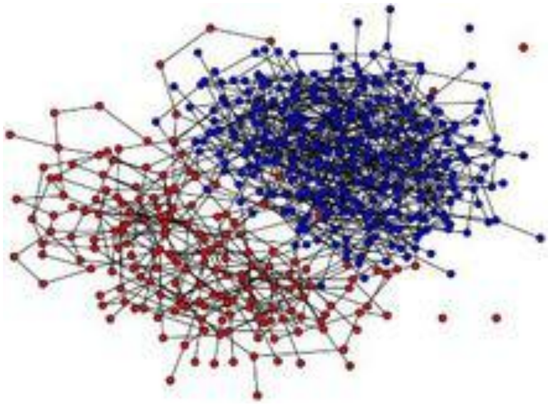
Coevolving voter model on a network – coupling of order parameters



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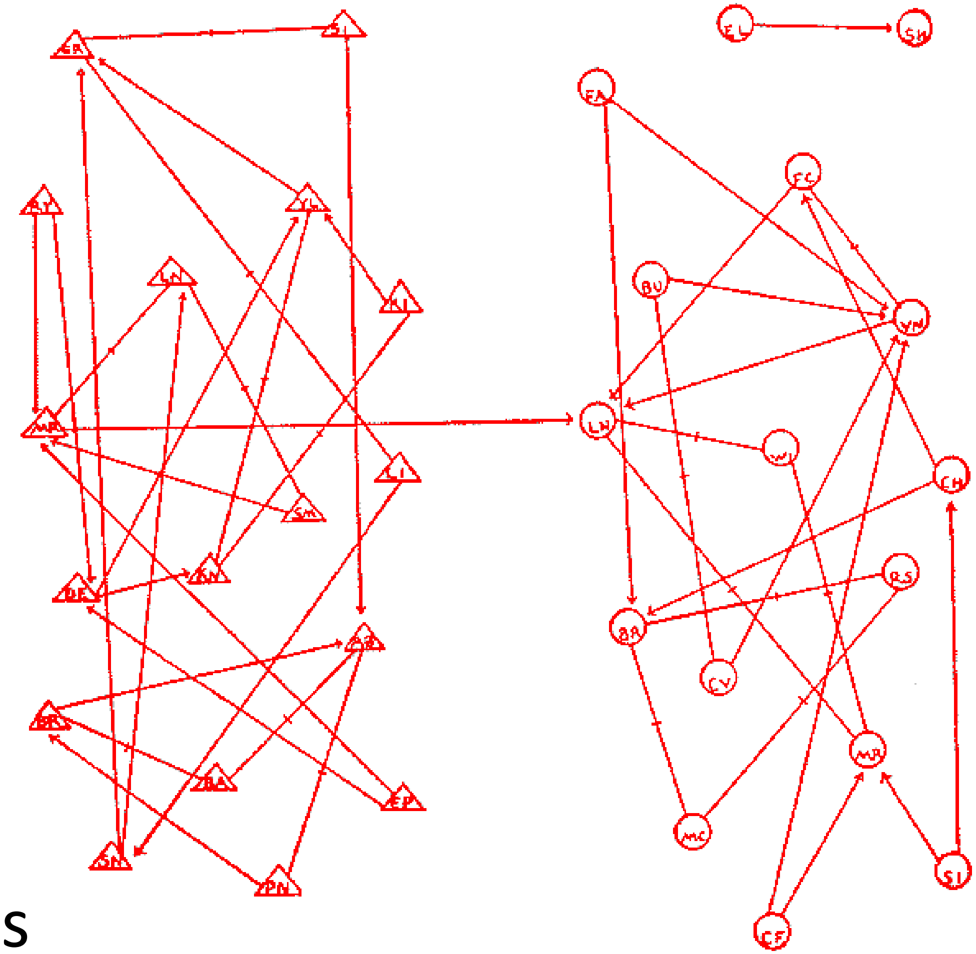
http://www.pnas.org/content/109/10/3682.full.pdf

Wrocław UST, 4.12.2017

outline

* A quote from Jacob Moreno
* Sociometric tests
* What are social networks like
* Homophily and social contagion
* Model evolution : single events
* Equations of motion after Vazquez et al
* More detailed equations of motion
* Analytical *vs* numerical results
* Summary

Jacob L. Moreno (1879-1974)



1928-1933: first sociograms

- Sing Sing prison,
- a reformatory for deliquent girls,
- public and private Brooklyn schools.

https://archive.org/stream/whoshallsurviven00jlmo#page/38/mode/2up
www.slate.com/articles/technology/future_tense/2014/10/j_l_moreno_a_psychologist_s_30s_experiments_invented_social_networking.html

…within a given more or less homogeneous group
members of an alien group may be introduced (…)
The group can assimilate, as it were, a certain number,
but beyond that point
assimilation is rendered difficult or impossible
and the group tends to break up
along the lines of cleavage created by the alien group
forming a minority group within the majority group.

[J. L. Moreno, Who shall survive? A new approach to the problem of human interrelations, Washington D.C., 1934]

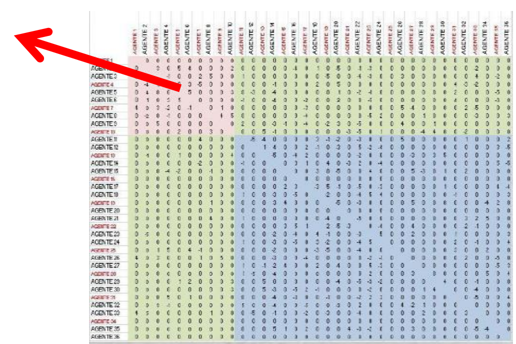
https://archive.org/stream/whoshallsurviven00jlmo#page/2/mode/2up

