Towards the Heider balance in complete graphs - long limit cycles

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Signed networks and their applications

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outline

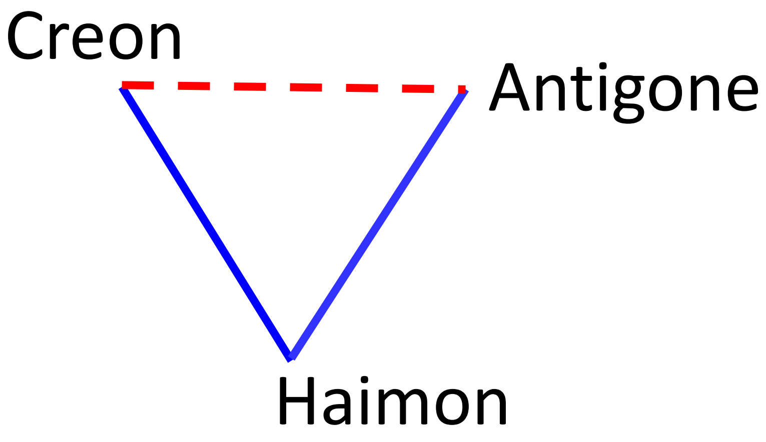
1. Cognitive dissonance – strategies of reduction
2. Heider balance: interpersonal relations, correlations of opinions
3. Balanced states, jammed states
4. **Evolution of complete graphs**
5. **Long limit cycles and their properties**
6. Conclusions

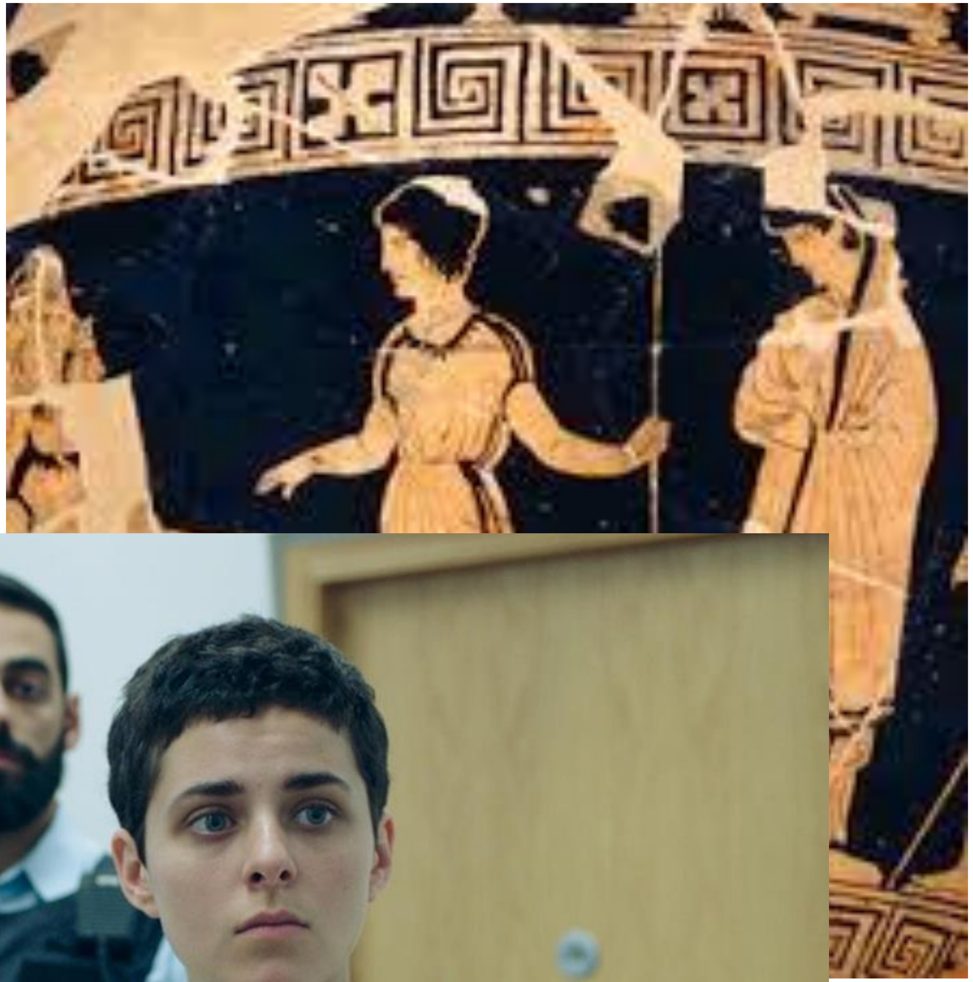
Strategies of dissonance reduction:

- **attitude change** („…they are unripe…”)
- **distraction and forgetting** („…ok, but what about my steak?”)
- **trivialization and self-affirmation** („…surely I was right a bit…”)
- **denial of responsability** („… we can't let in everyone…”)
- **adding consonant cognitions** („…I have seen a web page…”)
- **changing behavior** (after writing an assigned essay about the dangers of cheating)
- **act rationalization** („… agree to abstain from smoking for one day, then longer…”)

