Plastyczność

(opis ciągły)

Wstawka: Koło Mohra

Mamy tensor naprężenia:

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Wykonajmy obrót tego układu o kąt θ wokół osi x_3 :

$$[a] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Przetransformujmy tensor naprężeń do nowego układu:

$$\sigma_{11}' = (\cos\theta)^2\sigma_1 + (\sin\theta)^2\sigma_2 = \sigma_1\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_2\left(\frac{1-\cos 2\theta}{2}\right)$$

po przekształceniu:

$$\sigma'_{11} = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_2 - \sigma_1)\cos 2\theta$$

Podobnie:

$\sigma'_{22} = (\sin\theta)^2\sigma_1 + (\cos\theta)^2\sigma_2$, i po przekształceniu:

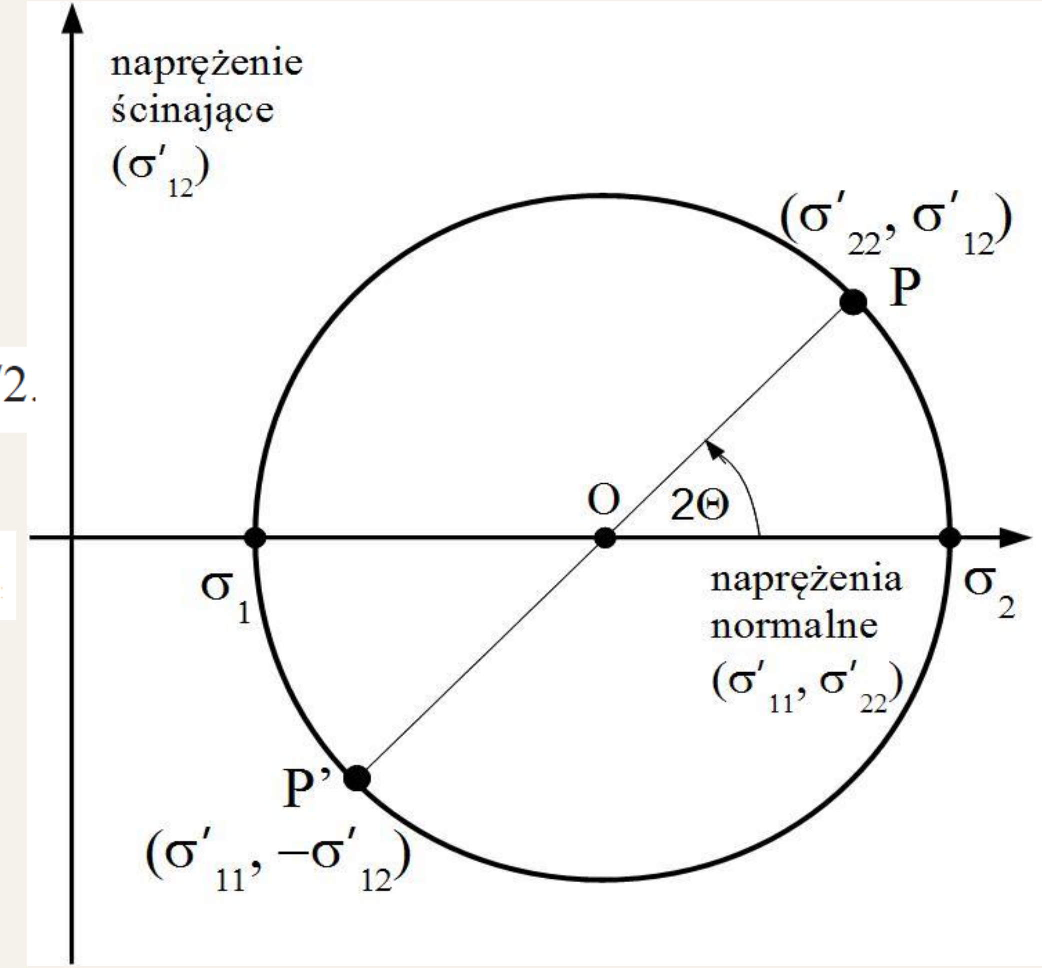
$$\sigma'_{22} = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}(\sigma_2 - \sigma_1)\cos 2\theta$$

oraz:

$\sigma'_{12} = -\cos\theta\sin\theta\,\sigma_1 + \sin\theta\cos\theta\,\sigma_2,$ czyli:

