THE NORMAL DISTRIBUTION

LAPLACE MODEL OF ERRORS:

n	-3ε	-2ε	$-\varepsilon$	0	ε	2ε	3ε
0				1			
1			1/2		1/2		
2		1/4		1/2		1/4	
3	1/8		3/8		3/8		1/8

this is again . . .

this is again . . .

a case of a binomial distribution with $n \gg 1$ and $p = q = 1/2$

$$W_k^n(p = q = 1/2) = \left(\begin{array}{c} n \\ k \end{array} \right) p^k q^{n-k} = \left(\begin{array}{c} n \\ k \end{array} \right) \left(\frac{1}{2}\right)^n$$

it can be shown that such distribution for large n and a RV of continuous $W_k^n \to \phi(x)$, where:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \frac{1}{b} \exp\left[-\frac{(x-a)^2}{2b^2}\right] \quad \text{or}$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} \exp\left[-\frac{(x-\hat{x})^2}{2\sigma^2}\right]$$

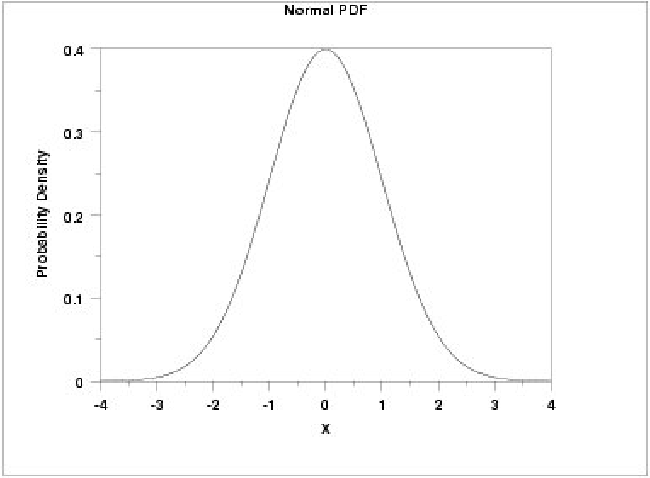
where: $E\{X\} = \hat{x} \equiv a; \quad VAR\{X\} = \sigma^2 \equiv b^2$ For a standardised variable

$$Z = \frac{X - \hat{x}}{\sigma_X}$$

we have *the standardised normal distribution*

$$\phi(x) \to \phi_0(z) \equiv N(0,1) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{z^2}{2}\right]$$

here comes a graph of the pdf for the standardised normal distribution:



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The cumulative standardised normal distribution. . .

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is given by:

$$F(z) \equiv \psi_0(z) = P(Z < z) = \int_{-\infty}^{z} \phi_0(z')dz'$$

The above interval cannot be expressed by ,,regular" functions; in fact, it constitutes a **new function** – erf(z) that is called *error function*.
For practical calculations one has to use tables or dedicated software to obtain $F(z)$ values. Some typical are listed in the table below:

$$P(|Z| > z) = 2\psi_0(-|z|) = 2[1 - \psi_0(|z|)]$$

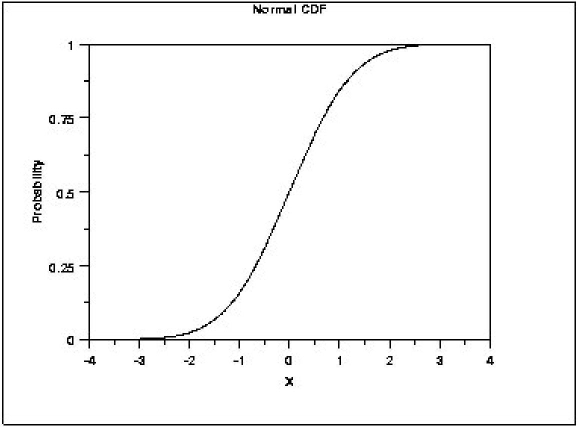
$$P(|Z| \leq z) = 1 - P(|Z| > z) = 2\psi_0(|z|) - 1$$

$$P(|Z| \leq 1) = 0,6827$$

$$P(|Z| \leq 2) = 0,9545$$

$$P(|Z| \leq 3) = 0,9973$$

here comes a graph of the pdf for the standardised cumulative normal distribution:



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and . . .

...a graph that illustrates the numerical values ftom the one-before-the-last slide:

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