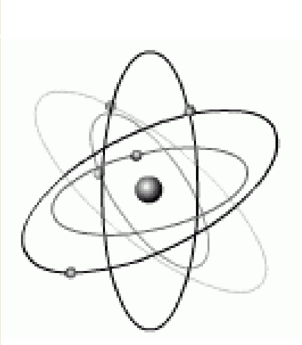
Looking for communities in networks



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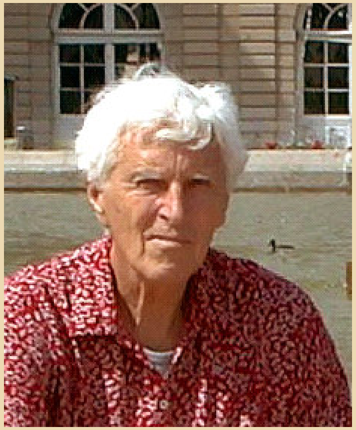
Overview

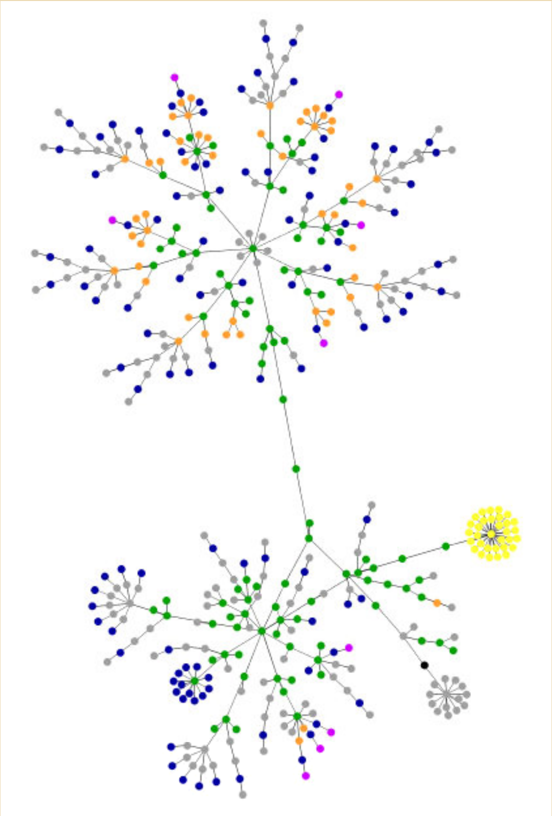
- The task
- A case study – the Southern Women
- The modularity after Newman
- A social problem – the Heider balance
- A continuous approach



Review:
S. Boccaletti et al.,
Complex networks: Structure and dynamics
Physics Reports 424 (2006) 175, section 7.1

