Nieprzewidywalność w fizyce

Krzysztof Kułakowski
Wydział Fizyki i Informatyki Stosowanej AGH

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- Przepis Laplace’a
- Rzut monetą
- Piłka na membranie
- Nieprzewidywalne wahadło

Laplace'a przepis na przewi-dywanie przyszłości

n	x(n)	x'(n)
1	0.5555550	0.5555560
2	0.9876545	0.9876541
3	0.0487720	0.0487738
4	0.1855734	0.1855797
5	0.6045438	0.6045596
6	0.9562823	0.9562691
7	0.1672258	0.1672739
8	0.5570454	0.5571733
9	0.9869832	0.9869248
10	0.0513891	0.0516168
11	0.1949930	0.1958102
12	0.6278831	0.6298742
13	0.9345836	0.9325307
14	0.2445483	0.2516687
15	0.7389777	0.7533263
16	0.7715585	0.7433030
17	0.7050238	0.7632145
18	0.8318608	0.7228724

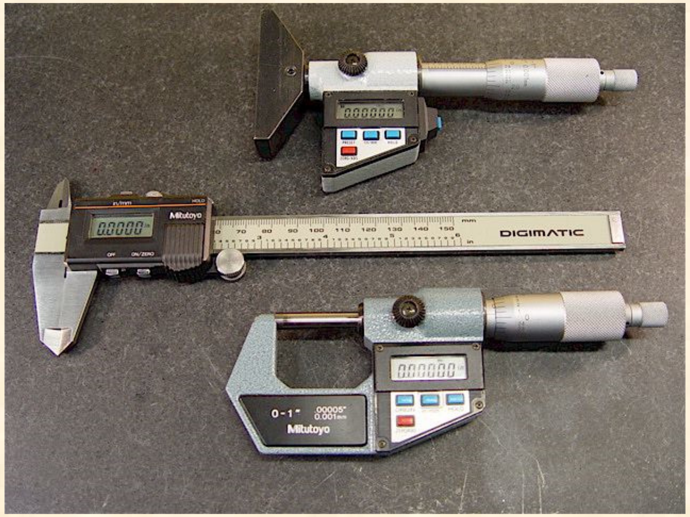
1. Określ stan początkowy
2. Użyj praw fizyki

na przykład

$$x(n+1)= 4x(n)[1-x(n)]$$


1
2
3
12
13


2X
Premiere


Mitutoyo
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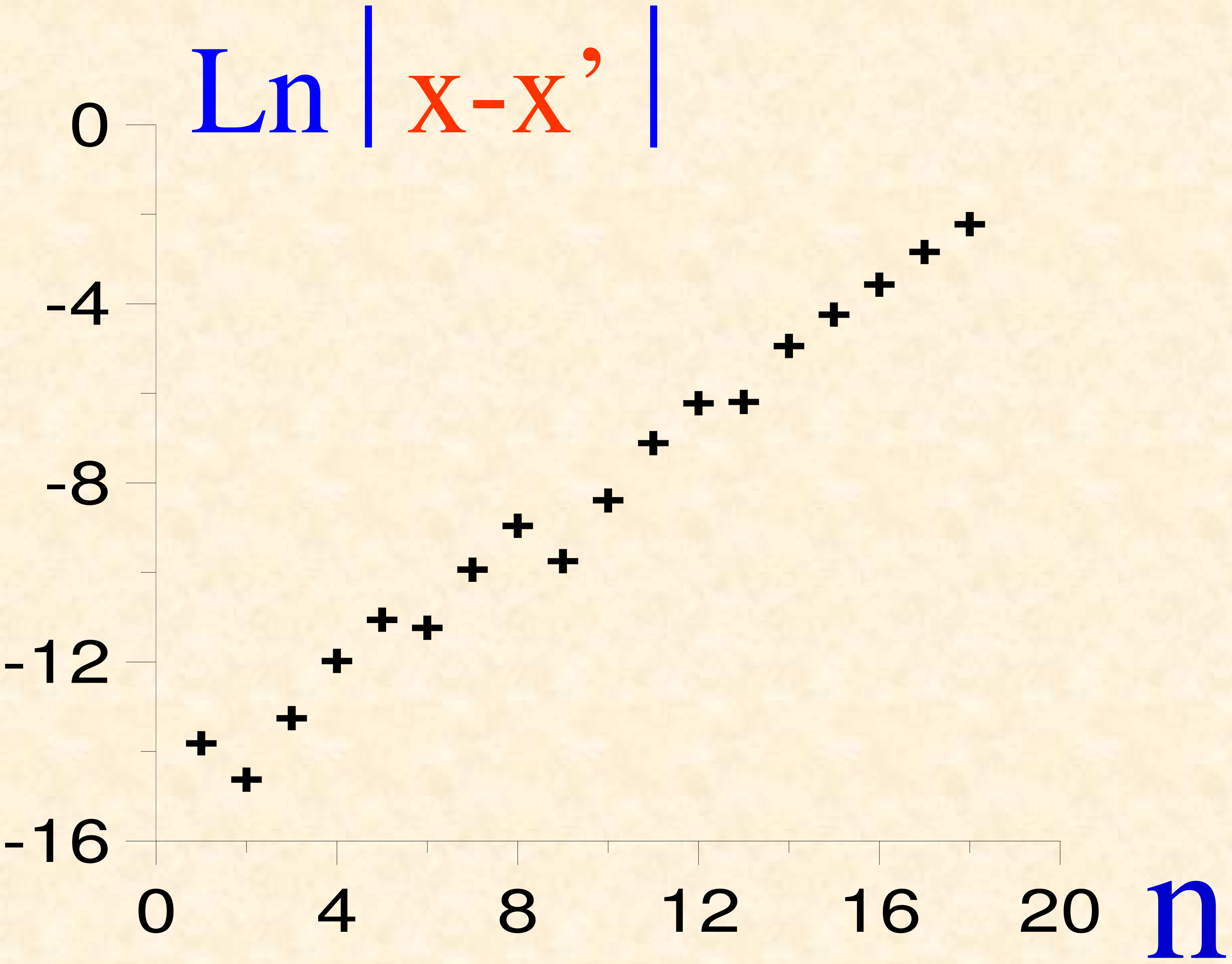
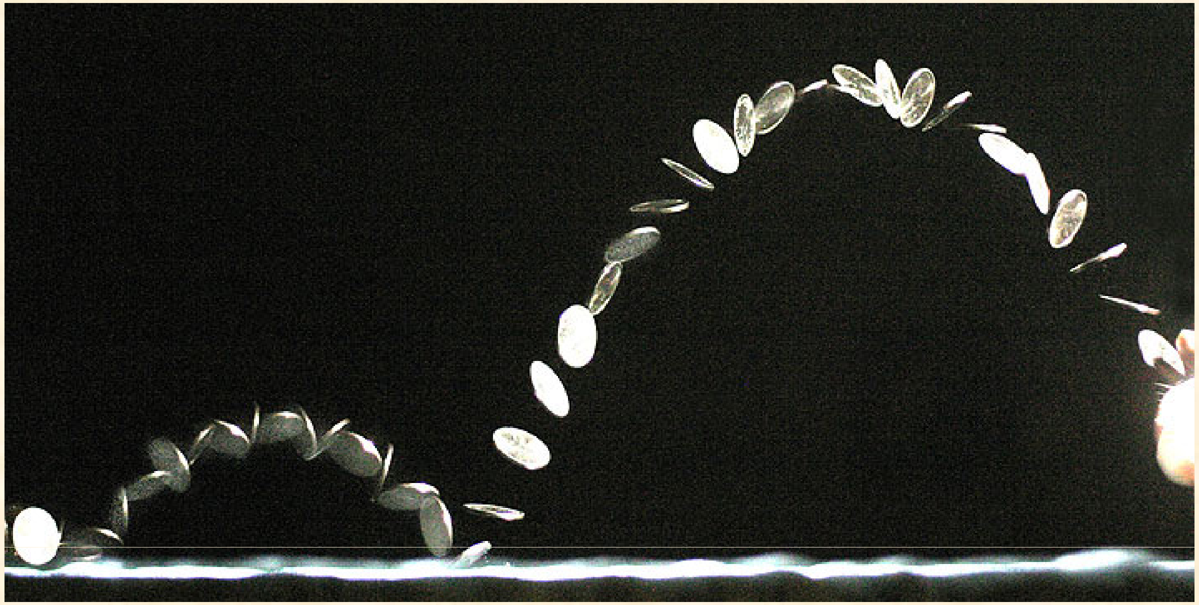


