If others jump to the queue front, how long I will wait?

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outline

- The Thomas theorem
- Queue as a social system
- Inference from incomplete data, role of context
- Examples:
  - a queue of pedestrians (taxi, post-office, xero..)
  - two lines of vehicles before a narrowing
- Conclusions and context again

The Thomas theorem [1928]

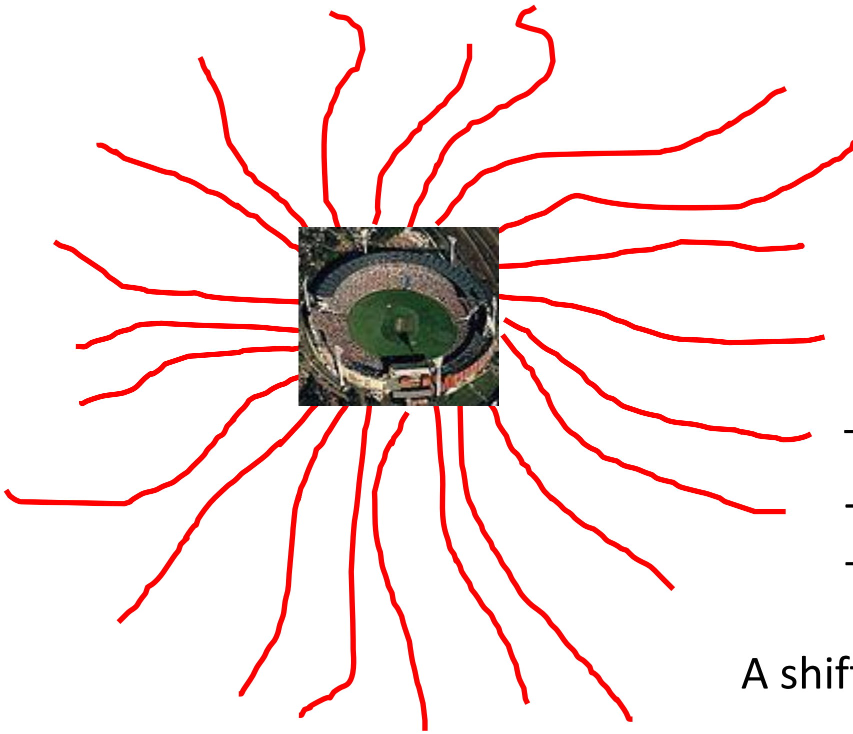
„If men define situations as real,
they are real in their consequences."

W. I. Thomas, 1863-1947

D. S. Thomas, 1899-1977

[R. K. Merton, The Thomas theorem and the Matthew effect, Social Forces 1995]

Queue as a social system – an example



Two competing rules:

- first come, first served (FIFO)
- who cares for the absent ?

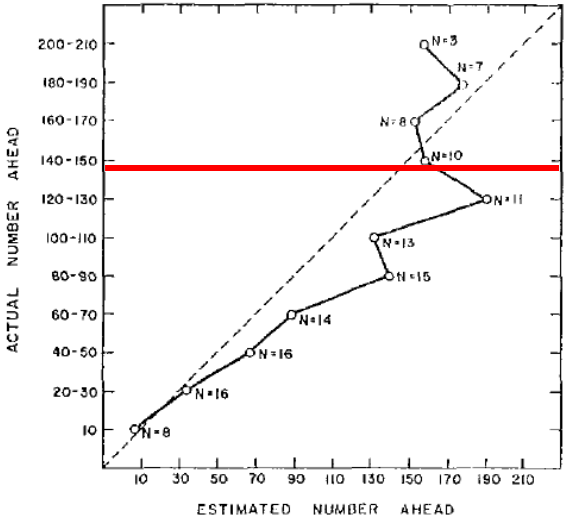
A shift system: one person can hold up to 4 places

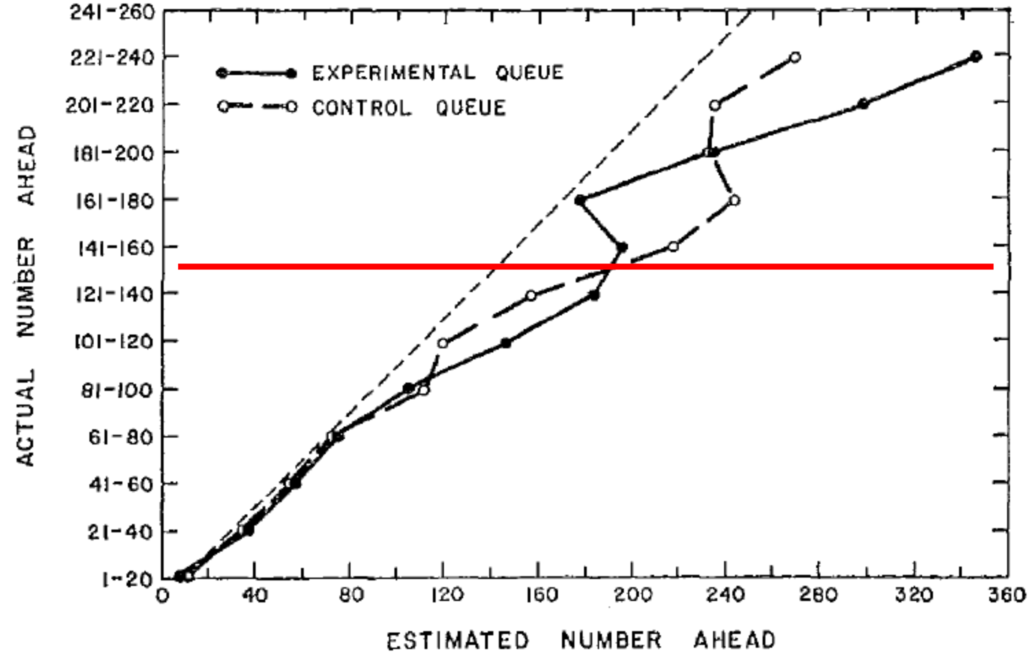
MCG stadium in Melbourne
15.08.1967, 6 A.M. - 8 A.M.
last chance to get tickets
≅10 000 persons, 22 queues
9 assistants, 216 interviews

