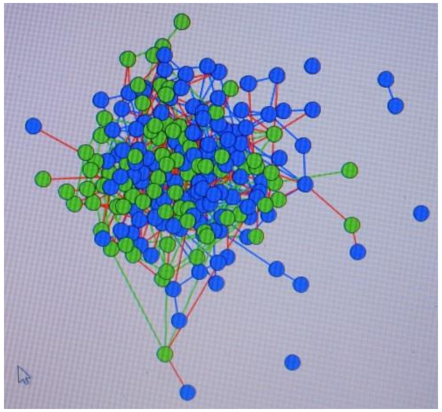
The coevolving voter model with spin-dependent rewiring probability – mean field approach



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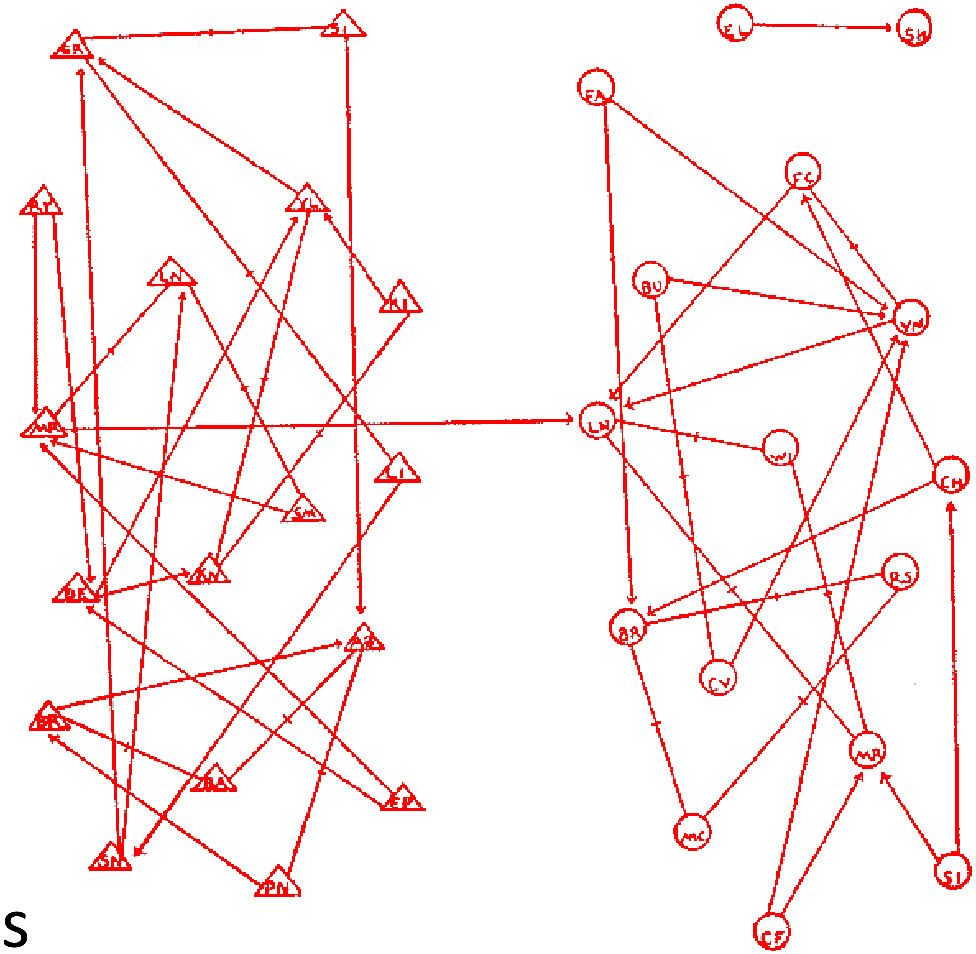
Summer Solstice, Gdańsk, June 25-27, 2018

Jacob L. Moreno (1879-1974)



1928-1933: first sociograms

- Sing Sing prison,
- a reformatory for deliquent girls,
- public and private Brooklyn schools.

https://archive.org/stream/whoshallsurviven00jlmo#page/38/mode/2up
www.slate.com/articles/technology/future_tense/2014/10/j_l_moreno_a_psychologist_s_30s_experiments_invented_social_networking.html

…within a given more or less homogeneous group
members of an alien group may be introduced (…)
The group can assimilate, as it were, a certain number,
but beyond that point
assimilation is rendered difficult or impossible
and the group tends to break up
along the lines of cleavage created by the alien group
forming a minority group within the majority group.

[J. L. Moreno, Who shall survive? A new approach to the problem of human interrelations, Washington D.C., 1934]

https://archive.org/stream/whoshallsurviven00jlmo#page/2/mode/2up

outline

* Homophily and social contagion
* Model evolution : two kinds of discrete events
* *Mean field equation of motion* after Vazquez et al., 2008
* *More detailed mean field equations of motion*
* Analytical results *vs* simulations
* *Even more detailed mean field equations of motion*
* More analytical results
* Are the conclusions meaningful for minorities?

homophily : similarity breeds connection
social contagion : connection breeds similarity

Do people befriend others who are similar to them, or do they become more similar to their friends over time?

movies, music → homophily

Exceptions: classical/jazz → contagion
indie/alternative → anti-contagion

[Lewis et al., PNAS 109 (2012)]

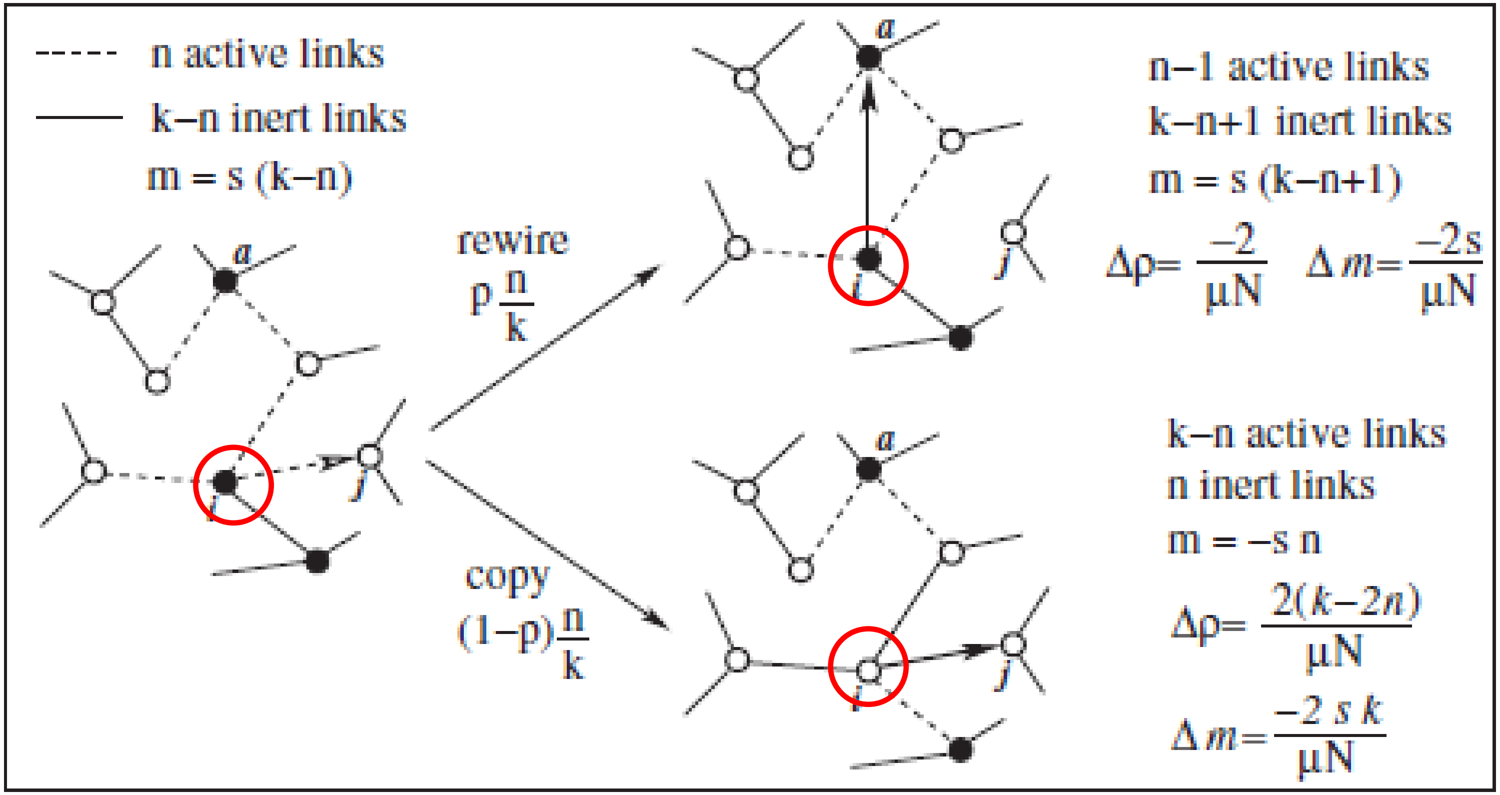
		women			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