$$\sigma'_{12} = \frac{1}{2}(\sigma_2 - \sigma_1)\sin 2\theta$$

I oczywiście: $\sigma'_{33} = \sigma_{33}$ $\sigma'_{13} = \sigma'_{23} = 0$



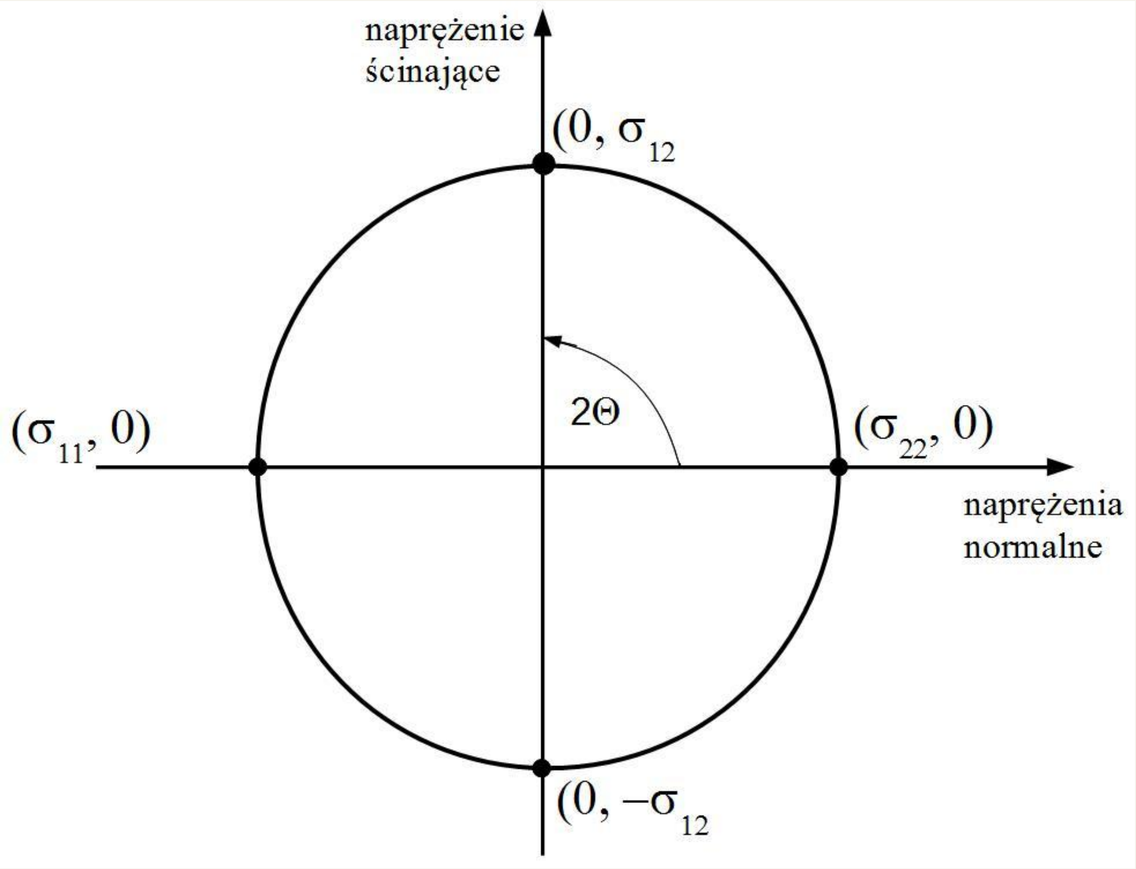
promień okręgu wynosi: $r = (\sigma_2 - \sigma_1)/2$.

punkt O ma współrzędne: $(\sigma_1 + \sigma_2)/2$

$$\sigma'_{11} = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_2 - \sigma_1)\cos 2\theta$$

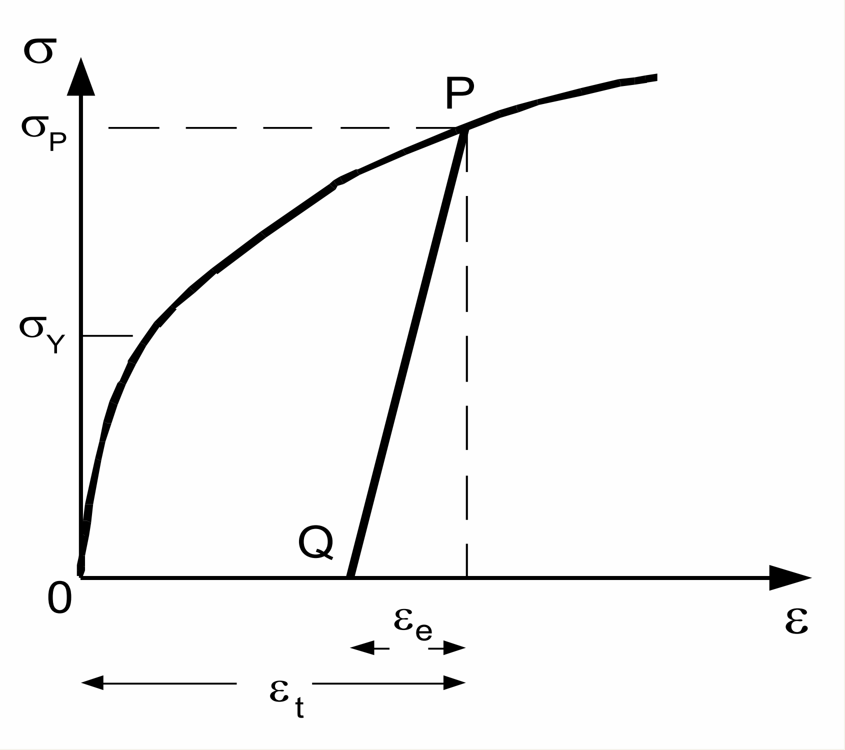
$$\sigma'_{22} = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}(\sigma_2 - \sigma_1)\cos 2\theta$$