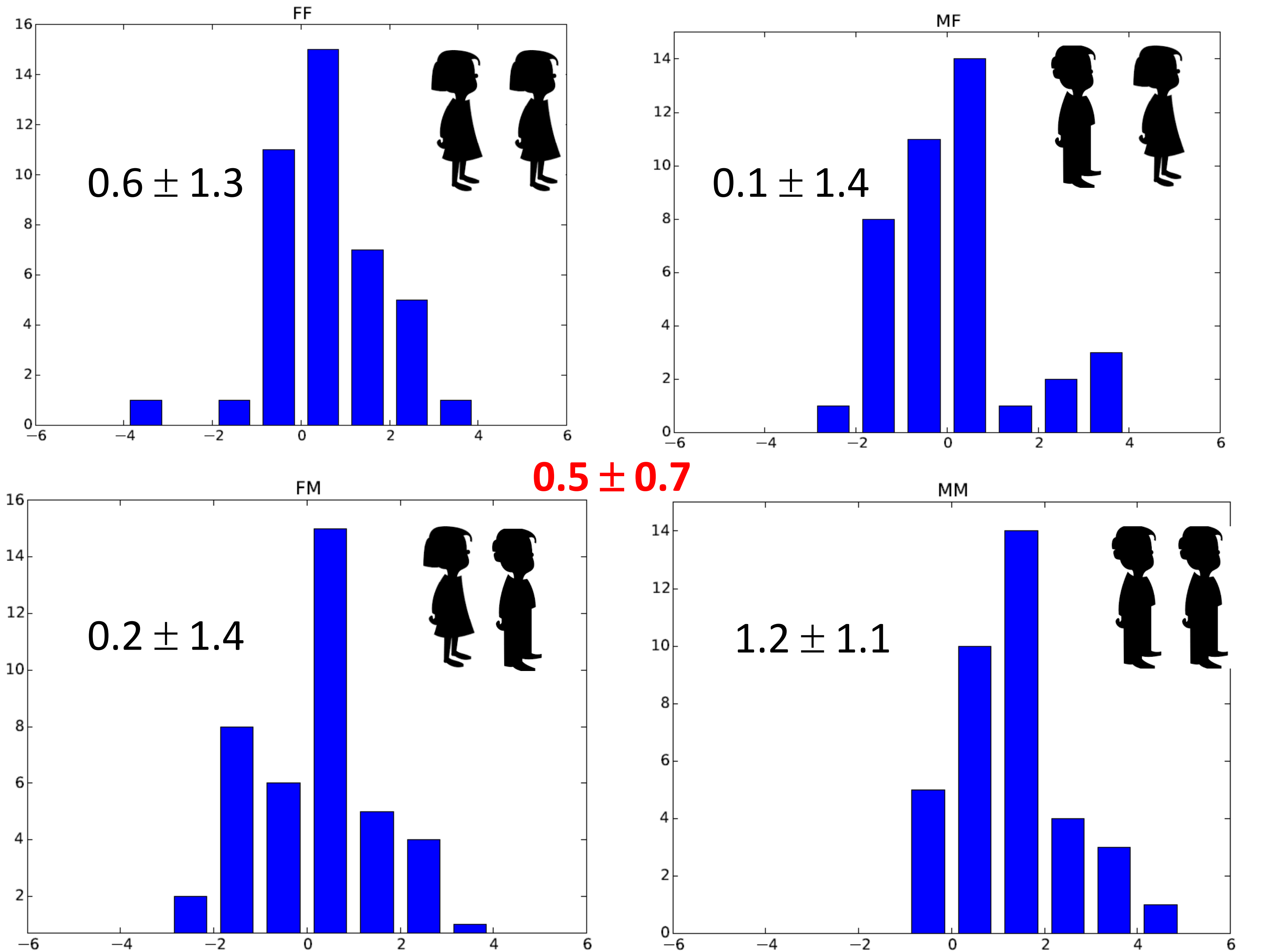
An example of a sociometric test : data from 37 Mexican school classes

1. Select 5 persons whom you like most, and assign them points from 1 to 5
2. Select 5 persons whom you dislike most, and assign them points from -5 to -1

example: girls on girls

			4	0	3	5			0	0	0	0	2	1
		0		-4		-3	5	0	0	0	0	-2	0	0
4	-2	0		0	-5		0	0	-3	0	5	-4	1	0
0	0		0		-5	0	0	3	0	4	-4			5
5			4	0		3	-2	0		0			2	1
5	0	1	0	3	0		-5	0	-2	0	0	-3	2	4
	5			0		-5		0		0	4	0		2
0			2	3		0	0		-5	4	0	-4	0	5
3	0		0	-4	5	1	0	-3		0	4	0	2	-1
	0	0		4	0	1	0	3	-4			0	0	5
2	0	0	5	-1	4	0		-2	3	0		0	1	0
-4	0	0	0	0	5		3		-5		4		0	0
		5	2	0		3		0		0		-4		0
		0	0	5		2	0	4	-5	3		-2	0	