The task

- Graph representation of a network



Jasmine's blog

N nodes

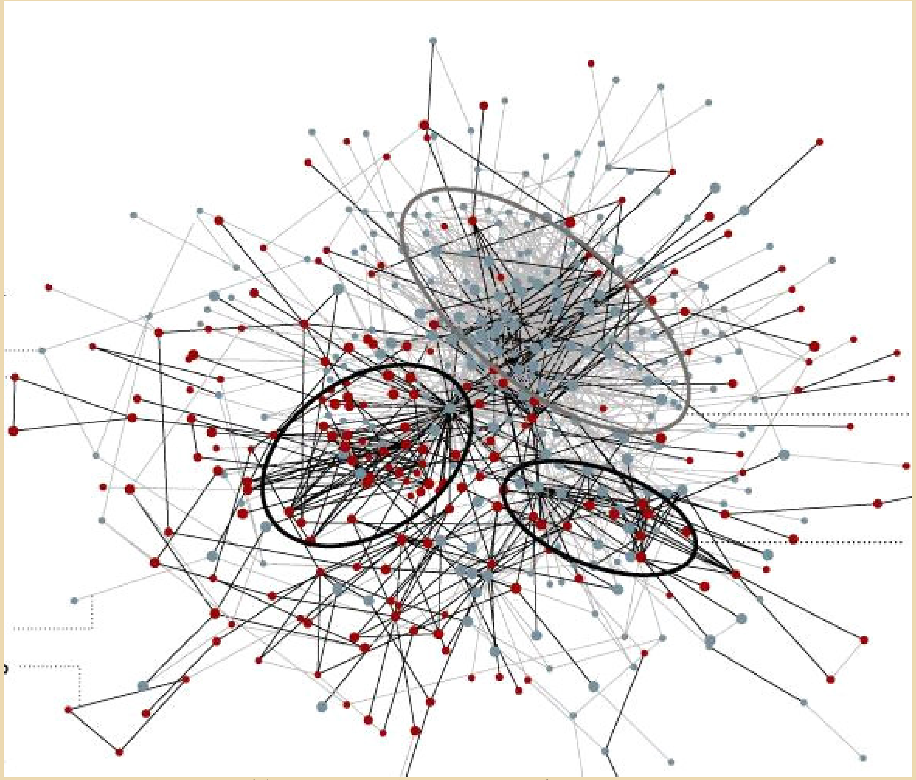
A(i,k)– connectivity matrix

A(i,k)= 0,1 or

A(i,k) ∈ $\mathcal{R}$ (weighted networks)

The task

- The communities:
  - subsets of nodes, internally connected more than with other parts of the network



Cornell > D.Constantine, NYT

Our aim is to find separated communities.

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The Southern Women, Natchez 1935

L. C. Freeman, *Finding social groups: a meta-analysis of the southern women data*, in Ronald Breiger, Kathleen Carley and Philippa Pattison (eds.) *Dynamic Social Network Modeling and Analysis.* Washington, D.C.:The National Academies Press, 2003.

Participation of the Southern Women in Events

Names of Participants of Group I	Code Numbers and Dates of Social Events Reported in *Old City Herald*													
	(1) 6/27	(2) 3/2	(3) 4/12	(4) 9/26	(5) 2/25	(6) 5/19	(7) 3/15	(8) 9/16	(9) 4/8	(10) 6/10	(11) 2/23	(12) 4/7	(13) 11/21	(14) 8/3
1. Mrs. Evelyn Jefferson	×	×	×	×	×	×		×	×					
2. Miss Laura Mandeville	×	×	×		×	×	×	×						
3. Miss Theresa Anderson		×	×	×	×	×	×	×	×					
4. Miss Brenda Rogers	×		×	×	×	×	×	×						
5. Miss Charlotte McDowd			×	×	×		×							
6. Miss Frances Anderson			×		×	×		×						
7. Miss Eleanor Nye					×	×	×	×						
8. Miss Pearl Oglethorpe						×		×	×					
9. Miss Ruth DeSand					×		×	×	×					
10. Miss Verne Sanderson							×	×	×			×		
11. Miss Myra Liddell								×	×	×		×		
12. Miss Katherine Rogers								×	×	×		×	×	×
13. Mrs. Sylvia Avondale							×	×	×	×		×	×	×
14. Mrs. Nora Fayette						×	×		×	×	×	×	×	×
15. Mrs. Helen Lloyd							×	×		×	×	×		
16. Mrs. Dorothy Murchison								×	×					
17. Mrs. Olivia Carleton									×		×			
18. Mrs. Flora Price									×		×			

Results of 21 methods

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	DGG41	W	W	W	W	W	W	W	W	WW	W	W	W	W	W	W	W	W	W
2	HOM50	W	W	W	W	W	W	W	WW			W	W	W	W	W		W	W
3	P&C72	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
4	BGR74	W	W	W	W	W	W	W		W	W	W	W	W	WW	WW		W	W
5	BBA75	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
6	BCH78	W	W	W	W	W	W				W	W	W	W	W	W			
7	DOR79	W	W	W	W	W	W	W		W	W	W	W	W	W	W			
8	BCH91	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
9	FRE92	W	W	W	W	W	W	W		W	W	W	W	W	W	W	W		
10	E&B93	W	W	W	W	W	W	W		W	W	W	W	W	W	W			
11	FR193	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
12	FR293	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
13	FW193	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	WW	W	W
14	FW293	W	W	W	W	W	W	W		W	W	W	W	W	W	W		W	W
15	BE197	W	W	W	W	W	W	W		W	W	W	W	W	W	W			
16	BE297	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
17	BE397	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
18	S&F99	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W		W	W
19	ROB00	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
20	OSB00	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W
21	NEW01	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W