Actual position	member of a shift system	observed hostile response to push-in attempt	consider place-keeping permissible
10-100 (N=95)	39 %	46 %	25 %
110-200 (N=82)	23 %	33 %	35 %
210-330 (N=39)	25 %	27 %	24 %

[L. Mann, Queue culture: the waiting line as a social system, Amer. J. Soc. 1969]

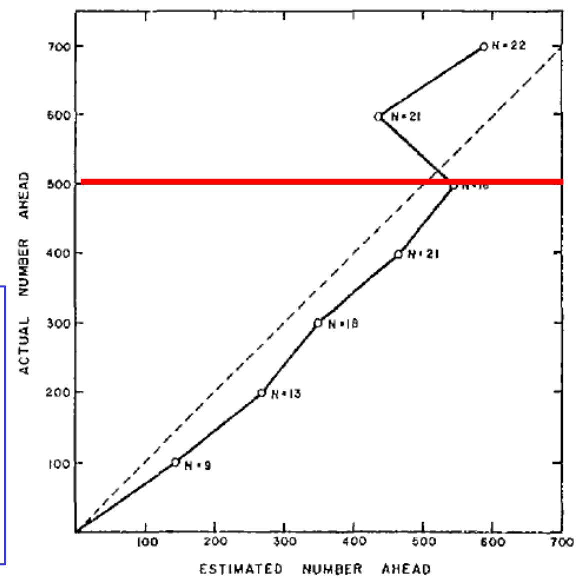
How many persons before me?





MCG Stadium, 12 500 tickets,
22 queues, 4 tickets/person
⇒ critical number = 142.
121 interviews, a day before

521 high school students,
2 queues,
130 chocolate bars
announced at
the experimental queue



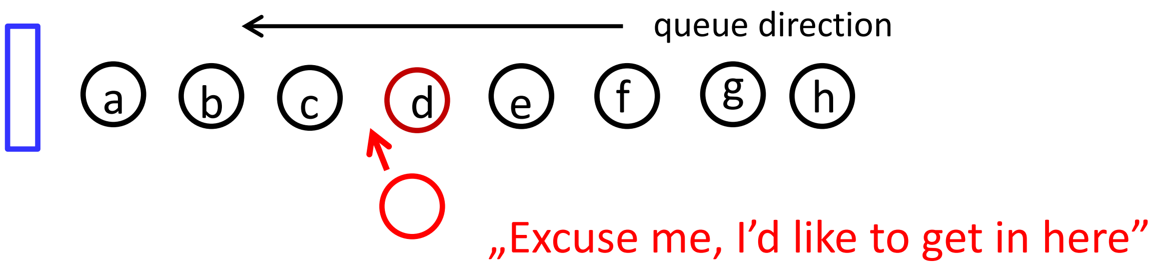
Collingwood stadium,
2 000 tickets, 1 queue,
4 tickets/person
⇒ critical number = 500.
134 interviews

[L. Mann, K. F. Taylor, Queue counting: the effect of motives upon estimates of numbers in waiting lines, J. Personality and Social Psychology, 1969]

Wattle Park High School, Victoria, Australia, 1972

[www.facebook.com/photo.php?fbid=10203444371172263&set=o.2420214391&type=3&theater]

How norms are executed?



NY, 120 waiting lines

- Physical action (10% of the lines) : from tapping the shoulder to pushing out the line
- Verbal objections (21%), from: - *Um … are you waiting to buy a ticket?*
  to : - *No way! The line's back there.*
- Nonverbal objections(15%): dirty looks, hostile stares, gestures to get to the line end.