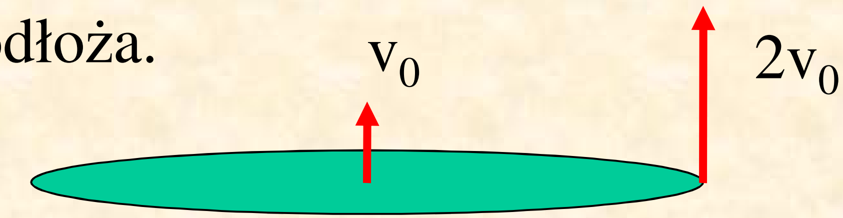
Ln|x-x'|
0
-4
-8
-12
-16
0
4
8
12
16
20
n

Przykład : rzut monetą



Założenia upraszczające: $\omega = v/R$, wtedy wysokość lotu $h = v_0^2/(2g)$

Moneta upada i przykleja się do podłoża.



Czas lotu $t = 2v_0/g$

Droga kątowa $\Delta\alpha = \omega t = 2v_0^2/(Rg)$

O czy R?

Jeżeli $\Delta\alpha \in (2n, 2n+1)\pi$, to O. Jeżeli $\Delta\alpha \in (2n+1, 2n)\pi$, to R

Jeśli część całkowita z $2v_0^2/(\pi Rg)$ jest parzysta, to O ; nieparzysta, to R.

Wartości graniczne: $\frac{2v_0^2}{\pi Rg} = n$ stąd $v_n = \sqrt{\frac{Rg\pi}{2}}\sqrt{n}$

Niech $R = 6.5\ cm$, wtedy $(Rg\pi/2)^{1/2} = 1\ m/s$.