Comparison of 21 methods

	Code	Analysis	Closeness to the Matching Criterion
1	DGG41	Davis, Gardner and Gardner, Ethnography	0.920
2	HOM50	Homans, Intuition	0.854
3	P&C72	Phillips and Conviser, Information Theory	0.968
4	BGR74	Breiger, Algebra	0.933
5	BBA75	Breiger, Boorman and Arabie, CONCOR	0.927
6	BCH78	Bonacich, Boolean Algebra	0.841
7	DOR79	Doreian, Algebraic Topology	0.923
8	BCH91	Bonacich, Correspondence Analysis	0.968
9	FRE92	Freeman, G-Transitivity	0.926
10	E&B93	Everett and Borgatti, Regular Coloring	0.916
11	FR193	Freeman, Genetic Algorithm 1	0.968
12	FR293	Freeman, Genetic Algorithm 2	0.842
13	FW193	Freeman and White, Galois Lattice	0.917
14	FW293	Freeman and White, Galois Sub-Lattice	0.954
15	BE197	Borgatti and Everett, Bi-Clique	0.916
16	BE297	Borgatti and Everett, Taboo Search	0.968
17	BE397	Borgatti and Everett, Genetic Algorithm	0.968
18	S&F99	Skvoretz and Faust, p* Model	0.957
19	ROB00	Roberts, SVD with Normalization	0.968
20	OSB00	Osbourn, VERI Algorithm	0.543
21	NEW01	Newman, Weighted Co-Attendance	0.932

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Basic tool - the modularity

$$Q = \frac{1}{w} \sum_{ij} \left(A(i,j) - \frac{k_i k_j}{w} \right) \delta(c_i, c_j)$$

where

$$w = \sum_{ij} A(i,j)$$

$$k_i = \sum_{j} A(i,j)$$

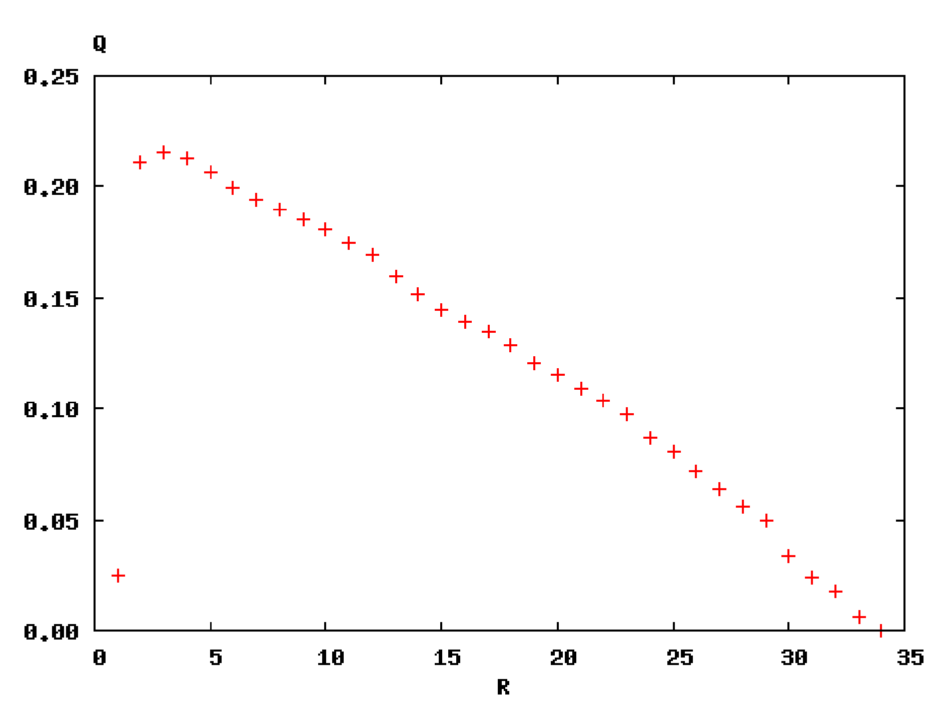
[M. E. J. Newman, *Analysis of weighted networks*, Phys. Rev. E 70, 056131 (2004)]