$$\sigma'_{12} = \frac{1}{2}(\sigma_2 - \sigma_1)\sin 2\theta$$

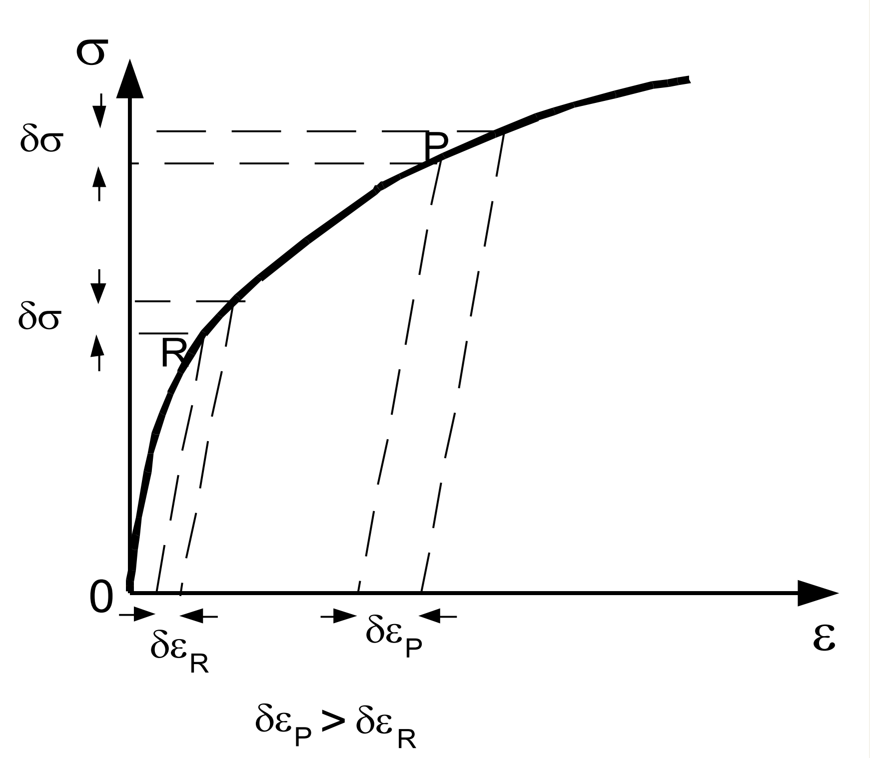


$$\sigma_{12\,(max)} = (\sigma_{22} - \sigma_{11})/2$$

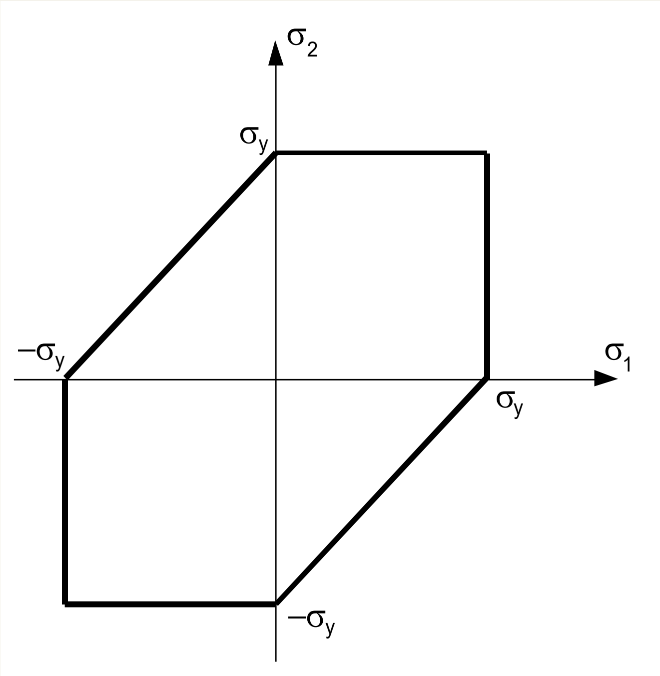
Odkształcenie plastyczne materiałów



σ
P
σP
σY
Q
0
ε
εe
εt

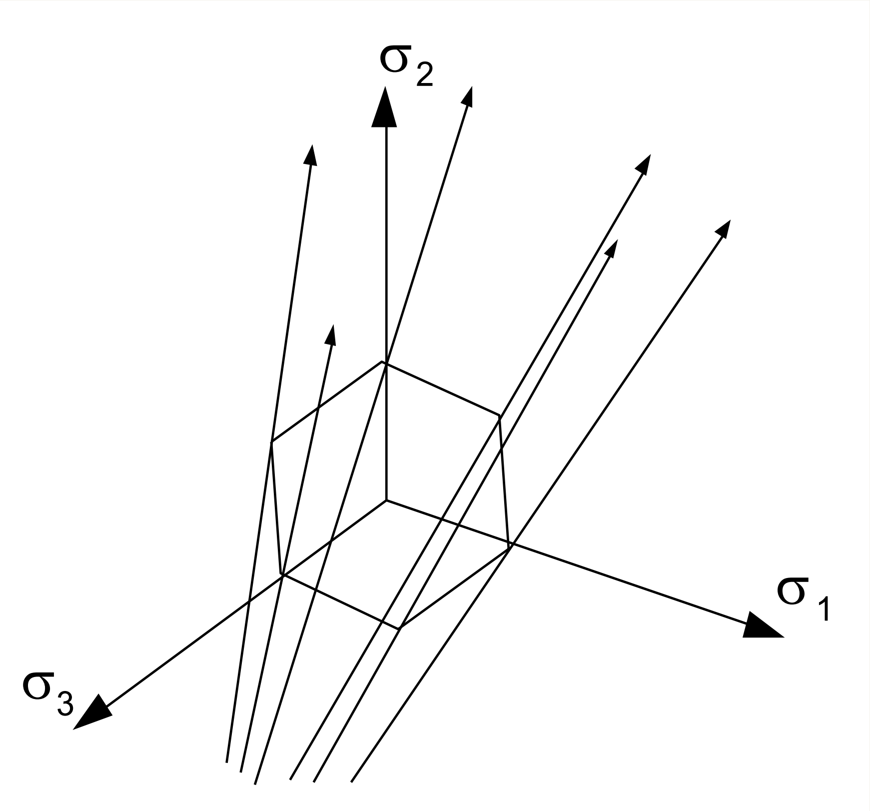

σ
δσ
δσ