[April McGrath, *Social and Personality Psychology Compass* 11 (2017) e12362]

Cognitive dissonance in interpersonal relations



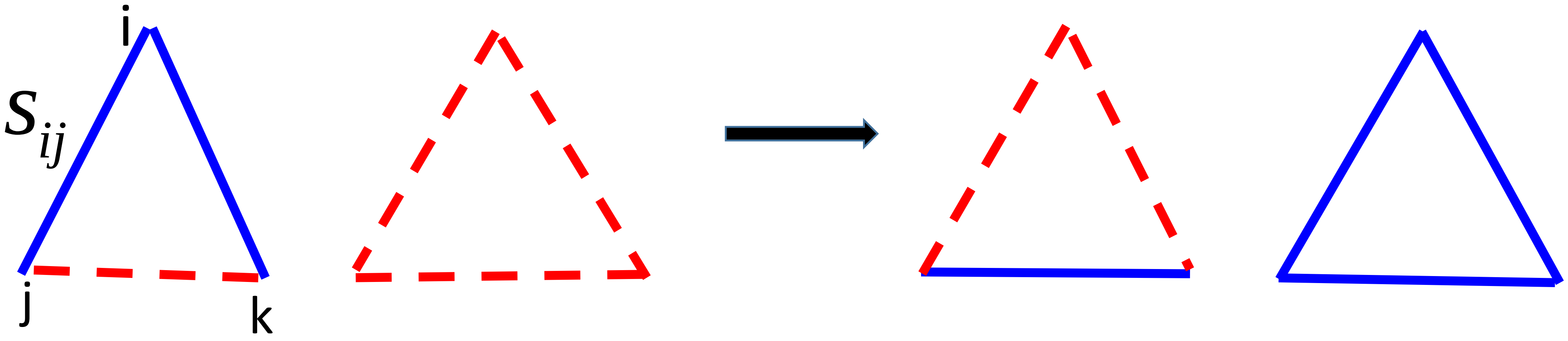


[Sophocles, Vth century BC;
Sophie Deraspe, 2019]

https://www.screendaily.com/reviews/antigone-busan-review/5143218.article

Dissonance about interpersonal relations is removed, if :

- A friend of my friend is my friend (+1)(+1)(+1)>0
- A friend of my enemy is my enemy (-1)(+1)(-1)>0
- An enemy of my friend is my enemy (+1)(-1)(-1)>0
- An enemy of my enemy is my friend (-1)(-1)(+1)>0



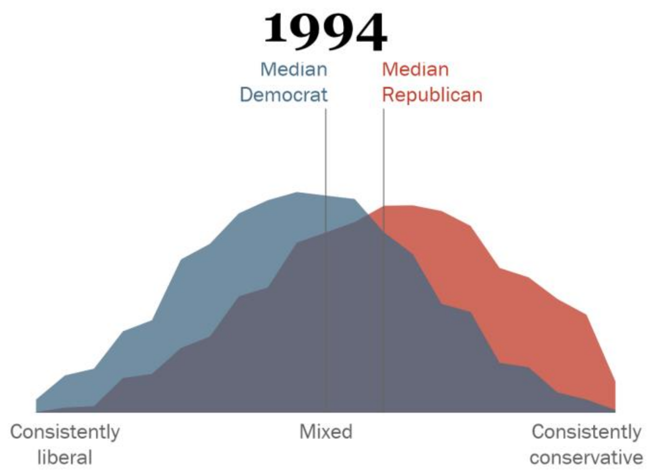
$$S_{ij}S_{jk}S_{ki} < 0$$

$$S_{ij}S_{jk}S_{ki} > 0$$

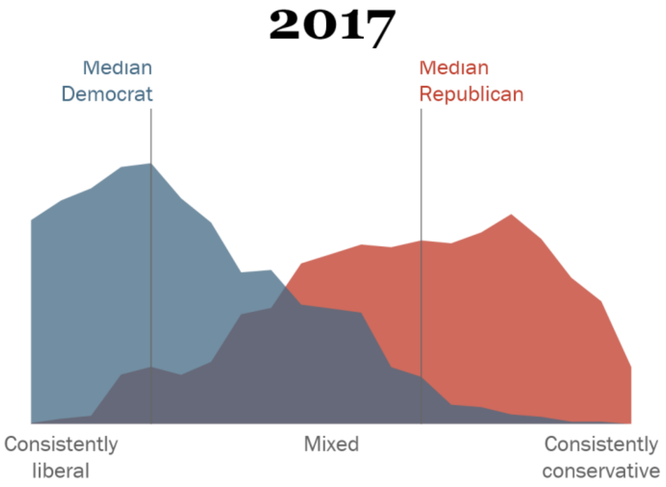
Relations with dissonance

Balanced states: dissonance removed

transition to balance in a complete graph



Source: Surveys conducted in 1994, 1999, 2004, 2011, 2015 and 2017.