frequency of objections

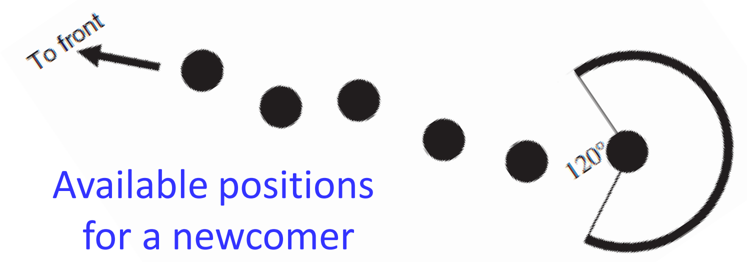
Intruder here intruder here

No. of intruders	No. of buffers	Position in line: **b**	**c**	**d**	**e**	**f**	**g**
1	0	4.5%	22.7%	36.4%	14.3%	0%	0%
1	1	0%	12.5%	*buffer*	16.7%	0%	0%
1	2	0%	5%	*buffer*	*buffer*	0%	0%
2	0	4.3%	21.7%	86.9%	43.5%	9.1%	0%
2	1	0%	10%	*buffer*	20%	0%	0%
2	2	0%	10%	*buffer*	*buffer*	15%	11.8%
Total		1.7%	14%	62.2%	24.7%	5%	2.7%

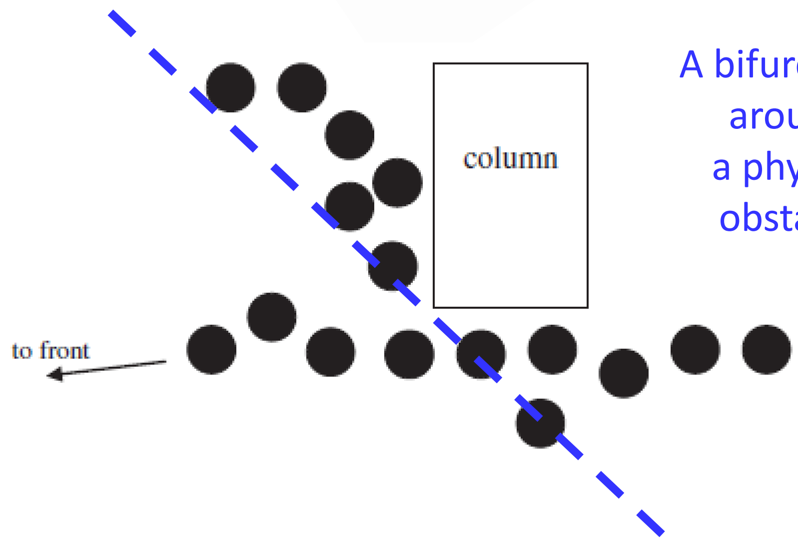
[S. Milgram et al., Response to intrusion into waiting lines, J. Personality and Social Psychology, 1986]

Other issues and findings:

- Delays caused by the service provider are accepted much easier than those caused by intruders *(B Schmitt et al., J. Personality &Social Psychology 1992)*



- Local geometry makes the rule FIFO more ambiguous *(D R Gibson, Soc Forum 2008)*



- Role of cultural context –> *next slide*

[www.dailymail.co.uk/debate/article-1300471/ What-queues-death-tell-British.html]

See also: D. Schweingruber, R.T.Wohlstein, *The madding crowd goes to school: myths about crowds in introductory sociology textbooks*, Teaching Sociology 33 (2005) 136

[cheezburger.com/6623126016]

Are we in autopilot mode?

- May I use the xerox machine? (60%)

- May I use the xerox machine,
  because I have to make copies? (93%)

- May I use the xerox machine, because I'm in a rush? (94%)

[E. Langer et al., The mindlessness of ostensibly thoughtful action: the role of "placebic" information in interpersonal interaction, J. Personality & Social Psychology, 1978; The construct of mindfulness, The Journal of Social Issues, 2000]

Inference from incomplete data

„German tank problem": Suppose an intelligence officer has spotted k = 4 tanks with serial numbers, 2, 6, 7, and 14, with the maximum observed serial number, m = 14. The unknown total number of tanks is called N.

Inference from context

Joshua Bell and his Stradivarius (4 milion USD)
Metro subway station, Washington,D.C.
45 minutes of rush hour, January 12, 2007.
1,097 people who passed by
seven stopped to listen to him => 32.17 USD
one recognized him => 20 USD

How long I will wait ?

P(waiting time/context) = P(waiting time/what I think of the context)

P(waiting time/no context)

How long I will wait ?

waiting time

$$\tau = <t> \left(n + \frac{\tau}{<t'>}\right)$$

mean time per person

number of persons before me

mean time between intruders

$$\tau = \frac{n <t>}{1 - \frac{<t>}{<t'>}}$$

finite iff $<t>$ shorter than $<t'>$

If an intruder appears before anybody leaves the office,
this „means” that intruders appear more frequently than people leave.

This queue goes backward!!!

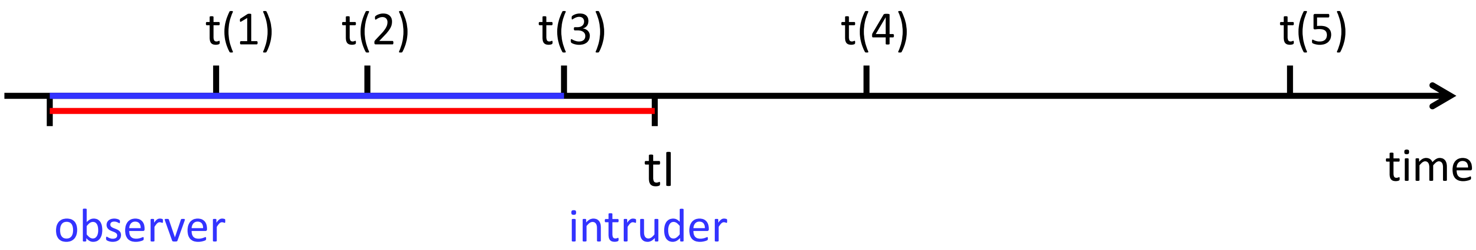
How frequently a person at the line end is made furious?