P
R
0
ε
δε_R
δε_P
δε_P > δε_R

Kryterium Treski

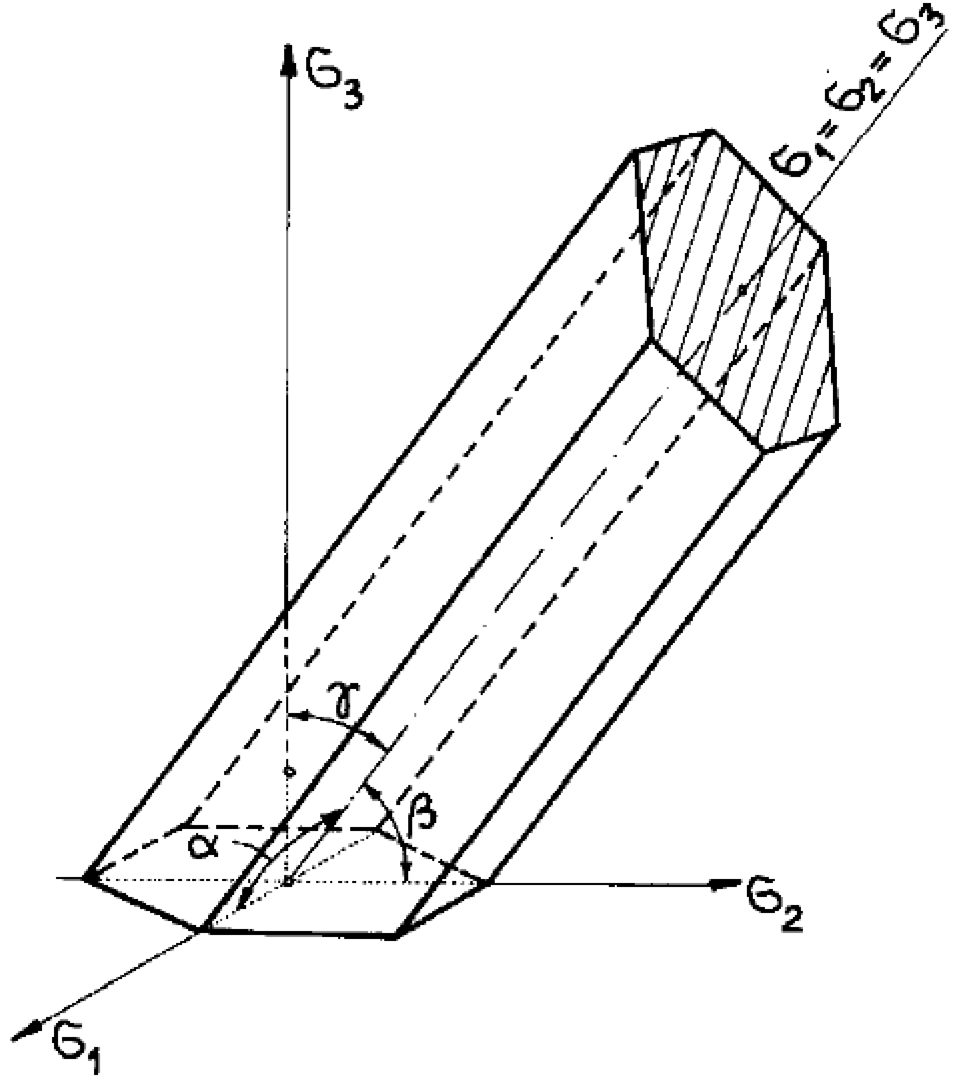


Płaski stan naprężeń

$$\sigma_{max} - \sigma_{min} = \sigma_y$$



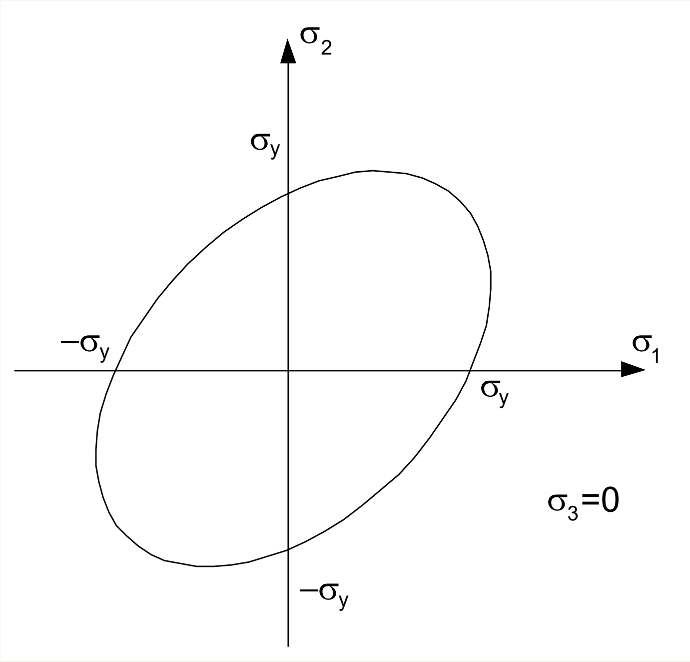
Pełny stan naprężeń



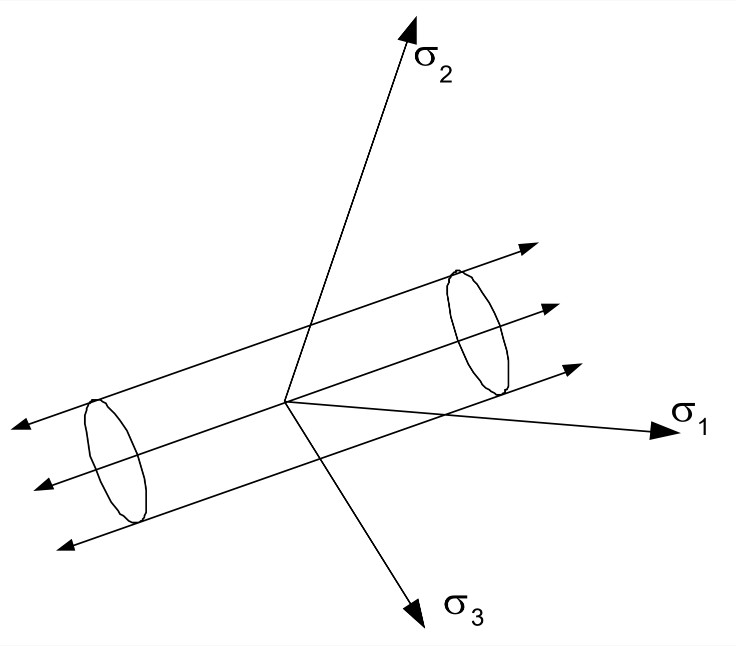
σ_3
$\sigma_1 = \sigma_2 = \sigma_3$
γ
α
β
σ_2
σ_1