Source: Surveys conducted in 1994, 1999, 2004, 2011, 2015 and 2017.

www.pewresearch.org/politics/interactives/political-polarization-1994-2017/

Data collected by National Opinion Research Center in 44 states of USA within 1974-1988 [P. Brace et al., Amer. J. Political Science, 46 (2002) 173]

1. More tolerance
2. Racism (less racist)
3. Abortion (more pro-choice)
4. Religiosity (more religious)
5. More acceptance of homosexuality
6. Public feminism (more accepting of women's rights)
7. Environment spending (support higher government spending on the environment)
8. Welfare spending (support higher government spending on welfare)
9. Death penalty (support for)

Support (US state)

->Pearson r(i,j)

1	0.907	0.686	-0.551	0.828	0.832	0.337	-0.151	0.163
	1	0.565	-0.443	0.828	0.887	0.414	-0.055	0.296
		1	-0.725	0.776	0.600	0.423	-0.219	-0.052
			1	-0.694	-0.571	-0.201	0.277	0.111
				1	0.760	0.354	-0.174	-0.155
					1	0.296	-0.277	0.266
						1	0.080	-0.140
							1	-0.042
								1

The symmetric matrix $r(i,j)$ of Pearson correlations of opinions on nine issues, collected in Ref.[26].

Correlation matrix = initial data

Result : portraits of voters:

4,8 *vs* 1,2,3,5,6,7,9

={religiosity + welfare} *vs* {others}

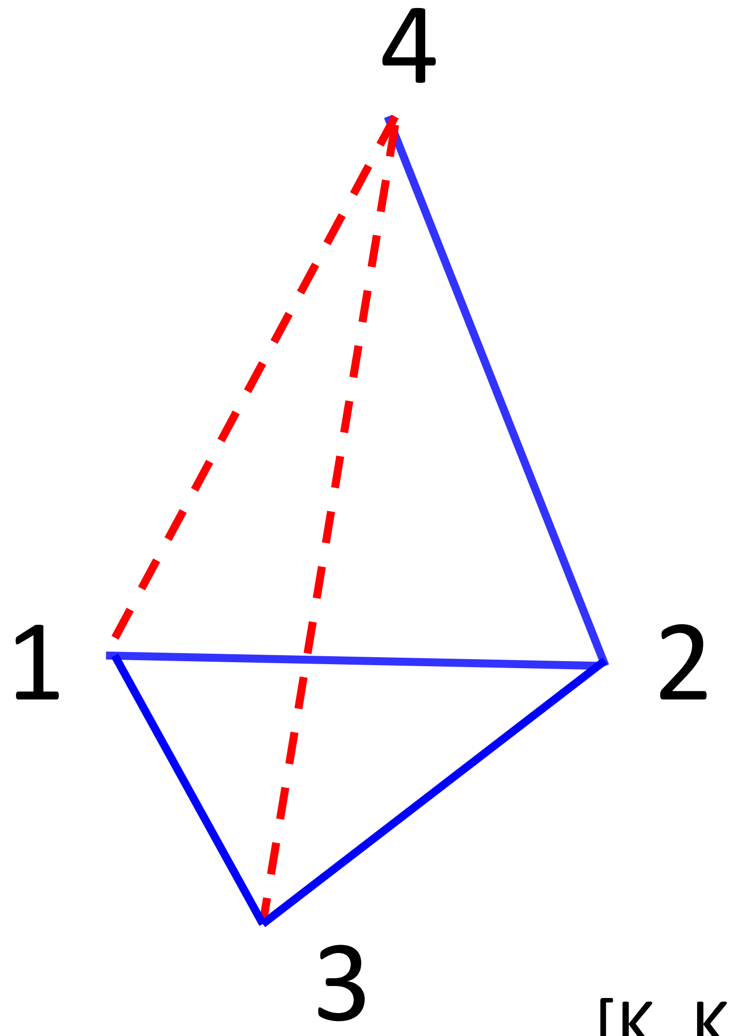
The Comfort Hypothesis:

“Parishioners whose life situations most deprive them of satisfaction and fulfillment in the secular society turn to the church for comfort and substitute rewards”

[M. J. Krawczyk, KK, Entropy 23 (2021) 1418]

[E. Babbie, The Practice of Social Research, 2007]

An example of a jammed state for symmetric links



conditions of stability:

N2+N3-2+N1-N4>0

N3+N1-2+N2+N4>0

N1+N2-2+N3-N4>0

N4+N3-2+N1-N2>0

N2+N4-2-N1-N3>0

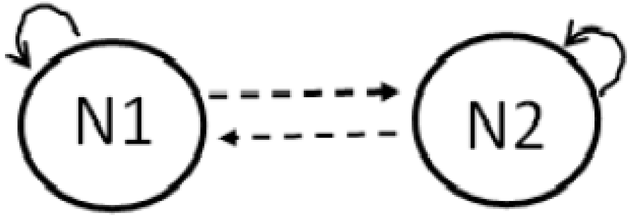
N1+N4-2-N2+N3>0

N*i* – size of community *i*

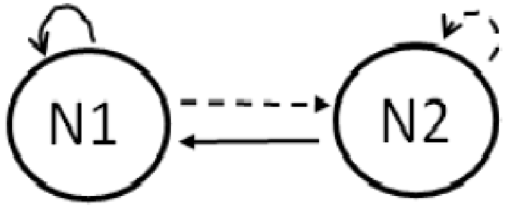
[K. K., M. Stojkow, D. Żuchowska-Skiba, J. Math. Soc. 2020]