Suppose that in the average, *a* persons per unit time leave the office.
Let the times they leave are *t*(*k*), each ruled by the exponential *pdf* with $\langle t \rangle = 1/a$.

The *pdf* of the time when an intruder appears is also exponential, with $\langle t_I \rangle = 1$
Then, the probability that an intruder appears before *t*(1) is

$$P(t_I < t) = \int_0^\infty \rho(t)dt \int_0^t \rho_I(t_I)dt_I = \int_0^\infty ae^{-at}dt \int_0^t e^{-t}dt = \frac{1}{a+1}$$

algorithm to get pdf of the imagined waiting time τ

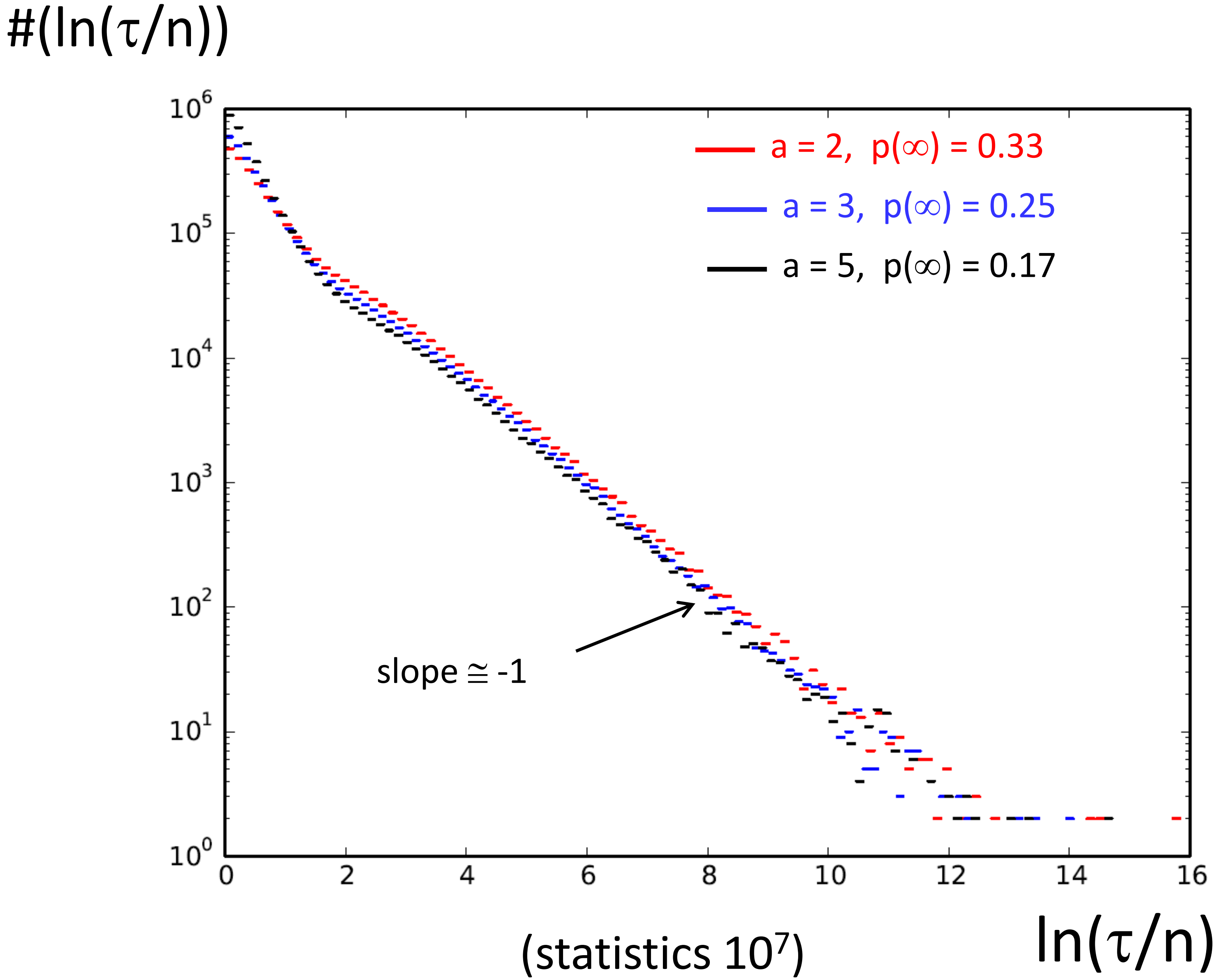


1. Find random moments $t(i)$ with exponential pdf with $1/\lambda = 1$ (times when people leave)
2. Set a moment $t^* < t(0)$ when an observer appears
3. Find a random moment $t(\mathrm{I})$ when an intruder appears, exponential pdf, $1/\lambda' = a > 1$
4. Find $< t >$ as the mean time between $t(i)$'s, from the time window $t(0) < t < t(\mathrm{I})$
5. Calculate τ/n as