Kryterium Von Mises'a

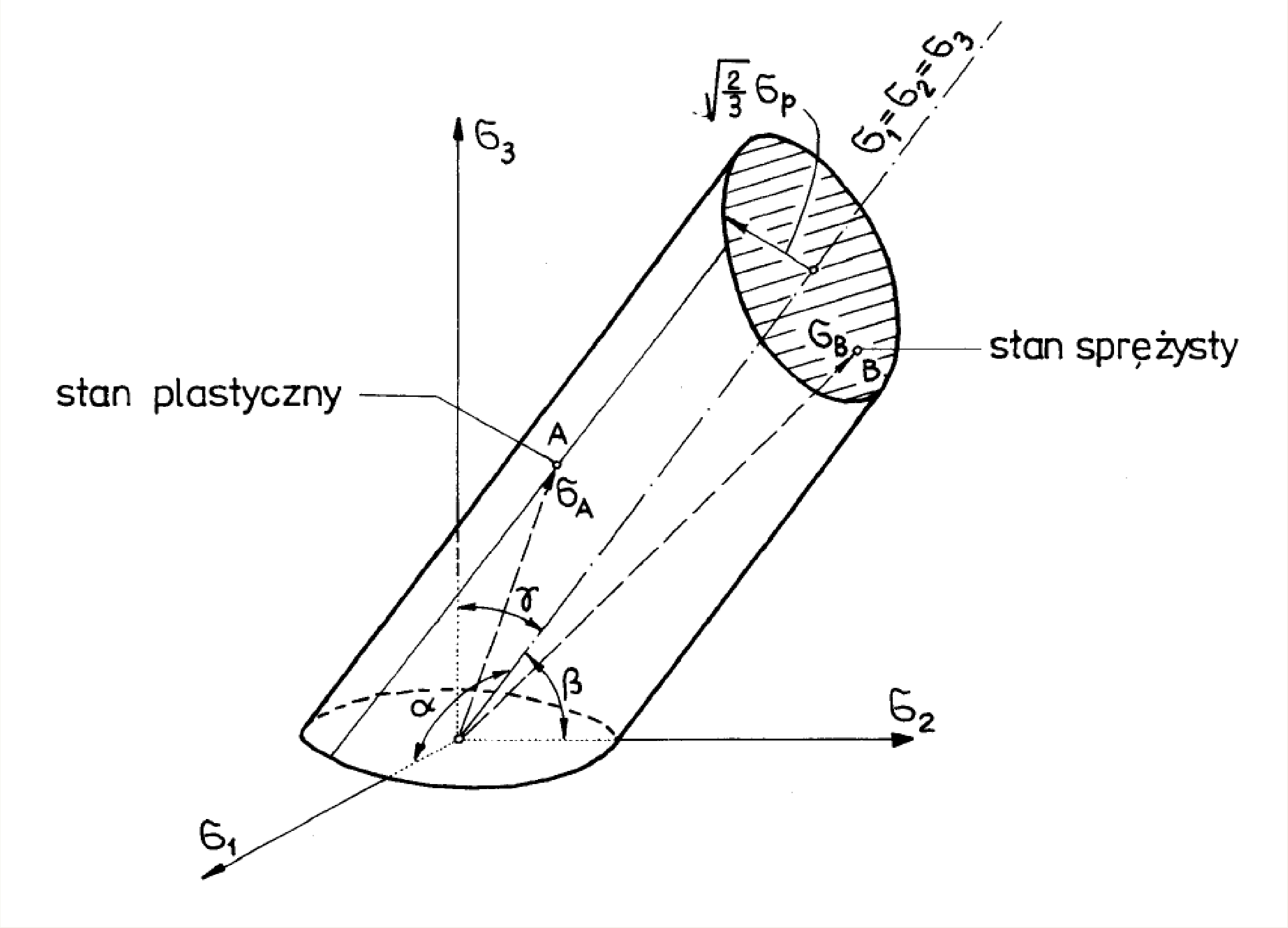
$$F = \left[\frac{1}{2} \left\{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right\} \right]^{\frac{1}{2}} = \sigma_y$$