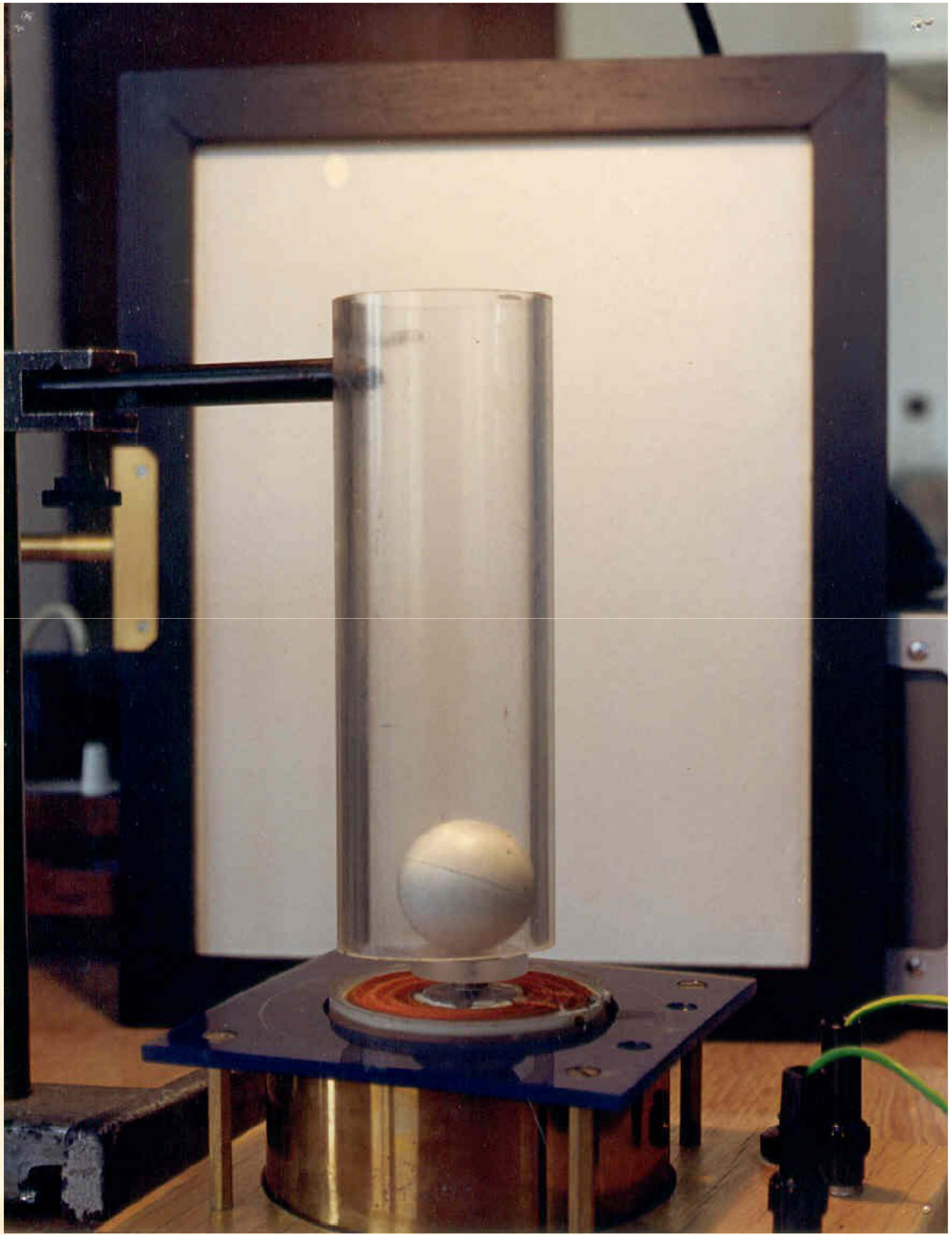
Z jaką dokładnością musimy określić prędkość początkową *v* ?

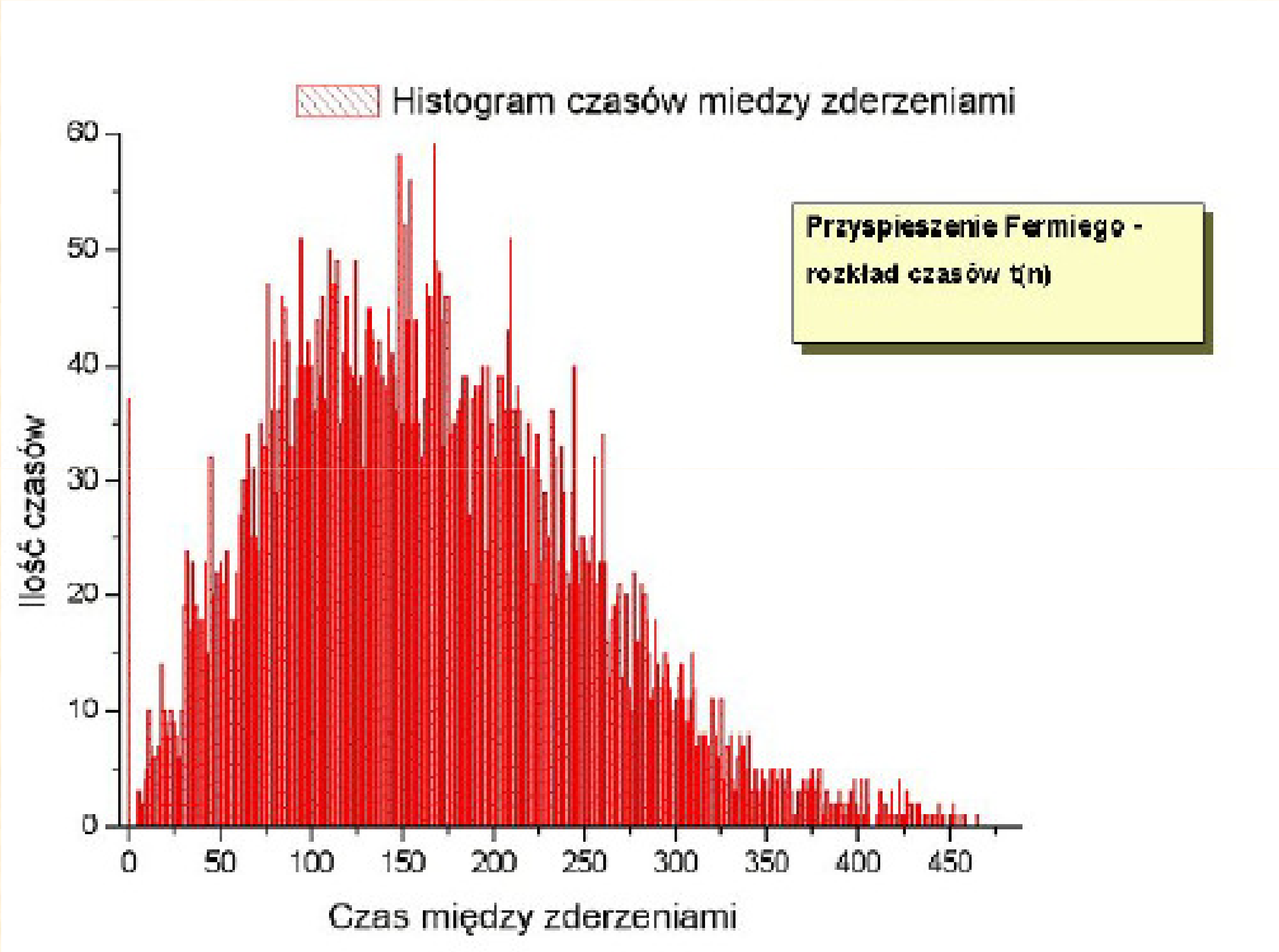
Kolejne wartości graniczne będą leżały w odległościach :

$$\Delta v_n = v_{n+1} - v_n = \sqrt{n+1} - \sqrt{n} = \sqrt{n}\left(\sqrt{1+\frac{1}{n}} - 1\right) \approx \sqrt{n}\left(1+\frac{1}{2n} - 1\right) = \frac{1}{2\sqrt{n}} \to 0$$