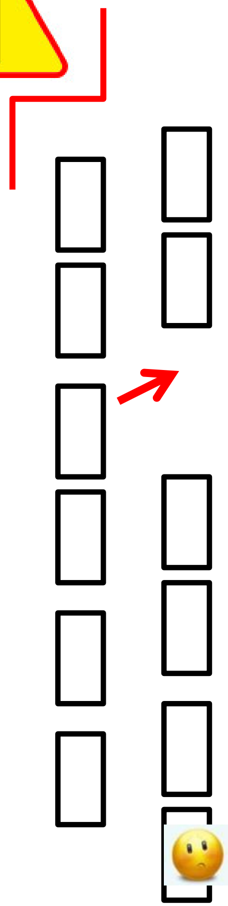
$$\frac{\tau}{n} = \frac{< t >}{1 - \dfrac{< t >}{t(I)}}$$

6. Separate out and count the cases when $\tau < 0$
7. Find the histogram $\#(\tau)$ for $\tau > 0$

Histogram of the evaluated waiting time



Two lines of cars



$P(n)$ – probability of n cars before me

$p=1-q$ – probability that a car enters to a moving hole.
This can happen $n+1$ times per a move

$$P_{t+1}(n) = P_t(n)(1-q)\sum_{k=0}^{n} q^k + P_t(n+1)q^{n+2}$$

entered to k-th hole

did not enter before

did not enter at all

We get

$$P_{t+1}(n) = P_t(n)(1-q^{n+1}) + P_t(n+1)q^{n+2}$$

A parallel, simpler problem:

$$P_{t+1}(n) = P_t(n)(1-q) + P_t(n+1)q$$

t=0	1	0	0	0
t=1	p	q	0	0
t=2	p^2	2pq	q^2	0
t=3	p^3	$3p^2q$	$3pq^2$	q^3
t=4	p^4	$4p^3q$	$6p^2q^2$	$4pq^3$
...				