Special cases

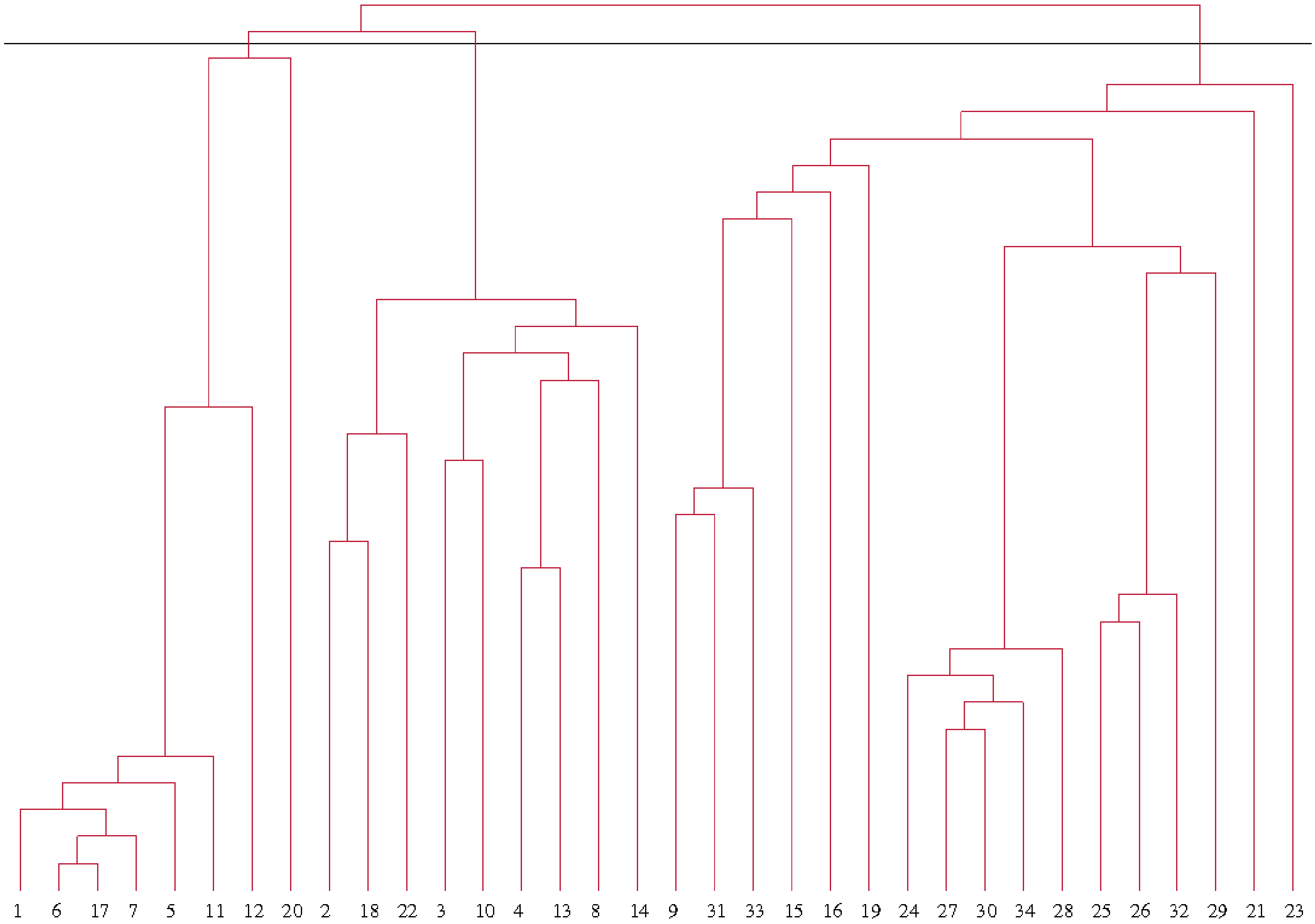
- One fully connected cluster of N nodes plus one separated node: $Q = 1/N$. When the node is added to the cluster, $Q=0$.
- R separated fully connected clusters, each of N nodes: $Q=1-(N-1)/(RN) \approx 1$ if $R>>1$. When one fully connected cluster is formed, $Q=1/(RN) \approx 0$.

