A small chance of paradise – equivalence of balanced states

Małgorzata Krawczyk, Sebastian Kałużny and Krzysztof Kułakowski
AGH Kraków

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abstract

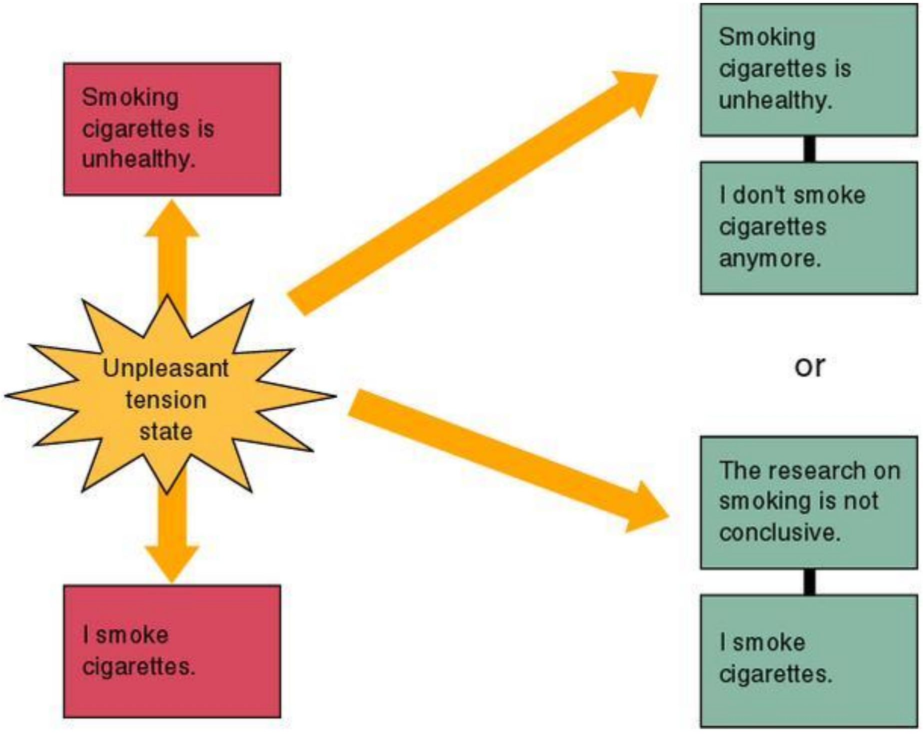
Two model dynamics are considered, continuous and discrete. Results of both indicate, that the state with all relations friendly – a paradise – is as (un)probable as any other balanced state ($P = 2^{1-N}$).

outline

* Cognitive dissonance and the Heider balance
* Continuous dynamics, N=3
* Discrete dynamics, $N \leq 7$
* Doubts & Conclusions

Removal of cognitive dissonance: examples

A. - Good morning.
B. - What you mean by „good morning”?
A. - Are you OK?



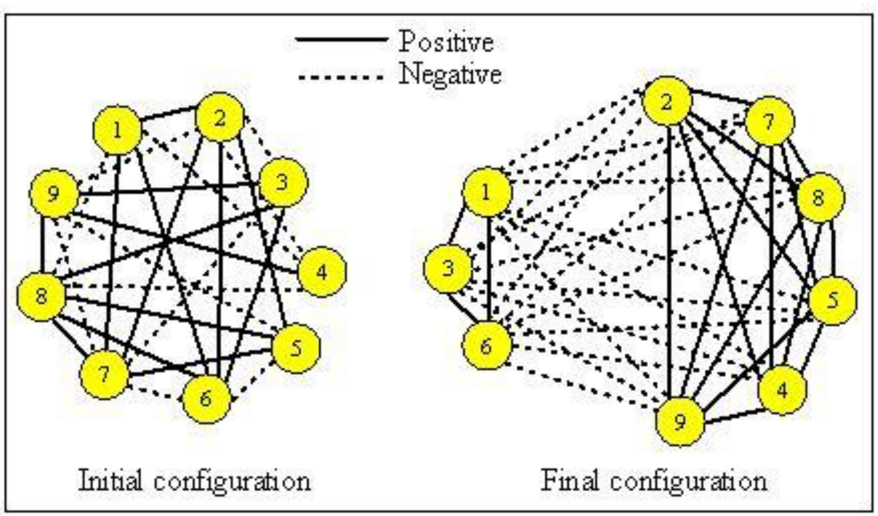
- a scientific conference: intellectual atmosphere. Suddenly somebody shouts. We start to observe each other: are they surprised? Is it really a lecture?

