# Jammed states, Heider balance and paradise

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The story is about interpersonal relations, friendly ($x_{ij}$ =+1, ——) or hostile ($x_{ij}$ =-1, – – ), as links in a fully connected social network.

## outline

- Cognitive dissonance
- The Heider balance
- Two algorithms
- Jammed states
- Asymmetry : jammed states are generic
- Direct reciprocity
- Phase diagram
- Application: gender segregation

# Cognitive dissonance
## (Festinger 1957)

- the mental conflict that occurs when beliefs or assumptions are contradicted by new information.

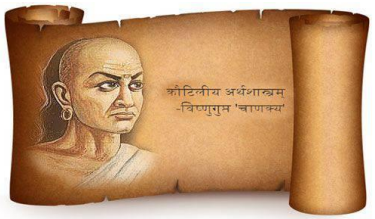
*[Encyclopædia Britannica]*

The condition of coherence of our beliefs helps also to solve problems as:

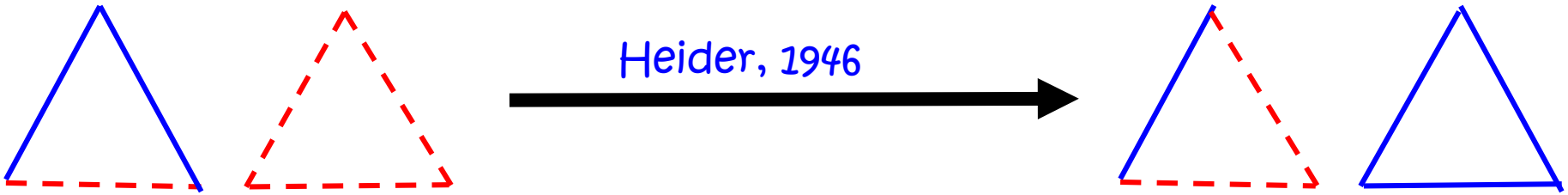
- what is the origin of the Universe?
- is Hamlet mad?
- an attempt on the life of the president or just an accident?

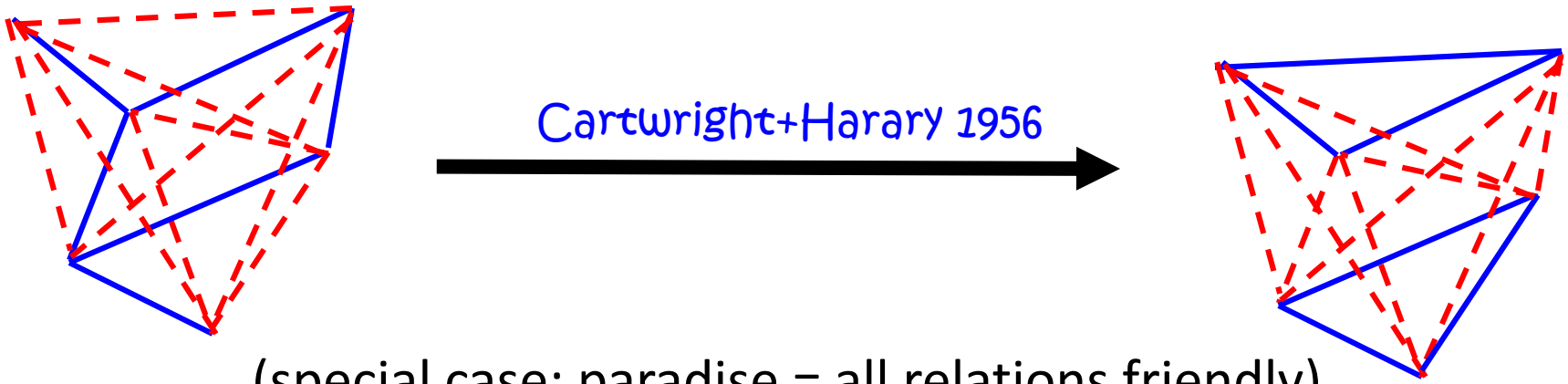
# A consequence of removal of cognitive dissonance: the Heider balance

a friend of my friend is my friend,
a friend of my enemy is my enemy,
an enemy of my friend is my enemy,
an enemy of my enemy is my friend.