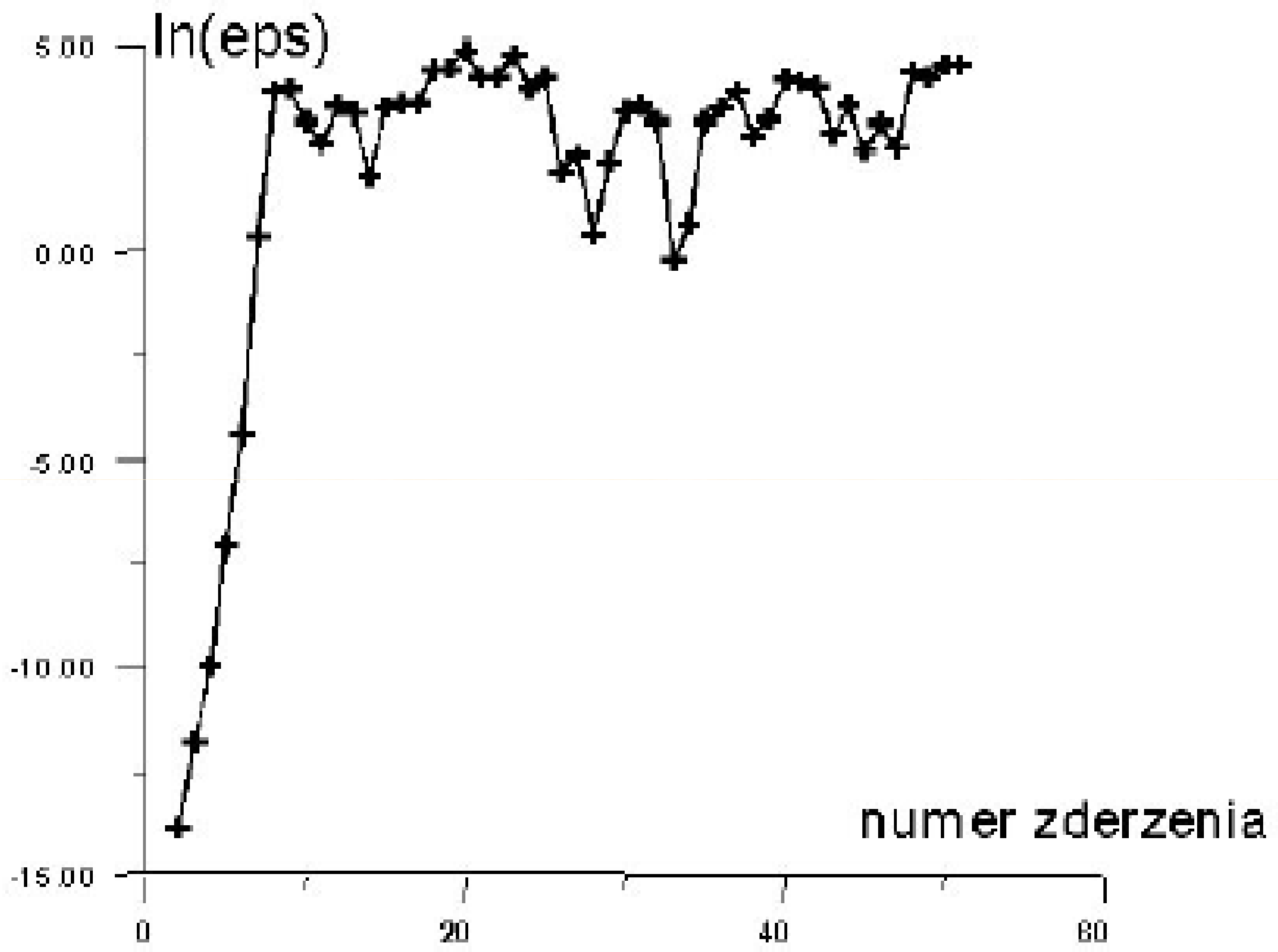
Jeżeli *n* wystarczająco duża, to Δv_n na pewno mniejsza, niż dokładność pomiaru prędkości poczatkowej *v*.