http://thesituationist.files.wordpress.com/2007/09/cognitive-dissonance.jpg

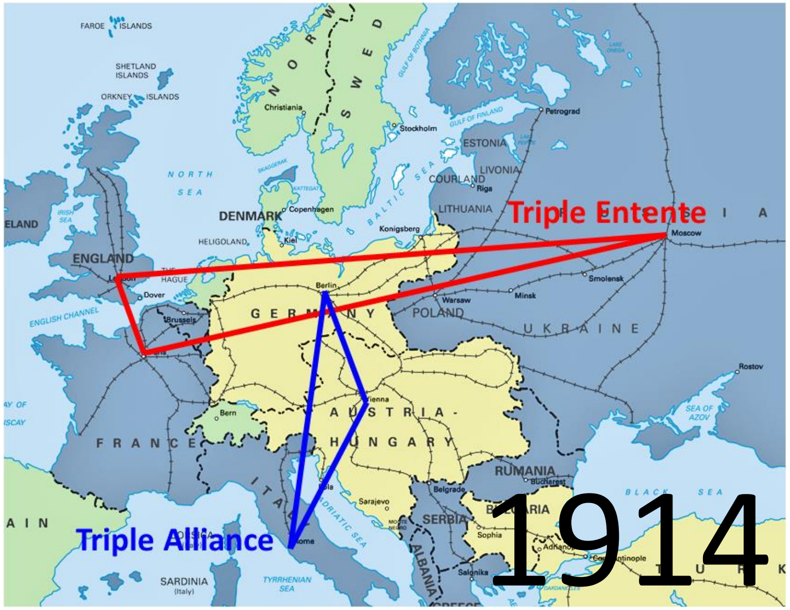
„Statesmen should, above all, have the ability to distinguish between friends from enemies”

[Irving Kristol]

In each ***balanced*** state,
a complete graph of N nodes
is divided
into **two**
internally friendly
and mutually hostile parts:
all links within each part are „+”,
and
all links between the parts are „-”.



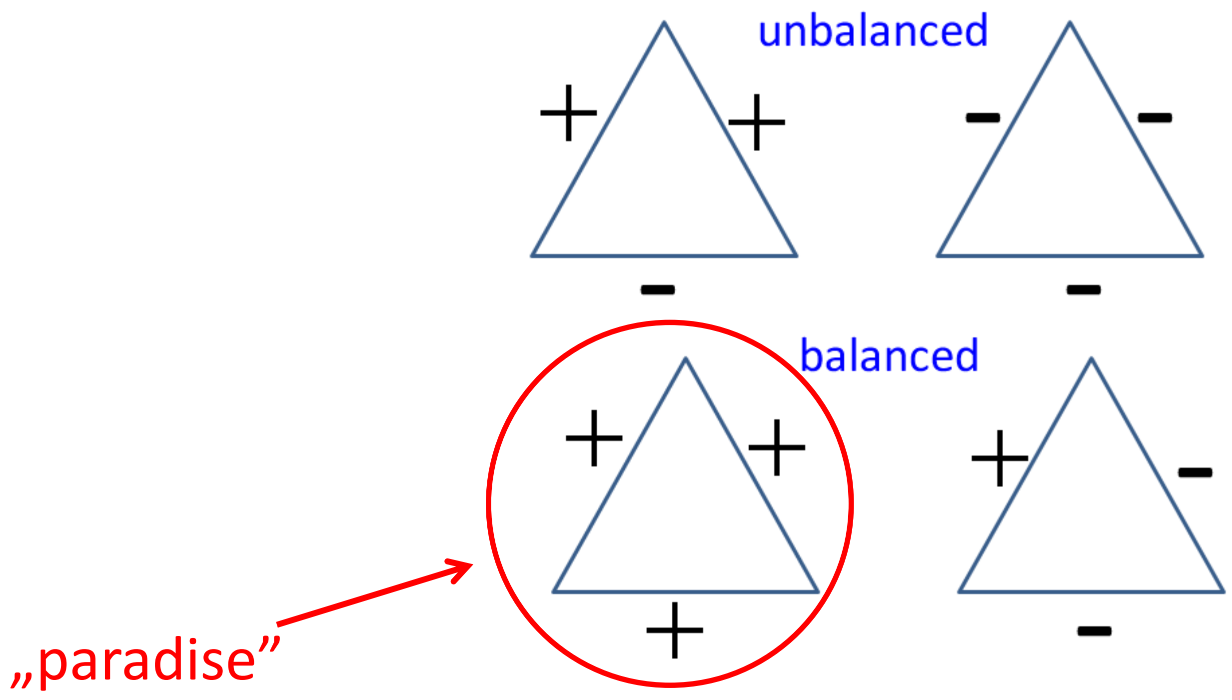
Z. Wang and W. Thorngate, JASSS (2003)



according to
T. Antal, P.L.Krapivsky, S. Redner,
Physica D (2006)