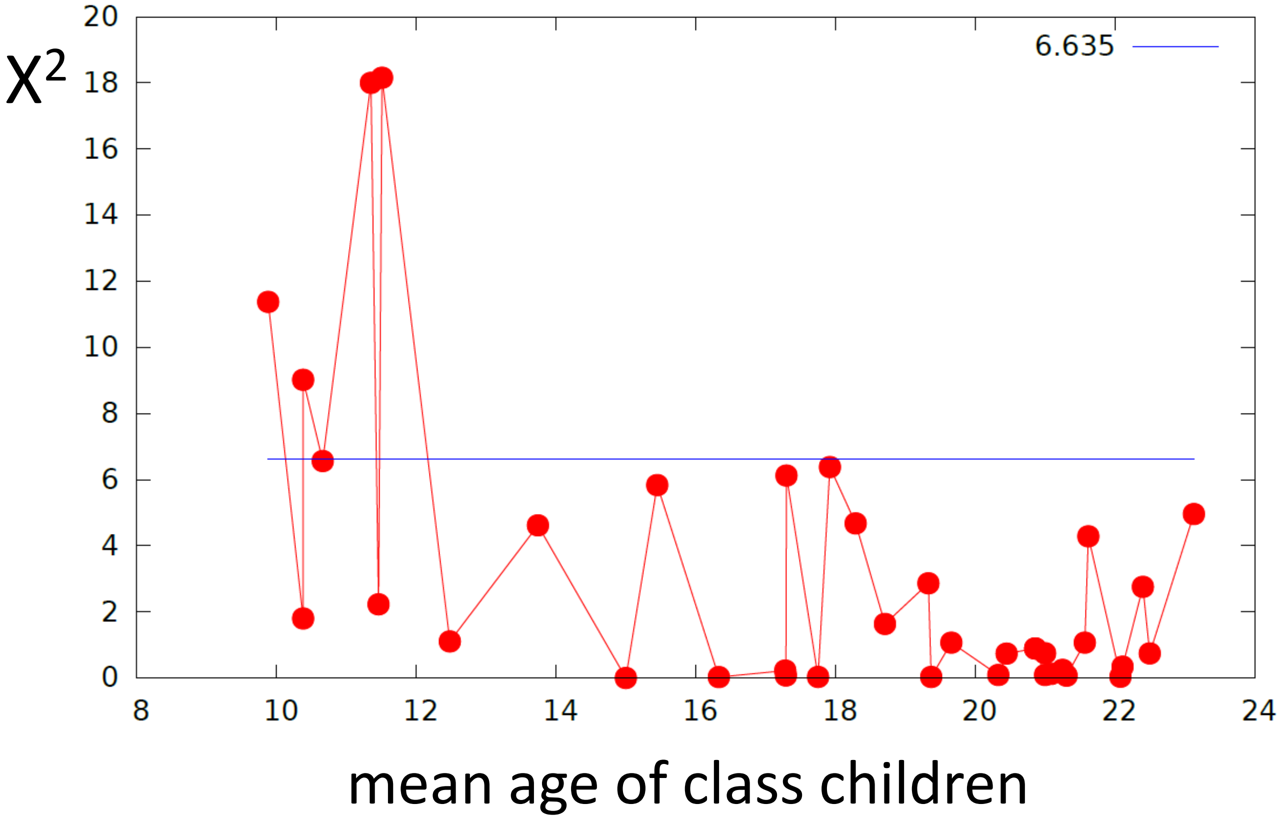



FF
0.6 ± 1.3
MF
0.1 ± 1.4
0.5 ± 0.7
FM
0.2 ± 1.4
MM
1.2 ± 1.1

A test of statistical significance

$$X^2 = \frac{(k+m)(k_1 m_2 - k_2 m_1)^2}{km(k_1+m_1)(k_2+m_2)} > 6{,}635 \quad ?$$

for statistical significance 0,99
(one degree of freedom)



[Krawczyk et al., Phys A 2017 (arXiv:1505.02539)]

[N. Mantel, J. of the Amer. Statistical Assoc. 58, 690 (1963);
S. Brandt, Statistical and Computational Methods in Data Analysis, Springer, 1997, Table I.7]

Top Ten Social Networking Websites

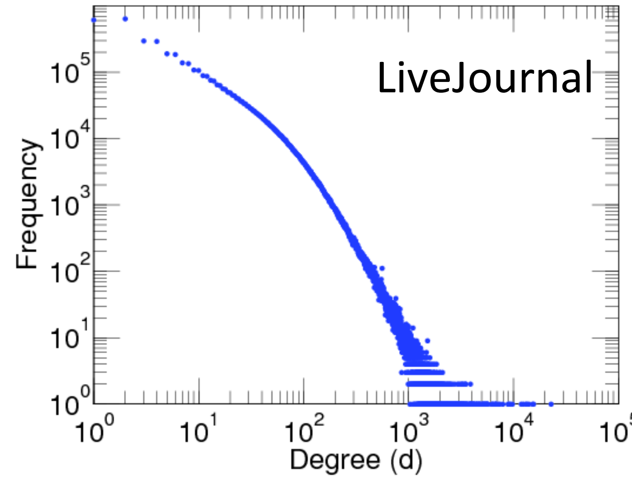
World social networking map shows the popularity of social networking websites across the world.
The map is based on the data collected using Alexa and Google trends for Website traffic.

www.mapsofworld.com/world-top-ten/social-networking-websites-popularity-map.html

Social networks are supposed to be:

- heterogeneous
- clustered
- assortative

...