Algorithm

- For a given set of data (N, $\{A(i,k)\}$) we have to construct a new connectivity matrix $\{c_{ik}=0,1\}$, which reveals the communities.
- start from $c_{ik}=0$ for all i,k.
- find the pair (n,m) such that when we put $c_{nm}=1$, Q increases most.
- repeat the search of such pairs, until Q starts to decrease. We accept the division where Q has a maximum.



Results – a dendrogram



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The Heider balance

- Let us transform the connectivity matrix as:

  $1 \rightarrow \ 1$ (friendly relation)

  $0 \rightarrow -1$ (hostile relation)

- Nodes = group members.
- In the balanced state, some rules are obeyed:

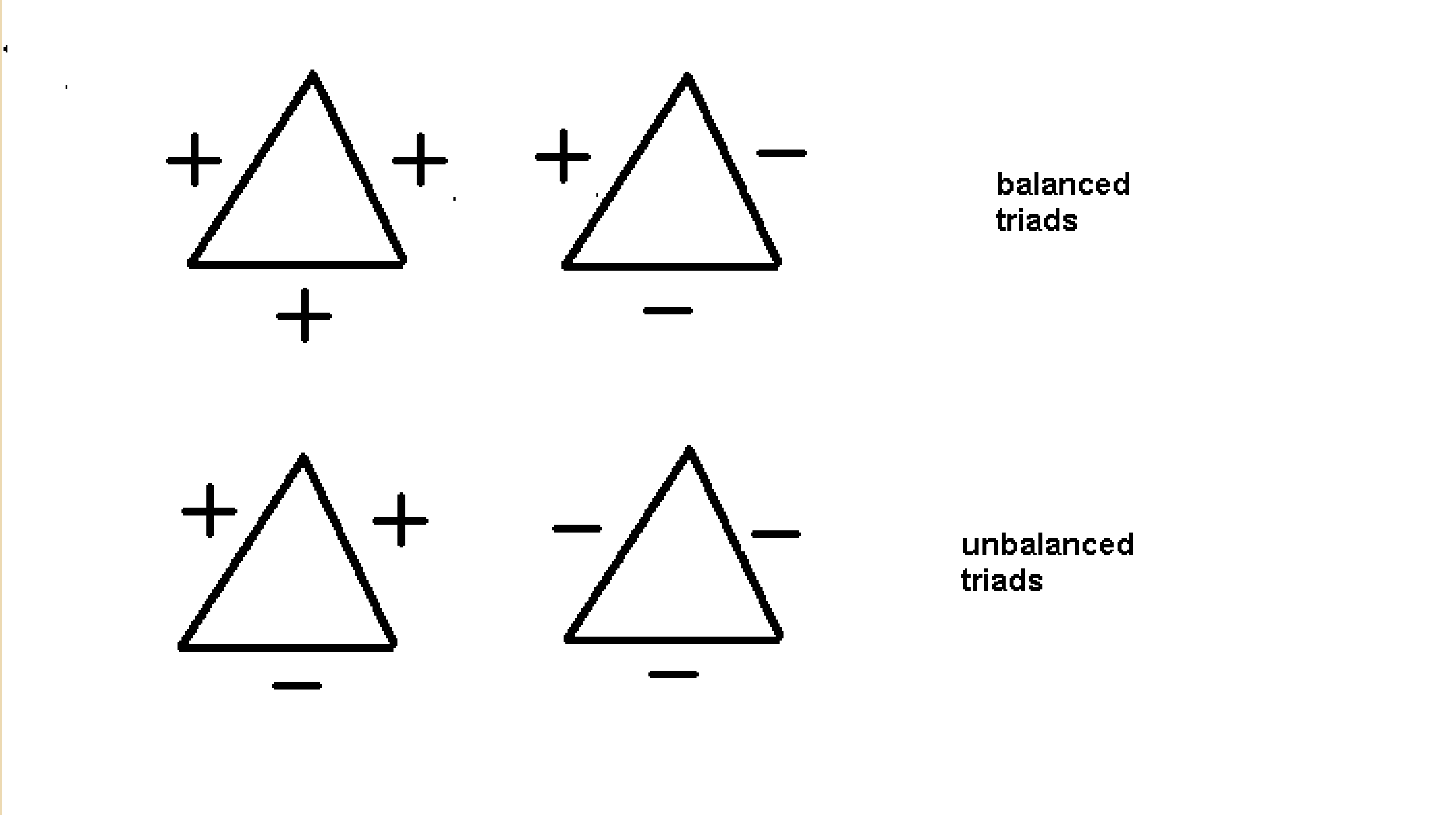
  Friend of my friend is my friend $A(i,k)>0 \ , \ A(k,n)>0 \ \Rightarrow A(i,n)>0$