The ***cognitive dissonance*** is removed if:

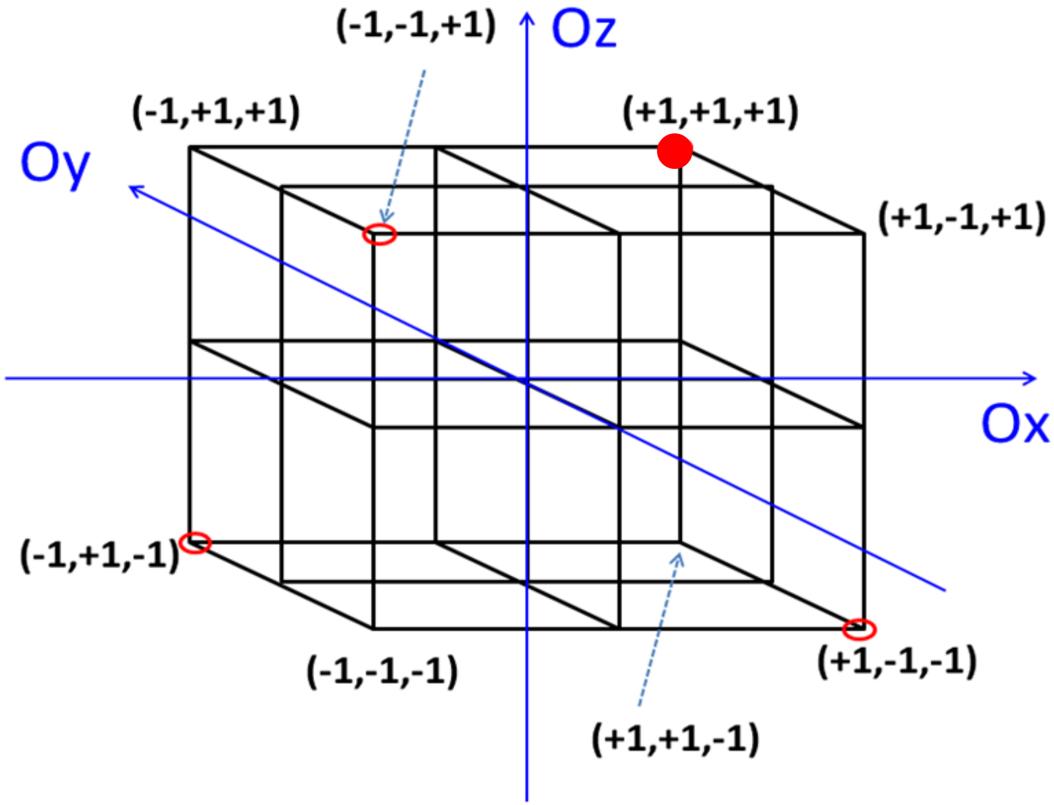
- a friend of my friend is my friend,
- a friend of my enemy is my enemy,
- an enemy of my friend is my enemy,
- an enemy of my enemy is my friend.



The continuous dynamics

$$\dot{s}_{ij} = (1 - s_{ij}{}^2)\sum_k s_{ik} s_{kj}$$

N=3:



$$\dot{x} = (1 - x^2)yz$$

$$\dot{y} = (1 - y^2)zx$$

$$\dot{z} = (1 - z^2)xy$$

How many trajectories end :

- at the paradise state $(+1,+1,+1)$?
- at any other balanced state?

those which start at:	P(+1,-1,-1)	P(-1,+1,-1)	P(-1,-1,+1)	P(+1,+1,+1)
$(x<0,y<0,z<0) \rightarrow$	1/24	1/24	1/24	0
$(x>0,y<0,z<0) \rightarrow$	1/8	0	0	0
$(x<0,y>0,z<0) \rightarrow$	0	1/8	0	0
$(x<0,y<0,z>0) \rightarrow$	0	0	1/8	0
$(x>0,y>0,z>0) \rightarrow$	0	0	0	1/8
$(x>0,y>0,z<0) \rightarrow$	$a/8$	$a/8$	0	(1-2a)/8
$(x>0,y<0,z>0) \rightarrow$	$a/8$	0	$a/8$	$(1-2a)/8$
$(x<0,y>0,z>0) \rightarrow$	0	$a/8$	$a/8$	$(1-2a)/8$

$b \equiv 1\text{-}2a = ?$

example: those which start at (x>0,y>0,z<0)

Invariant planes: if a trajectory starts there, it stays there. Hence, a trajectory cannot go through it.