Chanakya, 400 BC

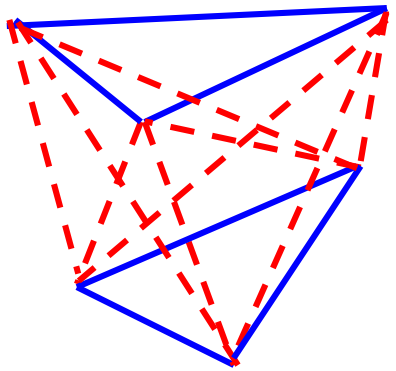


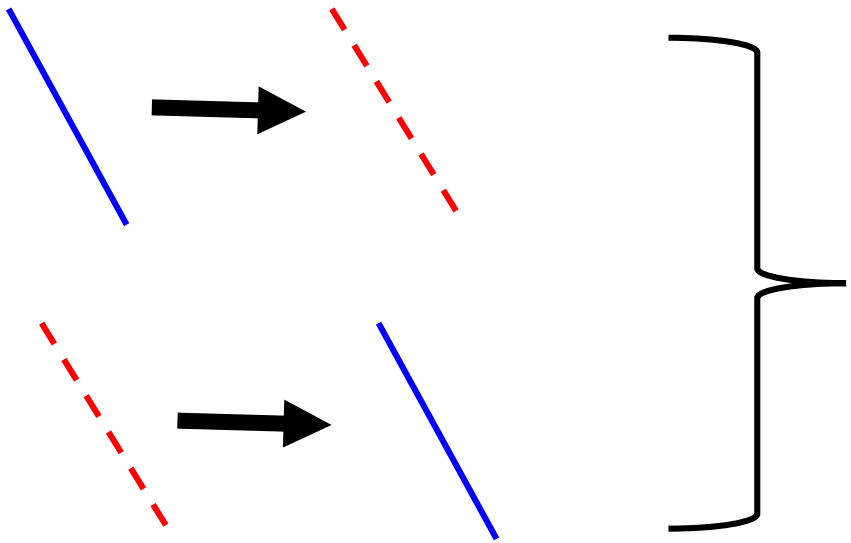




(special case: paradise = all relations friendly)