  Enemy of my friend is my enemy $A(i,k)<0 \ , \ A(k,n)>0 \ \Rightarrow A(i,n)<0$

  Friend of my enemy is my enemy $A(i,k)>0 \ , \ A(k,n)<0 \ \Rightarrow A(i,n)<0$

  Enemy of my enemy is my friend $A(i,k)<0 \ , \ A(k,n)<0 \ \Rightarrow A(i,n)>0$

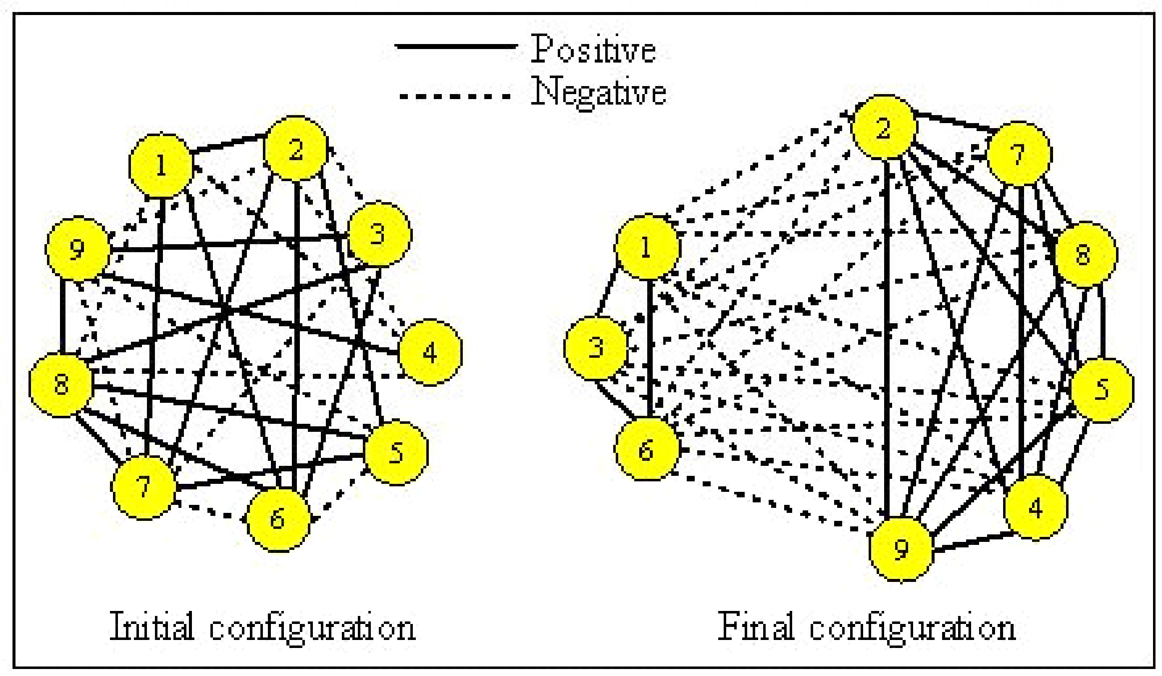
- If the state is not balanced, some members suffer the cognitive dissonance.