If $z+x = 0$, then $$\dot{z} + \dot{x} = [1-(-x)^2]xy + (1-x^2)y(-x) = 0$$

If $z+y = 0$, then $$\dot{z} + \dot{y} = [1-(-y)^2]xy + (1-y^2)x(-y) = 0$$

Hence b is the volume above the planes $z=-x$, $z=-y$:

$$b = \int_0^1 dx \int_0^1 dy \int_{\max(-x,-y)}^{0} dz = 1/3$$

$$\dot{x}_{12} = (1 - x_{12}{}^{2})(x_{13}x_{32} + x_{14}x_{42})$$

$$\dot{x}_{13} = (1 - x_{13}{}^{2})(x_{12}x_{32} + x_{14}x_{43})$$

*The Constrained Triad Dynamics**

[*T. Antal, P. Krapivsky, S. Redner, PRE (2005)]

„Energy”

$$U = -\sum_{ijk} s_{ij} s_{jk} s_{ki}$$

A. select a random link and change its sign if *U* decreases
B. Repeat step A

Here:

a) form a network of all possible states of the fully connected graph
b) assign ***U*** to each state, and weight ***w*** to each pair of nodes: ***w(U,U')***
c) compress** the network, grouping nodes (states) into classes.

All nodes (states) in the same class have the same probability.

[**M. J. Krawczyk, Physica A (2011)]

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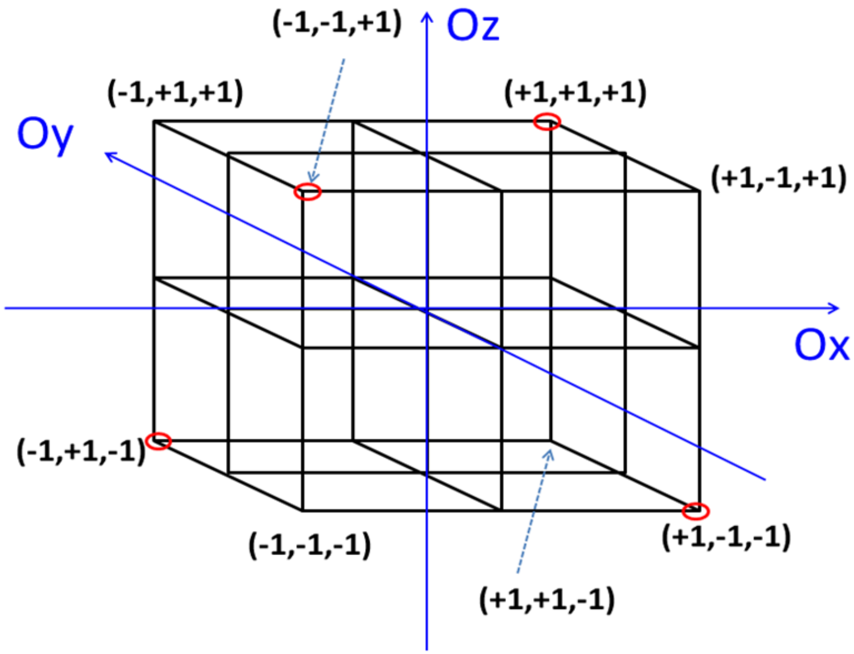
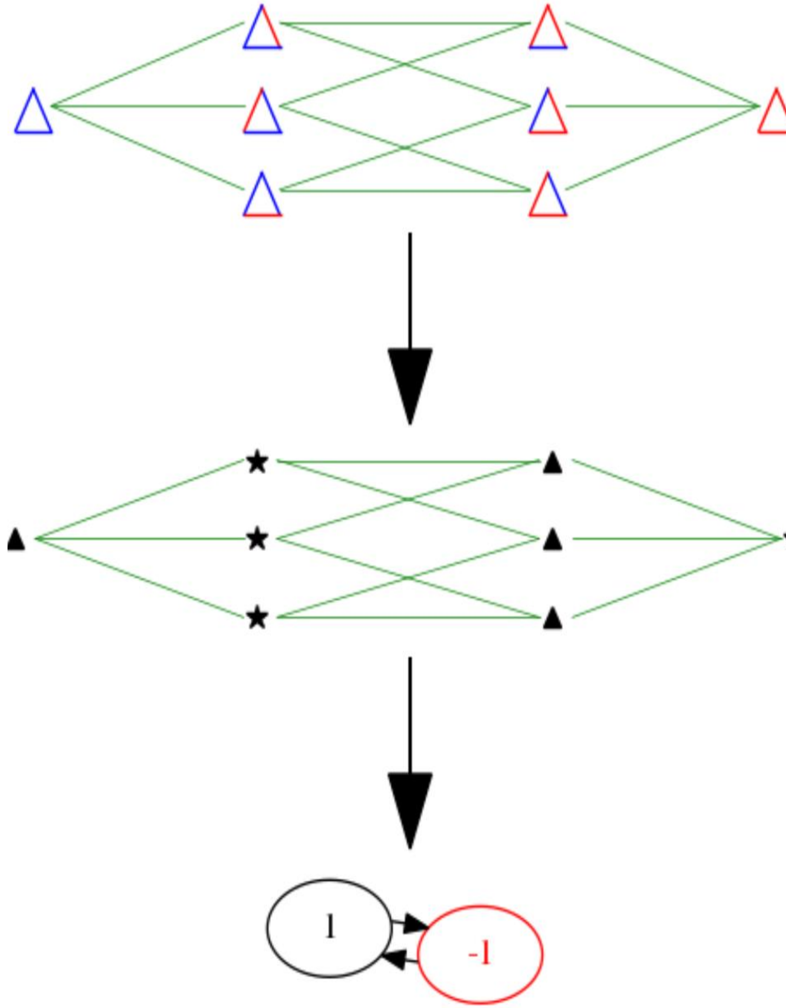
The compression procedure: N=3





