self-evaluation negative (*i* frustrated)

EXEMPLARY STATIONARY STATES

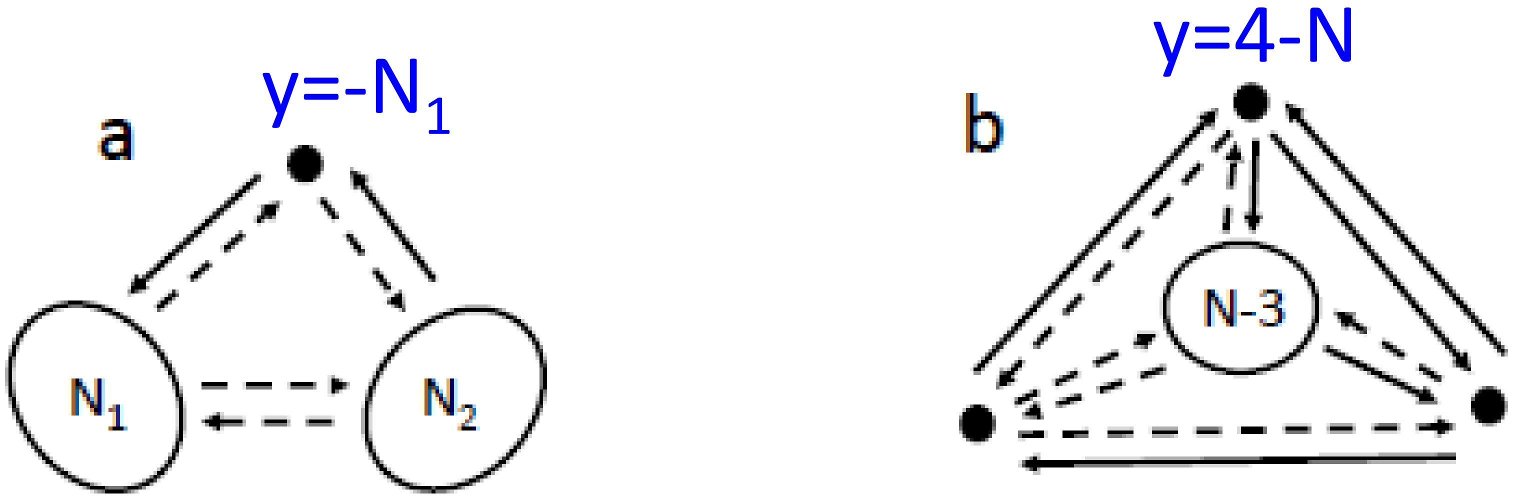
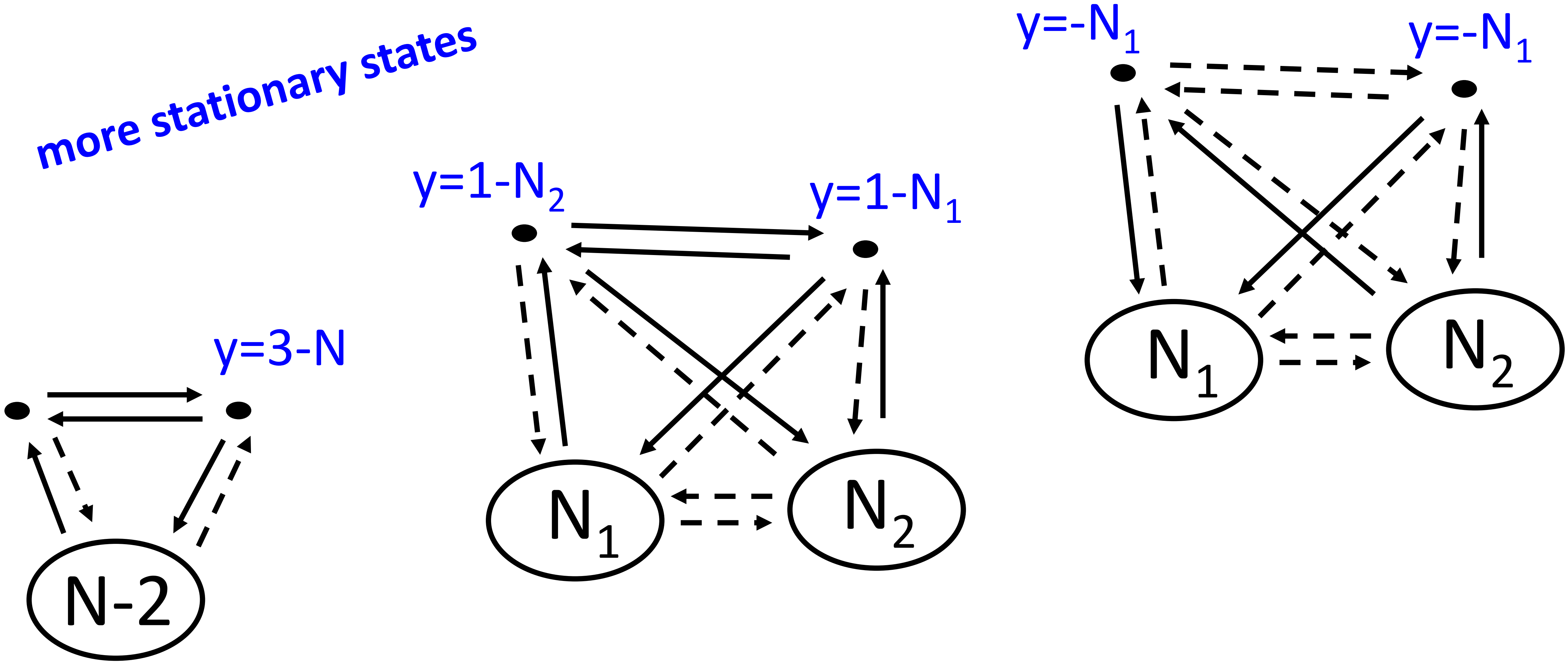