Network	C
Web [2]	0.081
Flickr	0.313
LiveJournal	0.330
Orkut	0.171
YouTube	0.136

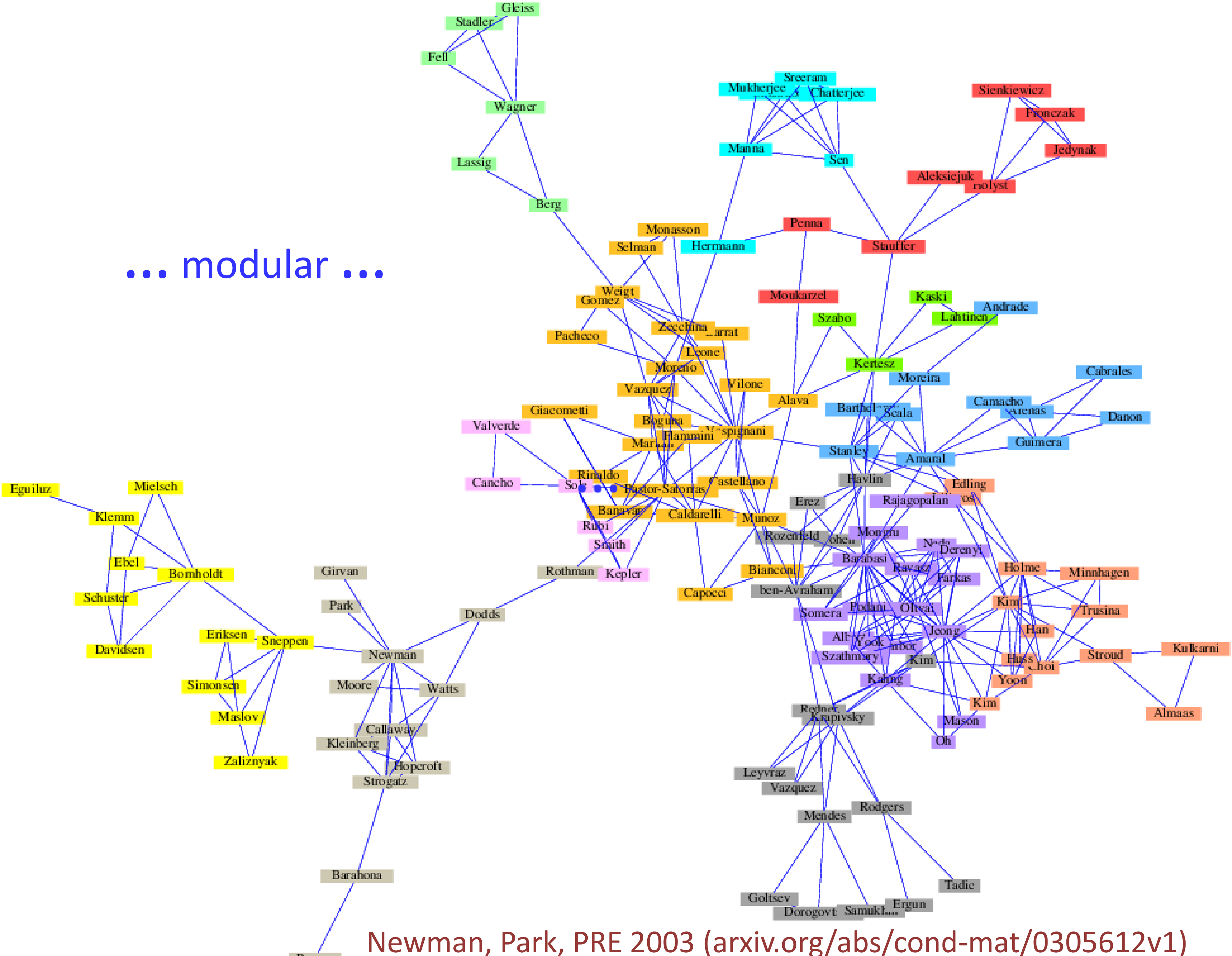
Newman, SIAM Rev 2003 (arXiv:cond-mat/0303516)

http://konect.uni-koblenz.de/networks/soc-LiveJournal1

	network	n	r
real-world networks	physics coauthorship[a]	52 909	0.363
	biology coauthorship[a]	1 520 251	0.127
	mathematics coauthorship[b]	253 339	0.120
	film actor collaborations[c]	449 913	0.208
	company directors[d]	7 673	0.276
	Internet[e]	10 697	−0.189
	World-Wide Web[f]	269 504	−0.065
	protein interactions[g]	2 115	−0.156
	neural network[h]	307	−0.163
	food web[i]	92	−0.276

Newman, PRL 2002 (arxiv.org/pdf/cond-mat/0205405.pdf)

... modular ...



Newman, Park, PRE 2003 (arxiv.org/abs/cond-mat/0305612v1)

homophily : similarity breeds connection
social contagion : connection breeds similarity

Do people befriend others who are similar to them, or do they become more similar to their friends over time?

movies, music → homophily

Exceptions: classical/jazz → contagion
indie/alternative → anti-contagion

[Lewis et al., PNAS 109 (2012)]

		women			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

Newman, SIAM Rev 2003 (arXiv:cond-mat/0303516)

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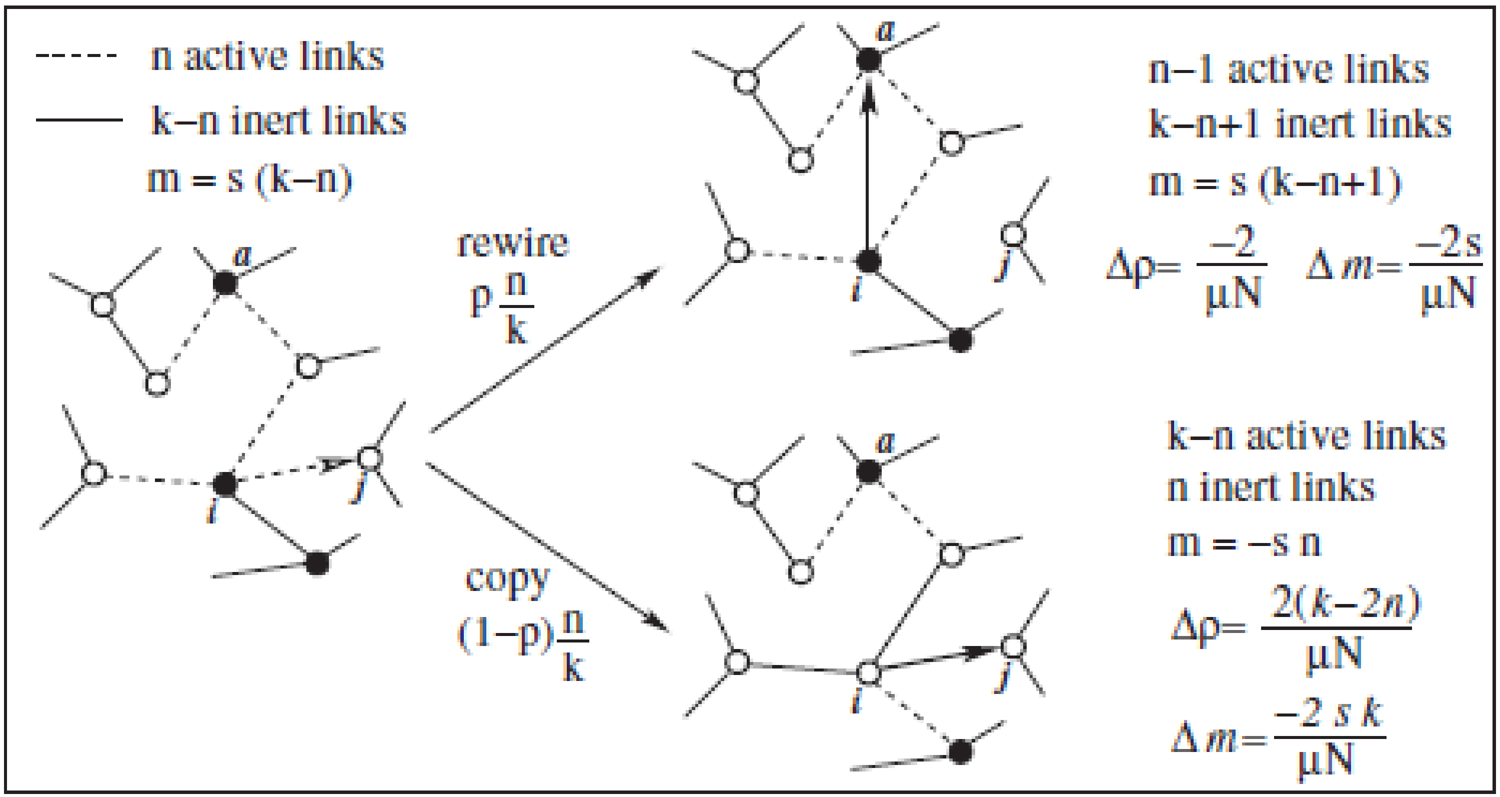
Coevolving voter model