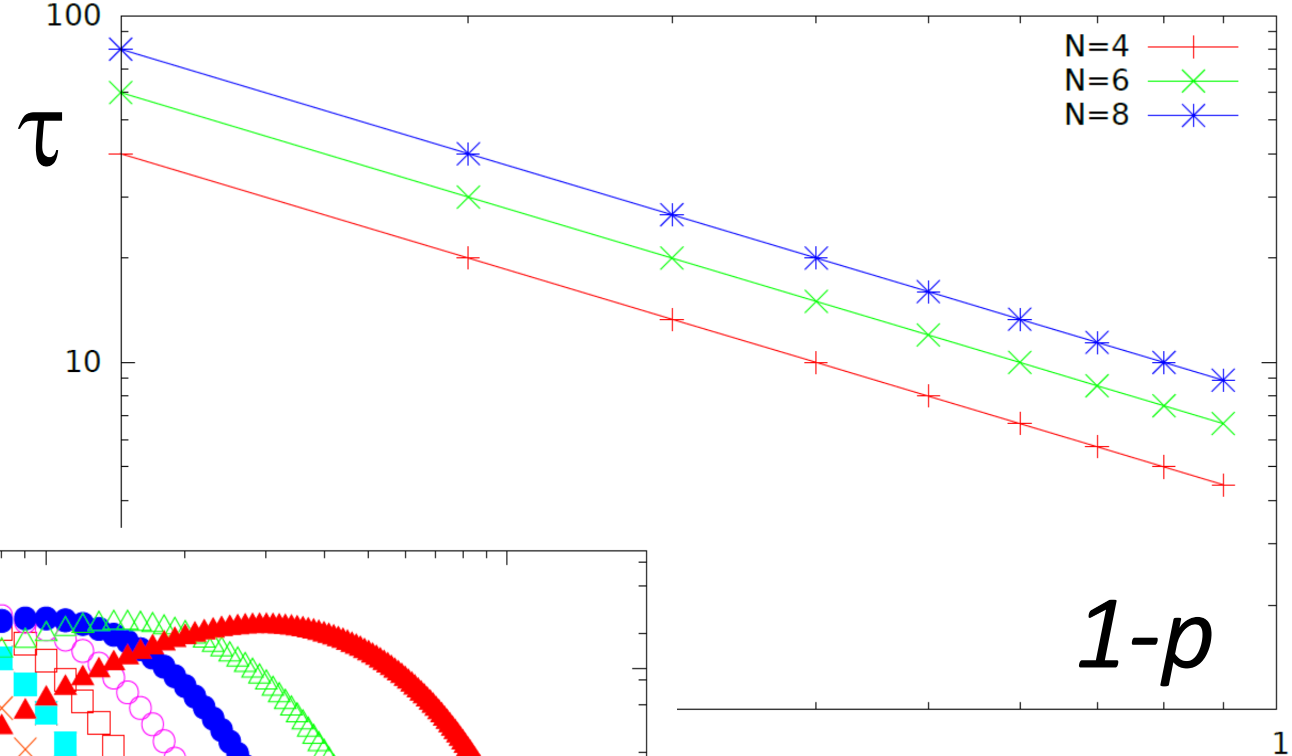
$\times p$ ↓ $\times q$

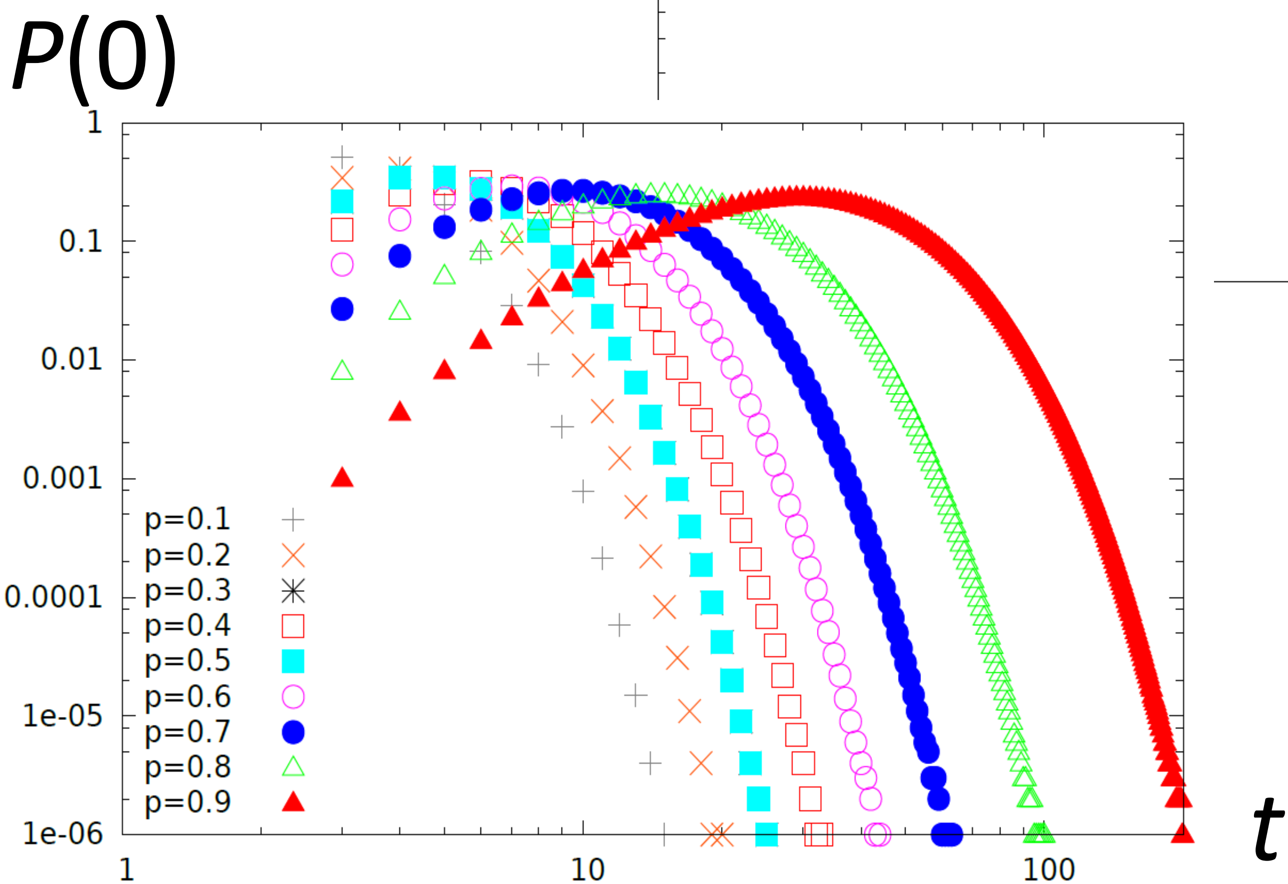
$$\binom{t}{k} p^{t-3} q^3$$

Hence

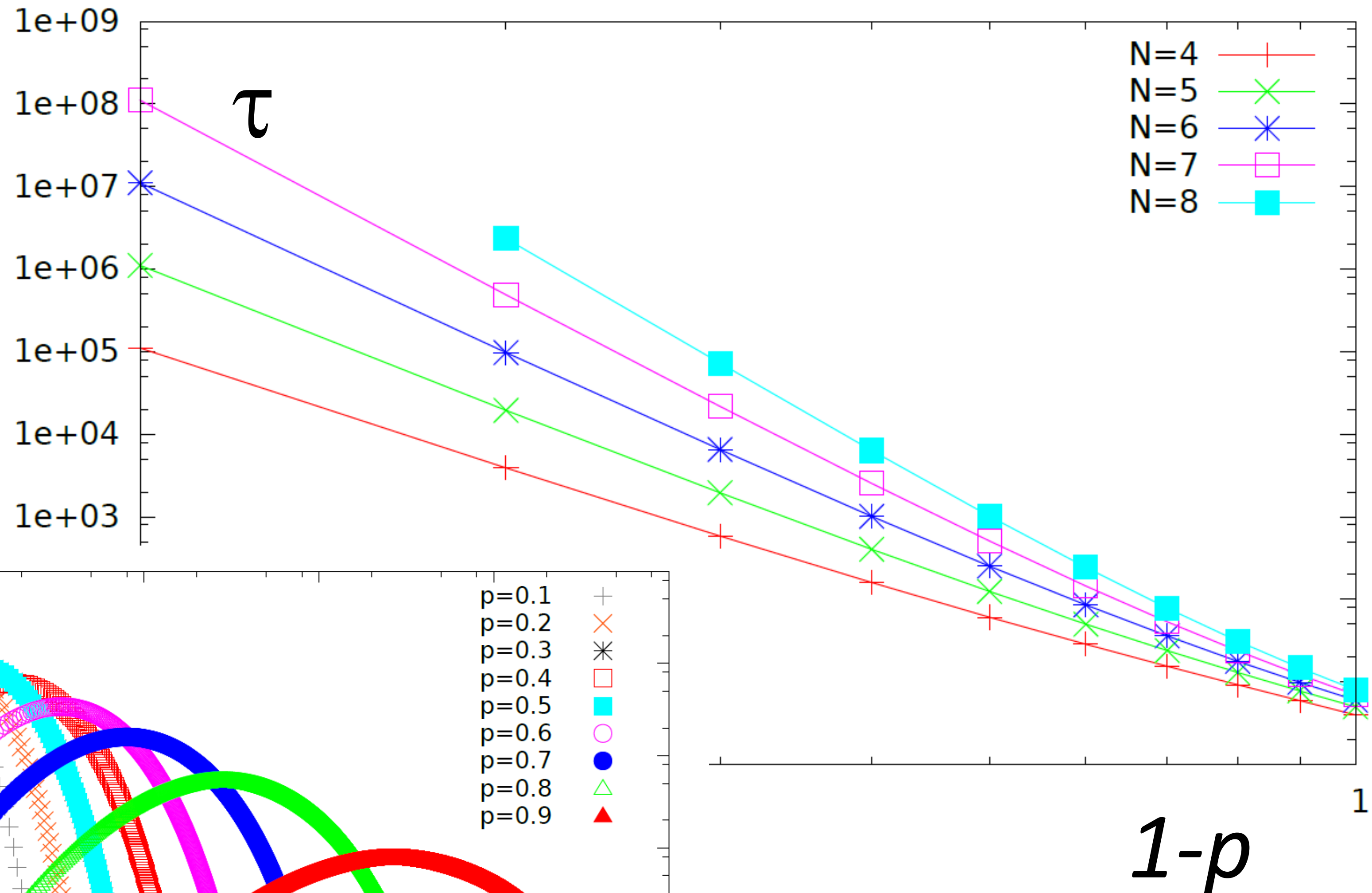