Fig. 1. Two examples of configurations of a fully connected network with asymmetric relations of some nodes. The ovals relate to clusters with identical signs of relations; positive (full lines) and negative (dotted lines). Both configurations (*a*) and (*b*) are unbalanced and stable.

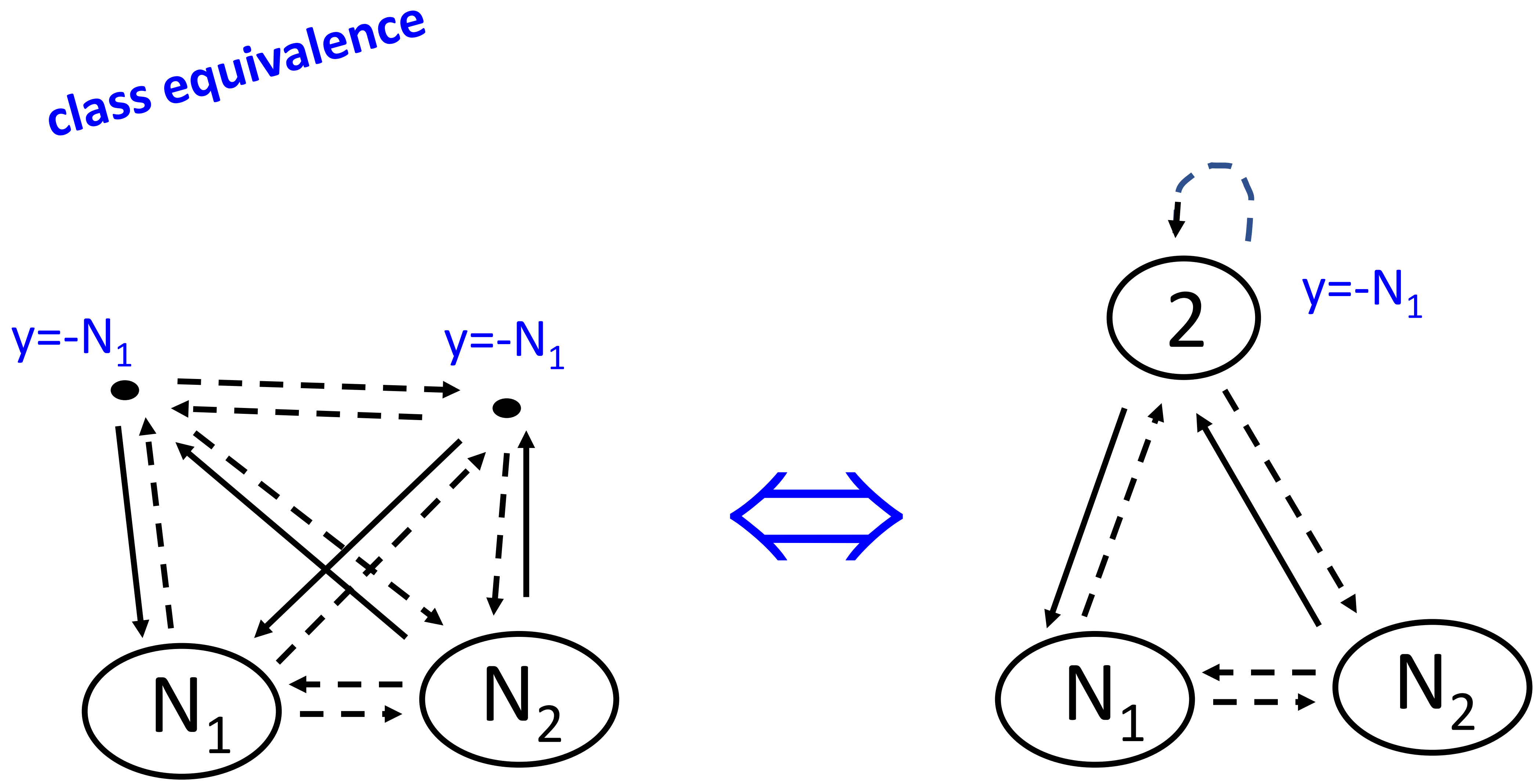

more stationary states
y=3-N
N-2
y=1-N2
y=1-N1
N1
N2
y=-N1
y=-N1
N1
N2

For higher *N*, we meet the problem of graph classification.

An algorithm for given *N* :

a) Set a random asymmetric initial state $\{x_{ij}(t=0)\}$
b) Find the stationary solution of the dynamics $\{x_{ij}(t=\infty)\}$