Piłka
na
membranie



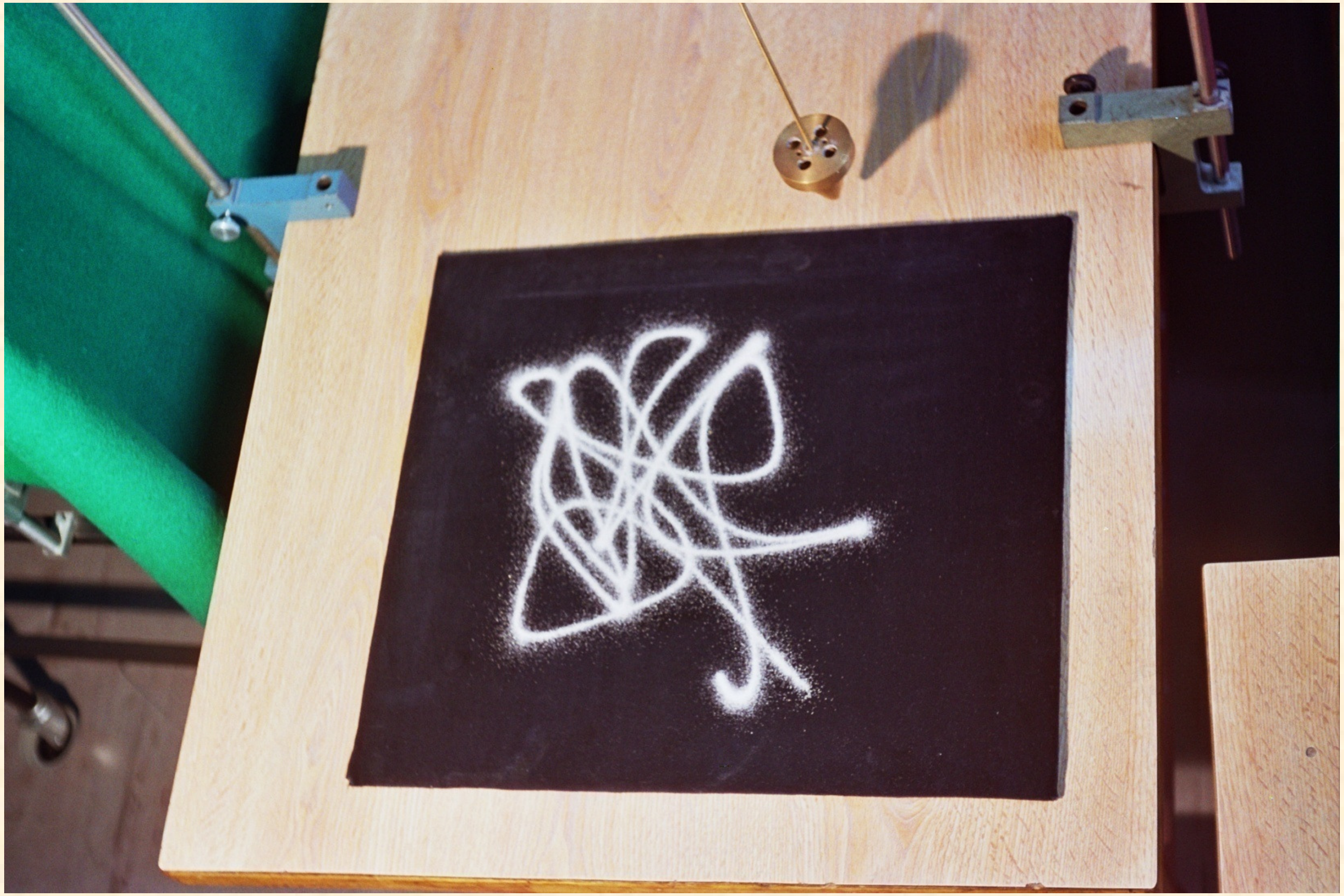


Histogram czasów miedzy zderzeniami
Przyspieszenie Fermiego - rozkład czasów t(n)
ilość czasów
Czas między zderzeniami
0
10
20
30
40
50
60
0
50
100
150
200
250
300
350
400
450