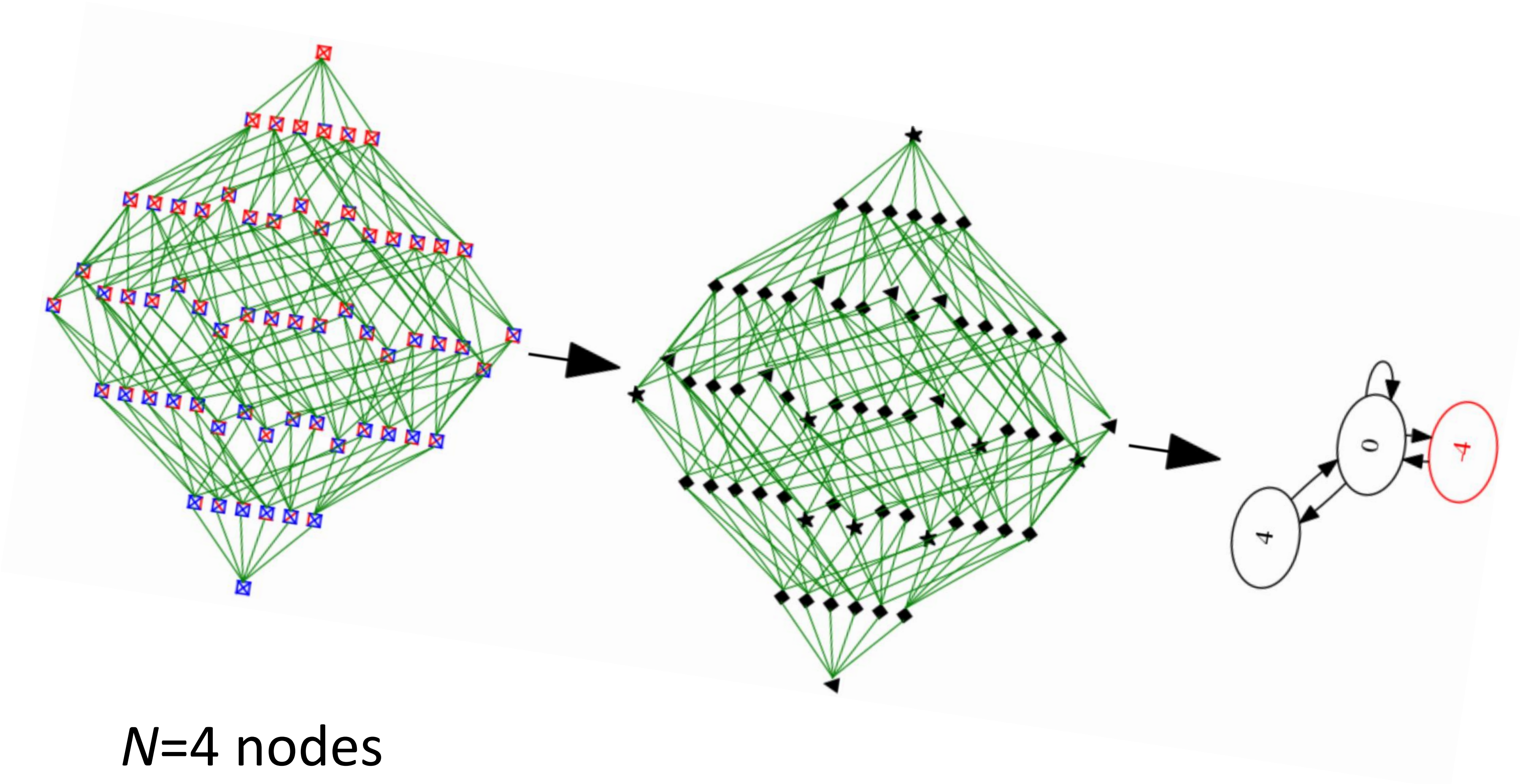
N=3 nodes

L=N(N-1)/2 = 3 links

2^L = 8 states

2 classes

The compression procedure: N=4



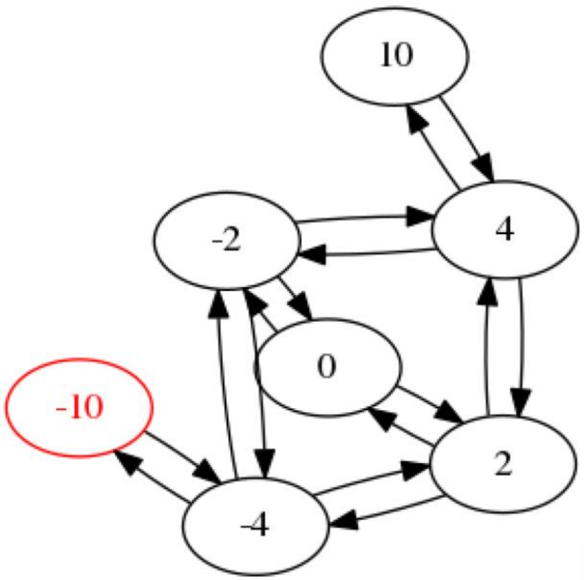
N=4 nodes

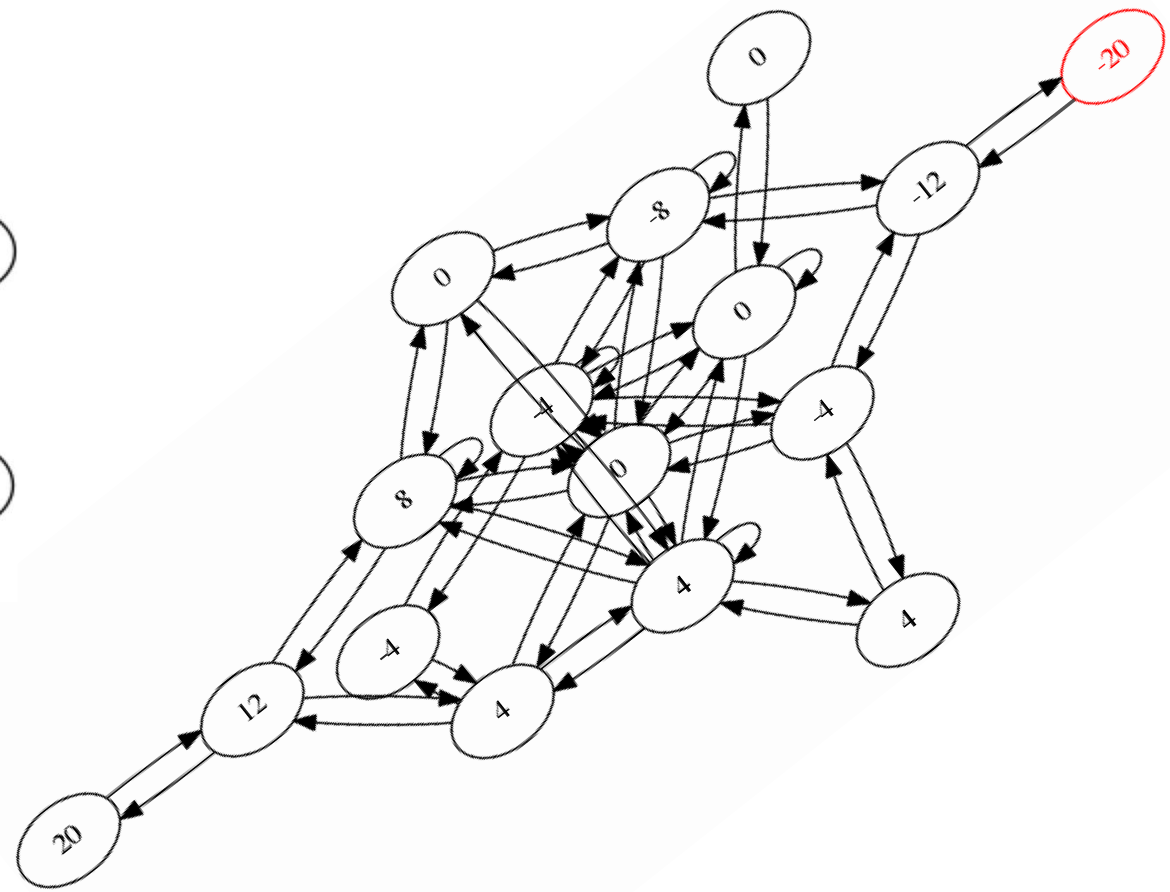
$L=N(N-1)/2$ = 6 links

2^L = 64 states

3 classes

The compression procedure: N=5,6





N=5 nodes

L=N(N-1)/2 = 10 links

2^L = 1024 states

7 classes

N=6 nodes

L=N(N-1)/2 = 15 links

2^L = 32678 states