MEJ Newman, SIAM Rev 2003 (arXiv:cond-mat/0303516)

Coevolving voter model

The model system is a network of nodes decorated with spins. Two processes compete here:

- rewiring (->homophily) with probability *p*
- flips (->contagion) with probability 1-*p*



[F. Vazquez, V. M. Eguíluz, M. San Miguel, PRL 100 (2008) 108702]

Equations of motion after Vazquez et al.

For the binomial distribution $\sum_{n=0}^{k} nB(n/k) = k\rho$

and $\sum_{n=0}^{k} n^2 B(n/k) = k^2\rho^2 + k\rho(1-\rho)$

then $\dfrac{d\rho}{dt} = \dfrac{2\rho}{\mu}[(1-p)(\mu-1)(1-2\rho)-1] \equiv f(\rho)$

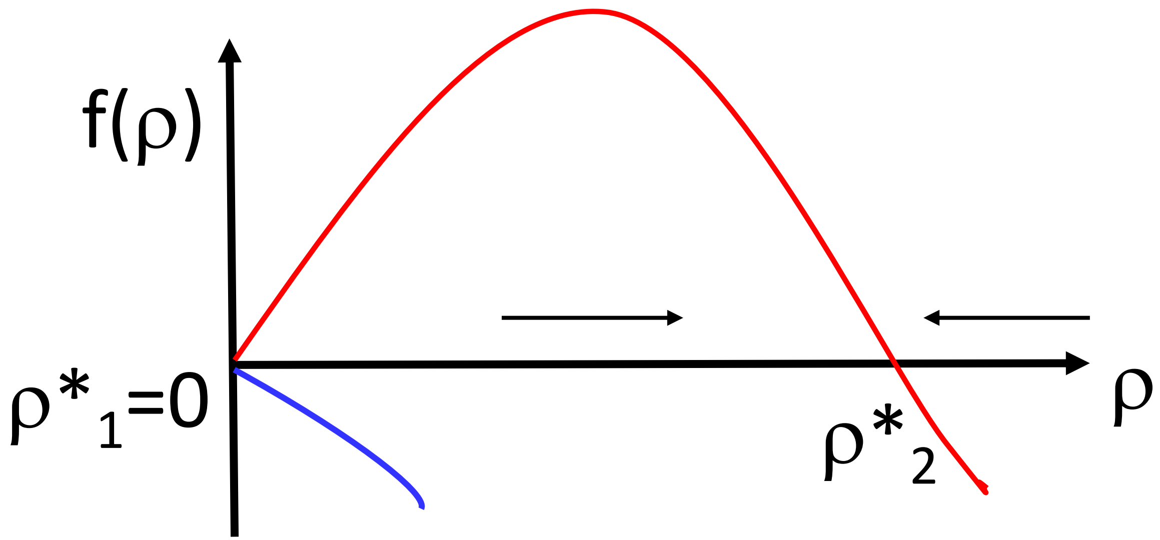
Fixed points: $f(\rho^*) = 0$, then $\rho^*_1 = 0$ (frozen phase)

or

$$\rho^*_2 = \frac{(1-p)(\mu-1)-1}{2(1-p)(\mu-1)} > 0$$

(active phase)

[F. Vazquez, V. M. Eguiluz, M. San Miguel, PRL 100 (2008) 108702]



ρ^*_1 is unstable, ρ^*_2 exists and is stable, iff $\left.\frac{df(\rho)}{d\rho}\right|_{\rho=0} > 0$

$$\left.\frac{df(\rho)}{d\rho}\right|_{\rho=0} = \frac{2}{\mu}[(1-p)(\mu-1)-1]$$

Hence, $\left.\frac{df(\rho)}{d\rho}\right|_{\rho=0} > 0$ for $p < p_c = \frac{\mu-2}{\mu-1}$

p is the probability of rewiring. If *p* is large enough, active links disappear.

[F. Vazquez, V. M. Eguiluz, M. San Miguel, PRL 100 (2008) 108702]

Parameterization: m, ρ, β, $\mu_\pm$

$$N_+ + N_- = N$$

$$N_+ - N_- = Nm$$

$$M_{+-} = M_{-+} = \frac{N\mu}{2}\rho$$

$$M_{+-} + M_{-+} + M_{++} + M_{--} = N\mu$$

$$M_{++} - M_{--} = N\mu\beta$$

hence

$$N_\pm = \frac{N}{2}(1 \pm m)$$

$$M_{++} = \frac{N\mu}{2}(1-\rho+\beta)$$

$$M_{--} = \frac{N\mu}{2}(1-\rho-\beta)$$

$\mu_\pm$ - mean degree of node ±

$$N_+\mu_+ = M_{++} + M_{+-}$$

$$N_-\mu_- = M_{--} + M_{-+}$$

hence

$$\mu_\pm = \mu\frac{1 \pm \beta}{1 \pm m}$$

More detailed equations of motion

$$\frac{d\rho}{dt} = -p\frac{2\rho(1-\beta m)}{\mu(1-\beta^2)} + (1-p)\frac{2\rho(1-\beta^2-2\rho)}{(1-\beta^2)} +$$

$$-(1-p)\frac{4\rho}{\mu(1-\beta^2)}\left[(1-\beta m) - \rho\frac{1+\beta^2-2\beta m}{1-\beta^2}\right]$$

$$\frac{dm}{dt} = (1-p)\frac{2\rho(\beta-m)}{1-\beta^2}$$

$$\frac{d\beta}{dt} = p\frac{2\rho(m-\beta)}{\mu(1-\beta^2)}$$

What could seem a bit strange:

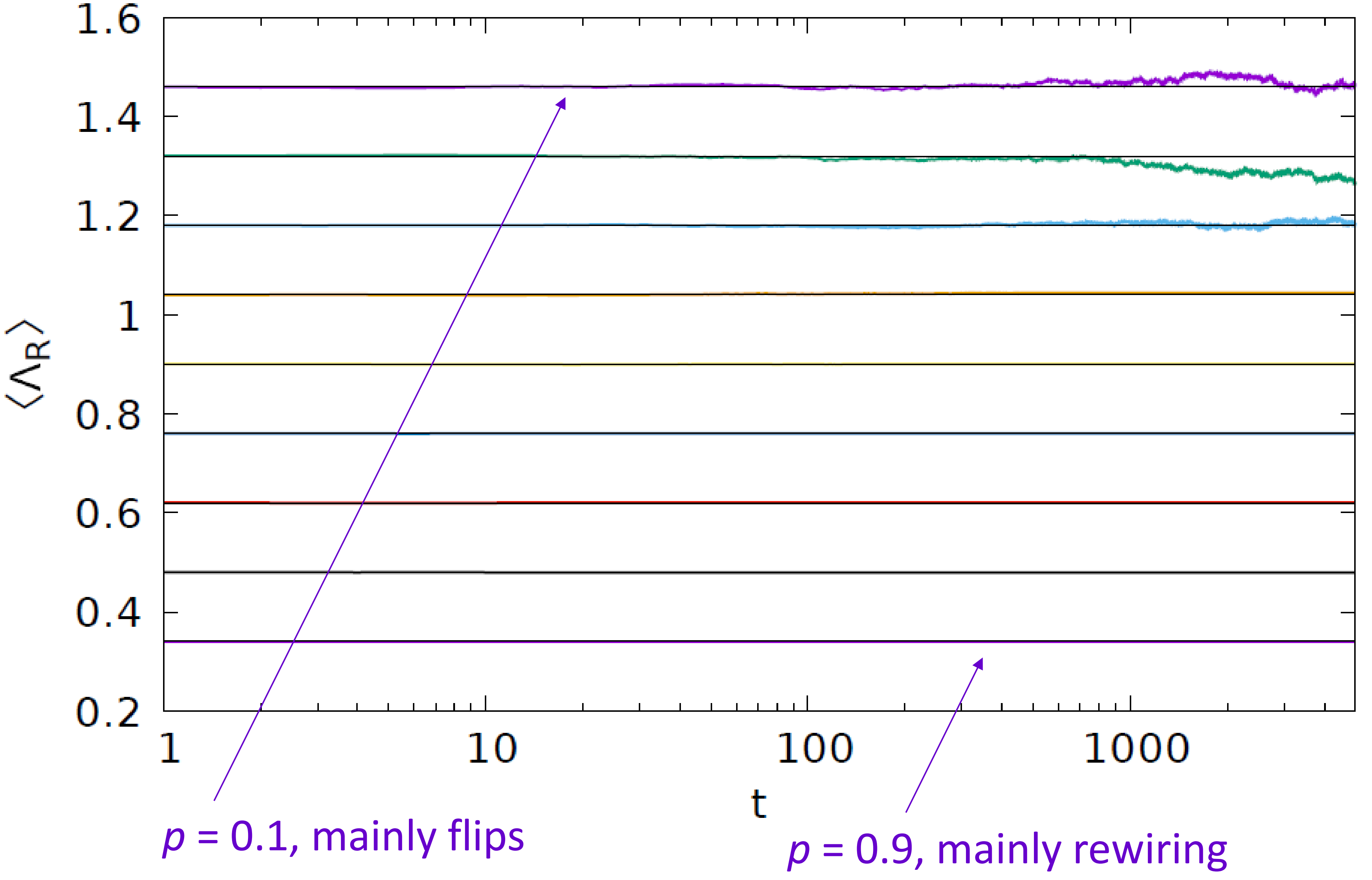
$$\Lambda_R \equiv (1-p)\mu\beta(t) + pm(t) = const$$

$p = 1$ only rewiring, m = const

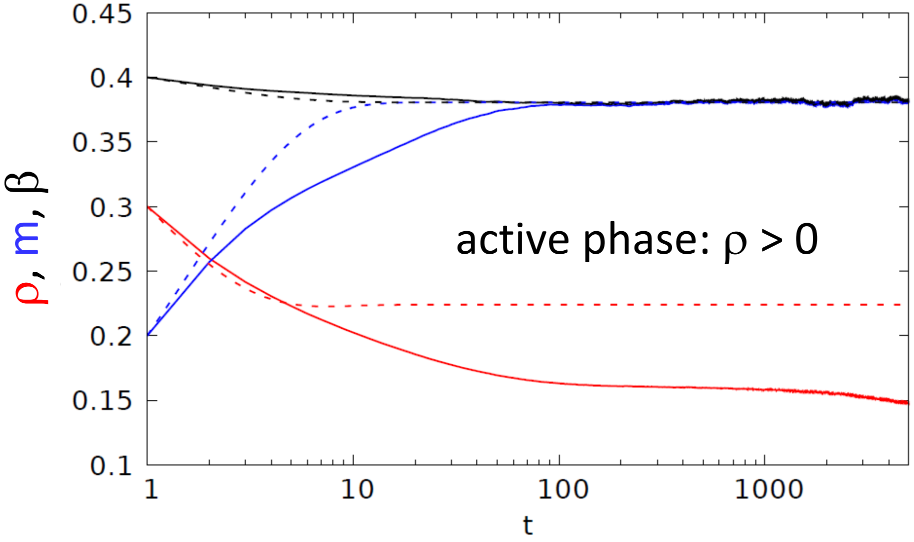
$p = 0$ only flips, β = const

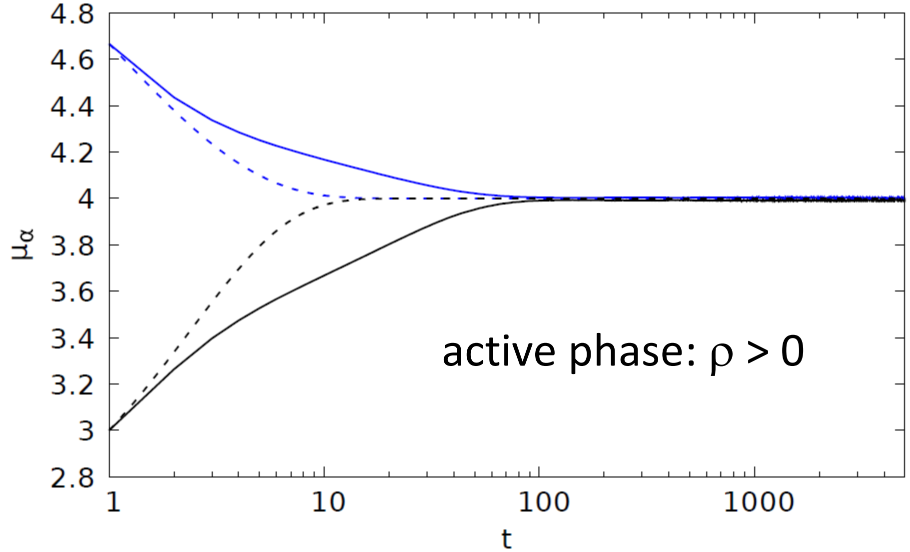
The final solution depends on the initial state.