c) Classify nodes according to their number of positive links
d) Do the neighhbors of nodes in the same class belong to the same classes?
If NOT, refine the classification. Continue until YES.
e) The graph is classified as the list of classes of nodes, with the numbers of nodes in each class, and the relations between the classes.

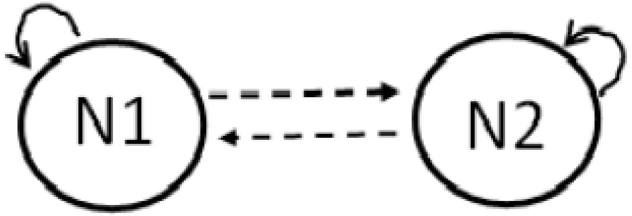
[M. J. Krawczyk, Physica A 390 (2011) 2181]



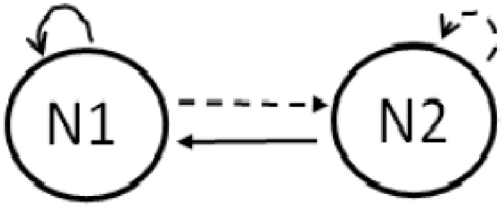
class equivalence
y=-N1
y=-N1
N1
N2
2
y=-N1
N1
N2

more stationary states

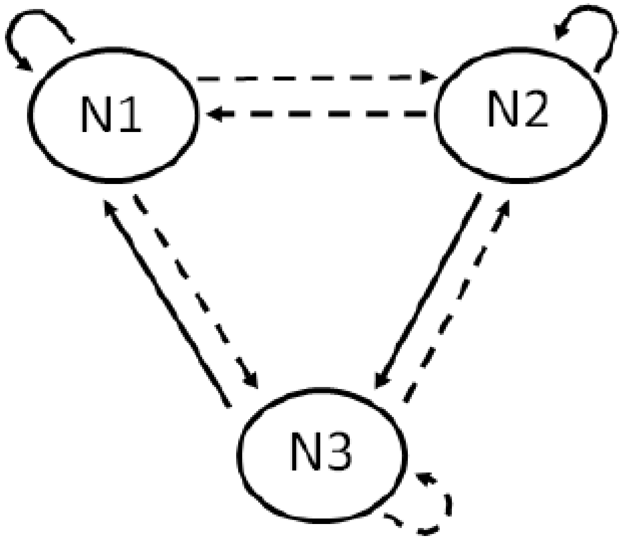
balanced, symmetric :