•They try to get the balanced state, changing some of their relations.

The Harary theorem

- At the balanced state, the group is divided into two mutually hostile subgroups. The relations within both subgroups are friendly.



[Z. Wang, W. Thorngate, *Sentiment and social mitosis: Implications of Heider's Balance Theory*, J. of Artificial Society and Social Simulation vol. 6, no. 3 (2003)]

Problems with the dynamics

- Sociological:

  Relations are more subtle than just friendly or hostile; more possibilities should be included.

- Computational:

  The final balanced state can depend on the order of modified relations.

A continuous dynamics

- The relations are described by real numbers $A(i,k) \in (-R,R)$
- Their time evolution is determined by differential equations

$$\frac{dA(i,k)}{dt} = \left[1 - \frac{A(i,k)^2}{R^2}\right] \sum_n A(i,n)A(n,k)$$

The results agree with the best six methods, reported by Freeman.

[More details in KK, *Some recent attempts to simulate the Heider balance problem*, Computing in Science and Engineering, July/August 2007, p.80.]

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Looking for communities in general– - the continuous approach

- In the Heider balance problem, the differential equations were motivated by the specific nature of the cognitive dissonance.
- In a general approach, for each pair we ask: do other connections confirm that this pair belongs to the same set?

The continuous approach

- The differential equations are

$$\frac{dA(i,k)}{dt} = \Theta(A(i,k))\Theta(1-A(i,k))\sum_{n}[A(i,n)A(n,k)-b]$$

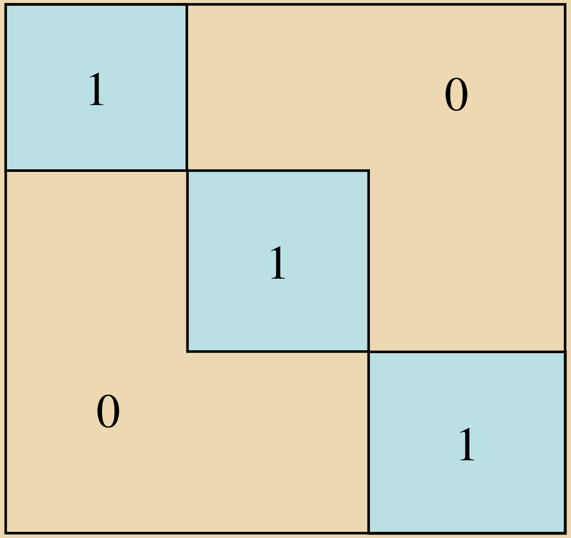
The product of the step functions ensures that once $A(i,k)$=0 or 1, it does not change any more.

Once the product $A(i,n)A(n,k)>b$, the connection of sites *i,k* via site *n* proves that *i,n* belong to the same cluster and so do *n,k*. Then *A(i.k)* should increase.

(more details in M. Krawczyk and K. K., arXiv:0709.0923)

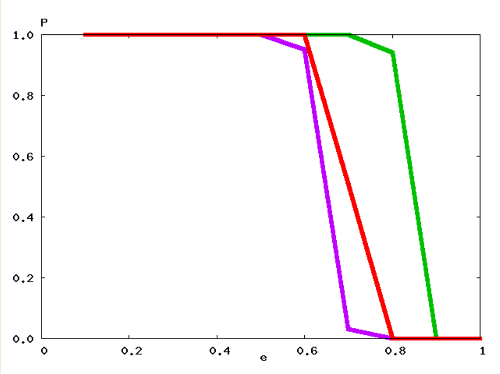
The continuous approach – a test

- For separated and fully connected clusters, the connectivity matrix has a block form