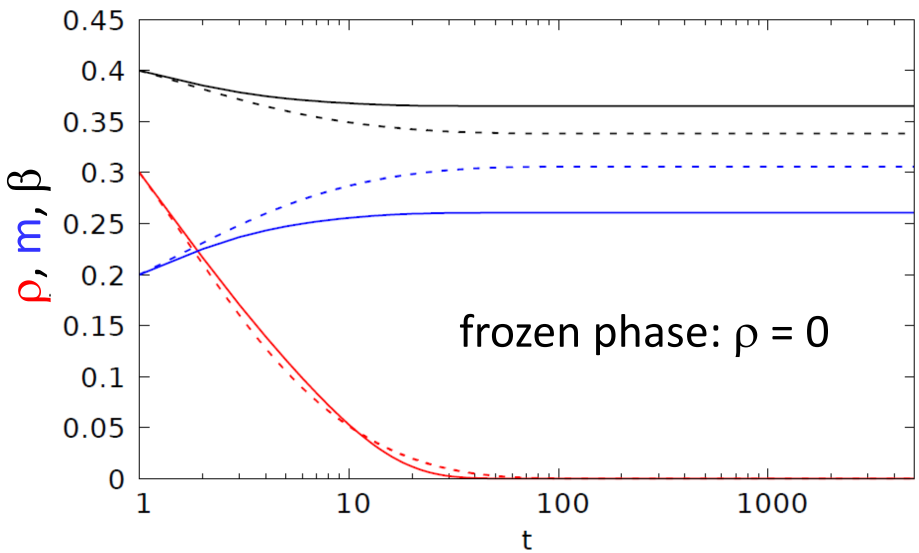
Numerical results: N=5×10^4, #=10^3

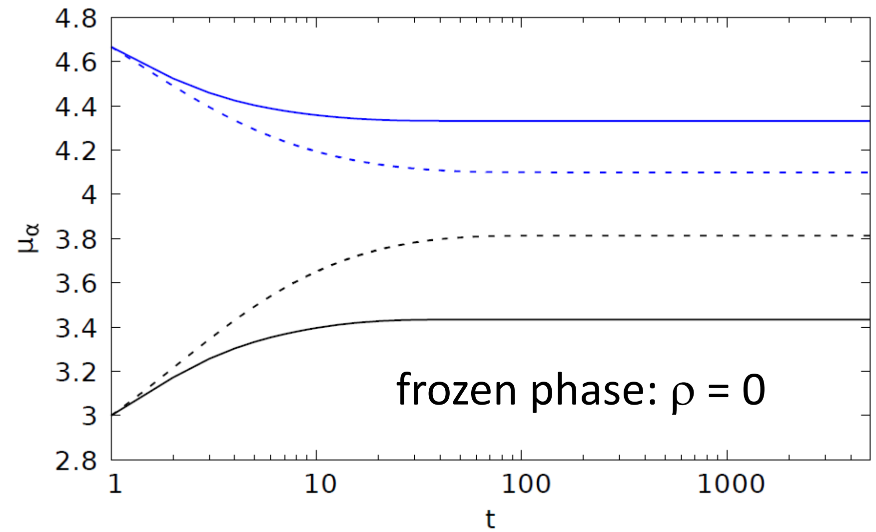


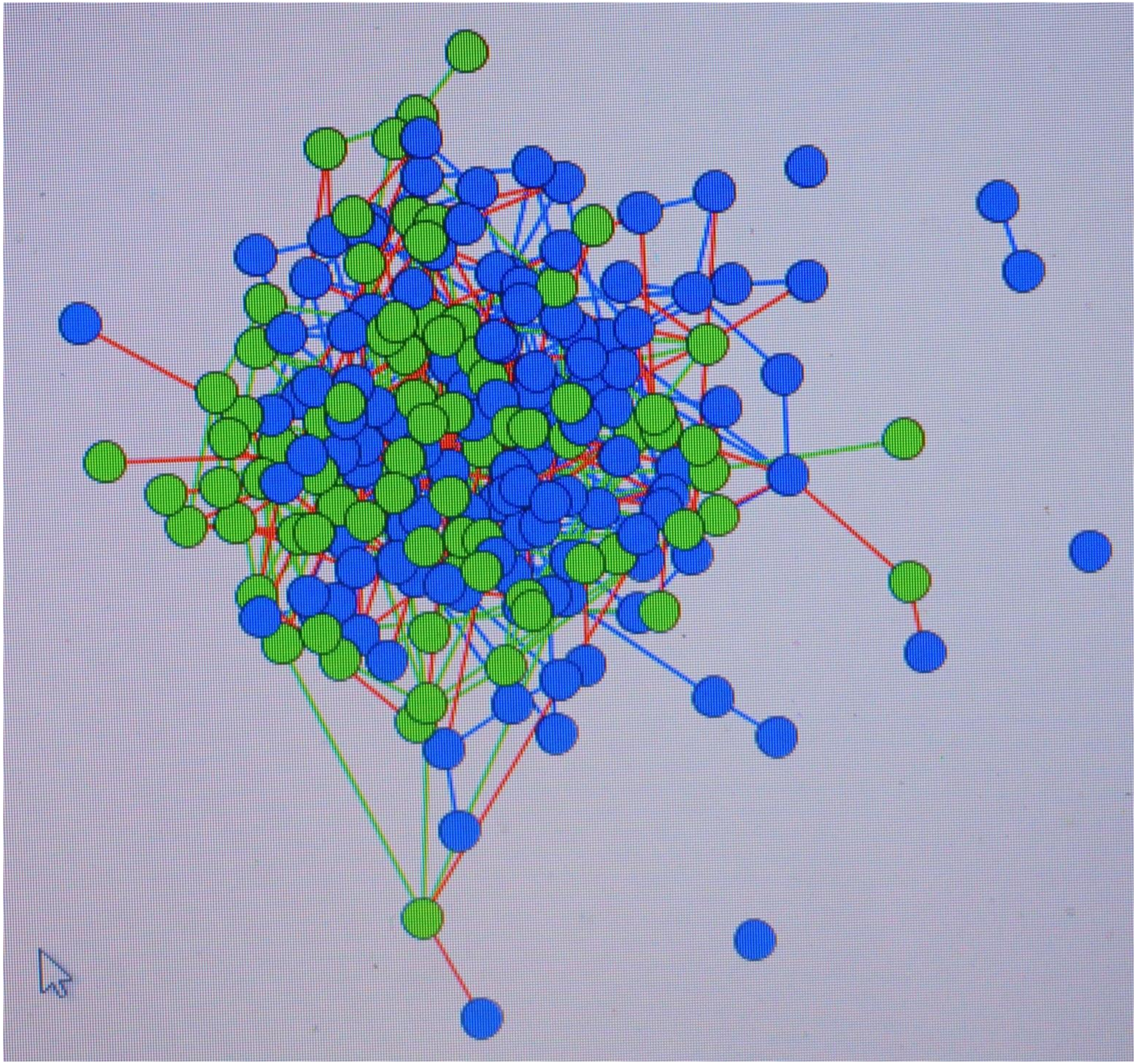
Analytical (- - -) vs numerical (——) results











In silico: N= 500, p=0.1   or N=200, p=0.9

Even more detailed equations of motion

$$\frac{d\rho}{dt} = \rho\frac{[-2+m(2-p_b)+(1-\beta)\mu(1-p_b)+p_b]}{1-\beta} +$$