More jammed states (M. J. Krawczyk et al., Sci. Rep. 2019)

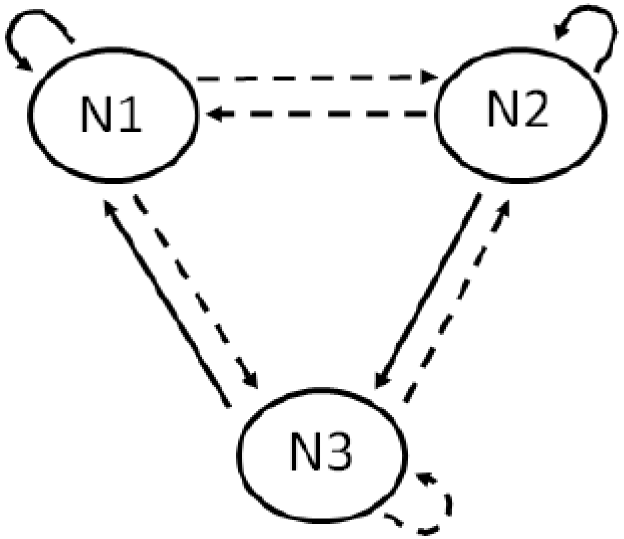
balanced, symmetric :



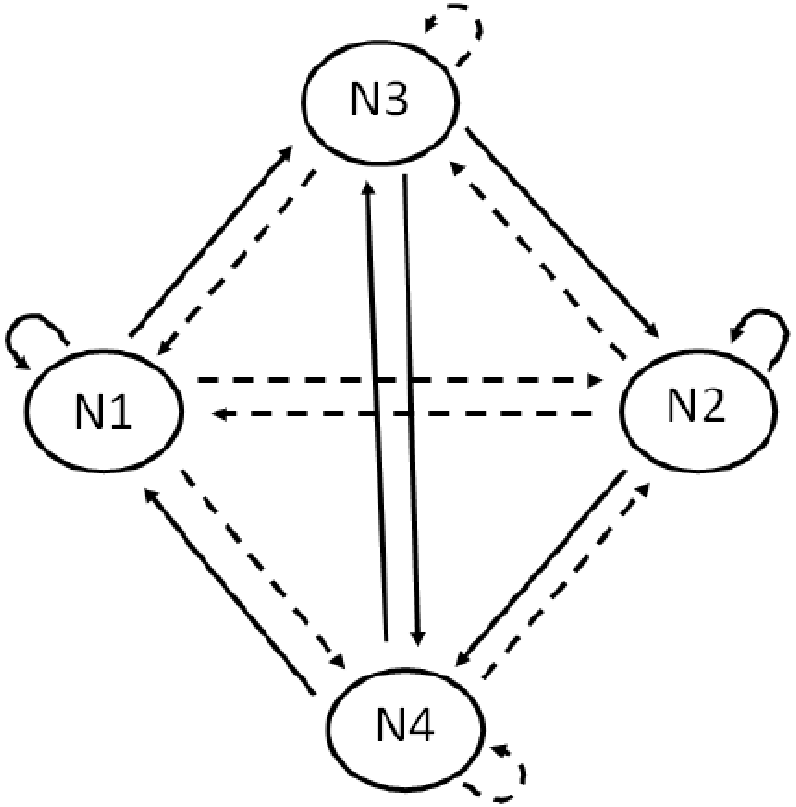
unbalanced, asymmetric :