The model system is a network of nodes decorated with spins. Two processes compete here: rewiring (->homophily) and flips (->contagion).



[F. Vazquez, V. M. Eguíluz, M. San Miguel, PRL 100 (2008) 108702]

Equations of motion after Vazquez et al.

density of active links

flip

prob of *n* active out of *k* links

$$\frac{d\rho}{dt} = N(1-p)\sum_k P(k)\sum_{n=0}^{k} B(n/k)\,\frac{n}{k}\,\frac{k-2n}{N\mu/2}$$

$$+\,Np\sum_k P(k)\sum_{n=0}^{k} B(n/k)\,\frac{n}{k}\left(\frac{-1}{N\mu/2}\right)$$

number of all links

rewire

[F. Vazquez, V. M. Eguiluz, M. San Miguel, PRL 100 (2008) 108702]

Equations of motion after Vazquez et al.

density of active links

flip

active links <-> frozen links

$$\frac{d\rho}{dt} = N(1-p)\sum_k P(k)\sum_{n=0}^{k} B(n/k)\frac{n}{k}\frac{k-2n}{N\mu/2}$$

$$+ Np\sum_k P(k)\sum_{n=0}^{k} B(n/k)\frac{n}{k}\left(\frac{-1}{N\mu/2}\right)$$

rewire

fraction of active links

active link -> frozen link

[F. Vazquez, V. M. Eguiluz, M. San Miguel, PRL 100 (2008) 108702]

Equations of motion after Vazquez et al.

For the binomial distribution $\sum_{n=0}^{k} nB(n/k) = k\rho$

and $\sum_{n=0}^{k} n^2 B(n/k) = k^2\rho^2 + k\rho(1-\rho)$

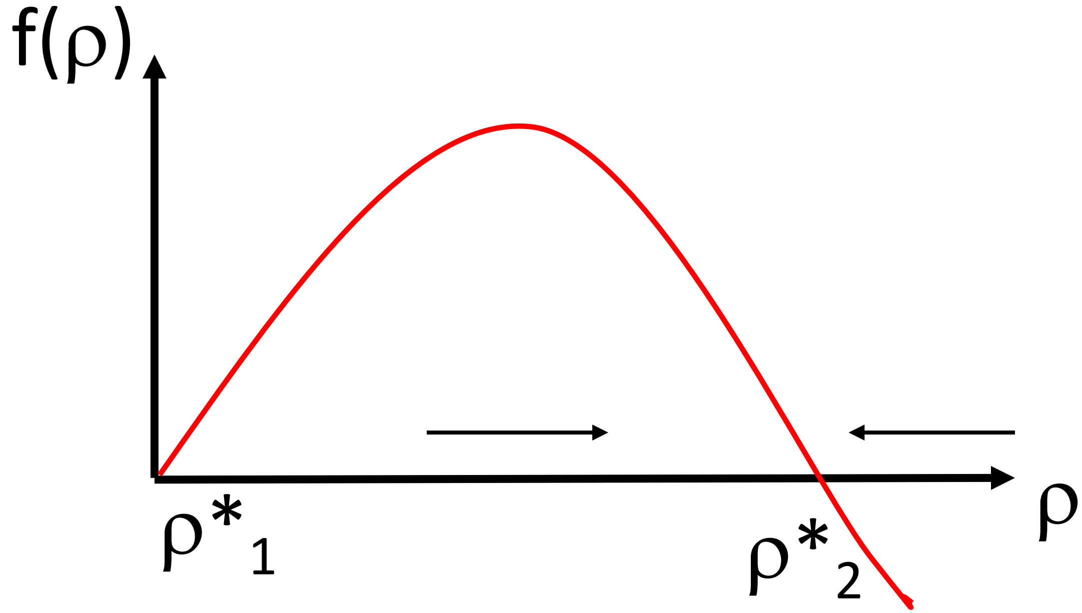
then $$\frac{d\rho}{dt} = \frac{2\rho}{\mu}[(1-p)(\mu-1)(1-2\rho)-1] \equiv f(\rho)$$

Fixed points: $f(\rho^*) = 0$, then $\rho^*_1 = 0$ (frozen phase)

or

$$\rho^*_2 = \frac{(1-p)(\mu-1)-1}{2(1-p)(\mu-1)}$$ (active phase)

[F. Vazquez, V. M. Eguiluz, M. San Miguel, PRL 100 (2008) 108702]



ρ^*_1 is unstable, ρ^*_2 exists and is stable, iff $\left.\frac{df(\rho)}{d\rho}\right|_{\rho=0} > 0$