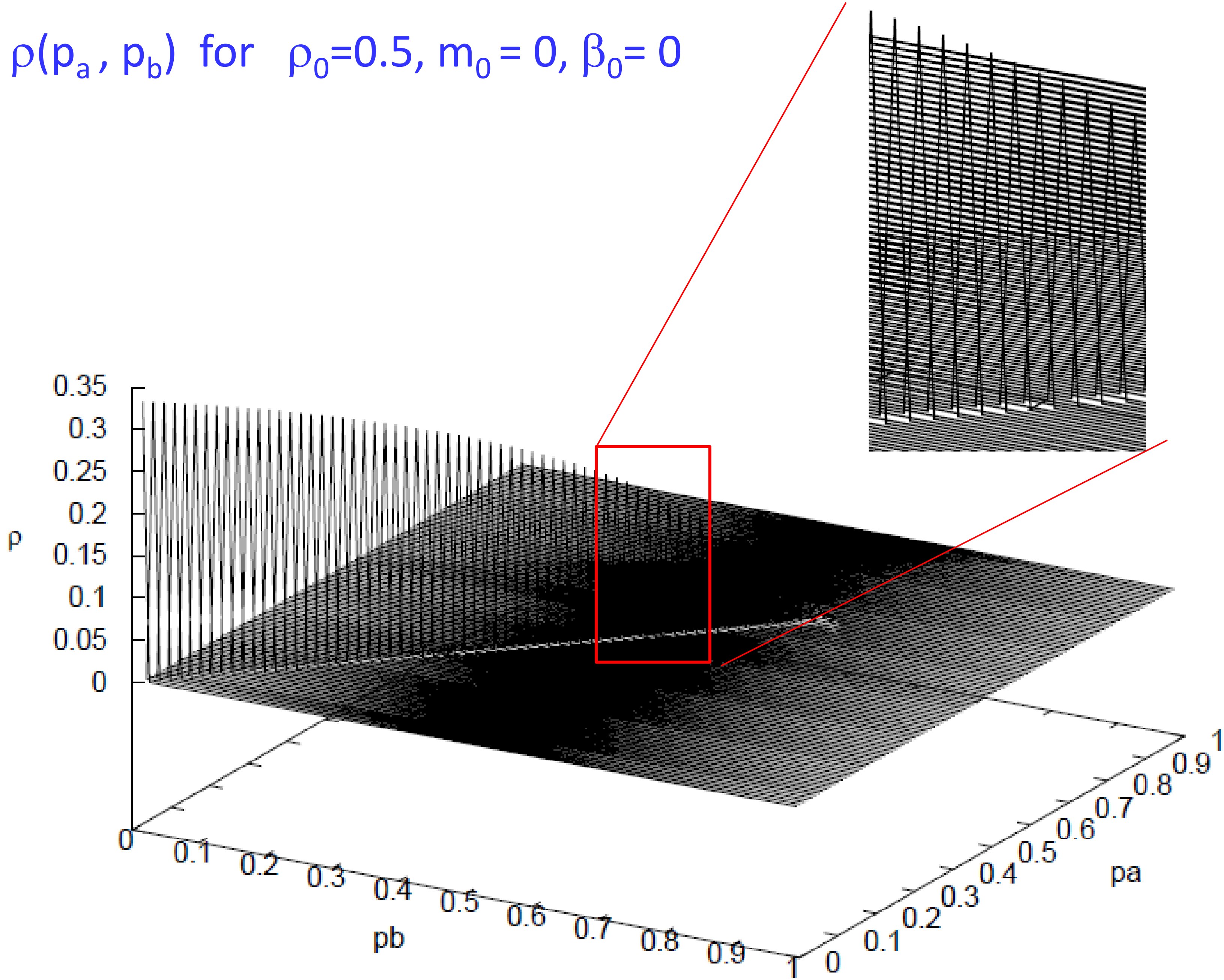
$$+\rho\frac{[-2-m(2-p_a)+(1+\beta)\mu(1-p_a)+p_a]}{1+\beta} +$$

$$+2\rho^2\frac{[1-m-\mu(1-\beta)](1-p_b)}{(1-\beta)^2} + 2\rho^2\frac{[1+m-\mu(1+\beta)](1-p_a)}{(1+\beta)^2}$$

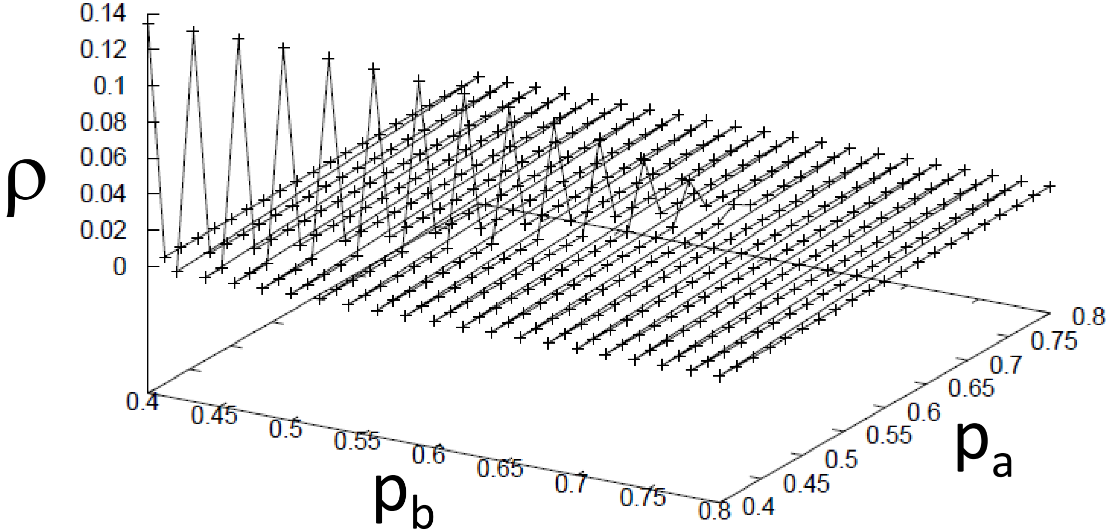
$$\frac{dm}{dt} = -\frac{(1+m)(1-p_a)\rho}{1+\beta} + \frac{(1-m)(1-p_b)\rho}{1-\beta}$$

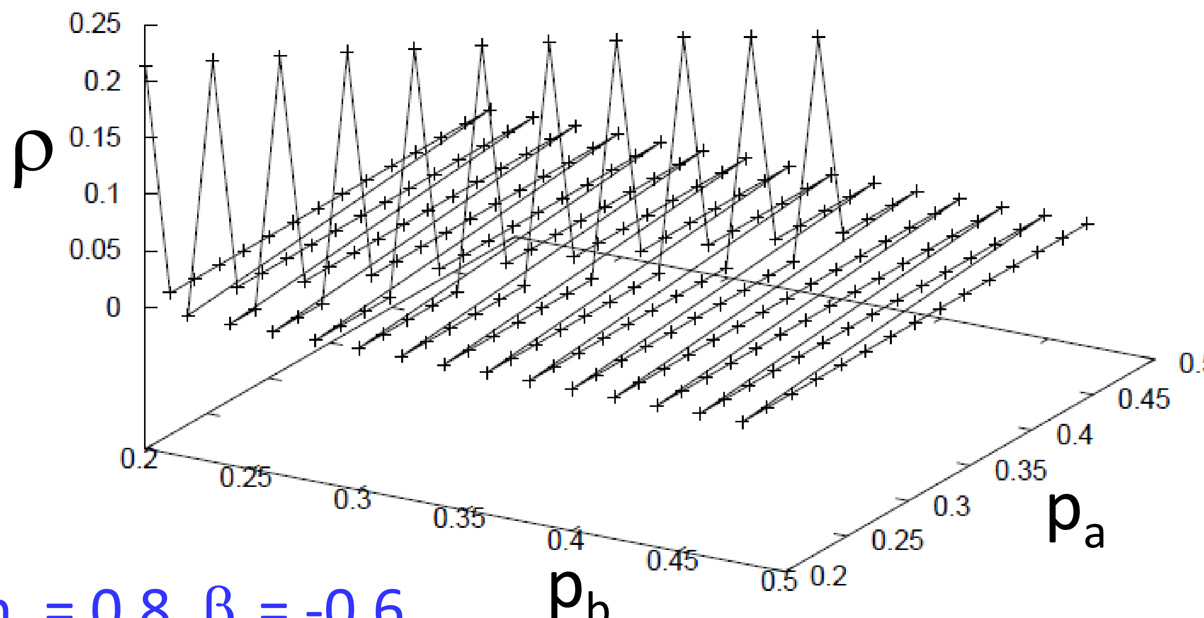
$$\frac{d\beta}{dt} = \rho\frac{-(1+\beta)[1-m+(1-\beta)\mu]p_b + (1-\beta)[1+m+(1+\beta)\mu]p_a}{\mu(1-\beta^2)}$$