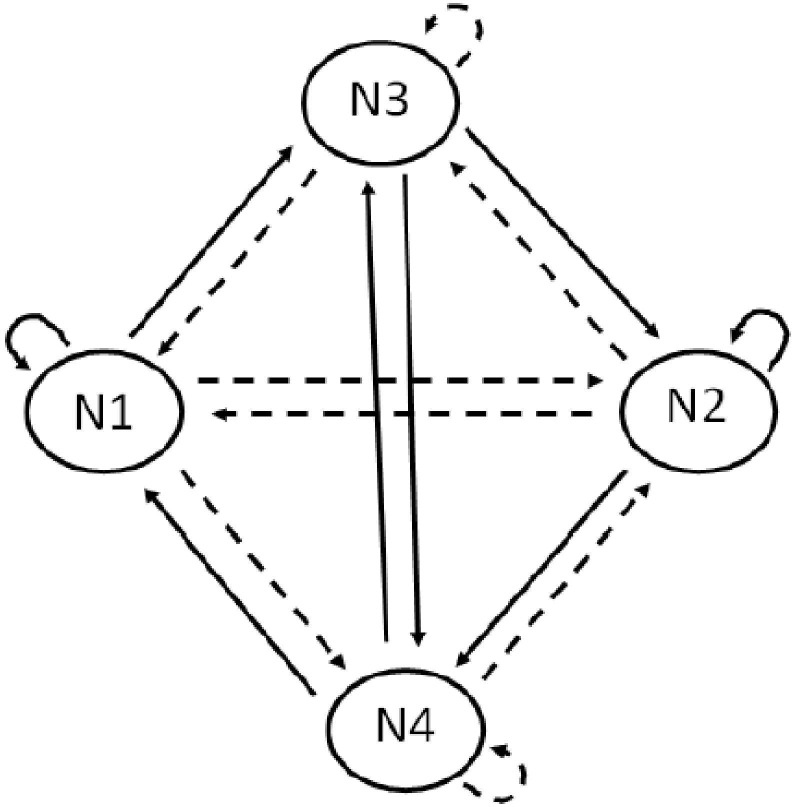
unbalanced, asymmetric :