Płaski stan naprężeń

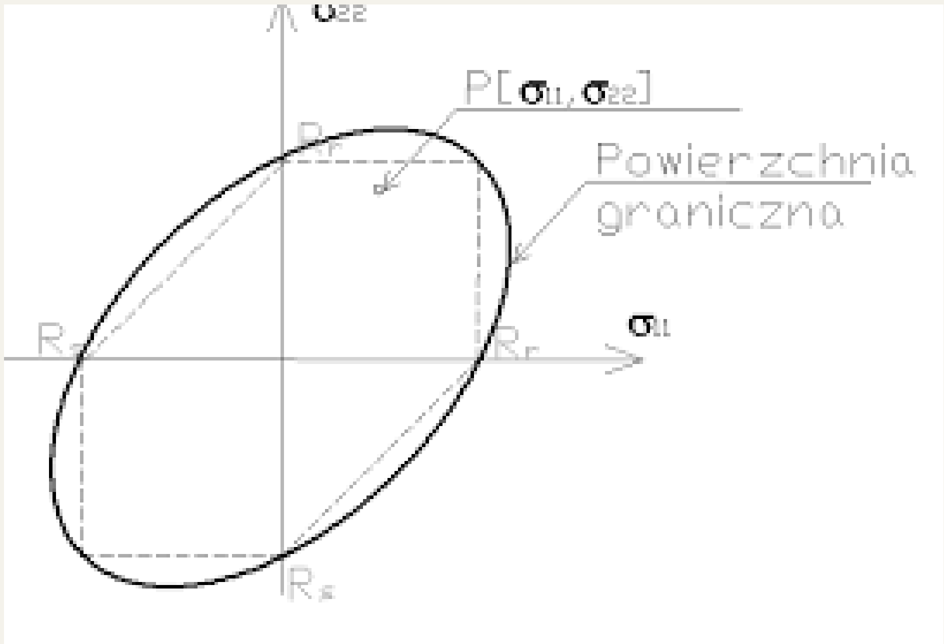


Pełny stan naprężeń



σ_3
$\sqrt{\frac{2}{3}}\sigma_p$
$\sigma_1=\sigma_2=\sigma_3$
stan plastyczny
A
σ_A
σ_B
B
stan sprężysty
γ
β
α
σ_2
σ_1

Porównanie:



Pojęcie naprężeń efektywnych

W układzie osi głównych:

$$\overline{\sigma} = \left[\frac{1}{2} \left\{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right\} \right]^{\frac{1}{2}}$$

Zaś w dowolnym układzie odniesienia:

$$\overline{\sigma} = \left[\frac{1}{2} \left\{ (\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 \right\} + 3 \left\{ \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2 \right\} \right]^{\frac{1}{2}}$$

Naprężenie efektywne ($\overline{\sigma}$) jest niezmiennikiem.

Relacje naprężenie - odkształcenie

$$\frac{d\gamma_A}{\tau_A} = \frac{d\gamma_B}{\tau_B}$$

albo:

$$\frac{\mathrm{d}\gamma_{xy}}{\sigma_{xy}} = \frac{\mathrm{d}\gamma_{yz}}{\sigma_{yz}} = \frac{\mathrm{d}\gamma_{zx}}{\sigma_{zx}} = \mathrm{d}\lambda$$

gdzie:

$$\gamma_{xy} = 2\varepsilon_{12}, \quad \gamma_{yz} = 2\varepsilon_{23}, \quad \gamma_{zx} = 2\varepsilon_{31}$$

lub ogólniej:

$$\frac{2\left(\mathrm{d}\varepsilon_x - \mathrm{d}\varepsilon_y\right)}{\sigma_{xx} - \sigma_{yy}} = \frac{2\left(\mathrm{d}\varepsilon_y - \mathrm{d}\varepsilon_z\right)}{\sigma_{yy} - \sigma_{zz}} = \frac{2\left(\mathrm{d}\varepsilon_z - \mathrm{d}\varepsilon_x\right)}{\sigma_{zz} - \sigma_{xx}} = \frac{\mathrm{d}\gamma_{xy}}{\sigma_{xy}} = \frac{\mathrm{d}\gamma_{yz}}{\sigma_{yz}} = \frac{\mathrm{d}\gamma_{zx}}{\sigma_{zx}} = \mathrm{d}\lambda$$

$$\frac{2\left(d\varepsilon_x - d\varepsilon_y\right)}{\sigma_{xx} - \sigma_{yy}} = \frac{2\left(d\varepsilon_y - d\varepsilon_z\right)}{\sigma_{yy} - \sigma_{zz}} = \frac{2\left(d\varepsilon_z - d\varepsilon_x\right)}{\sigma_{zz} - \sigma_{xx}} = \frac{d\gamma_{xy}}{\sigma_{xy}} = \frac{d\gamma_{yz}}{\sigma_{yz}} = \frac{d\gamma_{zx}}{\sigma_{zx}} = d\lambda$$

Biorąc pod uwagę: $d\varepsilon_x + d\varepsilon_y + d\varepsilon_z = 0$ (zerowa dylatacja)

otrzymujemy:

$$d\varepsilon_x = \frac{1}{3} d\lambda \left\{ \sigma_{xx} - \frac{1}{2}\left(\sigma_{yy} + \sigma_{zz}\right) \right\}$$

$$d\varepsilon_y = \frac{1}{3} d\lambda \left\{ \sigma_{yy} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{zz}\right) \right\}$$

$$d\varepsilon_z = \frac{1}{3} d\lambda \left\{ \sigma_{zz} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right) \right\}$$

Równ.(*)

$$d\gamma_{xy} = \sigma_{xy} d\lambda$$

$$d\gamma_{yz} = \sigma_{yz} d\lambda$$

$$d\gamma_{zx} = \sigma_{zx} d\lambda$$

Wróćmy do układu układu osi głównych:

$$\frac{2(d\varepsilon_1 - d\varepsilon_2)}{\sigma_1 - \sigma_2} = \frac{2(d\varepsilon_2 - d\varepsilon_3)}{\sigma_2 - \sigma_3} = \frac{2(d\varepsilon_3 - d\varepsilon_1)}{\sigma_3 - \sigma_1} = d\lambda$$