$\rho(p_a, p_b)$ for ρ_0=0.5, m_0 = 0, β_0= 0



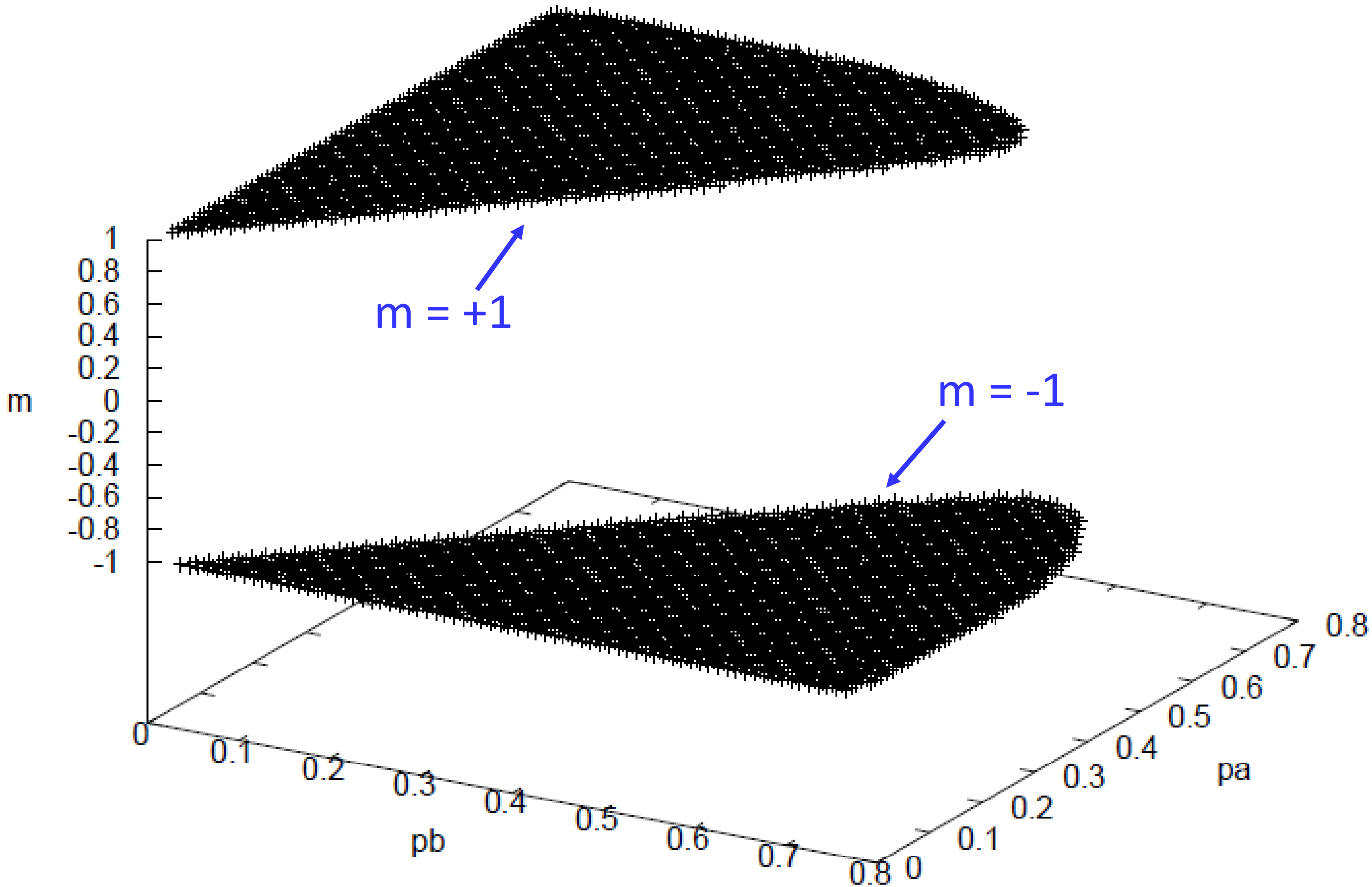
$\rho(p_a, p_b)$ for ρ_0=0.5, m_0 = 0.8, β_0= 0.6