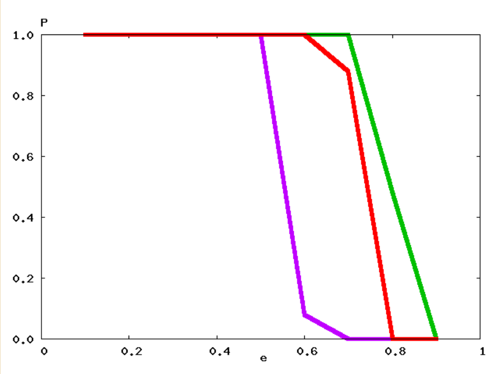


We add a noise: random numbers with the uniform distribution (0,*e*) are added to zeros and subtracted from units. The task is to reproduce the original structure.

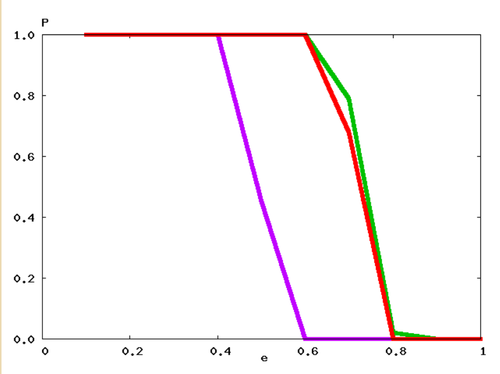
The continuous approach – a test



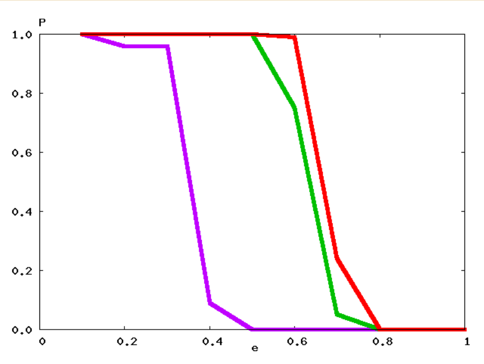
R=2



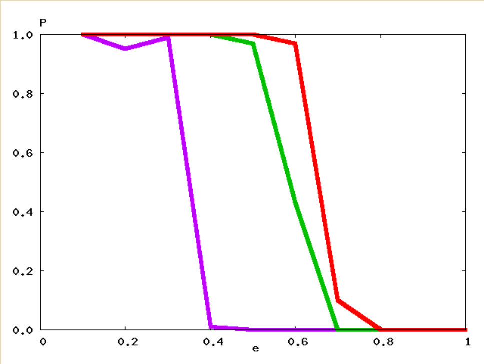
R=3



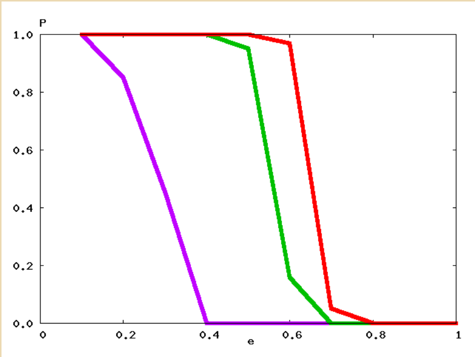
R=4



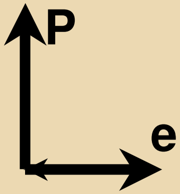
R=6



R=7



R=8



P – probability that the initial structure is reproduced
e – upper limit of noise

NR=60÷72, #=100, b=1/4, — modularity — random walk — continuous

Final remarks

- The method is deterministic. It produces some results even for a purely random graph.

- Direct comparisons with social experiments (Southern women, Zachary karate club) can be dubious.

Thank you

Control questions

- To calculate the modularity, one takes into account:
  A. Only existing connections
  B. Only a proposed division into clusters
  C. Both A and B

- Differential equations raise difficulties because
  A. high numerical precision is needed
  B. the order of modified relations does matter