Wykorzystajmy tożsamość:

$$\mathrm{a/d = b/e = c/f} = \sqrt{\left(\mathrm{a}^2 + \mathrm{b}^2 + \mathrm{c}^2\right) / \left(\mathrm{d}^2 + \mathrm{e}^2 + \mathrm{f}^2\right)}$$

$$\mathrm{d}\lambda = \frac{\sqrt{2\left\{(\mathrm{d}\varepsilon_1 - \mathrm{d}\varepsilon_2)^2 + (\mathrm{d}\varepsilon_2 - \mathrm{d}\varepsilon_3)^2 + (\mathrm{d}\varepsilon_3 - \mathrm{d}\varepsilon_1)^2\right\}}}{\sqrt{\frac{1}{2}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}}}$$

$$d\lambda = \frac{\sqrt{2\left\{(d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2\right\}}}{\sqrt{\frac{1}{2}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}}}$$

Zauważmy, że:

$$\overline{\sigma} = \left[\frac{1}{2}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}\right]^{\frac{1}{2}}$$

Wprowadźmy:

$$d\bar{\varepsilon} = \left[\frac{2}{9}\{(d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2\}\right]^{1/2}$$

wtedy:

$$\frac{1}{3} d\lambda = \frac{d\overline{\varepsilon}}{\overline{\sigma}}$$

podstawmy: $$\frac{1}{3}d\lambda = \frac{d\bar{\varepsilon}}{\bar{\sigma}}$$ Do Równ. *

$$d\varepsilon_x = \left\{\sigma_{xx} - \frac{1}{2}\left(\sigma_{yy} + \sigma_{zz}\right)\right\} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

$$d\varepsilon_y = \left\{\sigma_{yy} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{zz}\right)\right\} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

$$d\varepsilon_z = \left\{\sigma_{zz} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right)\right\} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

Równania LEVY-MISES'a –

(w układzie osi głównych)

Jeśli w dowolnym układzie, to:

$$d\bar{\varepsilon} = \left[\frac{2}{9}\left\{\left(d\varepsilon_x - d\varepsilon_y\right)^2 + \left(d\varepsilon_y - d\varepsilon_z\right)^2 + \left(d\varepsilon_z - d\varepsilon_x\right)^2\right\} + \frac{1}{3}\left\{d\gamma_{xy}^2 + d\gamma_{yz}^2 + d\gamma_{zx}^2\right\}\right]^{1/2}$$

$$\bar{\sigma} = \left[\frac{1}{2}\left\{\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + \left(\sigma_{zz} - \sigma_{xx}\right)^2\right\} + 3\left\{\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2\right\}\right]^{\frac{1}{2}}$$

$$d\varepsilon_x = \left\{\sigma_{xx} - \frac{1}{2}\left(\sigma_{yy} + \sigma_{zz}\right)\right\} \cdot d\bar{\varepsilon}/\bar{\sigma}$$

$$d\varepsilon_y = \left\{\sigma_{yy} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{zz}\right)\right\} \cdot d\bar{\varepsilon}/\bar{\sigma}$$

$$d\varepsilon_z = \left\{\sigma_{zz} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right)\right\} \cdot d\bar{\varepsilon}/\bar{\sigma}$$

$$d\gamma_{xy} = 3\sigma_{xy} \cdot d\bar{\varepsilon}/\bar{\sigma}$$

$$d\gamma_{yz} = 3\sigma_{yz} \cdot d\bar{\varepsilon}/\bar{\sigma}$$

$$d\gamma_{zx} = 3\sigma_{zx} \cdot d\bar{\varepsilon}/\bar{\sigma}$$

Równania LEVY-MISES'a

$$d\varepsilon_x = \left\{\sigma_{xx} - \frac{1}{2}\left(\sigma_{yy} + \sigma_{zz}\right)\right\} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

$$d\varepsilon_y = \left\{\sigma_{yy} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{zz}\right)\right\} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

$$d\varepsilon_z = \left\{\sigma_{zz} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right)\right\} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

$$d\gamma_{xy} = 3\sigma_{xy} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

$$d\gamma_{yz} = 3\sigma_{yz} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

$$d\gamma_{zx} = 3\sigma_{zx} \cdot d\bar{\varepsilon} / \bar{\sigma}$$

Po scałkowaniu:

$$\varepsilon_x = \bar{\varepsilon}\left\{\sigma_{xx} - \frac{1}{2}\left(\sigma_{yy} + \sigma_{zz}\right)\right\} / \bar{\sigma}$$

$$\varepsilon_y = \bar{\varepsilon}\left\{\sigma_{yy} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{zz}\right)\right\} / \bar{\sigma}$$

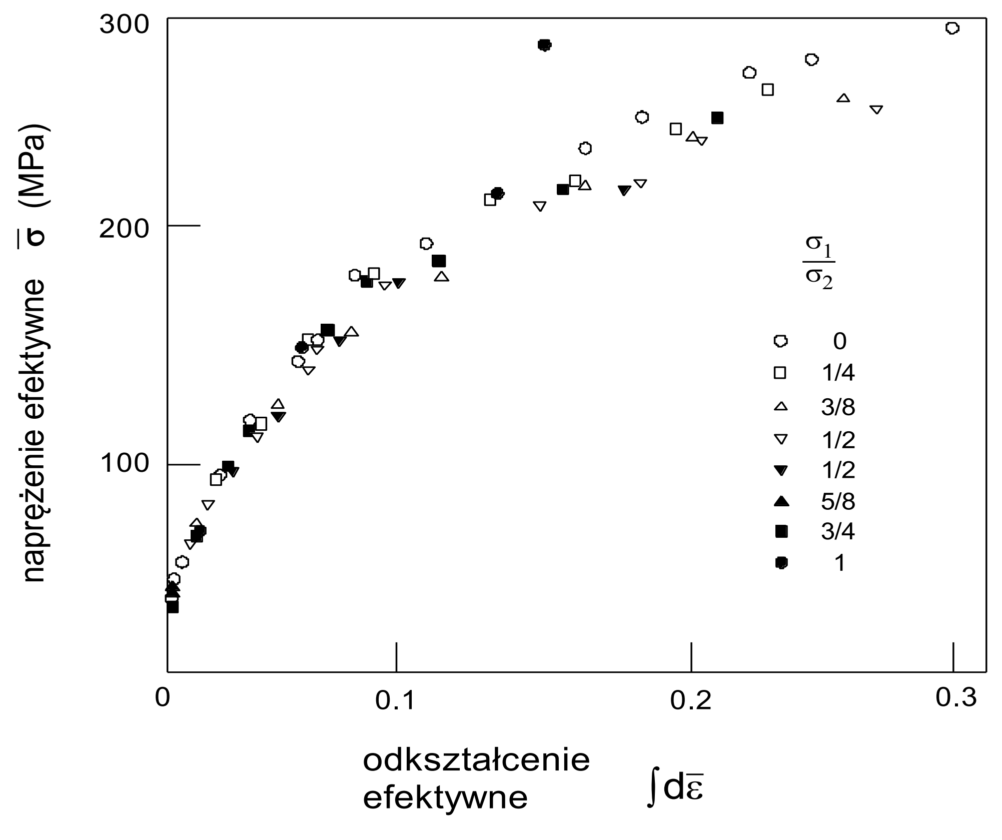
$$\varepsilon_z = \bar{\varepsilon}\left\{\sigma_{zz} - \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right)\right\} / \bar{\sigma}$$