ln(eps)
5.00
0.00
-5.00
-10.00
-15.00
numer zderzenia
0
20
40
60

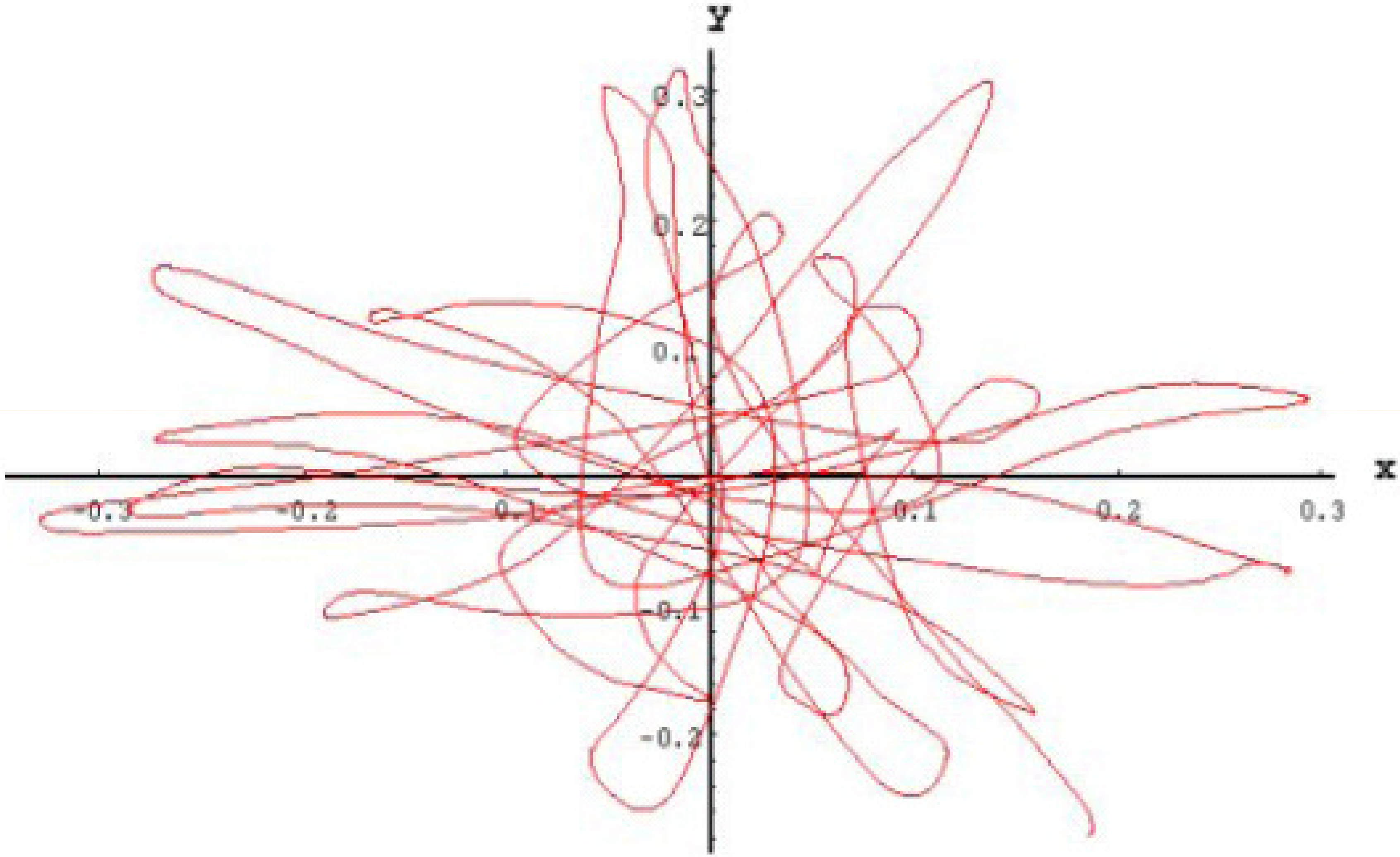
Nieprzewi-
dywalne
wahadło



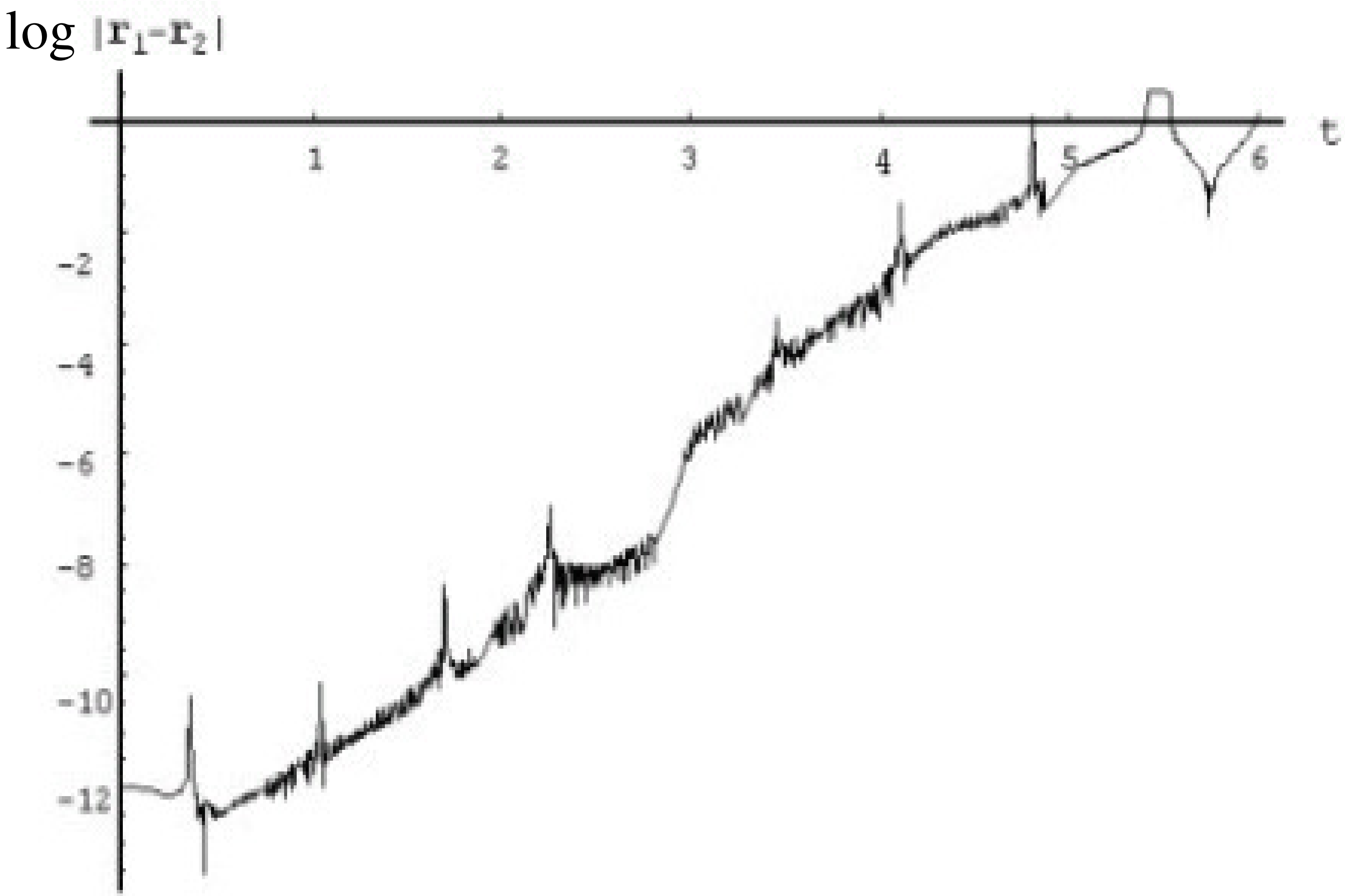
Nie można otrzymać tego samego rysunku …