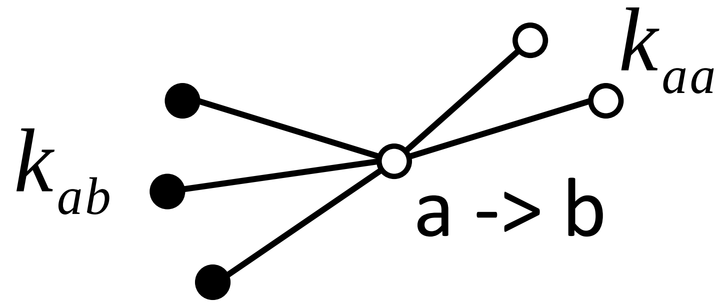
$$\left.\frac{df(\rho)}{d\rho}\right|_{\rho=0} = \frac{2}{\mu}[(1-p)(\mu-1)-1]$$

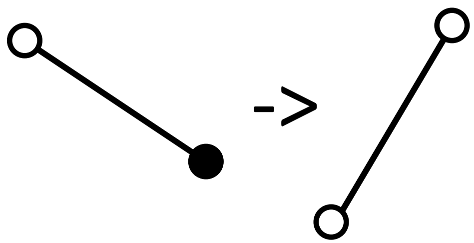
Hence, $\left.\frac{df(\rho)}{d\rho}\right|_{\rho=0} > 0$ for $p < p_c = \frac{\mu-2}{\mu-1}$

p is the probability of rewiring. If *p* is large enough, active links disappear.

[F. Vazquez, V. M. Eguiluz, M. San Miguel, PRL 100 (2008) 108702]

M_{ab} – number of links from a to b





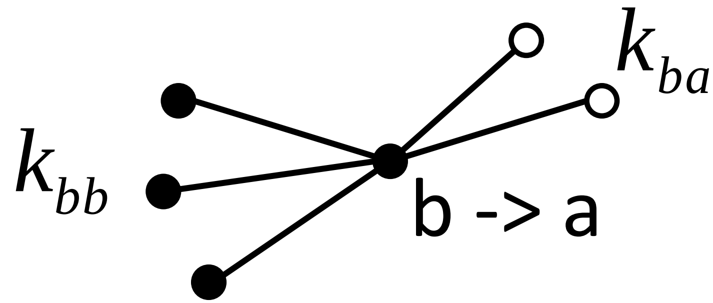
flip, probability *1-p*

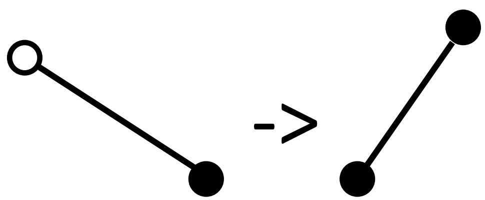
$$N_a \to N_a - 1$$
$$N_b \to N_b + 1$$
$$M_{aa} \to M_{aa} - 2k_{aa}$$
$$M_{ab} \to M_{ab} - k_{ab} + k_{aa}$$
$$M_{ba} \to M_{ba} - k_{ab} + k_{aa}$$
$$M_{bb} \to M_{bb} + 2k_{ab}$$

rewiring, probability *p*

$$M_{ab} \to M_{ab} - 1$$
$$M_{ba} \to M_{ba} - 1$$
$$M_{aa} \to M_{aa} + 2$$

M_{ba} – number of links from b to a





flip, probability *1-p*

$$N_a \rightarrow N_a + 1$$

$$N_b \rightarrow N_b - 1$$

$$M_{aa} \rightarrow M_{aa} + 2k_{ba}$$

$$M_{ab} \rightarrow M_{ab} + k_{bb} - k_{ba}$$

$$M_{ba} \rightarrow M_{ba} - k_{ba} + k_{bb}$$

$$M_{bb} \rightarrow M_{bb} - 2k_{bb}$$

rewiring, probability *p*

$$M_{ab} \rightarrow M_{ab} - 1$$

$$M_{ba} \rightarrow M_{ba} - 1$$

$$M_{bb} \rightarrow M_{bb} + 2$$

Parameterization: m, ρ, β, $\mu_\pm$

$$N_+ + N_- = N$$

$$N_+ - N_- = Nm$$

$$M_{+-} = M_{-+} = \frac{N\mu}{2}\rho$$

$$M_{+-} + M_{-+} + M_{++} + M_{--} = N\mu$$

$$M_{++} - M_{--} = N\mu\beta$$

hence

$$N_\pm = \frac{N}{2}(1 \pm m)$$

$$M_{++} = \frac{N\mu}{2}(1 - \rho + \beta)$$

$$M_{--} = \frac{N\mu}{2}(1 - \rho - \beta)$$

$\mu_\pm$ - mean degree of node ±

$$N_+\mu_+ = M_{++} + M_{+-}$$

$$N_-\mu_- = M_{--} + M_{-+}$$

hence

$$\mu_\pm = \mu\frac{1 \pm \beta}{1 \pm m}$$

Equations of motion

$$\frac{d\rho}{dt} = -p\frac{2\rho(1-\beta m)}{\mu(1-\beta^2)} + (1-p)\frac{2\rho(1-\beta^2-2\rho)}{(1-\beta^2)} +$$

$$-(1-p)\frac{4\rho}{\mu(1-\beta^2)}\left[(1-\beta m) - \rho\frac{1+\beta^2-2\beta m}{1-\beta^2}\right]$$

$$\frac{dm}{dt} = (1-p)\frac{2\rho(\beta-m)}{1-\beta^2}$$

$$\frac{d\beta}{dt} = p\frac{2\rho(m-\beta)}{\mu(1-\beta^2)}$$

What could seem a bit strange:

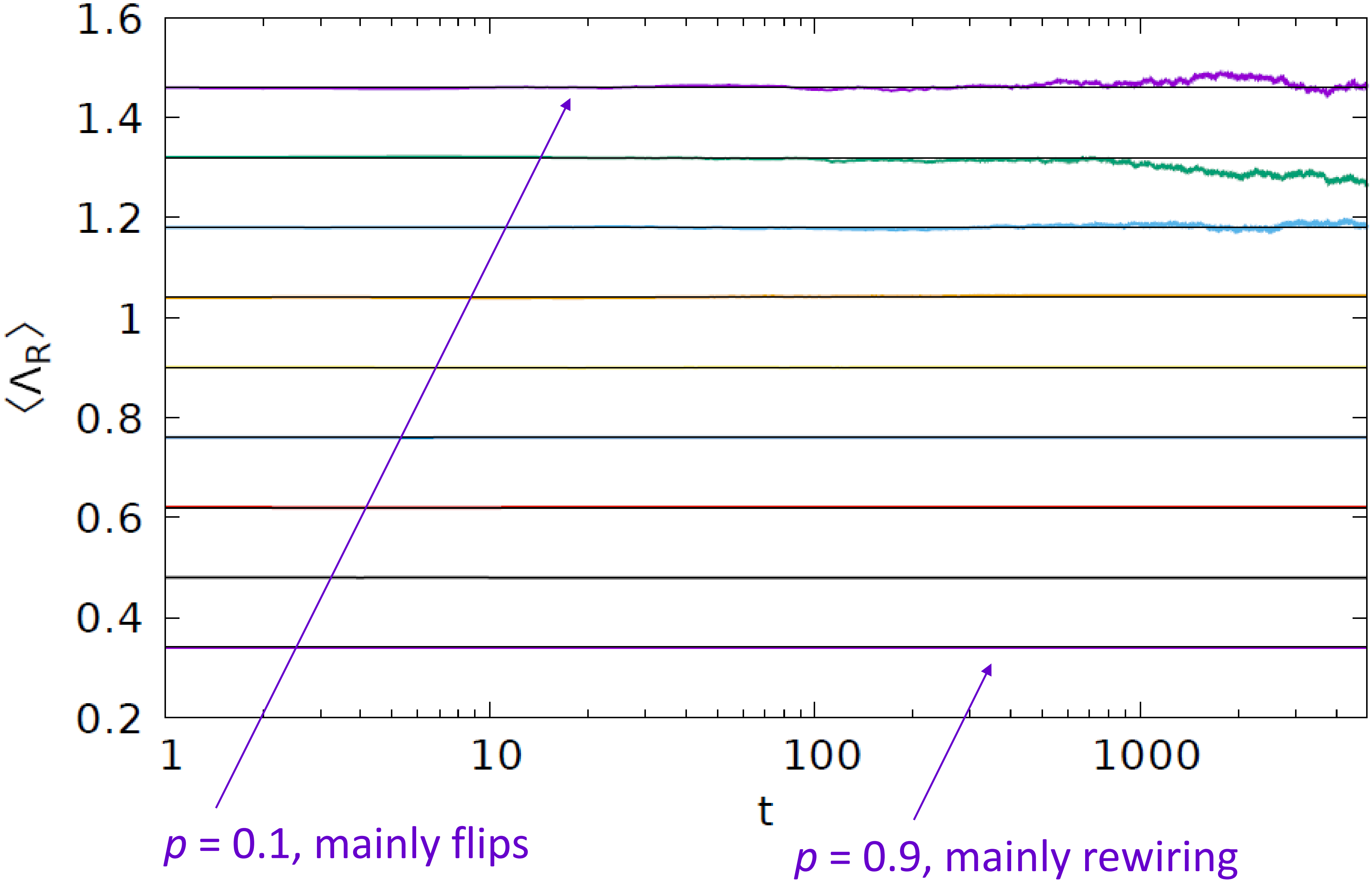
$$\Lambda_R \equiv (1-p)\mu\beta(t) + pm(t) = const$$