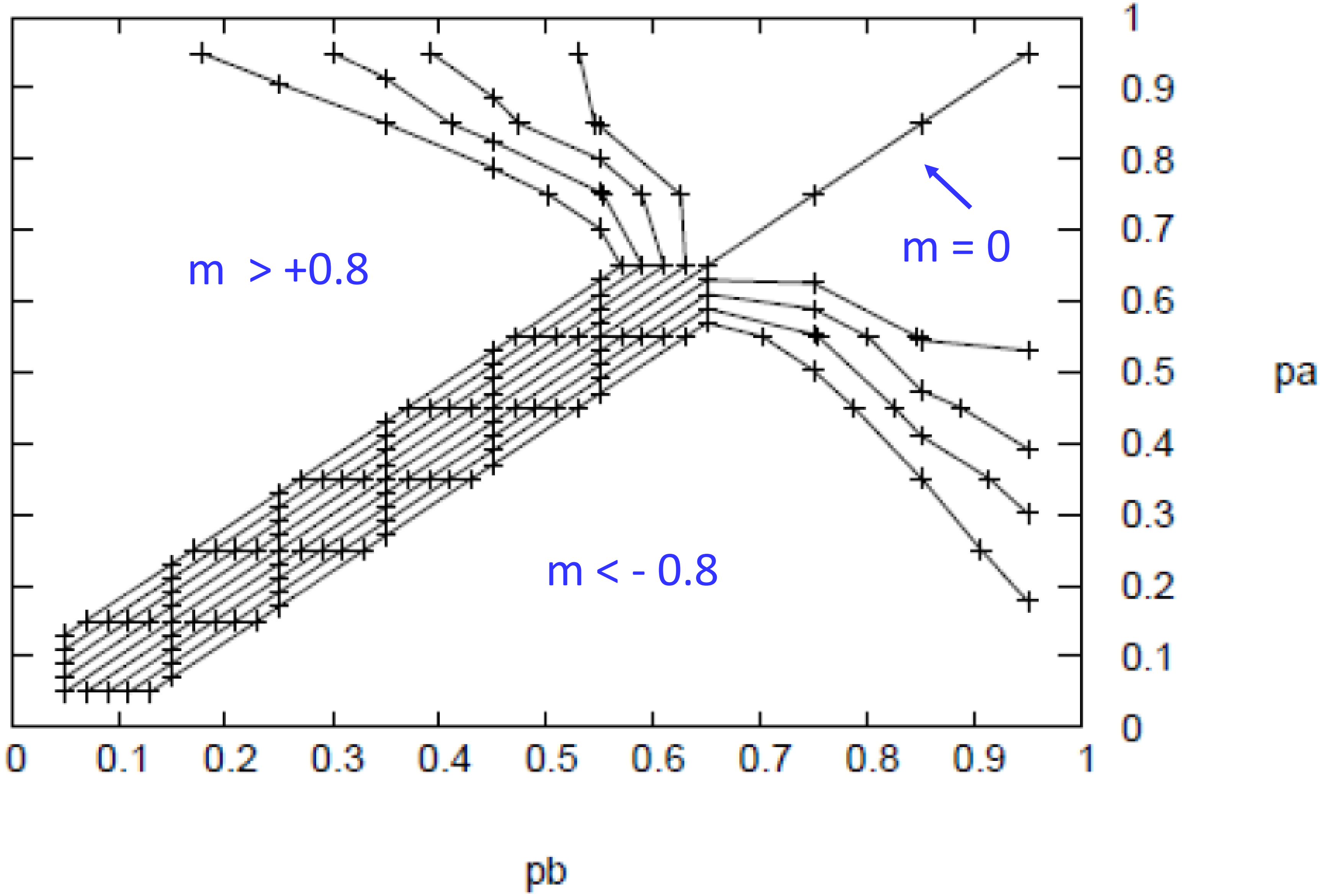


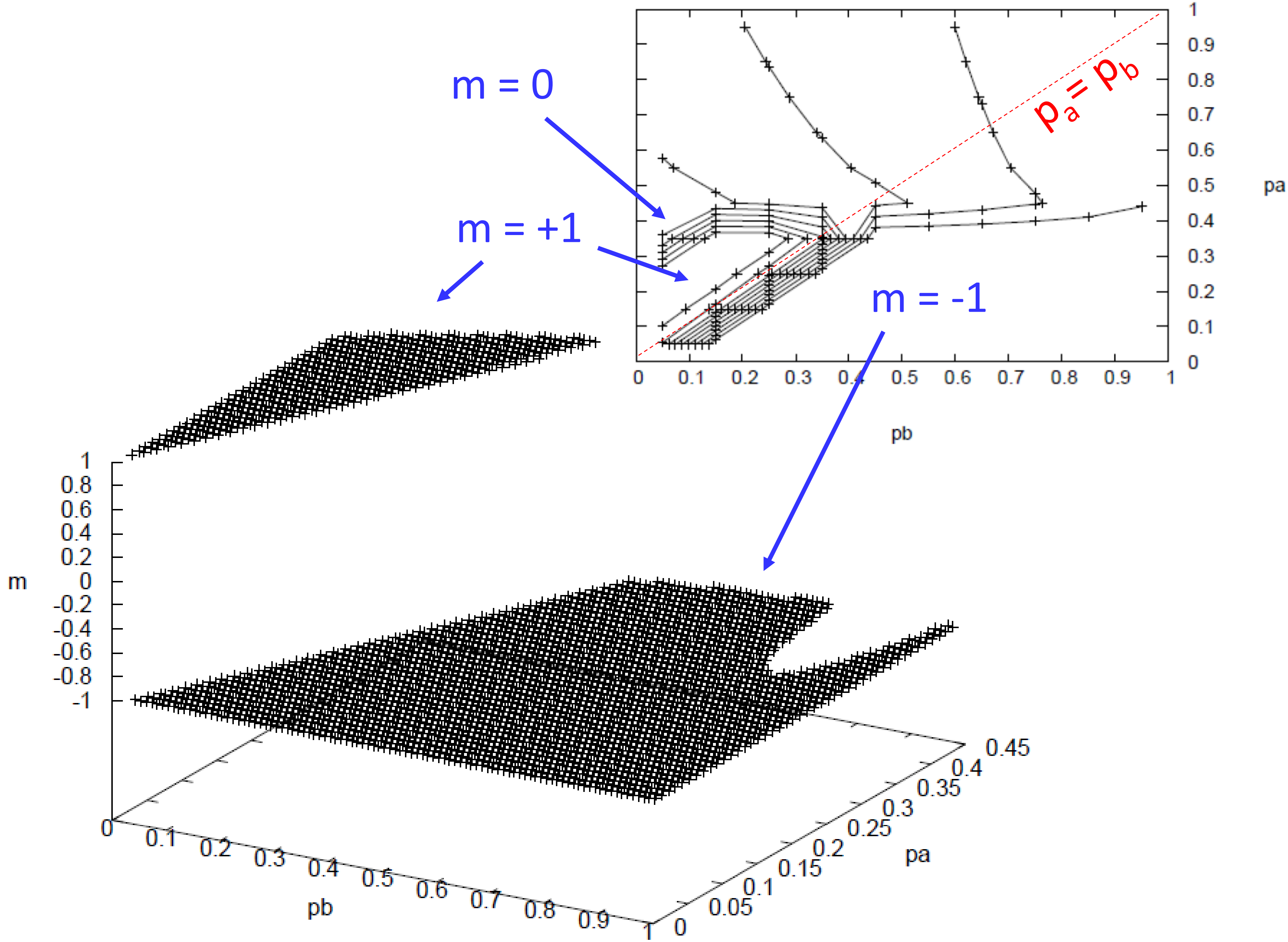
$\rho(p_a, p_b)$ for ρ_0=0.5, m_0 = 0.8, β_0= -0.6