…dwa razy

Położenie końca wahadła – symulacja

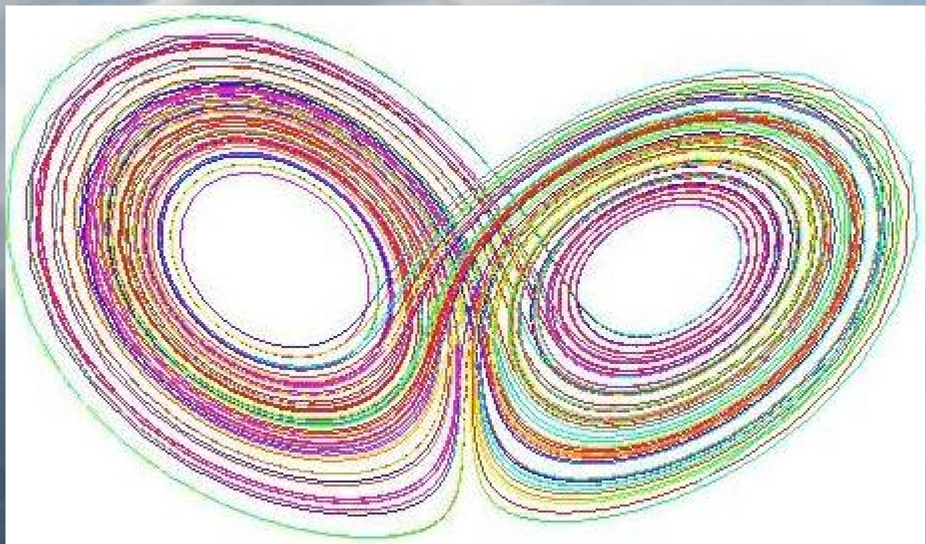


Różnica między dwoma, poczatkowo bliskimi, torami



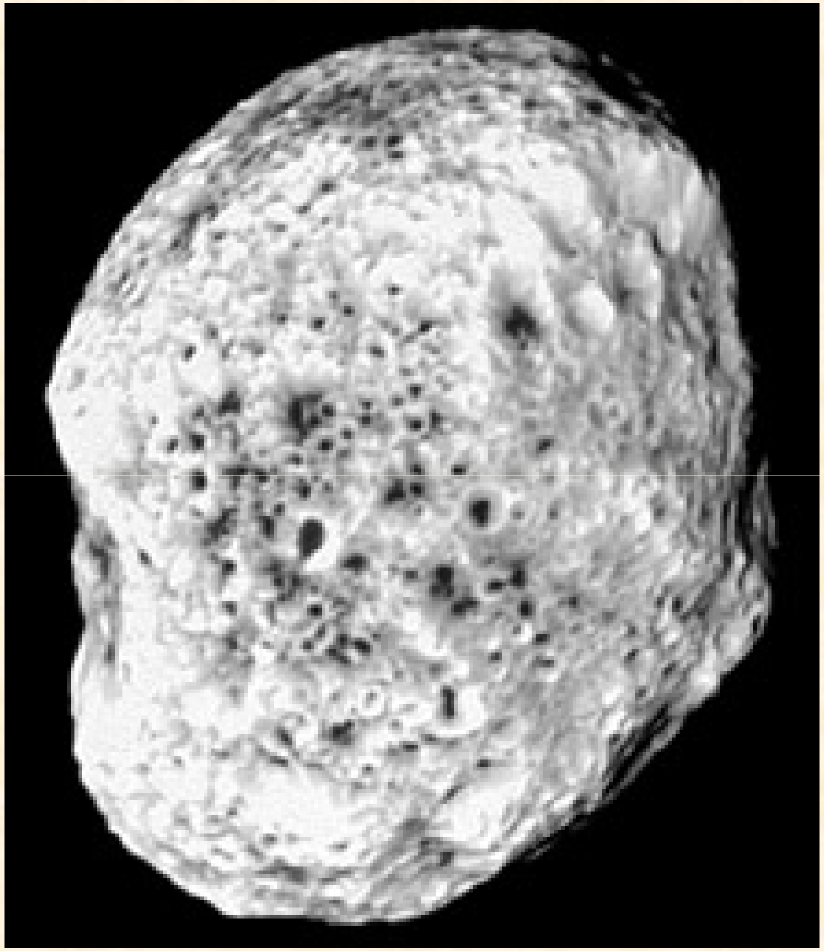
Przykład: **pogoda np. w Augustowie**

$$\frac{dx}{dt} = \sigma(y-x)$$
$$\frac{dy}{dt} = rx-y-xz$$
$$\frac{dz}{dt} = -bz+xy$$