CII: N1>N2+2



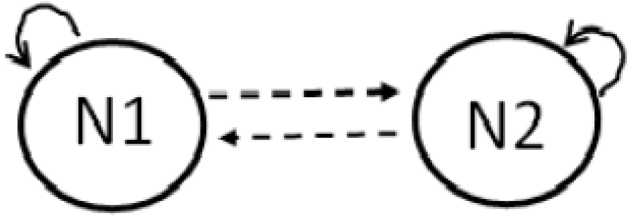
CIII: N1+N2>N3+2



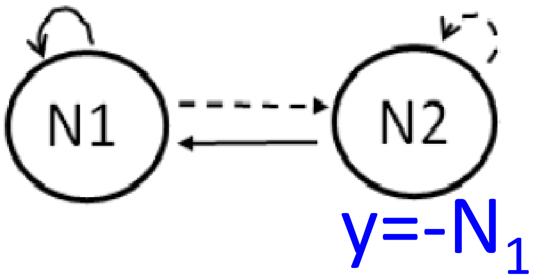
CIV: N1+N2>N3+N4+2

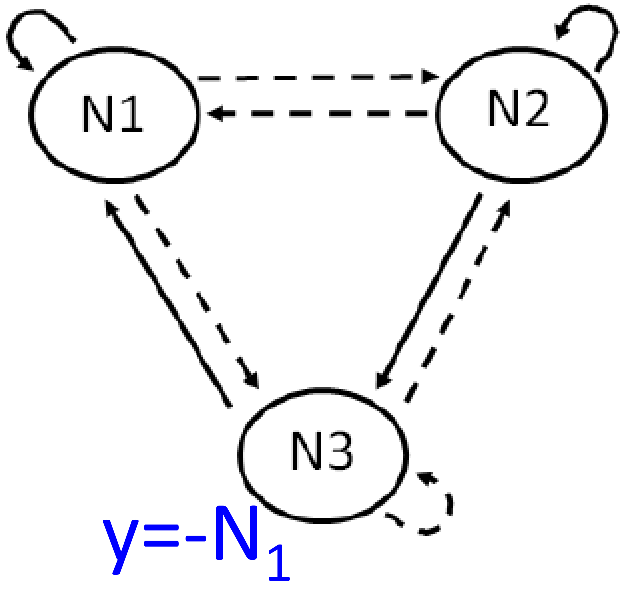
The self-evaluation index

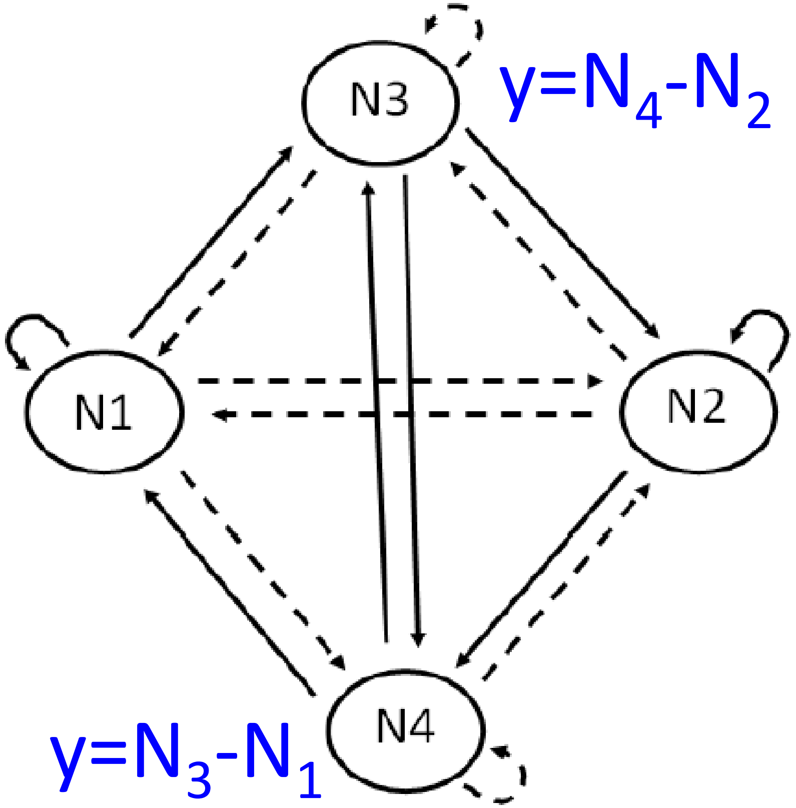
balanced, symmetric



unbalanced, asymmetric







N=7

N1	N2	N3	N4	#
3	3	1	0	1465
4	2	1	0	1125
2	4	1	0	1111
3	2	2	0	474
2	3	2	0	462
1	5	1	0	443
5	1	1	0	422
1	4	2	0	226
4	1	2	0	224
2	3	1	1	929
4	1	1	1	442

CIII

CIV

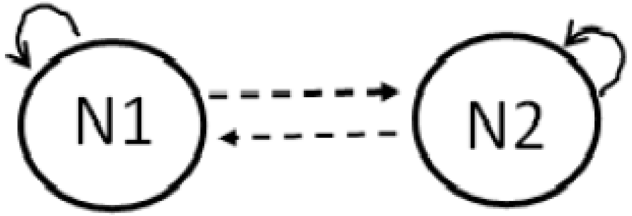
N=9

N1	N2	N3	N4	#
4	4	1	0	721
5	3	1	0	610
3	5	1	0	597
4	3	2	0	547
3	4	2	0	545
5	2	2	0	347
2	5	2	0	327
2	6	1	0	294
6	2	1	0	276
3	3	3	0	172
3	4	1	1	1084
5	2	1	1	664
3	3	1	2	435
4	2	1	2	375
4	2	2	1	353
1	6	1	1	196
5	1	1	2	139
5	1	2	1	138

The frequencies of CIII and CIV for *N*=7 and *N*=9.