$m(p_a, p_b)$ for $\rho_0=0.5$, $m_0 = 0$, $\beta_0 = 0$

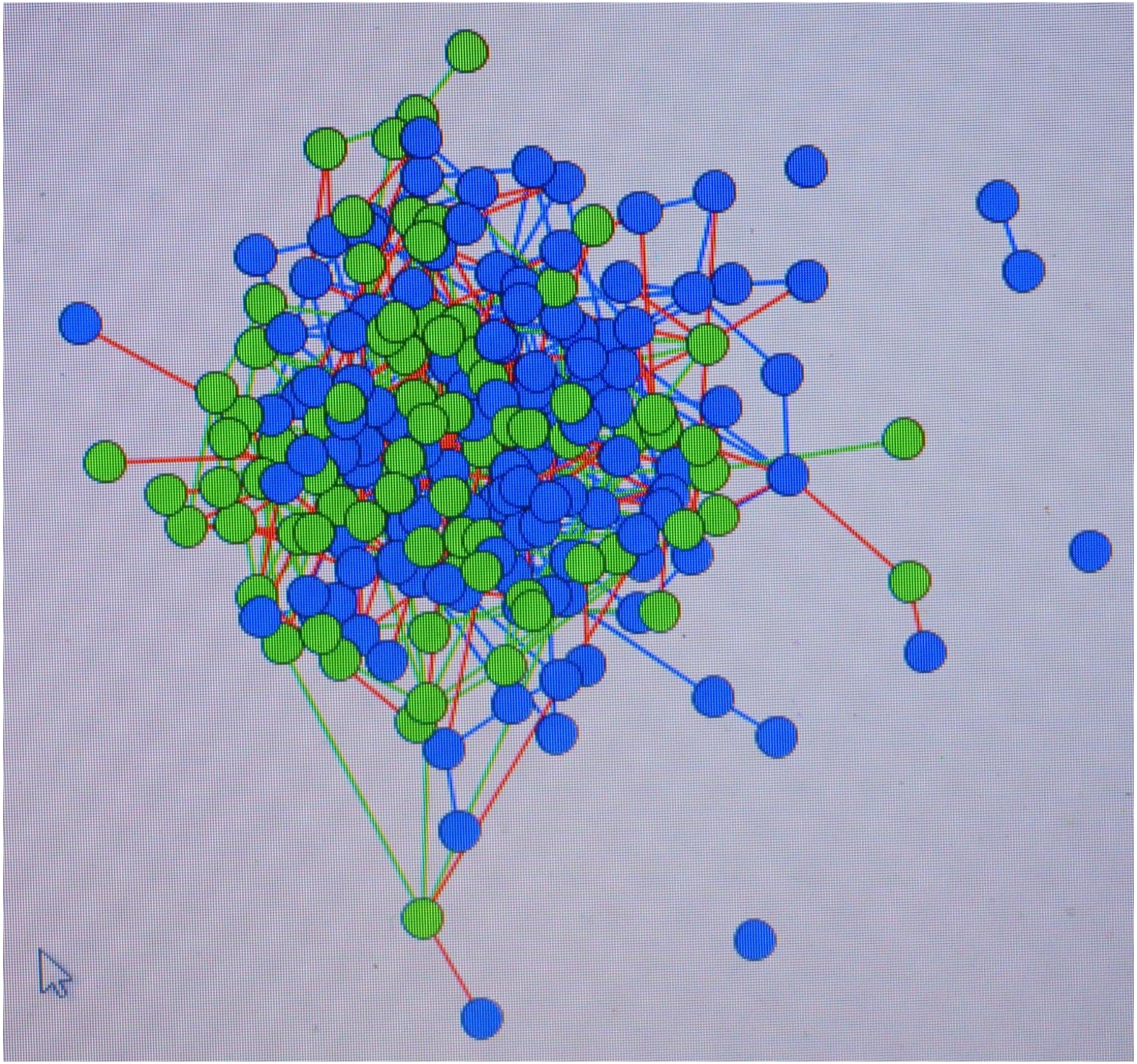


contour map $m(p_a, p_b)$ for $\rho_0=0.5$, $m_0 = 0$, $\beta_0= 0$





$m(p_a, p_b)$ for $\rho_0=0.5$, $m_0=-0.8$, $\beta_0=0.6$



In silico again: N=100, p=0.1

Conclusions

The model by Vazquez *et al* has been generalized for the case where:
- the mean number of neighbors of an actor
- and the probability $p_{a,b}$ of rewiring

depend on the group (a,b).

Once $p_a \neq p_b$, the active phase vanishes.
Either the groups mutually separate, or one of them is absorbed.

The group a cannot be absorbed as long as $p_a > p_b$. *A competition?*

Two bad news…

[J. Toruniewska et al., PRE 96 (2017) 042306
KK et al., IJMPC (2018), in print (arXiv:1804.06650)]

Thank you

Equations of motion after Vazquez et al.

density of active links

flip

prob of n active out of k links

$$\frac{d\rho}{dt} = N(1-p)\sum_k P(k)\sum_{n=0}^{k} B(n/k)\,\frac{n}{k}\,\frac{k-2n}{N\mu/2}$$

$$+\,Np\sum_k P(k)\sum_{n=0}^{k} B(n/k)\,\frac{n}{k}\left(\frac{-1}{N\mu/2}\right)$$

number of all links

rewire

[F. Vazquez, V. M. Eguiluz, M. San Miguel, PRL 100 (2008) 108702]

Equations of motion after Vazquez et al.

density of
active links

flip

active links <-> frozen links

$$\frac{d\rho}{dt} = N(1-p)\sum_k P(k)\sum_{n=0}^{k} B(n/k)\,\frac{n}{k}\,\frac{k-2n}{N\mu/2}$$

$$+\,Np\sum_k P(k)\sum_{n=0}^{k} B(n/k)\,\frac{n}{k}\left(\frac{-1}{N\mu/2}\right)$$

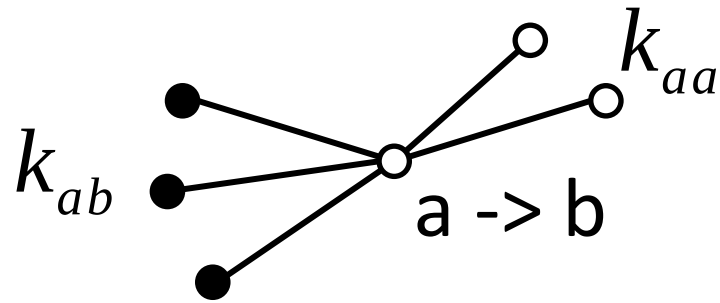
rewire

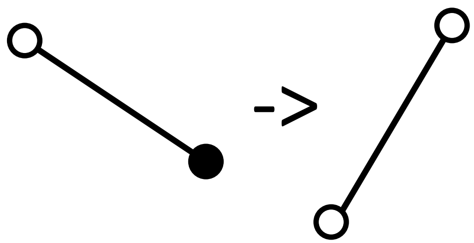
active link
-> frozen link

fraction of
active links

[F. Vazquez, V. M. Eguiluz, M. San Miguel, PRL 100 (2008) 108702]

M_{ab} – number of links from a to b





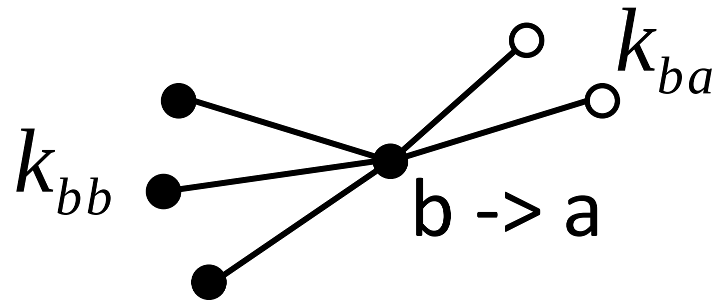
rewiring, probability *p*

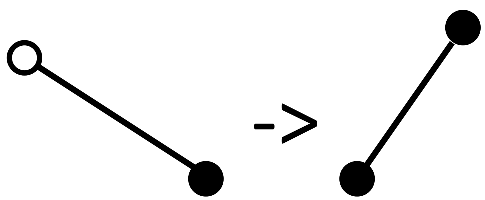
$$M_{ab} \to M_{ab} - 1$$
$$M_{ba} \to M_{ba} - 1$$
$$M_{aa} \to M_{aa} + 2$$

flip, probability *1-p*

$$N_a \to N_a - 1$$
$$N_b \to N_b + 1$$
$$M_{aa} \to M_{aa} - 2k_{aa}$$
$$M_{ab} \to M_{ab} - k_{ab} + k_{aa}$$
$$M_{ba} \to M_{ba} - k_{ab} + k_{aa}$$
$$M_{bb} \to M_{bb} + 2k_{ab}$$

M_{ba} – number of links from b to a





rewiring, probability *p*

$$M_{ab} \rightarrow M_{ab} - 1$$

$$M_{ba} \rightarrow M_{ba} - 1$$

$$M_{bb} \rightarrow M_{bb} + 2$$

flip, probability *1-p*

$$N_a \rightarrow N_a + 1$$

$$N_b \rightarrow N_b - 1$$

$$M_{aa} \rightarrow M_{aa} + 2k_{ba}$$

$$M_{ab} \rightarrow M_{ab} + k_{bb} - k_{ba}$$

$$M_{ba} \rightarrow M_{ba} - k_{ba} + k_{bb}$$

$$M_{bb} \rightarrow M_{bb} - 2k_{bb}$$

If Rabbit

Was bigger
And fatter
And stronger,
Or bigger
Than Tigger,
If Tigger was smaller,
Then Tigger's bad habit
Of bouncing at Rabbit
Would matter
No longer,
If Rabbit
Was taller.

A. A. Milne

http://disney.wikia.com/wiki/File:Winniethepooh-disneyscreencaps.com-8218.jpg