$$\tau = \frac{q^{n+1}}{n!} \sum_{k=n}^{\infty} \frac{(t+1)!}{(t-n)!} (1-q)^{t-n} = \frac{n}{q}$$

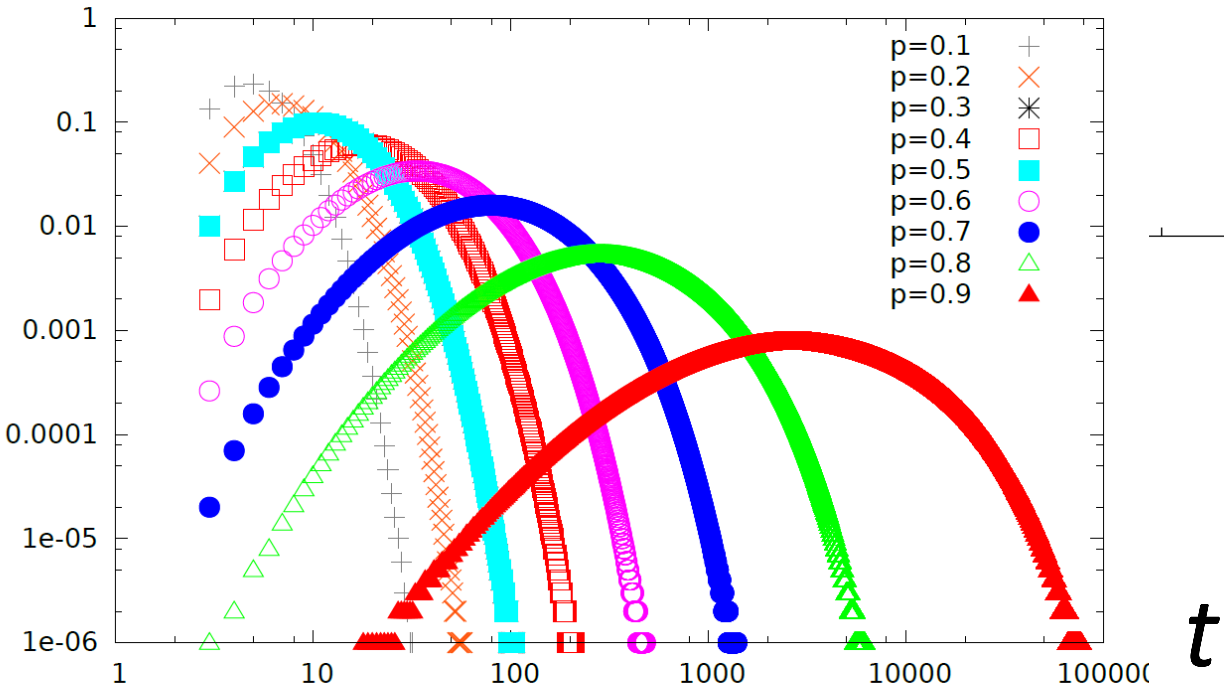
Yet the simpler problem:



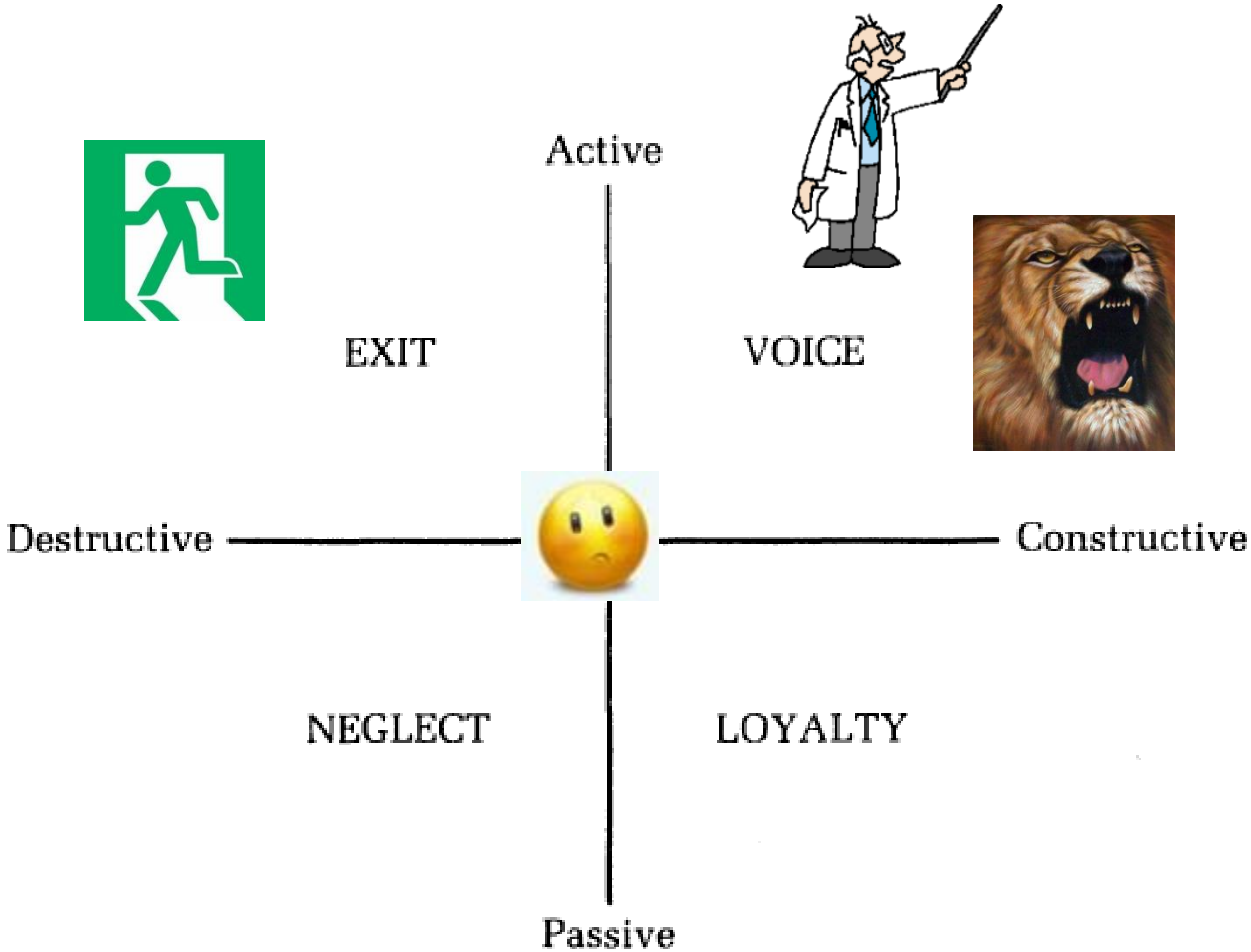


The actual problem:





What will be the reaction ?



[A. O. Hirschman,Exit, Voice, and Loyalty: Responses to Decline in Firms, Organizations and States, Harvard UP 1970/1995; C. E. Rusbult et al., Acad. of Management J. , 1988]

Conclusions

In a human queue, the distribution of perceived waiting time τ is scale-free

In a car queue, for small probability of a single intrusion p the mean waiting time τ varies with p approximately as

$$\tau=n(1-p)^{-a(n)}$$

Inference from limited statistics is deceptive. Mathematical formulations can provide a very pessimistic solution.

…

not the end yet!

Last conclusion: role of context again

Usually we do not believe such a pessimistic solution, because we think: „if it had looked like that, someone would have reacted a long time ago!”

or even: „more pessimistic people will leave and the queue will be shorter”

Beginners in business:
- see only positive examples
- illusion of control

when experienced:
- „if I had known …”

[D. M. De Carolis, P. Saparito, Entrepreneurship Theory and Practice, January 2006, 41-56]

Thank you