$$\gamma_{xy} = 3\bar{\varepsilon}\sigma_{xy} / \bar{\sigma}$$

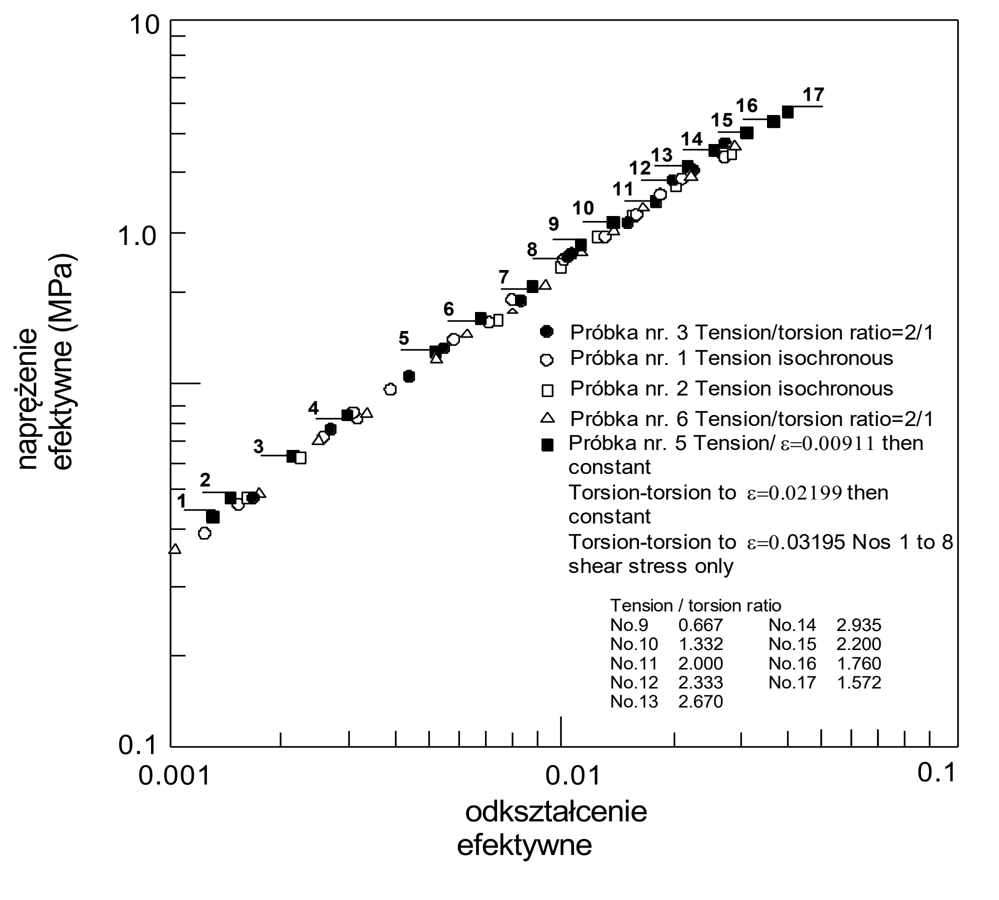
$$\gamma_{yz} = 3\bar{\varepsilon}\sigma_{yz} / \bar{\sigma}$$

$$\gamma_{zx} = 3\bar{\varepsilon}\sigma_{zx} / \bar{\sigma}$$

Równania HENCKY'ego



300
200
100
0
0.1
0.2
0.3
naprężenie efektywne $\bar{\sigma}$ (MPa)
odkształcenie efektywne $\int d\bar{\varepsilon}$
$\frac{\sigma_1}{\sigma_2}$
0
1/4
3/8
1/2
1/2
5/8
3/4
1



10
1.0
0.1
0.001
0.01
0.1
naprężenie efektywne (MPa)
odkształcenie efektywne
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
Próbka nr. 3 Tension/torsion ratio=2/1
Próbka nr. 1 Tension isochronous
Próbka nr. 2 Tension isochronous
Próbka nr. 6 Tension/torsion ratio=2/1
Próbka nr. 5 Tension/ ε=0.00911 then constant
Torsion-torsion to ε=0.02199 then constant
Torsion-torsion to ε=0.03195 Nos 1 to 8 shear stress only
Tension / torsion ratio
No.9 0.667
No.10 1.332
No.11 2.000
No.12 2.333
No.13 2.670
No.14 2.935
No.15 2.200
No.16 1.760
No.17 1.572