CONDITIONAL PROBABILITY; BAYES THEOREM

1. The manufacturing process of a detail can be done in two ways. The first one consists of 3 stages with the probability of some technological mishap at a given stage being equal to: 0.05, 0.1 and 0.3, respectively. The second consists of two stages and the probability of mishap at each of them is 0.25. (a) Which way is a "better" one ?

   (b) One of the 2 ways has been adopted. What is the $\mathcal{P}$ that a picked-at-random detail will be without any fault?
   Hint: *easy. By the way: "better" does not have to mean that the chosen way is more advantageous economically.*

2. The plants A, B and C produce computer mice. The market structure is:

plant	share in the market	1st choice	2nd choice	faulty items
A	45%	88%	11.2%	0.8%
B	35%	90%	8.8%	1.2%
C	20%	91%	7.5%	1.5%

   What are the probabilities that: (a) a mouse bought is a faulty one?; (b) that it is under the 1st choice category? (c) among two mice bought one will be in the 1st choice category and the second — in the 2nd choice category?
   Hint: *revise: total probability and the conditional probability. It's an easy problem.*

3. A pair of dice is thrown twice. What is the probability of getting totals of 7 and 11 ?

4. A circuit consists of two active elements: A and B. Observing the operation of the circuit during a given time (number of operations) one can state that the probability of the failure of A is 6%, that of B – 8% and the probability that both fail is 4%. Are the events: ,,failure of A" and ,,failure of B" independent? What is the probability that A failed if we know that B did?
   Hint: *the basic formulae for the conditional probability answer the 2 questions.*

5. A pair of dice is thrown. If it is known that one die shows a 4, what is the probability that
   (a) the other die shows a 5?
   (b) the total of both dice is greater than 7?

6. An experiment is being documented with 3 photographic cameras acting independently. Each of the cameras guarantees a 60% probability of a correct picture. What is the probability that the experiment has been documented? How many cameras should we use if we want to document the experiment with the probability of the success equal to (at least) 99 per cent?

7. The probability of a disease is one in thousand persons. A routine screening test is positive in 100% of "true" cases and gives an erroneous positive result for 0.5% of healthy persons. A randomly chosen person is tested and the result is positive What is the probability that the person is really sick? Hint: *Bayes theorem.*

8. Two taxi companies operate in a town: the "blue cab" (170 cars) and the "green cab" (30 cars). One evening, a pedestrian is knocked over by a taxi whose driver runs away. A witness of the accident claims that he has seen a green taxi. According to some tests the reliability of the

witness statement (ability of distinguishing correctly colours under conditions similar to those during the accident) is 80%. What does the probability say about the possible colour of the faulty taxi? Hint: *Bayes theorem.*

The following two problems are for those of you who find special pleasure in solving ,,statistical questions. I shall not interfere (by showing the solutions to you), but I can give some hints if the persons who are interested ask me for them.

- The probability of the child having the genotype OA, with the genotype of mother being determined but unknown, is equal to: (a) 0.5 if the father's genotype is OA; (b) 1.0 if the father's genotype is AA; (c) 0.5 if the father's genotype is AB; (d) 0 if the father's genotype other than the mentioned three. 35.2% of men have the genotype OA, 6.9% – genotype AA and 1.5% – AB. What is the probability that the father of an OA-genotype child has the genotype AB? Hint: *Bayes theorem.*

- Two persons decide on a meeting at a coffee shop between 1 p.m. and 2 p.m. The rule is that the first person to come shall wait 15 minutes — if the second person does not show during that time the first goes away. What is the chance for the meeting to take place?
  Hint: *The moments of the arrival of both persons can be picked at random (with equal probability) any time between 1 p.m. and 2 p.m. The most efficient way of solving this problem is by using the "geometrical probability" idea. The event space can be represented as a square with the length of the side being equal to 1 hour. Each point (x, y) of the area inside the square represents the event of two persons coming at the meeting at the time x (first person) and y (second person). Find the shape and its area that guarantee that the meeting will take place. This area divided by the (unit) area of the whole square is the probability we are looking for. The answer is: 7/16.*