CII: N1>N2+2



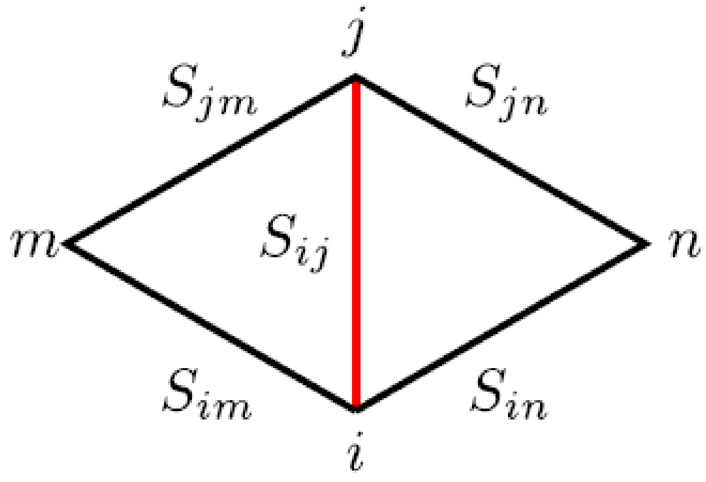
CIII: N1+N2>N3+2

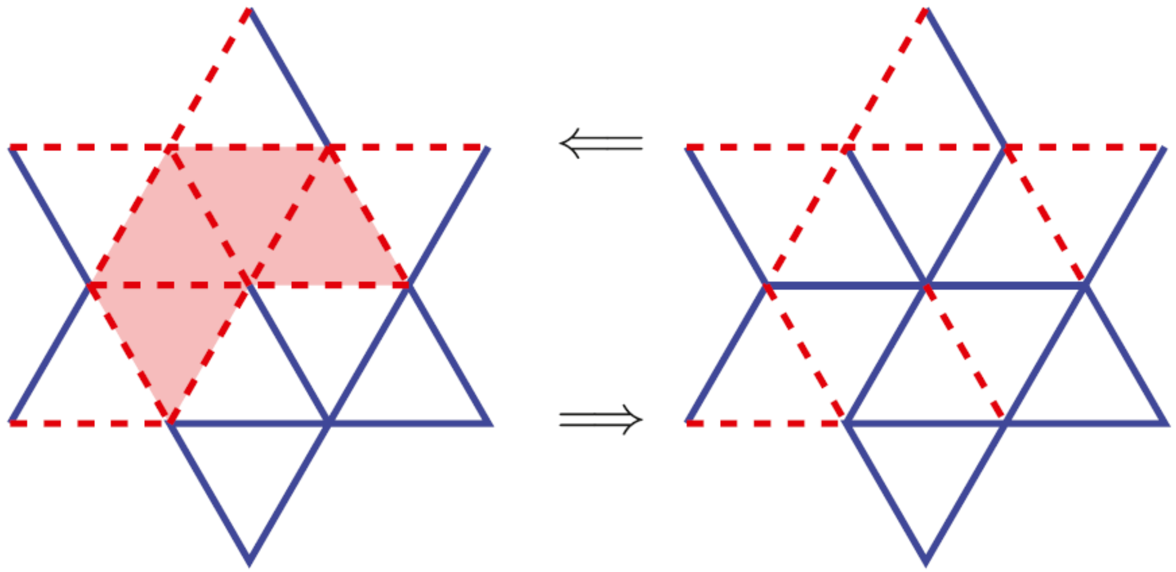


CIV: N1+N2>N3+N4+2

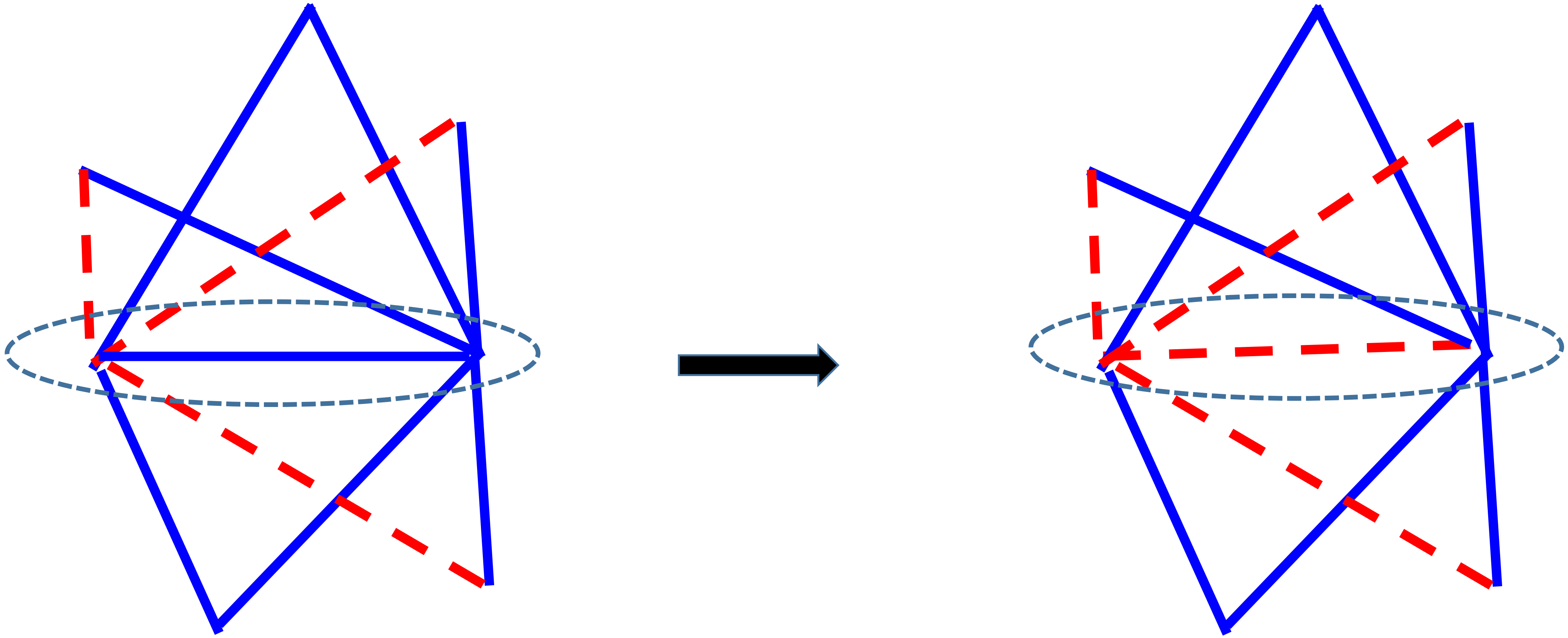
To remove cognitive dissonance – a deterministic map