# Discrete algorithm (Antal *et al*, 2005) :

$$U = -\sum_{ijk}^{N} x_{ij}x_{jk}x_{ki}$$





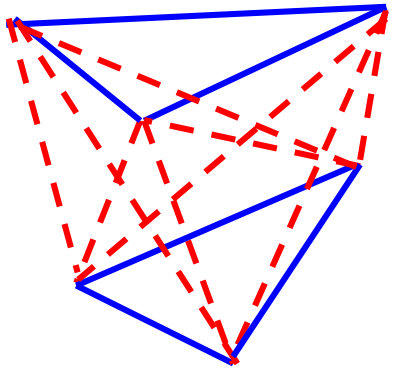
with probability 1 if *U* decreases

with probability 1/2 if *U=const*

# Continuous algorithm (KK *et al*, 2005) :

$$\frac{dx_{ij}}{dt} = H(1 - |x_{ij}|) \sum_{k}^{N} x_{ik} x_{kj}$$

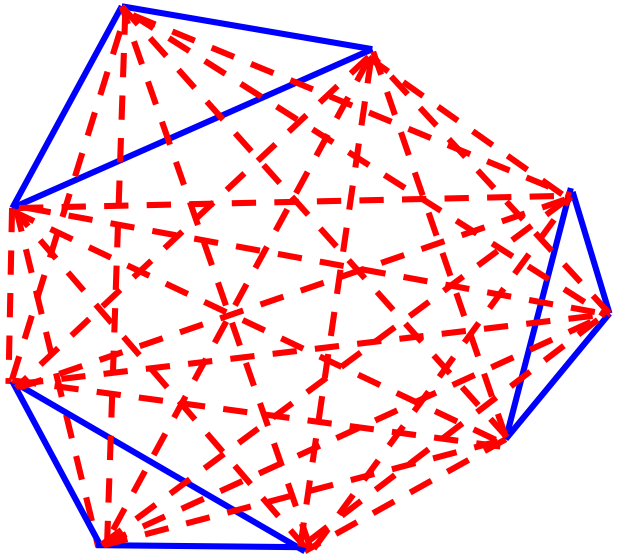
keeps $x_{ij}$ in the range [-1,1]



the relation $x_{ij}$ improves if *i* likes *k* and *k* likes *j*
and if *i* dislikes *k* and *k* dislikes *j*,
otherwise relation $x_{ij}$ deteriorates

# The jammed states

an example for $N = 9$ (Antal *et al*, 2005)



- a change of any link enhances $U$
- for all links $x_{ij}$

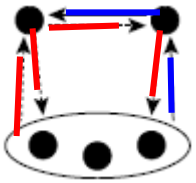
$$x_{ij} \sum_k^N x_{ik} x_{kj} > 0$$