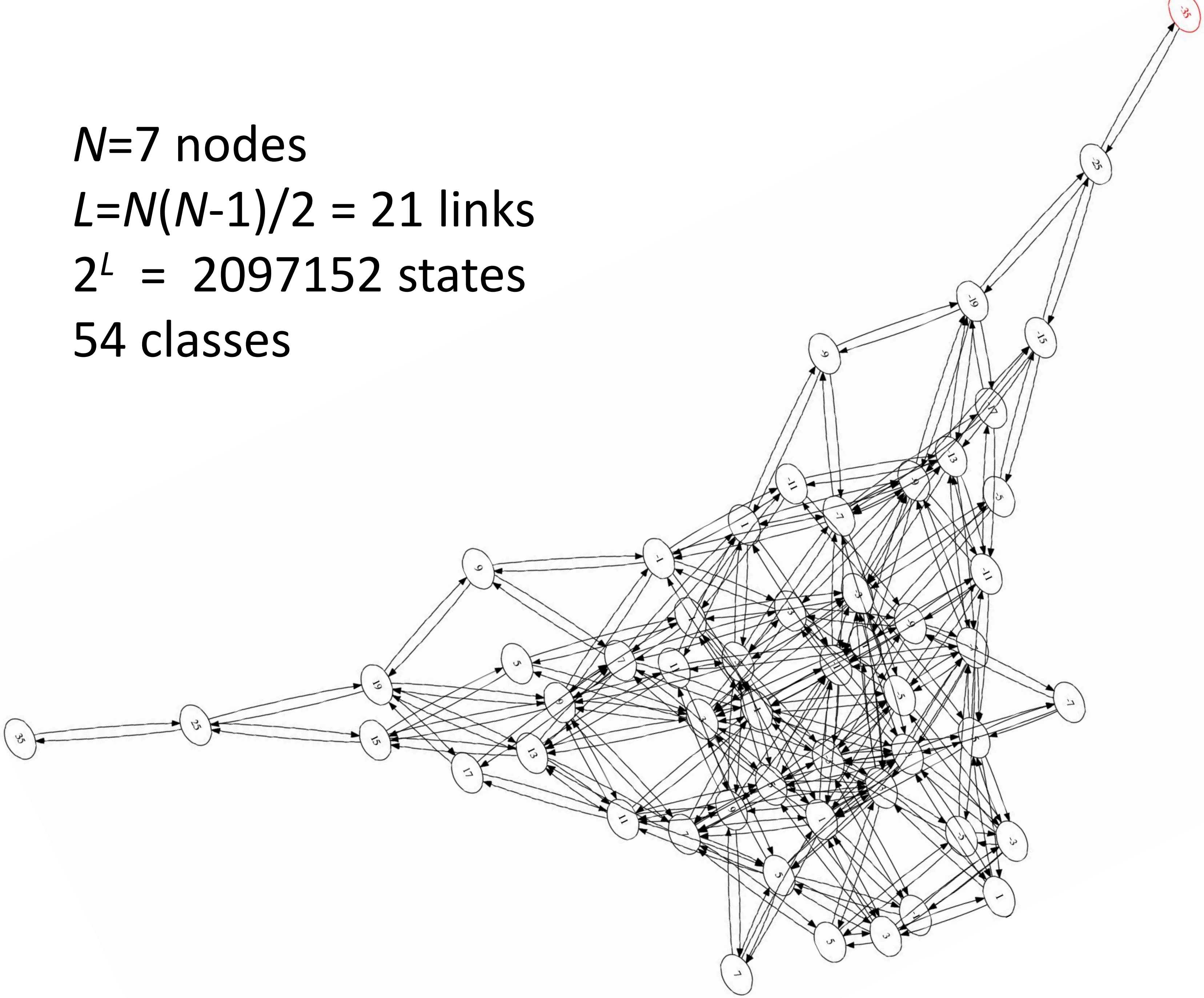
16 classes

N=7 nodes

L=N(N-1)/2 = 21 links

2^L = 2097152 states

54 classes



N=8 nodes

L=N(N-1)/2 = 28 links

2^L = 268435456 states

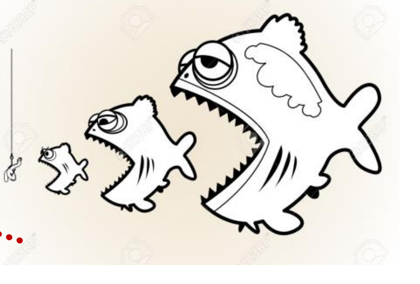
? classes

More important :

for $2 < N \leq 7$

the paradise state is as unlikely as each other balanced state:

$$\boxed{P = 2^{1-N}}$$



... a pity...

What for $N > 7$?

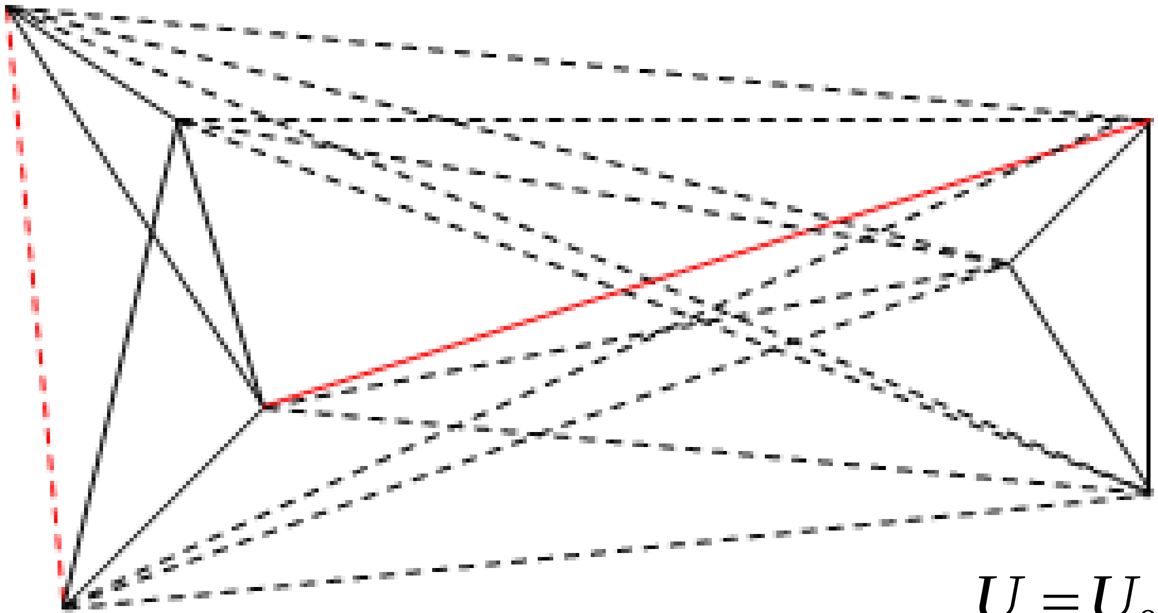
It seems that our conjecture is true as :

1. Energy U_0 of each balanced state is the same
2. The number of neighbors of each balanced state is the same
3. Energy of each neighbor of each balanced state is the same

Ad.1 . Yes, $U_0 = -\binom{N}{3}$

Ad.2. Yes, $k = \frac{N(N-1)}{2}$

Ad.3 . Yes,



$$U = U_0 + 2(N-2)$$

Doubts

Not true for any *N* ?

For *N*>8, jammed states are possible*,
which are absent for smaller systems.
Then, odd things can happen for large *N*.
Yet, the jammed states and the balanced states
do not fall into the same class.

Trivial ?

Diffusion in weighted networks:
A.Baronchelli, R.Pastor-Satorras,
mean field, PRE 2010;
G. Siudem, J. Hołyst,
adiabatic approx., arXiv 2013
Here the weights can depend on *U,U'*.

Conclusions

A general agreement is pretty far from generic.

Fortunately, in fact
the division into enemies
and friends remains
disputable.

* Antal et al., ibid