$$s_{ij}(t+1) = sign[s_{ik}(t)s_{jk}(t) + s_{im}(t)s_{jm}(t)]$$





K. Malarz,
M. Wołoszyn, K.K.,
Physica D 411
(2020) 132506



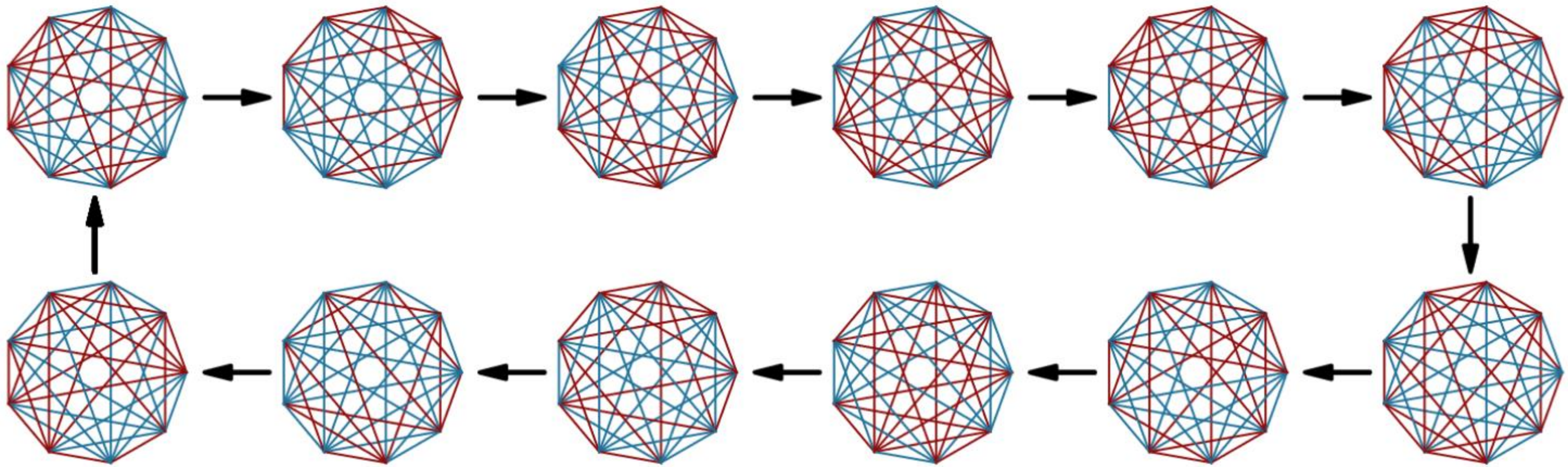
$$s_{ij}(t+1) = sign \sum_{k \neq i,j}^{N-2} s_{ik}(t) s_{jk}(t)$$

sometimes we get a limit cycle

$$N = 9; \quad 2^{N(N-1)/2} \approx 10^{11}$$

N	B	J	C
6	0.584	0.238	0.177
9	0.647	0.015	0.338
20	0.946	0.045	0.009
100	0.947	0.051	0.003

$c(N=9)$	1	2	3	4	6	12
C	0.662	0.280	0.033	0.008	0.010	0.004



For 967680 cycles of length 12, all of them have the following properties:

For the Hamming distance between the network states

$$d_H(A,B) = \frac{1}{4}\sum_{i<j}[s_{ij}(A) - s_{ij}(B)]^2$$

we have

$$d_H(A_t, A_{t+1}) = 18$$

$$d_H(A_t, A_{t+2}) = 22$$

$$d_H(A_t, A_{t+3}) = 20$$

$$d_H(A_t, A_{t+4}) = 10$$

$$d_H(A_t, A_{t+5}) = 18$$

$$d_H(A_t, A_{t+6}) = 20$$

and

$$d_H(A_t, A_{t+s}) = d_H(A_t, A_{t-s})$$

For 967680 cycles of length 12, all of them have the following properties:

For the total energy $U = -\sum_{i<j<k} s_{ij} s_{jk} s_{ki} = \sum_{i<j<k} u_{ijk}$

and the Hamming distance between the triad energies

$$D_H(A,B) = \frac{1}{4} \sum_{i<j<k} [u_{ijk}(A) - u_{ijk}(B)]^2$$

we have

$$D_H(A_t, A_{t\pm1}) = 36$$
$$D_H(A_t, A_{t\pm2}) = 32$$
$$D_H(A_t, A_{t\pm3}) = 16$$
$$D_H(A_t, A_{t\pm4}) = 32$$
$$D_H(A_t, A_{t\pm5}) = 36$$
$$D_H(A_t, A_{t\pm6}) = 0$$

For 967680 cycles of length 12, all of them have the following properties:

For the link energy defined as $u_{ij} = \sum_k u_{ijk} = -\sum_k s_{ij} s_{jk} s_{ki}$

we can define the spectrum of energy. Here it is

u	-7	-5	-3	-1	$+1$	$+3$	$+5$	$+7$
$n_e(u)$	3	2	4	9	12	4	2	0

This spectrum is the same in each state of each cycle.