As $N$ increases, the probability of jammed states decreases both for the discrete algorithm (Antal *et al*, 2005) and for the continuous one (Marvel *et al*, 2011).

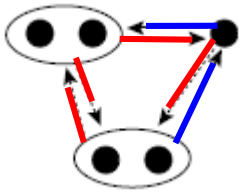
# Asymmetry: $x_{ij} \neq x_{ji}$ , then the jammed states generic

## Number of jammed states in an asymmetric matrix

*N* ways to choose a node, 2*N* states



*N* ways to choose a node,
divide the remaining *N*-1
into 2 non-empty parts, $2^{N-2}$-1 ways



Summing up, we get

$$2N + 2N(2^{N-2} - 1) = N2^{N-1}$$

what is *N* times larger than the numer of balanced states

More lengthily in Hassanibesheli *et al*, submitted (arXiv:1609.03358)

# Direct reciprocity
(Krawczyk *et al.*, 2015)

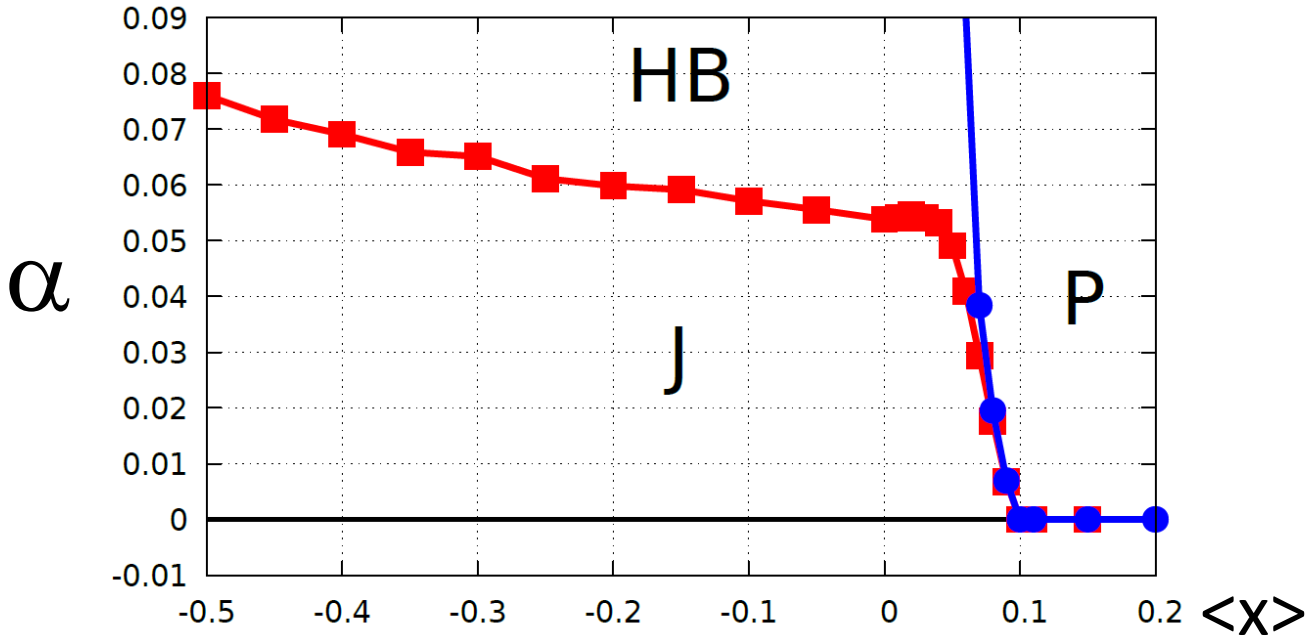
$$\frac{dx_{ij}}{dt} = (1 - {x_{ij}}^2)\left[\alpha\left(x_{ji} - x_{ij}\right) + \frac{(1-\alpha)}{N-2}\sum_{k}^{N} x_{ik}x_{kj}\right]$$