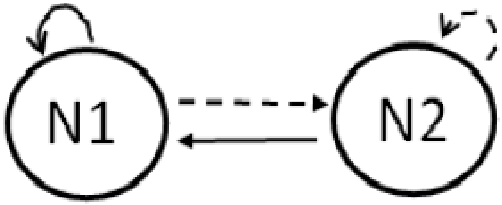
On the frequency of the stationary states

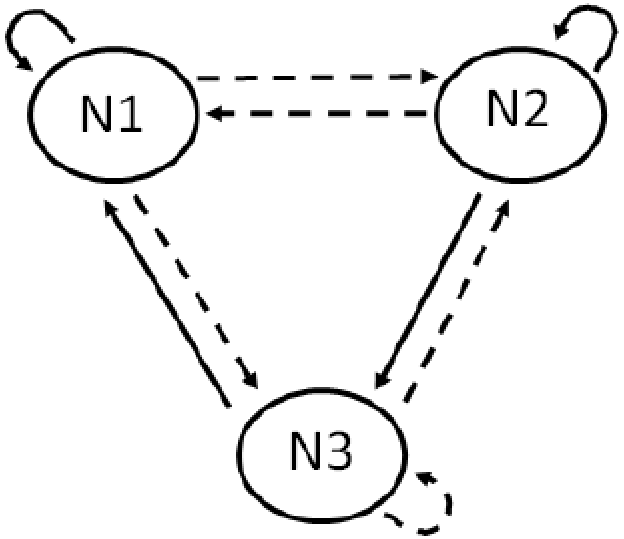
balanced, symmetric

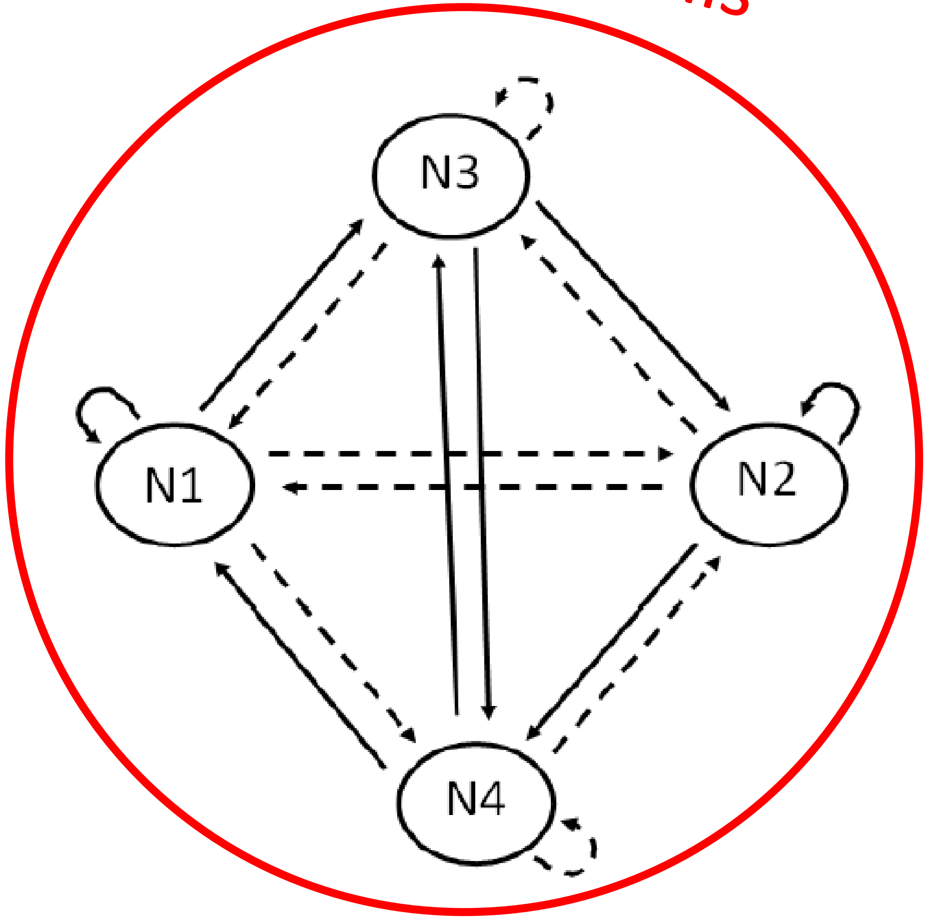


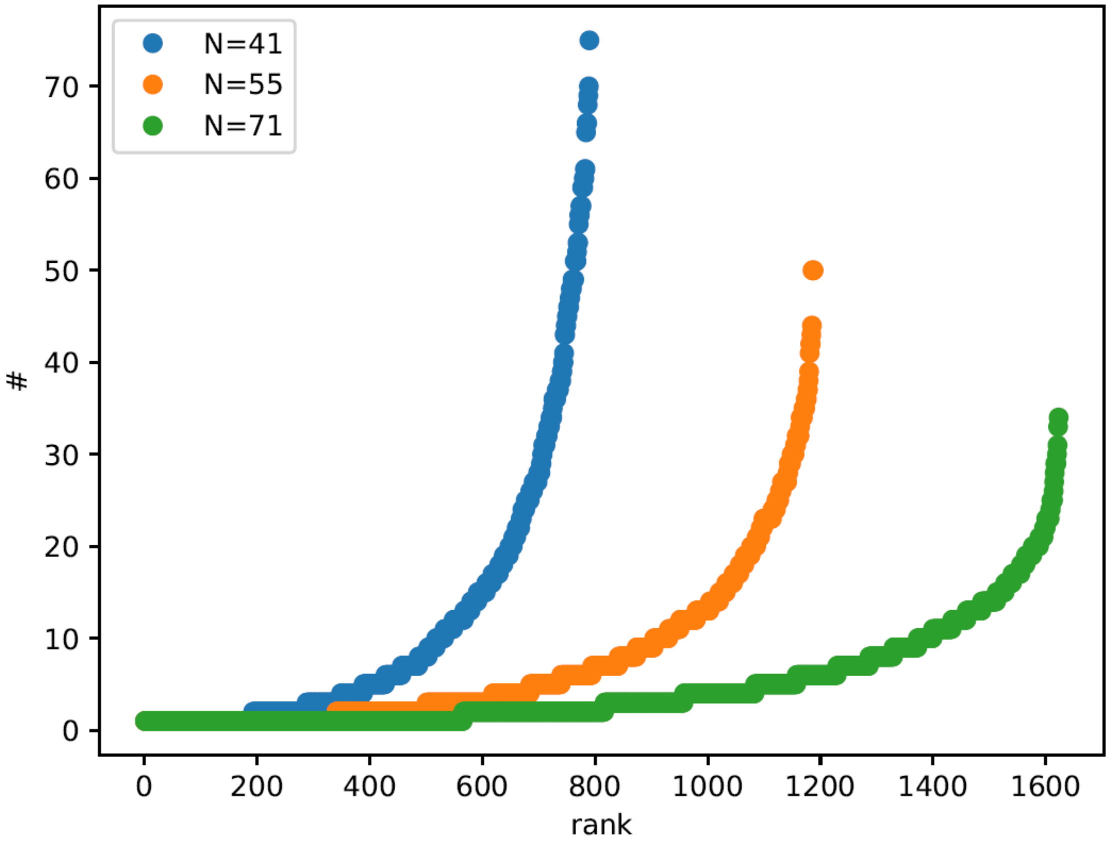
For large *N*: only this

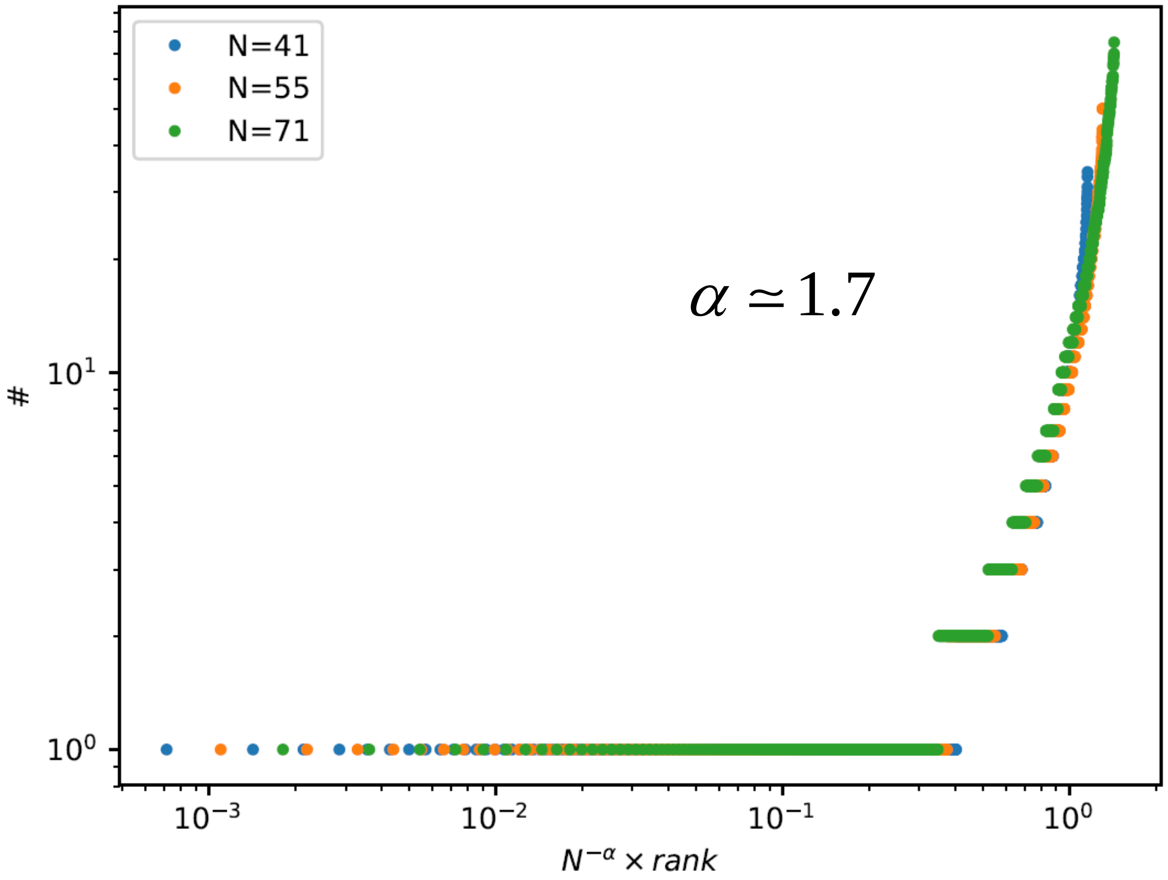
unbalanced, asymmetric











The frequencies of CIV for N=41, 55 and 71.

The number of ways of partitioning a set of *n* distinguishable elements into *k* nonempty sets

Stirling numbers of 2-nd kind *S(n,k)*

$$S(n,2) = 2^{n-1} - 1$$

$$S(n,3) = \frac{1}{6}(3^n - 3 \cdot 2^n + 3)$$

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(10,3) = 9330$$

$$S(10,4) = 34105$$