The evolution changes all links where $u_{ij} > 0$, hence

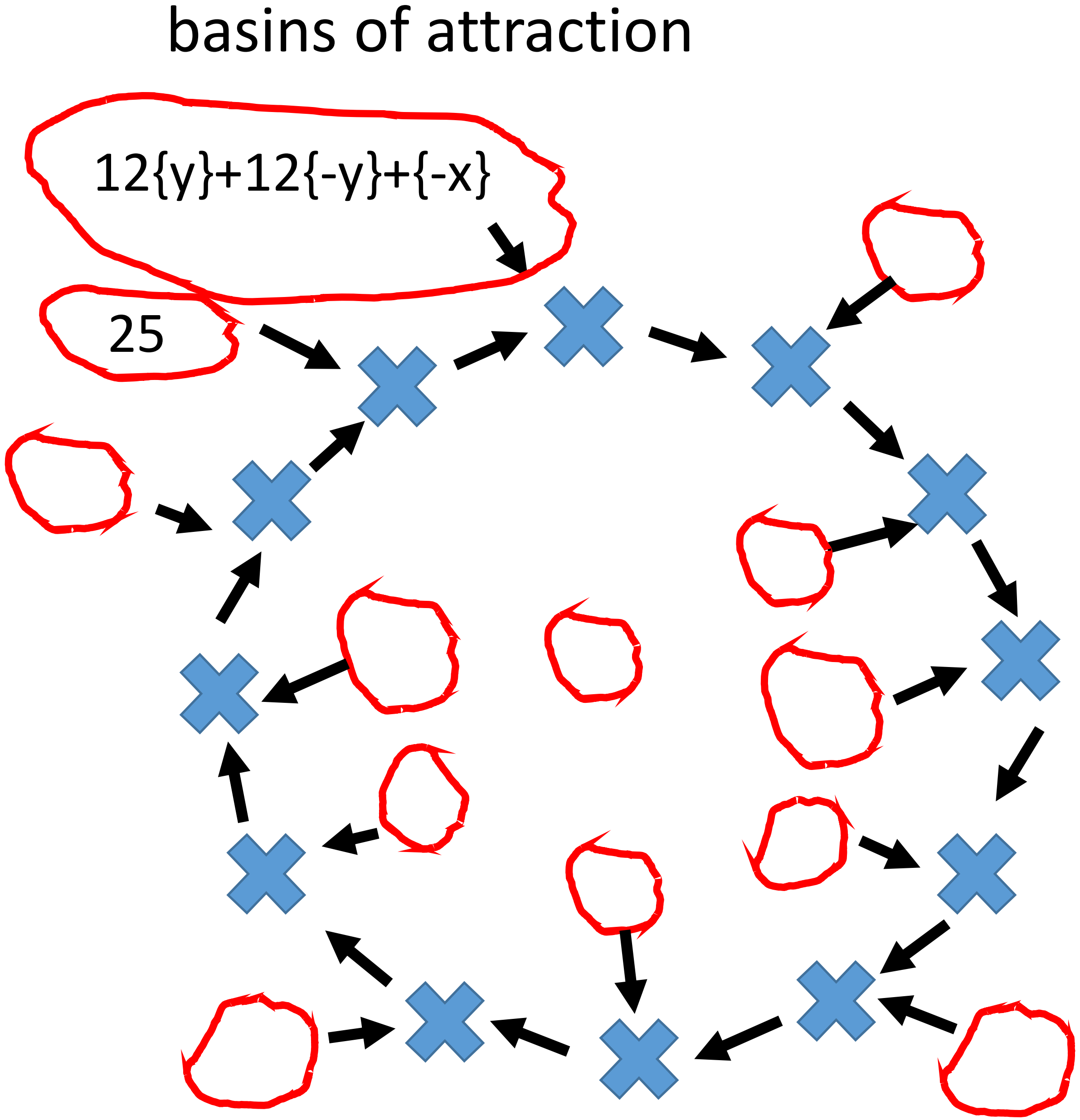
$$\forall t : d_H(A_t, A_{t+1}) = 18$$

For 967680 cycles of length 12, all of them have the following properties:

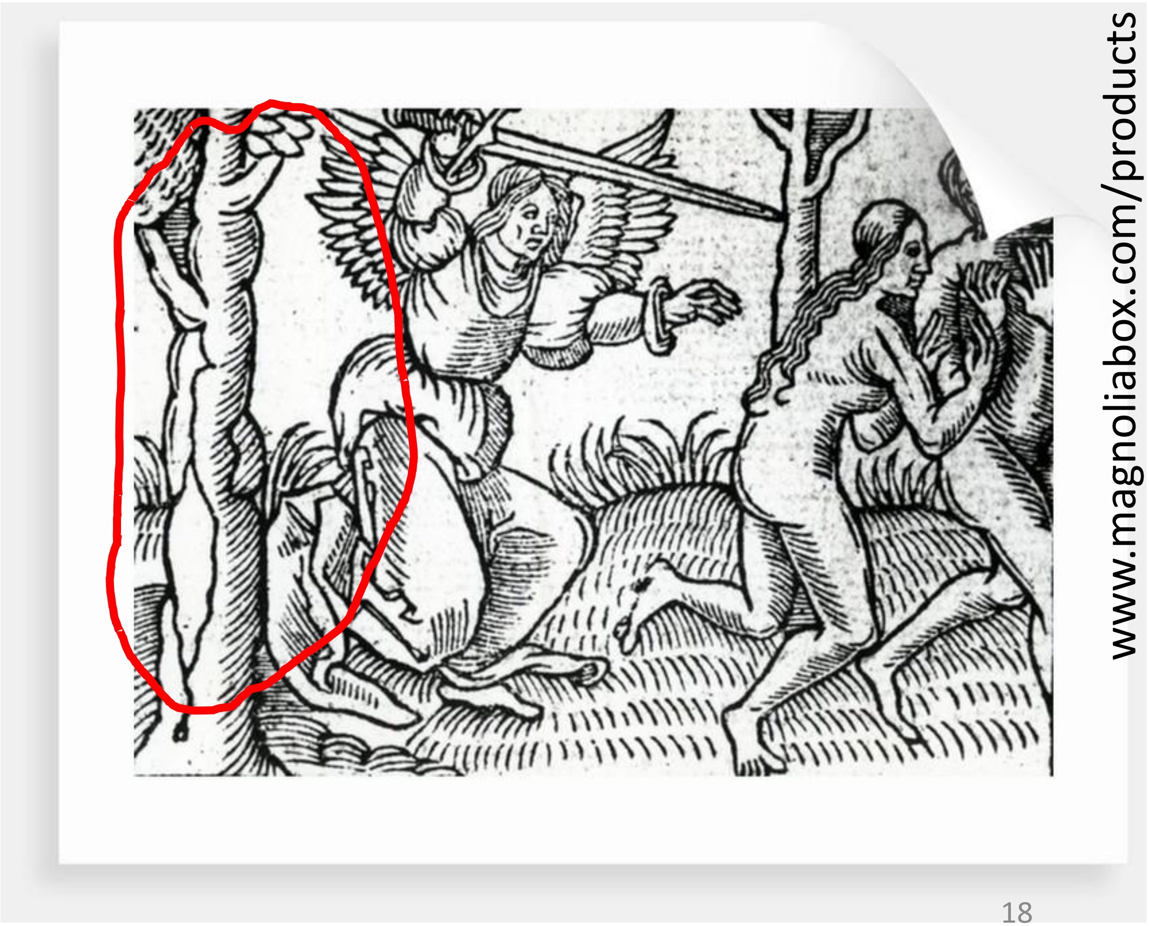
In each cycle we have only 6 kinds of links, with different sequences of states:

- 3 constant links (s=±1), of energy u= -7;
- 4 links which change 12 times, of energy u=+1;
- 6 links which change 4 times (s=1,1,1,-1,-1,-1 and cyclically), of energy u=-1,-5,+1 and cyclically;
- 3 links which change 8 times (s=-1,-1,+1 and cyclically), of energy u=-1,+3,+3 and cyclically;
- 8 links which change 6 times (s=1,1,-1,-1 and cyclically), of energy u=-1,+1 and cyclically;
- 12 links which change 6 times (s=-1,-1,1,1,-1,1,1,1,-1-1,1,-1), of energy u= -1,5,-3,1,3,-3 and cyclically .

For 967680 cycles of length 12, all of them have the following properties:



$$s_{ij}(t+1) = sign \sum_{k \neq i,j} [-s_{ik}(t)][-s_{jk}(t)]$$



conclusions

As each link has its own trajectory, it can be stabilized
and remain unchanged even if the system is damaged.

It seems that for larger systems, the observed symmetries
can be broken. For N=13, we have seen a cycle of length 48,
where the energy spectra change with period 3.

As a fairly simple map, the same for each link, can produce
such a complex behaviour of a fairly small system,
it is tempting to take any other nontrivial map and see what happens.

Małgorzata J. Krawczyk, K. K., Zdzisław Burda, Phys. Rev. E 104 (2021) 024307
Zdzisław Burda, Małgorzata J. Krawczyk, K. K., Phys. Rev. E 105 (2022) 054312

谢谢

THANK YOU