the relation $x_{ij}$ improves if *j* likes *i* more than *i* likes *j*,
otherwise relation $x_{ij}$ deteriorates

The parameter $\alpha$ is a measure of mutual weights of two processes.

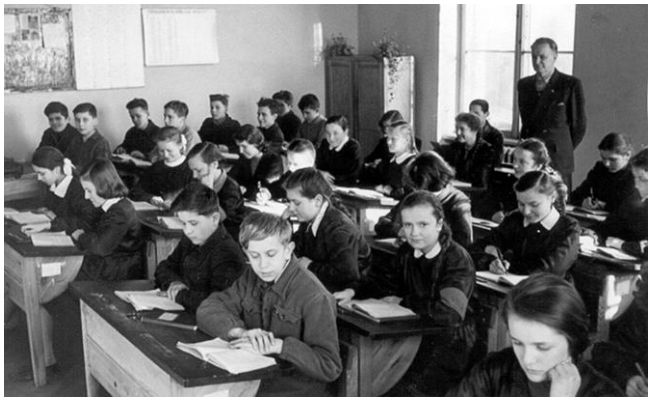
# The phase diagram (Hassanibesheli et al., arXiv:1609.03358)

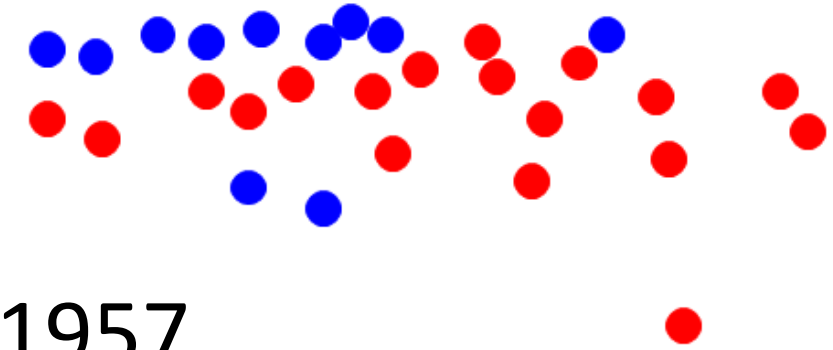
Initial values of $\{x_{ij}\}$ selected randomly from the range [$\langle x \rangle$-0.3, $\langle x \rangle$+0.3]




- in the phase [HB], the relations are symmetric: $x_{ij} \neq x_{ji}$ ;
- there, the proces of direct reciprocity is fast enough ($\alpha$ is sufficiently large) to even the mutual relations out;
- if most of the initial relations are positive, the system tends to [P]
- the phase [J] is confirmed to be generic.