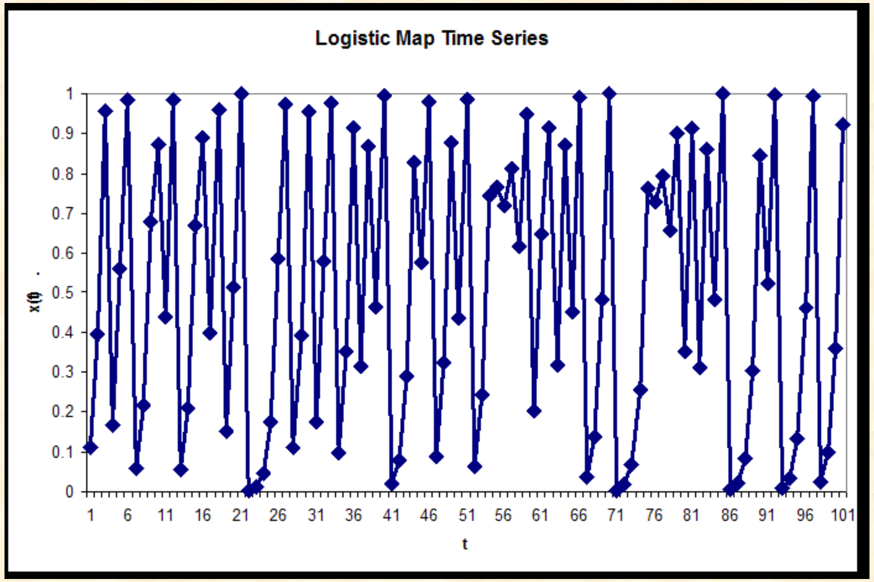
Inny przykład: **HIPERION**

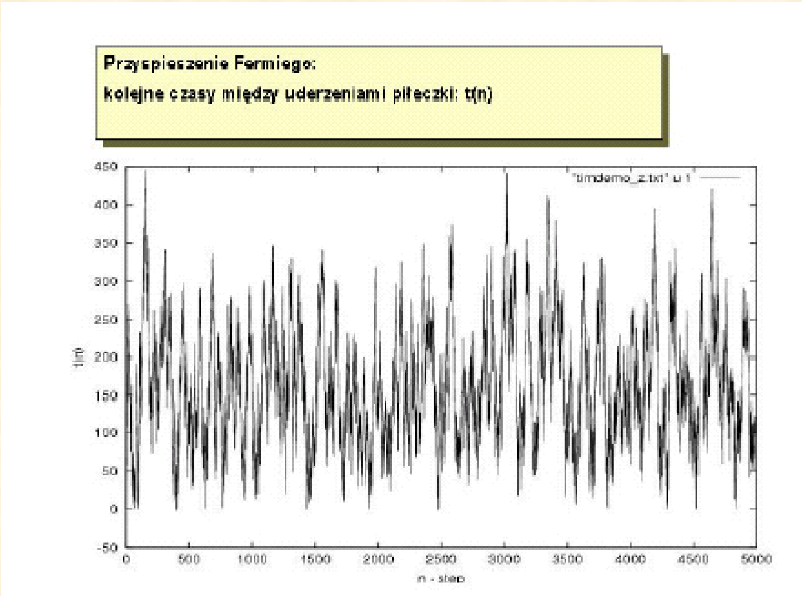
Księżyc Saturna
410 x 260 x 220 km
Okres orbity 21 dni 7 godzin
Odkryty 1848
Zdjęcie 26.09.2005



Hiperion koziołkuje chaotycznie pod wpływem innego księżyca, Tytana:

$\lambda \approx 10^{-7}$ /sek





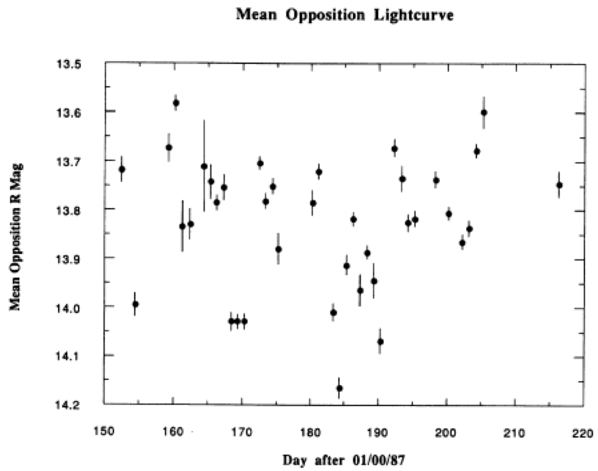


FIG. 4. Hyperion light curve corrected to zero solar phase angle ($\alpha = 0$) and mean opposition distance ($r_s = 9.54$ AU and $\Delta_s = 8.54$ AU). Note the change in scale from Figs. 2 and 3.

The absolute minimum value of the pdm plot shown in Fig. 7 agrees with that value using the *H,G* model phase correction, shown in Fig. 5, and the overall structure is similar. No well-defined period was found for any of the minima. All of the phase plots for these trials looked as scattered as Fig. 6. No period from 1 hr to 50 days even comes close to fitting the dataset satisfactorily.

IV. DISCUSSION

Voyager information on Hyperion's shape led Wisdom *et al.* (1984) to predict that Hyperion would be in a rotation state of chaotic tumbling. All previous published observations (Andersson 1974; Goguen *et al.* 1983; Thomas and Veverka 1985; Binzel *et al.* 1986) were too undersampled to make any definitive conclusions concerning Hyperion's rotation state. Peale (1986), Wisdom *et al.* (1984), Wisdom and Peale (1984), and Peale and Wisdom (1984) warn of the dangers of traditional methods of folding back the light curve and performing least-squares analysis on data sampled less than about once every 1.5 days for a chaotically rotating Hyperion. Goguen *et al.* (1983) and Thomas and Veverka (1985) find best-fit periods using this technique. Both fits, however, produce inconsistent results that are possibly consistent with chaotic rotation. None of the previous datasets have been able to constrain Hyperion's rotation state.

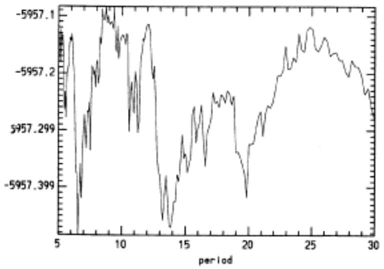


FIG. 5. Phase-dispersion-minimization (pdm) plot of the light curve with *H,G* phase correction corresponding to Fig. 4. Best period is 6.6 days. Ordinate is a relative measure of the dispersion significance of the period; values near zero are good fits.

With an average of nine independent Hyperion observations per night, I obtained a light curve using the MHO 1.3 and 2.4 m telescopes with the MASCOT/MIS detector at the required sampling rate. My final light curve contains 38 nightly means over a period of 64 days (Fig. 4). I found that Hyperion was essentially at a constant brightness over a period of one night and that its color is $V - R = 0.41 \pm 0.02$. The light-curve amplitude, after correction to mean opposition magnitude (Fig. 4), is ≈ 0.6 mag. This is close to what would be expected from the *Voyager 2* knowledge of shape and albedo.

With pdm analysis, I demonstrated that no period from 1 hr to 7 weeks fits the light curve. Although a large part of the discussion in Sec. III concentrated on fitting the *H,G* phase function to my light curve, the essential point is that the analysis is insensitive to the phase correction. This is demonstrated by comparing the pdm plots with phase correction (Fig. 5) and without phase correction (Figs. 7), noting that there is no well-fit period for any of the minima.

CCD photometry of Hyperion over an interval of 64 days shows no evidence of periodic modulation in the light curve. Is this conclusion equivalent to Hyperion being chaotic? Chaos has a very specific definition: chaotic motion is deterministic but unpredictable motion due to exponential divergence of nearby initial conditions (Hénon and Heiles 1964; Wisdom 1987). While this work does not, and cannot, *prove*


Dziękuję
za uwagę.
Dobranoc