RANDOM VARIABLE - BASIC CONCEPTS

1. A drunk wanders out of a bar thoroughly inebriated. At each step he will move to the right one (1) meter with probability p or to the left one meter with $\mathcal{P} = 1 - p$. How many meters from the entrance to the bar do you expect the drunk to be after n steps?

   Hint: let X_n – # of meters after n steps; $E(X)$ – # of meters of a single step is (two-way RV)

2. Show that $E(aX) = aE(X)$ if a is a constant and X is a continuous or discrete RV (consider both cases).

   Show also $VAR(aX) = a^2 VAR(X)$.

   Once you perform the necessary (and simple) calculations think – were they really necessary? The results can be justified by simple argumentation.

3. A sample consists of results: $2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 7$. Compute: (a) μ; (b) σ^2.

   Hint: assume that all results occur with the same probability.

4. A discrete random variable X has the probability function :

x_i	-2	-1	2	5
$P(X = x_i)$	0.3	0.1	0.2	0.4

   (a) sketch the cumulative distribution of X ; (b) calculate the mean and variance for two random variables: $U = 2X - 3$ and $V = 2X^2$. Comment on $p(U = u_i)$ and $p(V = v_k)$ values.

5. The probability density function of the random variable X is given by:

$$f(x) = \begin{cases} 0 & \text{for} \quad -\infty < x < 0 \\ \sin x & \text{for} \quad 0 \le x \le \frac{1}{2}\pi \\ 0 & \text{for} \quad \frac{1}{2}\pi < x < \infty \end{cases}$$

   Find the cumulative distribution $F(x)$ and calculate $P(1/6\pi < x < 1/3\pi)$.

   Sketch the graphs of probability density function and cumulative distribution function and read out the calculated P from the graphs.

6. A continuous random variable has a *uniform* distribution, i.e. its probability density function is constant in the whole interval. For example

$$f(x) = C \qquad \text{for} \quad x \in [0., 2.]$$

   (a) find the value of the constant C.

   (b) Calculate the expected value $E(X)$ and variance $VAR(X)$.

7. Given that the RV X has the probability density function

$$f(x) = \frac{1}{2}(x+1) \quad \text{when} \quad -1 < x < 1$$

and $f(x) = 0$ elsewhere, calculate the expected value of X and the variance of X.

8. Given that the RV X has the density probability function

$$f(x) = \frac{1}{2\sqrt{3}} \quad \text{when} \quad -\sqrt{3} < X < \sqrt{3}$$

and $f(x) = 0$ elsewhere, compute $\mathcal{P}\left(|X| \geq \frac{3}{2}\right)$ and $\mathcal{P}\left(|X| \geq 2\right)$. Compare these probabilities with the upper bounds given by the Chebyshev's inequality.

9. Use the Chebyshev's inequality to find the lower bound on $\mathcal{P}(-4 < X < 20$ where X is a RV with mean $\mu = 8$ and variance $\sigma^2 = 9$.

10. iF X follows a *discrete uniform distribution* i.e. $\mathcal{P}(X = i) \quad i = 1, 2, \ldots, N-1, N = \frac{1}{N}$ find $E(X)$ and $\sigma^2(X)$.

    Hint: use the formula $\sum_{X=1}^{N} = \frac{N(2N+1)(N+1)}{6}$ (by the way, would you know how to justify this formula?).

11. Please, download the le salaries.xlsx from the directory (zipped archive, at the very end). Analyse the calculations of the expected value, variance (standard mean deviation), lower- and upper-quartile and median (in the 1st sheet). Repeat the estimation of median and quartiles for the 2nd and 3rd plot.