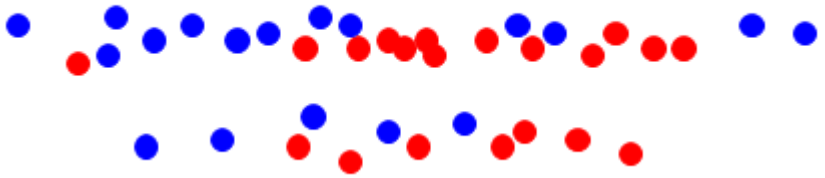
# An application: research on gender segregation





1957

2012

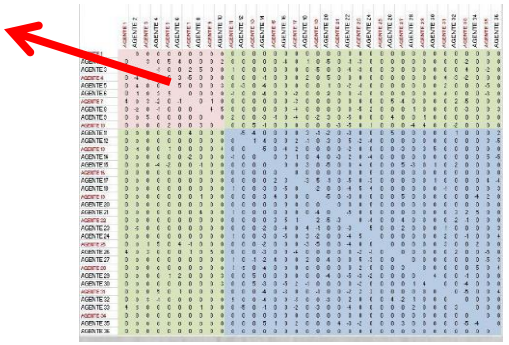


# Data from 37 Mexican school classes

1. Select 5 persons whom you like most, and assign them points from 1 to 5
2. Select 5 persons whom you dislike most, and assign them points from -5 to -1

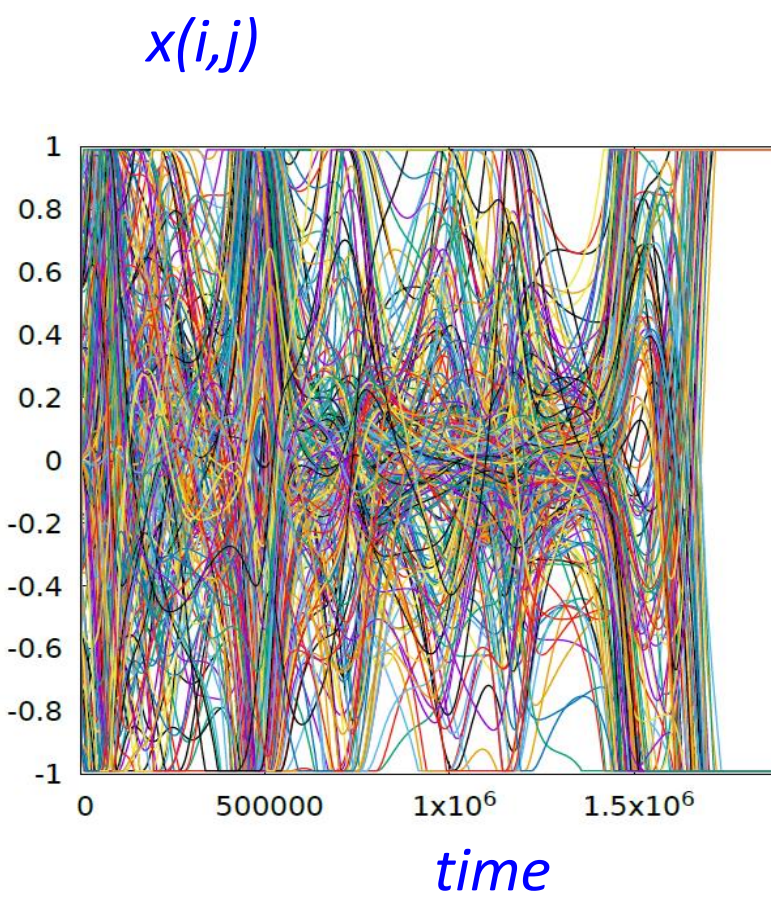
| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 4 | 0 | 3 | 5 | | | 0 | 0 | 0 | 0 | 2 | 1 |
| | | 0 | | -4 | | -3 | 5 | 0 | 0 | 0 | 0 | -2 | 0 | 0 |
| | | | | | | | | | | | | | | |
| 4 | -2 | 0 | | 0 | -5 | | 0 | 0 | -3 | 0 | 5 | -4 | 1 | 0 |
| 0 | 0 | | 0 | | -5 | 0 | 0 | 3 | 0 | 4 | -4 | | | 5 |
| 5 | | | 4 | 0 | | 3 | -2 | 0 | | 0 | | | 2 | 1 |
| 5 | 0 | 1 | 0 | 3 | 0 | | -5 | 0 | -2 | 0 | 0 | -3 | 2 | 4 |
| | 5 | | | 0 | | -5 | | 0 | | 0 | 4 | 0 | | 2 |
| 0 | | | 2 | 3 | | 0 | 0 | | -5 | 4 | 0 | -4 | 0 | 5 |
| 3 | 0 | | 0 | -4 | 5 | 1 | 0 | -3 | | 0 | 4 | 0 | 2 | -1 |
| | 0 | 0 | | 4 | 0 | 1 | 0 | 3 | -4 | | | 0 | 0 | 5 |
| 2 | 0 | 0 | 5 | -1 | 4 | 0 | | -2 | 3 | 0 | | 0 | 1 | 0 |
| -4 | 0 | 0 | 0 | 0 | 5 | | 3 | | -5 | | 4 | | 0 | 0 |
| | | 5 | 2 | 0 | | 3 | | 0 | | 0 | | -4 | | 0 |
| | | 0 | 0 | 5 | | 2 | 0 | 4 | -5 | 3 | | -2 | 0 | |