$$S(20,3) = 580606446$$

$$S(20,4) = 45232115901$$

$$S(30,3) = 34314651811530$$

$$S(30,4) = 48004081105038305$$

mathworld.wolfram.com/StirlingNumberoftheSecondKind.html

austinmohr.com/home/?page_id=431

the Sampson data: 18×18 non-symmetric, valued rankings

SAMPLK/SAMPDLK	: whom you like/dislike most?	3,2,1
SAMPES/SAMPDES	: whom you estimate most/least?	3,2,1
SAMPIN/SAMPNIN	: positive/negative influence	3,2,1
SAMPPR/SAMPNPR	: whom you praise/blame most?	3,2,1

vlado.fmf.uni-lj.si/pub/networks/data/ucinet/ucidata.htm#sampson

matrices too thin : if we remove the 1-st monk, then:

(SAMPIN - SAMPNIN)/5 → CIII

(SAMPLK – SAMPDLK)/ 5 → CIV

(SAMPES – SAMPDES)/ 5 → CIV

(SAMPPR – SAMPNPR)/ 5 → CIV

SO WHAT ?

Following the „looking-glass self" theory, we infer on low self-evaluation from asymmetric interpersonal relations.

The proces of removal of cognitive dissonance drives interpersonal relations to stationary configurations. Some of these configurations appear to be frustrating for selected individuals.

Our results allow to classify these configurations and – in principle – to identify them in social groups.

more details in arXiv:1903.12464

Not merely little devils: a still from the 1990 Columbia film Lord of the Flies. Photograph: Pictorial Press Ltd/Alamy

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