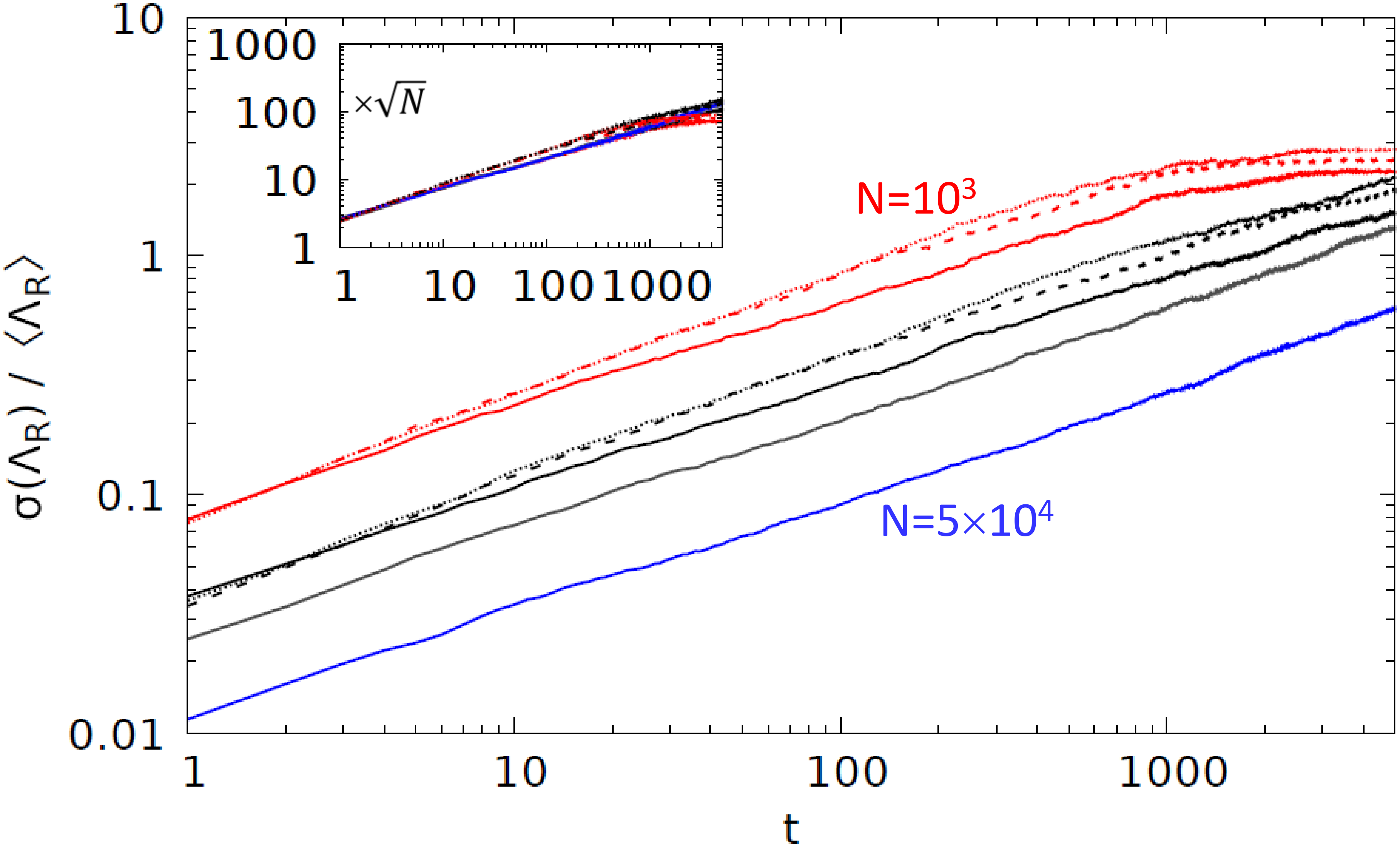
$p = 1$ only rewiring, m = const

$p = 0$ only flips, β = const

The final solution depends on the initial state.

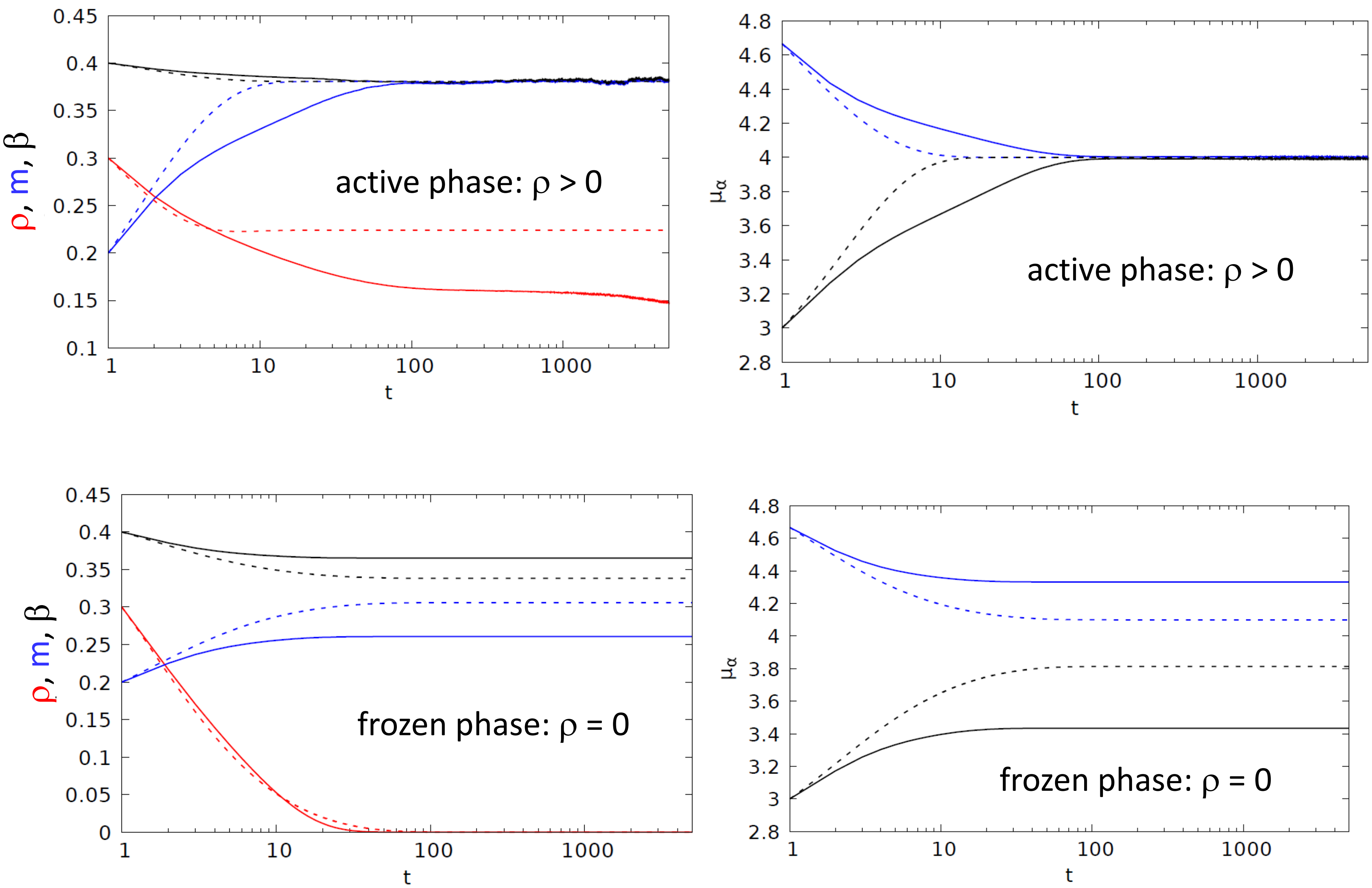
Numerical results: $N=5\times10^4$, $\#=10^3$





solid lines: μ = 4; dashed : μ = 20; dotted: μ = 40

Analytical (- - -) vs numerical (——) results



Summary

The simulations confirm the mean-field result Λ_R = const.

The model has been generalized for the case when
the mean numer of neighbors of a node depends on its spin.

Technical details in Toruniewska et al., PRE 96 (2017) 042306.

In terms of social networks, the numbers of acquaintances
can be different for members of different groups.

In the active phase, this difference disappears.
In the frozen phase the group is split ($\rho = 0$),
and the numbers of acquaintances remain different.


Thank you