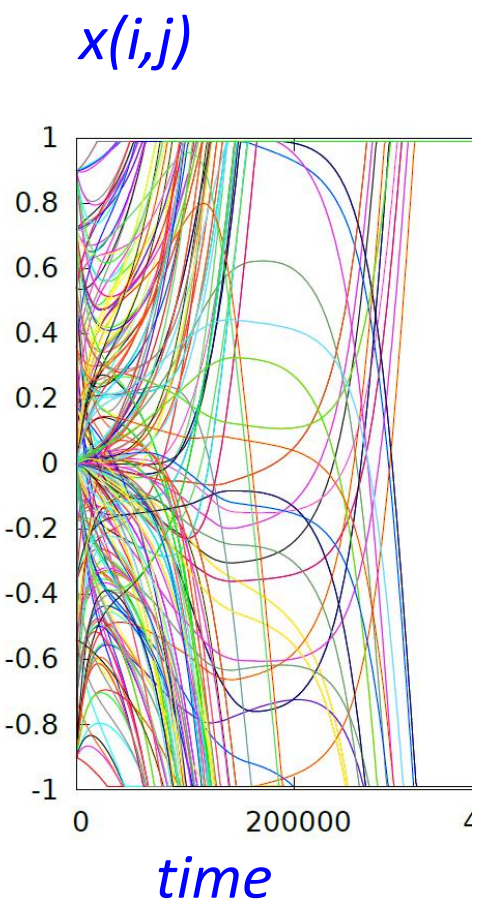
an example:
girls on girls

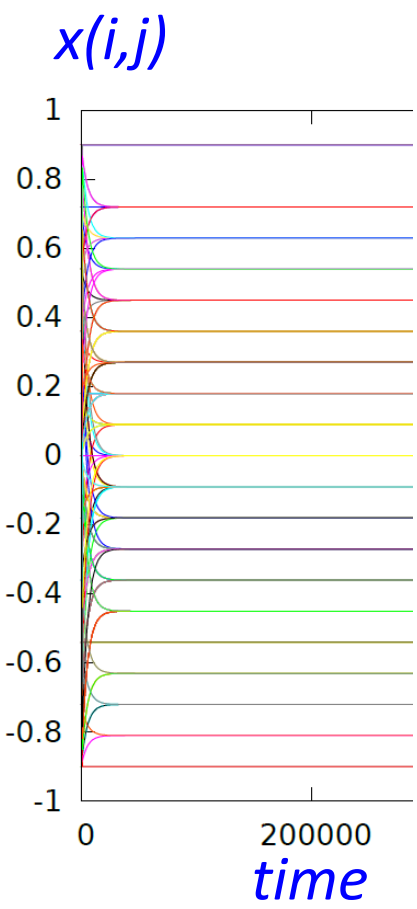


# Exemplary runs

$\alpha = 0.0$ $\alpha = 0.5$ $\alpha = 1.0$



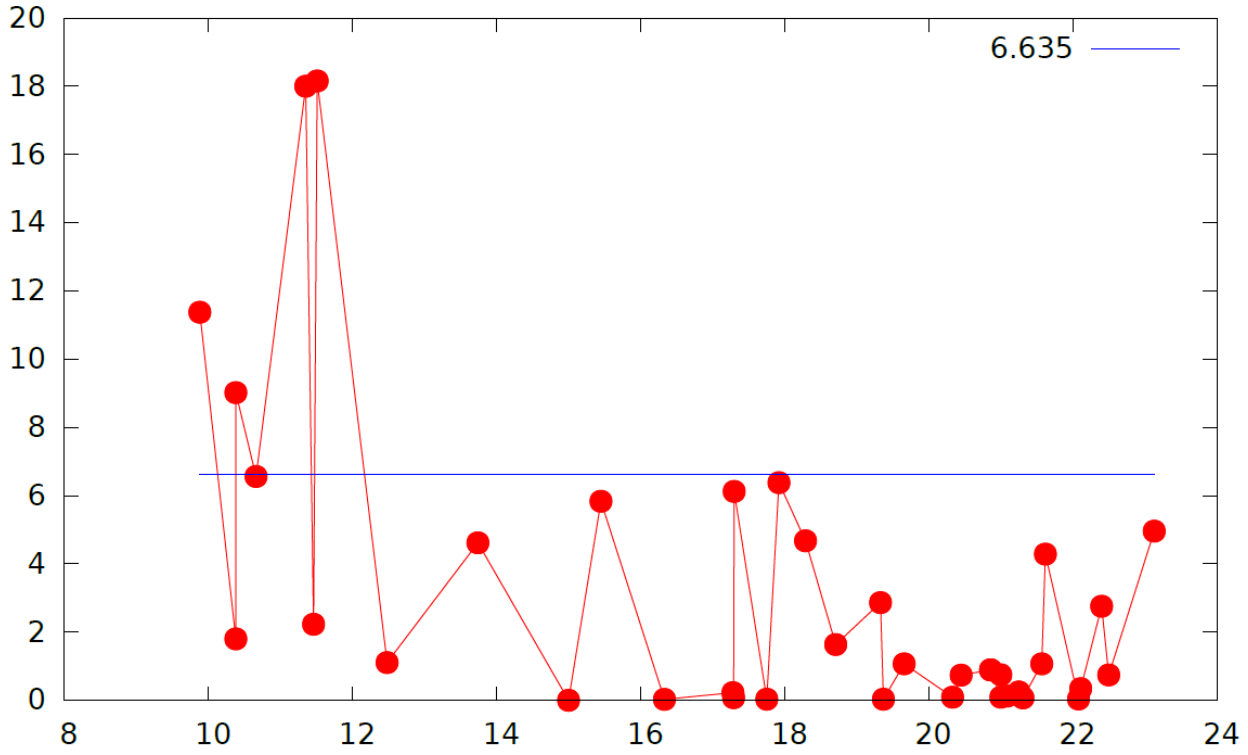



# A test of statistical significance

$$X^2 = \frac{(k+m)(k_1 m_2 - k_2 m_1)^2}{km(k_1+m_1)(k_2+m_2)}$$

>6,635 ? for statistical significance 0,99
(one degree of freedom)




mean age of class children

(Krawczyk et al., 2015)

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# summary

In the case of asymmetric relations (which is ubiquitous), the jammed phase is generic.

The effect of direct reciprocity drives the system to Heider balance with symmetric relations.